

Threshold corrections to baryon number violating operators in supersymmetric $SU(5)$ GUTs

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Received 13 May 2015; received in revised form 19 June 2015; accepted 25 June 2015

Available online 26 June 2015

Editor: Hong-Jian He

Abstract

The nucleon decay is a significant phenomenon to verify grand unified theories (GUTs). For the precise prediction of the nucleon lifetime induced by the gauge bosons associated with the unified gauge group, it is important to include the renormalization effects on the Wilson coefficients of the dimension-six baryon number violating operators. In this study, we have derived the threshold corrections to these coefficients at the one-loop level in the minimal supersymmetric $SU(5)$ GUT and the extended one with additional $SU(5)$ vector-like pairs. As a result, it is found that the nucleon decay rate is suppressed about 5% in the minimal setup, and then the suppression could be $O(10)\%$ in the vector-like matter extensions.

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1. Introduction

The supersymmetric grand unified theories (SUSY GUTs) are attractive extensions of the Standard Model (SM). The three gauge groups of the SM are unified into one, and the SM fermions are embedded into the fields charged under the unified gauge group in the GUT. The minimal candidate for the gauge symmetry is $SU(5)$, and we may understand the origin of the

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hypercharge assignment according to the group structure of $SU(5)$. SUSY also plays a crucial role in the gauge coupling unification as well as the natural explanation of the gauge hierarchy problem, and we are looking forward to the discovery of the SUSY particles at the LHC experiment. In 2012, it was reported that a scalar particle, which may be consistent with the SM Higgs boson, was discovered around 126 GeV [1,2]. The SM is firmly established and we expect that new physics predicted by the SUSY GUT is also discovered near future, although it has not been found yet at the LHC [3–9].

On the other hand, it is true that there are several issues which should be carefully studied in the SUSY GUTs. One of the issues is how to achieve the 126-GeV scalar boson. The low-energy effective field theory (EFT) for the SUSY GUT is considered to be the minimal supersymmetric standard model (MSSM). It is known that the MSSM predicts the upper bound on the Higgs mass, and the observed Higgs mass may require high-scale SUSY [10–12], or very specific SUSY mass spectrums [13], unless the MSSM is further extended, for instance, introducing extra vector-like fields [14].

Another big issue is from the experimental constraints on baryon number violation, such as nucleon decay. The GUTs unify quarks and leptons, so that the baryon-number-violating processes are introduced through the gauge interaction. The processes are strongly suppressed by the GUT scale, but it is possible to test the models through the nucleon decay search. The current status of the nucleon decay experiments is as follows: the partial lifetime limit on $p \rightarrow \pi^0 e^+$ is $\tau(p \rightarrow \pi^0 e^+) > 1.4 \times 10^{34}$ years [15,16], and the partial lifetime limit on $p \rightarrow K^+ \bar{\nu}$ is $\tau(p \rightarrow K^+ \bar{\nu}) > 5.9 \times 10^{33}$ years [17]. The prediction of the GUT depends on the scenario between the electroweak (EW) and the GUT scale ($\sim 10^{16}$ GeV). In the minimal SUSY $SU(5)$ GUT, the color-triplet Higgs exchange induces dangerous dimension-five operators to cause baryon number violation [18,19]. It is a serious problem, if the SUSY scale is close to the EW scale. If the SUSY scale is much higher, the constraint from the color-triplet Higgs becomes mild and the dominant decay mode $p \rightarrow K^+ \bar{\nu}$ may be detected at the future detectors [20,21]. Furthermore, the heavy gaugino masses make the GUT scale lower, so that the decay rate for $p \rightarrow \pi^0 e^+$, induced by a massive gauge boson (X boson), may be also large enough to be detected at the future detectors [22]. If we introduce additional SM-charged fields, the gauge coupling constants would become larger at the GUT scale since the extra fields contribute to the running of the gauge coupling constants [14]. Then the nucleon decay through the X -boson exchange is enhanced [23]. Note that the lifetime of proton is very sensitive to the X -boson mass, because the decay width is suppressed by the fourth power of the X -boson mass. This means that we need careful analysis to draw the constraint on the X boson.

In this paper we derive the threshold corrections to the Wilson coefficients of the baryon-number violating dimension-six operators induced by the X boson in the minimal setup of the $SU(5)$ GUT and the extended one with extra $SU(5)$ vector-like pairs. In particular, since the unified gauge coupling at the GUT scale becomes large in the vector-like extensions, it is important to evaluate quantum corrections via gauge interaction in these models. The two-loop order corrections to the dimension-six operators have been investigated, including the long-distance effect [24] and the short-distance effect [25]. However, the threshold corrections to the dimension-six operators at the GUT scale have never been discussed. The correction will not be non-negligible, especially when the gauge coupling constants at the GUT scale are large. We evaluate the corrections at the one-loop level analytically.

This paper is organized as follows: in Section 2, we introduce the minimal SUSY $SU(5)$ GUT to summarize our notations. In Section 3, we show the radiative corrections such as the wave function renormalizations, vertex corrections, and box-like corrections, using supergraph

techniques. The definition of covariant derivatives on superfields in this paper is the same as in Ref. [26] though we use the metric signature $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$. We adopt the $\overline{\text{DR}}$ scheme [27] for the gauge coupling constants while we impose the on-shell condition to the X boson mass M_X . For simplicity, we choose the Feynman gauge ($\xi = 1$) through this paper. In the next section, we estimate the threshold corrections to the Wilson coefficients of the dimension-six operators at the GUT scale, and we evaluate the numerical results for these finite corrections in the minimal SUSY $SU(5)$ GUT and its vector-like matter extensions. Finally, we summarize our paper in Section 5. We introduce the gauge interactions relevant to our analysis in Appendix A. Our explicit results on the one-loop corrections are shown in Appendix B, and the renormalization group equations (RGEs) of gauge couplings, Yukawa couplings and the Wilson coefficients for dimension-six operators are discussed in Appendix C.

2. SUSY $SU(5)$ GUTs

In the SUSY extensions of the SM, it is useful to use the superfield formalism in order to describe the fundamental interactions. Matter fields, Higgs fields, and their superpartners are embedded in chiral superfields and their conjugation. Gauge bosons and gauginos are described by vector superfields.

In the SUSY extension [28] of the minimal $SU(5)$ GUT [29], the matter fields are given by the $\bar{\mathbf{5}}$ and $\mathbf{10}$ representational superfields which are denoted by Φ and Ψ as follows:

$$\Phi_{iA}(\bar{\mathbf{5}}) = \begin{pmatrix} D_{i\alpha}^C \\ \epsilon_{rs} L_i^s \end{pmatrix}, \quad \Psi_i^{AB}(\mathbf{10}) = \frac{1}{\sqrt{2}} \begin{pmatrix} \epsilon^{\alpha\beta\gamma} e^{-i\varphi_i} U_{i\gamma}^C & Q_i^{r\alpha} \\ -Q_i^{s\beta} & \epsilon^{sr} V_{ij} E_j^C \end{pmatrix}, \quad (2.1)$$

where $A, B, \dots = 1, 2, \dots, 5$ are the indices of the $SU(5)$, $\alpha, \beta, \dots = 1, 2, 3$ and $r, s, \dots = 1, 2$ are the indices of the $SU(3)_C$ and $SU(2)_L$, respectively. $i, j = 1, 2, 3$ denote the generations. All the chiral superfields include the left-handed fermions in the flavor basis. φ_i and V_{ij} correspond to additional phases in the minimal SUSY GUT and the CKM matrix with the constraint $\sum_i \varphi_i = 0$. Q and L denote the weak-doublet chiral superfields for left-handed quarks and left-handed leptons, respectively:

$$Q_i = \begin{pmatrix} U_i \\ V_{ij} D_j \end{pmatrix}, \quad L_i = \begin{pmatrix} N_i \\ E_i \end{pmatrix}, \quad (2.2)$$

where U, D, E , and N are the chiral superfields for left-handed up-type and down-type quarks, and left-handed charged and neutral leptons, respectively. U^C, D^C , and E^C denote the chiral superfields for the charge-conjugation of right-handed up-type and down-type quarks, and right-handed charged lepton, respectively. In the Higgs sector, there are $\mathbf{5}$, $\bar{\mathbf{5}}$, and $\mathbf{24}$ representational superfields,

$$H_{\mathbf{5}}^A(\mathbf{5}) = \begin{pmatrix} H_C^\alpha \\ H_u^r \end{pmatrix}, \quad H_{\bar{\mathbf{5}}A}(\bar{\mathbf{5}}) = \begin{pmatrix} H_{\bar{C}\alpha} \\ \epsilon_{rs} H_d^s \end{pmatrix},$$

$$\Sigma^A_B(\mathbf{24}) = \begin{pmatrix} \Sigma_8 & \Sigma_{(3,2)} \\ \Sigma_{(3^*,2)} & \Sigma_3 \end{pmatrix} + \frac{1}{\sqrt{60}} \begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix} \Sigma_{\mathbf{24}}. \quad (2.3)$$

$H_{\mathbf{5}}(\mathbf{5})$ and $H_{\bar{\mathbf{5}}}(\bar{\mathbf{5}})$ include the MSSM Higgs doublets, H_u and H_d . In order to embed the MSSM Higgs multiplets in the $SU(5)$ multiplets, we have to introduce the color-triplet Higgs multiplets H_C and $H_{\bar{C}}$. The adjoint Higgs multiplet $\Sigma(\mathbf{24})$ is introduced to cause the spontaneous symmetry

breaking of the $SU(5)$ gauge symmetry according to the non-zero vacuum expectation value (VEV) of Σ_{24} .

The Lagrangian for the minimal SUSY $SU(5)$ GUT is given by

$$\mathcal{L} = \int d^4\theta \mathcal{K}_{\text{MSGUT}} + \left[\int d^2\theta \left(W_{\text{MSGUT}} + \frac{1}{8g_5^2} \text{tr} \mathcal{W}^\alpha \mathcal{W}_\alpha \right) + \text{h.c.} \right] + \mathcal{L}_{\text{ghost}} + \mathcal{L}_{\text{gauge-fixing}}, \quad (2.4)$$

where $\mathcal{K}_{\text{MSGUT}}$ and W_{MSGUT} are the Kähler potential and the superpotential, respectively. g_5 denotes the unified gauge coupling constant. The field strength chiral superfield $\mathcal{W}^\alpha$ consists of vector superfield $\mathcal{V}_5 = \mathcal{V}_5^A T^A$, where T^A is the generator of $SU(5)$ with $\text{tr} T^A T^B = \frac{1}{2} \delta^{AB}$:

$$\mathcal{W}^\alpha = -\frac{1}{4} \bar{\mathcal{D}}^2 (e^{-2g_5 \mathcal{V}_5} \mathcal{D}^\alpha e^{2g_5 \mathcal{V}_5}). \quad (2.5)$$

Here, $\mathcal{D}$ and $\bar{\mathcal{D}}$ denote the covariant derivatives on superspace. The vector superfield $\mathcal{V}_5$ is decomposed in terms of the SM gauge group:

$$\mathcal{V}_5 = \begin{pmatrix} G^\alpha_\beta - \frac{2}{\sqrt{60}} B \delta^\alpha_\beta & \frac{1}{\sqrt{2}} X^{\dagger\alpha}_r \\ \frac{1}{\sqrt{2}} X^s_\beta & W^s_r + \frac{3}{\sqrt{60}} B \delta^s_r \end{pmatrix}. \quad (2.6)$$

G , W , and B are the vector superfields for $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$, respectively, and they are defined as

$$G^\alpha_\beta = G^a (T^a)^\alpha_\beta, \quad W^s_r = W^a (t^a)^s_r, \quad (2.7)$$

using the generators T^a and t^a of $SU(3)_C$ and $SU(2)_L$, respectively. X is the vector superfield for the X boson, which induces baryon-number violating operators. It acquires the heavy mass by eating the Nambu–Goldstone (NG) modes, $\Sigma_{(3,2)}$ and $\Sigma_{(3^*,2)}$, after the $SU(5)$ symmetry breaking. M_X denotes the mass for the X boson in this paper.

In the minimal SUSY $SU(5)$ GUT, the Kähler potential and the superpotential in the flavor basis of matter superfields are written as

$$\begin{aligned} \mathcal{K}_{\text{MSGUT}} &= \Phi_i^{\dagger A} (e^{-2g_5 \mathcal{V}_5})^B{}_A \Phi_{iB} + \Psi_i^{\dagger} (e^{2g_5 \mathcal{V}_5})^A{}_C (e^{2g_5 \mathcal{V}_5})^B{}_D \Psi_i^{CD} \\ &\quad + 2 \Sigma^{\dagger A}{}_B (e^{-2g_5 \mathcal{V}_5})^C{}_A (e^{2g_5 \mathcal{V}_5})^B{}_D \Sigma^D{}_C + H_5^{\dagger A} (e^{-2g_5 \mathcal{V}_5})^B{}_A H_5^B \\ &\quad + H_5^{\dagger} (e^{2g_5 \mathcal{V}_5})^A{}_B H_5^B, \\ W_{\text{MSGUT}} &= \frac{y}{3} \text{tr} \Sigma^3 + \frac{y v_\Sigma}{2} \text{tr} \Sigma^2 + y_H H_5^A (\Sigma_B^A + 3\mu_0 \delta_B^A) H_5^B \\ &\quad + \frac{y_u^i}{4} e^{i\varphi_i} \epsilon_{ABCDE} \Psi_i^{AB} \Psi_i^{CD} H_5^E + \sqrt{2} V_{ij}^* y_d^j \Psi_i^{AB} \Phi_{jA} H_5^B. \end{aligned} \quad (2.8)$$

y denotes the cubic coupling constant of the adjoint Higgs multiplet and v_Σ is the VEV of the adjoint Higgs multiplet. y_u^i and y_d^i denote the diagonalized Yukawa matrices.

The adjoint Higgs multiplet and the color-triplet Higgs multiplets acquire heavy masses through the interactions in the superpotential. The doublet–triplet splitting is achieved by tuning μ_0 in the minimal SUSY $SU(5)$ GUT. $M_{H_C} (= 5y_H v_\Sigma)$ denotes the mass of the color-triplet Higgs multiplets. The masses of the adjoint Higgs multiplets are also split after the $SU(5)$ symmetry breaking. The triplet Σ_3 and the octet Σ_8 have a common mass denoted as $M_\Sigma (= \frac{5}{2} y v_\Sigma)$,

and the mass for Σ_{24} is $M_{\Sigma_{24}} (= \frac{1}{2} y v_\Sigma)$. The X boson mass M_X is $5\sqrt{2} g_5 v_\Sigma$. Note that y and y_H should be large, if the color-triplet Higgs and adjoint Higgs multiplets are much heavier than the X boson.

In the minimal setup of the SUSY $SU(5)$ GUT, the X -boson interactions with the matter superfields are given by the following terms,

$$\mathcal{L}_X = \int d^4\theta \left(\mathcal{K}_{V_1}^{(0)} + \mathcal{K}_{V_2}^{(0)} + \mathcal{K}_{V_3}^{(0)} \right), \quad (2.9)$$

$$\mathcal{K}_{V_1}^{(0)} = -\sqrt{2} g_5 \epsilon^{rs} L_{si}^\dagger D_{\alpha i}^C X_r^{\dagger\alpha} + \text{h.c.}, \quad (2.10)$$

$$\mathcal{K}_{V_2}^{(0)} = -\sqrt{2} g_5 \epsilon_{\alpha\beta\gamma} e^{i\varphi_i} U_i^{C\dagger\gamma} Q_i^{r\beta} X_r^{\dagger\alpha} + \text{h.c.}, \quad (2.11)$$

$$\mathcal{K}_{V_3}^{(0)} = -\sqrt{2} g_5 \epsilon^{sr} V_{ij}^\dagger Q_{s\alpha i}^C E_j^C X_r^{\dagger\alpha} + \text{h.c.}, \quad (2.12)$$

and the baryon-number violating operators are effectively induced by integrating out the X boson at the low energy. The effective dimension-six operators are written as follows at the tree level¹;

$$\mathcal{L}_{\text{dim.6}} = \int d^4\theta \left(\mathcal{K}_1^{(0)} + \mathcal{K}_2^{(0)} \right) \quad (2.13)$$

$$\mathcal{K}_1^{(0)} = -e^{i\varphi_i} \frac{g_5^2}{M_X^2} \epsilon_{\alpha\beta\gamma} \epsilon_{rs} U_i^{C\dagger\alpha} D_j^{C\dagger\beta} Q_i^{r\gamma} L_j^s + \text{h.c.} \quad (2.14)$$

$$\mathcal{K}_2^{(0)} = -e^{i\varphi_i} V_{kj}^* \frac{g_5^2}{M_X^2} \epsilon_{\alpha\beta\gamma} \epsilon_{rs} E_j^{C\dagger} U_i^{C\alpha\dagger} Q_k^{\beta r} Q_i^{s\gamma} + \text{h.c.} \quad (2.15)$$

Below, we investigate the one-loop correction to the 4-Fermi interactions and especially estimate how large the threshold correction is according to the heavy particles decoupling around the GUT scale. We focus on the operators relevant to nucleon decay in not only the minimal SUSY $SU(5)$ GUT but also its vector-like extensions, where $SU(5)$ vector-like chiral superfields are additionally introduced. In the later case, we simply assume that the vector-like pairs have supersymmetric masses without the mixing between the extra fields and the MSSM fields. We only discuss the gauge interactions in our calculation. The gauge interactions in the minimal SUSY $SU(5)$ GUT, which are relevant to the evaluation of the threshold correction to the baryon-number violating operators, are summarized in [Appendix A](#). For simplicity, we omit the generation indices ($i, j, \dots$) below.

3. Radiative correction to the baryon-number violating operators

In the supersymmetric theories, effective Kähler potentials are useful to derive the radiative corrections. In order to evaluate the corrections to the baryon-number violating dimension-six operators induced by the X boson, we discuss the effective Kähler potentials at the one-loop level, and evaluate the threshold corrections to the operators.

First of all, let us discuss a general effective supersymmetric action $\Gamma[\Phi, \Phi^\dagger]$, which is the function of chiral superfield Φ , antichiral superfield $\Phi^\dagger$, and their derivatives. The general form of the effective supersymmetric action would be as follows,

¹ Notice that the propagators of the vector superfields differ from those of canonically normalized gauge bosons by a factor 1/2 under our convention for the kinetic terms of the vector superfields.

$$\Gamma[\Phi, \Phi^\dagger] = \int d^4x d^4\theta \mathcal{L}_{\text{eff}}(\Phi, \mathcal{D}_A \Phi, \mathcal{D}_A \mathcal{D}_B \Phi, \dots, \Phi^\dagger, \mathcal{D}_A \Phi^\dagger, \mathcal{D}_A \mathcal{D}_B \Phi^\dagger, \dots) + \left\{ \int d^4x d^2\theta \mathcal{L}_{\text{eff}}^{(c)}(\Phi, \mathcal{D}_A \Phi, \mathcal{D}_A \mathcal{D}_B \Phi, \dots) + \text{h.c.} \right\}, \quad (3.1)$$

where $\mathcal{D}_A$ is the superspace covariant derivative which consists of ∂_μ , $\mathcal{D}_\alpha$, and $\bar{\mathcal{D}}_{\dot{\alpha}}$. Here, we do not include vector superfields for simplify. The perturbative corrections appear only in the D term due to the non-renormalization theorem. The effective supersymmetric Lagrangian $\mathcal{L}_{\text{eff}}$ is divided into two parts under $\partial_\mu \Phi = 0$,

$$\mathcal{L}_{\text{eff}} = \mathcal{K}(\Phi, \Phi^\dagger) + \mathcal{F}(\mathcal{D}_\alpha \Phi, \mathcal{D}^2 \Phi, \bar{\mathcal{D}}_{\dot{\alpha}} \Phi^\dagger, \bar{\mathcal{D}}^2 \Phi^\dagger; \Phi, \Phi^\dagger), \quad (3.2)$$

where $\mathcal{K}(\Phi, \Phi^\dagger)$ is the effective Kähler potential and $\mathcal{F}(\mathcal{D}_\alpha \Phi, \mathcal{D}^2 \Phi, \bar{\mathcal{D}}_{\dot{\alpha}} \Phi^\dagger, \bar{\mathcal{D}}^2 \Phi^\dagger; \Phi, \Phi^\dagger)$ is called the effective auxiliary potential. While some diagrams may generate the terms including superfields on which more than three covariant derivatives act, we may always obtain the above form by using algebra of super-covariant derivatives ($\mathcal{D}$ algebra). The effective auxiliary potential vanishes in the limit that $\mathcal{D}_\alpha \Phi = 0$ and $\bar{\mathcal{D}}_{\dot{\alpha}} \Phi^\dagger = 0$, so that the effective Kähler potential is identified by taking the limit.

Below, we study the threshold corrections to the baryon-number violating dimension-six operators at the GUT scale with the effective Kähler potential. First, we calculate the effective actions for constant fields in both full and effective theories at the one-loop level with the supergraph technique [30]. We adopt the modified dimensional reduction ($\overline{\text{DR}}$) scheme [27] as the renormalization scheme of the gauge coupling constants while we impose the on-shell condition for the X boson mass. We also introduce the IR cut off in order to control fictitious IR singularities. Then, we identify the effective Kähler potential for the baryon-number violating operators by taking $\mathcal{D}_\alpha \Phi = 0$ and $\bar{\mathcal{D}}_{\dot{\alpha}} \Phi^\dagger = 0$ together with the $\mathcal{D}$ algebra. By matching the effective Kähler potentials in full and effective theories, we derive the one-loop threshold corrections to the Wilson corrections of the dimension-six operators.

3.1. Radiative corrections in the full theory

In this subsection, we show the radiative corrections to the baryon-number violating dimension-six operators in the full theory, where the X boson is activated. The radiative corrections consist of the wave function renormalization of quarks and leptons, the vacuum polarization of the X boson, the vertex correction, and the box-like corrections. In this section, we show only the results of the supergraph calculation. Details of the calculations are given in [Appendix B](#).

3.1.1. Two-point functions for matter fields

First we study two-point functions for matter fields at the one-loop level. The functions generally include UV divergences which are renormalized by the wave function renormalization factors. We estimate the factors in the $\overline{\text{DR}}$ scheme, ignoring the contributions from the Yukawa interactions. The radiative corrections to the two-point functions via the gauge interactions are determined by the gauge groups, in the both of the full theory and the EFT.

In general, the one-loop renormalized two-point function for chiral superfield Φ is defined as $\Gamma_\Phi^{2\text{-pt}} = \tilde{\Gamma}_\Phi^{2\text{-pt}} + Z_\Phi - 1$. The wave function renormalization constant for the matter superfield Z_Φ absorbs the UV divergent terms proportional to $1/\epsilon'$ in the $\overline{\text{DR}}$ scheme²:

² $2/\epsilon' \equiv 2/\epsilon - \gamma + \ln 4\pi$ is defined and ϵ satisfies $\epsilon = 4 - d$ in the d -dimension momentum space.

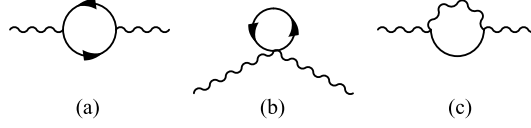


Fig. 1. Diagrams of chiral multiplets for radiative correction to two-point function of superfield for X boson.

$$Z_\Phi = 1 + c_5^\Phi \frac{g_5^2}{4\pi^2} \times \frac{1}{\epsilon'}, \quad Z_\Phi^{\text{EFT}} = 1 + \sum_{n=1}^3 c_n^\Phi \frac{g_n^2}{4\pi^2} \times \frac{1}{\epsilon'}. \quad (3.3)$$

Z_Φ and Z_Φ^{EFT} denote the wave function renormalization factors in the full and the effective theories, respectively. g_3 , g_2 , and g_1 are the gauge couplings of $SU(3)_C$, $SU(2)_L$, and unified $U(1)_Y$ gauge symmetries. c_5^Φ and c_n^Φ ($n = 3, 2, 1$) are the quadratic Casimir of Φ in $SU(5)$, $SU(3)_C$, $SU(2)_L$, and GUT normalized $U(1)_Y$ gauge symmetries.³

Then, the one-loop renormalized two-point function in the full theory is given by

$$\Gamma_\Phi^{2\text{-pt}; 1\text{-loop}} = \left(1 + a_\Phi f(M_X^2) + b_\Phi f(\mu_{\text{IR}}^2) \right) \Gamma_{\Phi 0}^{2\text{-pt}}. \quad (3.4)$$

$\Gamma_{\Phi 0}^{2\text{-pt}}$ is the tree-level two-point function, and a_Φ and b_Φ are the constants obtained from the one-loop calculations,

$$a_\Phi = -(c_5^\Phi - \sum_{n=1}^3 c_n^\Phi) \frac{g_5^2}{8\pi^2}, \quad b_\Phi = -\sum_{n=1}^3 c_n^\Phi \frac{g_n^2}{8\pi^2}. \quad (3.5)$$

We set the mass of the MSSM vector superfields to be a non-zero value which is denoted by μ_{IR} in order to regularize the IR divergence, as mentioned above. The function f in Eq. (3.4) is defined as

$$f(M^2) \equiv 1 - \ln \frac{M^2}{\mu^2}, \quad (3.6)$$

where μ denotes the renormalization scale in the $\overline{\text{DR}}$ scheme. The two-point function in the effective theory is derived by removing the X boson contribution in Eq. (3.4) when $g_5 = g_3 = g_2 = g_1$.

3.1.2. Vacuum polarization

Next, we estimate the radiative corrections to the propagator for the X boson. Not only the MSSM fields but also the GUT-scale fields such as the $SU(5)$ -adjoint field contribute to the vacuum polarization of the X boson.

The chiral superfields have three kinds of the contributions which are described in Fig. 1. The diagrams (a) and (b) are induced by the supergauge interaction $\Phi^\dagger V \Phi$ and $\Phi^\dagger V^2 \Phi$, respectively. The diagram (c) is generated by the $SU(5)$ -breaking adjoint Higgs superfield, which has interactions $\langle \Sigma^\dagger \rangle V^2 \Sigma$ and $\Sigma^\dagger V^2 \langle \Sigma \rangle$ after acquiring the VEV.

For the gauge sector, we have the four-type diagrams to contribute to the two-point function of the X boson. The diagrams (a) and (b) in Fig. 2 arise from the self interactions of the vector superfields. If the internal vector superfields in the diagram (b) are massless, the diagrams have

³ c_1^Φ is given by $c_1^\Phi = (Q_Y^\Phi)^2 \times (3/5)$, where Q_Y^Φ is hypercharge of Φ .

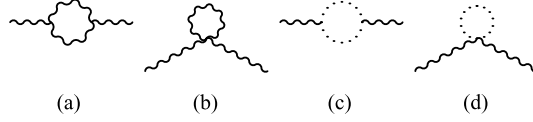


Fig. 2. Diagrams of gauge and ghost superfields for radiative correction to two-point function of superfield for X boson.

no contribution to the two-point function in the $\overline{\text{DR}}$ scheme. The diagrams (c) and (d) show the ghost loop contribution.

Finally, the two-point function of the X boson is in the form as below:

$$\Gamma_X^{(2)}(k^2) = k^2 - M_X^2 - \Sigma_X(k^2), \quad (3.7)$$

where $\Sigma_X(k^2)$ is the renormalized vacuum polarization for X boson. The UV divergence in the one-loop corrections is absorbed by the wave function factor (Z_X) and mass (M_X) of the X boson. In this paper the on-shell condition for the X boson mass is imposed so that this leads the equation $\Sigma_X(M_X^2) = 0$. This is because heavy particles are decoupled from $\Sigma_X(0)$ under the on-shell condition, if they have $SU(5)$ symmetric masses much larger than the X boson mass.⁴ $\Sigma_X(0)$ will appear in the threshold correction to the baryon-number violating operators.

The counter term δZ_X is determined to absorb the UV divergence which arise from the gauge contributions and the matter contributions such as Figs. 1, 2. We obtain

$$\delta Z_X = Z_X - 1 = \frac{g_5^2}{8\pi^2} \left(3C_2(G) - \sum_R T(R) \right) \times \frac{1}{\epsilon'}, \quad (3.8)$$

where $T(R)\delta^{ab} = \text{tr}(T_R^a T_R^b)$ and $C_2(G)\delta_i^j = \sum_a (T_G^a T_G^a)_i^j$ are defined. As expected, δZ_X is proportional to the one-loop beta function for the $SU(5)$ gauge coupling constant. In the SUSY $SU(5)$ GUT models with $\mathbf{5} + \bar{\mathbf{5}}$ vector-like matter superfields and $\mathbf{10} + \bar{\mathbf{10}}$ vector-like matter superfields, we find

$$\sum_R T(R) = \frac{1}{2}(N_f + 2 + 2n_5) + \frac{3}{2}(N_f + 2n_{10}) + 5, \quad (3.9)$$

where N_f , n_5 , and n_{10} are the number of generations, $\mathbf{5} + \bar{\mathbf{5}}$ and $\mathbf{10} + \bar{\mathbf{10}}$ vector-like pairs, respectively.

In the SUSY $SU(5)$ GUT with extra vector-like matters, the vacuum polarization $\Sigma_X(p^2) (= \bar{\Sigma}_X(p^2) - \bar{\Sigma}_X(M_X^2))$ is given by

$$\begin{aligned} \frac{16\pi^2}{2g_5^2} \bar{\Sigma}_X(p^2) = & \left[\frac{1}{2}(N_5 + N_{\bar{5}}) + \frac{3}{2}(N_{10} + N_{\bar{10}}) \right] B(p^2, 0, 0) + \frac{25}{6} B(p^2, M_\Sigma^2, M_X^2) \\ & + \frac{5}{6} B(p^2, M_{\Sigma_{24}}^2, M_X^2) + B(p^2, M_{H_C}^2, 0) \\ & + \frac{5}{24} M_X^2 \left[7A(p^2, M_X^2, 0) + 20A(p^2, M_X^2, M_\Sigma^2) + 2A(p^2, M_X^2, M_{\Sigma_{24}}^2) \right] \end{aligned}$$

⁴ The GUT-scale mass spectrum may be constrained using the gauge coupling unification [31,32]. In the works, they use the threshold correction to the gauge coupling constants at the GUT scale at the one-loop level so that the renormalization condition for the X boson mass does not appear there. We need the threshold correction at the two-loop level in order to get the constraint on the on-shell X boson mass.

$$\begin{aligned}
& -5C_2(G)\frac{p^2}{4}A(p^2, M_X^2, 0) - \frac{1}{2}C_2(G)B(p^2, M_X^2, 0) \\
& + (p^2\text{-independent terms}),
\end{aligned} \tag{3.10}$$

where N_r ($r = \mathbf{5}, \bar{\mathbf{5}}, \mathbf{10}, \bar{\mathbf{10}}$) denotes the number of the massless superfields in r representation. The loop functions A and B are defined as

$$\begin{aligned}
A(p^2, M_1^2, M_2^2) & \equiv \int_0^1 dx \ln \frac{\Delta}{\mu^2}, \\
B(p^2, M_1^2, M_2^2) & \equiv \int_0^1 dx \left[\Delta - (2\Delta + x(x-1)p^2) \ln \frac{\Delta}{\mu^2} \right],
\end{aligned} \tag{3.11}$$

where $\Delta = x(x-1)p^2 + xM_2^2 + (1-x)M_1^2$ is defined.

The first and second lines in Eq. (3.10) correspond to the contributions of the massless and massive fields in Fig. 1(a). The third line is for diagram (c) in Fig. 1, in which the VEV of the adjoint Higgs multiplet is included in the vertices. In the fourth line, we show the contributions from the gauge sector: The first term in the fourth line is induced by the three-vector interactions (Fig. 2(a)), while the second term corresponds to the ghost diagrams (Fig. 2(c)). The p^2 -independent terms come from the diagrams Fig. 1(b), Fig. 2(b), and Fig. 2(d).

We finally obtain the full one-loop corrections to the two-point function by summing of the contributions from the chiral superfields, the vector superfield, and the ghost superfields. The resummed propagator $D_{XX}(p^2)$ of X superfield in terms of the superfield notation is given by $D_{XX}(p^2) = -i/(2\Gamma_X^{(2)}(p^2))$.

After the spontaneous symmetry breaking of the GUT gauge symmetry, the baryon-number violating dimension-six operators are induced by the X boson, and the coefficients are proportional to $1/M_X^2$. In order to match the full and the effective theories at the one-loop level, we need to take into account the one-loop corrections to the propagator of the X boson. Since the momenta of external fields in the baryon-number violating dimension-six operators are negligible compared with the X boson mass, we may set the momentum of internal X boson zero.

3.1.3. Vertex corrections

Next, we show the one-loop vertex corrections to the X boson interactions with quarks and leptons. The tree-level interactions are given in Eq. (2.9).

Several one-loop diagrams in Fig. 3 contribute to the vertex corrections. Since the supersymmetric gauge interactions in terms of the superfield formalism have the form $\Phi^\dagger e^{2gV} \Phi$ (Φ is a matter chiral superfield, and V and g are a vector superfield and its gauge coupling, respectively), there exist diagrams which do not appear in component calculation. The diagram (a) has only the vertex $2g\Phi^\dagger V \Phi$, and the diagrams (b) and (c) include the vertex $2g^2\Phi^\dagger V^2 \Phi$. The diagrams (d) and (e) include the three-point self interactions of vector superfields. Since the external vector superfield is for the broken gauge symmetry, two internal vector superfields must be massive and massless ones. The contribution from the diagram (f) is vanishing due to the superspace integral.

Thus, we calculate the contributions from the diagrams (a)–(e) in Fig. 3. The momenta of all the external superfields are set to be $p^2 = 0$, for simplicity. In some diagrams, since they contain IR divergent contributions in this momentum assignment, the non-zero masses of the MSSM vector superfields (μ_{IR}) are introduced as IR regulators. Under this momentum assignment, we

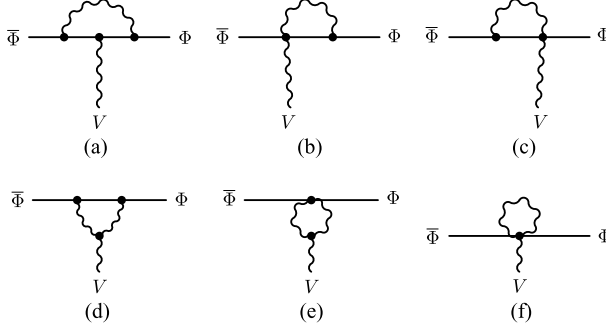


Fig. 3. Diagrams for vertex correction.

carry out loop momentum integrals and Grassmann integrals, and we discard the auxiliary terms. We expand the one-loop Kähler terms around $p^2 = 0$, and then we extract the dominant contributions around $p^2 = 0$. The vertex corrections to the gauge interactions between the MSSM matter fields and the X boson are as follows:

$$\begin{aligned}\mathcal{K}_{V_1}^{(1)} &= \left[-\frac{2}{5}C_1^{(v)}(\mu_{\text{IR}}) + \frac{21}{5}C_2^{(v)}(\mu_{\text{IR}}) + 5C_2^{(v)}(M_X) \right] \mathcal{K}_{V_1}^{(0)}, \\ \mathcal{K}_{V_2}^{(1)} &= \left[\frac{12}{5}C_1^{(v)}(\mu_{\text{IR}}) - 2C_1^{(v)}(M_X) + \frac{49}{5}C_2^{(v)}(\mu_{\text{IR}}) + 9C_2^{(v)}(M_X) \right] \mathcal{K}_{V_2}^{(0)}, \\ \mathcal{K}_{V_3}^{(1)} &= \left[\frac{2}{5}C_1^{(v)}(\mu_{\text{IR}}) - 4C_1^{(v)}(M_X) + \frac{29}{5}C_2^{(v)}(\mu_{\text{IR}}) + 13C_2^{(v)}(M_X) \right] \mathcal{K}_{V_3}^{(0)}.\end{aligned}\quad (3.12)$$

The contributions from the diagrams (d) and (e) in Fig. 3 are canceled each other. The coefficients $C_1^{(v)}$ and $C_2^{(v)}$ correspond to the correction from the diagram (a), and the ones from the diagrams (b) and (c) in Fig. 3, respectively. After the loop momentum and superspace integrals, we find that $C_1^{(v)}$ and $C_2^{(v)}$ are given by the functions of the mass of the internal vector superfield M ,

$$C_1^{(v)}(M) = -C_2^{(v)}(M) \equiv \frac{1}{2} \frac{g_5^2}{16\pi^2} \left[\frac{2}{\epsilon'} + 1 - \ln \frac{M^2}{\mu^2} \right]. \quad (3.13)$$

These loop functions are the coefficients of the effective Kähler potential $\mathcal{K}_1^{(v)}$ and $\mathcal{K}_2^{(v)}$ defined in Appendix B in the limit that p^2 vanishes.

Now, we determine the renormalization constants for the vertices. One-loop renormalized vertex functions are given by

$$\begin{aligned}\mathcal{K}_{V_1} &= \mathcal{K}_{V_1}^{(0)} + \mathcal{K}_{V_1}^{(1)} + \left(Z_L^{1/2} Z_D^{1/2} Z_X^{1/2} Z_{C_{V_1}} - 1 \right) \mathcal{K}_{V_1}^{(0)}, \\ \mathcal{K}_{V_2} &= \mathcal{K}_{V_2}^{(0)} + \mathcal{K}_{V_2}^{(1)} + \left(Z_U^{1/2} Z_Q^{1/2} Z_X^{1/2} Z_{C_{V_2}} - 1 \right) \mathcal{K}_{V_2}^{(0)}, \\ \mathcal{K}_{V_3} &= \mathcal{K}_{V_3}^{(0)} + \mathcal{K}_{V_3}^{(1)} + \left(Z_Q^{1/2} Z_E^{1/2} Z_X^{1/2} Z_{C_{V_3}} - 1 \right) \mathcal{K}_{V_3}^{(0)}.\end{aligned}\quad (3.14)$$

When $\mathcal{K}_{V_n}$ ($n = 1, 2, 3$) are described as $\mathcal{K}_{V_n} = C_{V_n} \mathcal{O}_{V_n}$ with the operators $\mathcal{O}_{V_n}$ and the Wilson coefficients C_{V_n} , $Z_{C_{V_n}}$ are defined to renormalize the UV divergences in C_{V_n} . Then we find

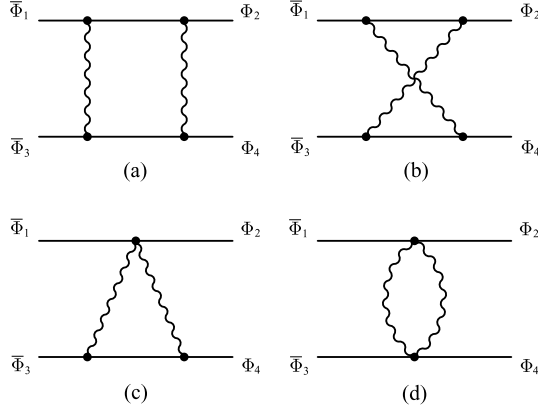


Fig. 4. Box-like diagrams.

$$Z_{C_{V_1}} = Z_{C_{V_2}} = Z_{C_{V_3}} = 1 - \frac{g_5^2}{16\pi^2} \left(3C_2(G) - \sum_R T(R) \right) \times \frac{1}{\epsilon'}, \quad (3.15)$$

which are consistent with the one-loop beta function for the gauge coupling.

3.1.4. Box-like corrections

The box-like diagrams contribute to the radiative corrections of the dimension-six operators. Fig. 4 shows all type of the box-like diagrams; we refer to the diagram (a) as the box diagram, the diagram (b) as the crossing box diagram, and the diagram (c) as the triangle diagram. The diagram (d) vanishes due to the superspace integral. Thus, it is sufficient that we evaluate the diagrams (a)–(c) in Fig. 4. In these figures, one of two internal gauge superfield lines must be massive since we focus on the baryon-number violating operators. As is the case in the vertex corrections, we set all momenta of the external superfields to be $p^2 = 0$ and the fictitious masses of the MSSM vector superfields to be μ_{IR} , and we remove the auxiliary terms. For the momentum assignment, we find that the box diagram (a) vanishes while the crossing box diagram (b) and the triangle diagram (c) are given by the following functions:

$$C_{\text{cross}}(M_X) = -C_{\text{triangle}}(M_X) \equiv -\frac{1}{2} \frac{g_5^2}{16\pi^2} \ln \frac{M_X^2}{\mu_{\text{IR}}^2}. \quad (3.16)$$

These loop functions correspond to the coefficients in the effective Kähler potential $\mathcal{K}_{\text{cross}}$ and $\mathcal{K}_{\text{triangle}}$ defined in Appendix B in the limit that p^2 vanishes.

In SUSY $SU(5)$ GUTs, the baryon-number violating dimension-six operators are generated at the tree level in Eq. (2.13). The one-loop radiative corrections from the box-like diagrams are written by C_{cross} and C_{triangle} :

$$\begin{aligned} \mathcal{K}_1^{(\text{Box})} &= \left[-\frac{18}{5} C_{\text{cross}}(M_X) + \frac{14}{5} C_{\text{triangle}}(M_X) \right] \mathcal{K}_1^{(0)}, \\ \mathcal{K}_2^{(\text{Box})} &= \left[-\frac{14}{5} C_{\text{cross}}(M_X) + \frac{22}{5} C_{\text{triangle}}(M_X) \right] \mathcal{K}_2^{(0)}. \end{aligned} \quad (3.17)$$

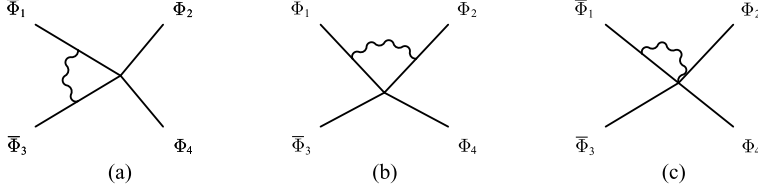


Fig. 5. Radiative corrections in EFT.

3.2. Radiative corrections in EFT

Now we consider the radiative correction to the higher-dimensional Kähler terms in the EFT. There are three kinds of contributions to the radiative correction. The first one is the diagram (a) in Fig. 5, where a vector superfield is attached to two chiral superfields or two antichiral superfields. The second is the diagram (b), in which a vector superfield is attached to both a chiral and an antichiral superfield. The third one is the radiative corrections induced by the gauge interaction of the composite operators.

We adopt the same momentum assignment which we used in the full theory. After the loop momentum and the superspace integrals, we derive the one-loop corrections as

$$\begin{aligned}\mathcal{K}_1^{(1):\text{eff}} &= \left[\left(\frac{16}{3}g_3^2 + \frac{4}{15}g_1^2 \right) C_1^{\text{EFT}}(\mu_{\text{IR}}) + \left(\frac{32}{3}g_3^2 + \frac{8}{15}g_1^2 \right) C_2^{\text{EFT}}(\mu_{\text{IR}}) \right] \mathcal{K}_1^{(0)}, \\ \mathcal{K}_2^{(1):\text{eff}} &= \left[\left(\frac{16}{3}g_3^2 + \frac{4}{15}g_1^2 \right) C_1^{\text{EFT}}(\mu_{\text{IR}}) + \left(\frac{32}{3}g_3^2 + \frac{8}{15}g_1^2 \right) C_2^{\text{EFT}}(\mu_{\text{IR}}) \right] \mathcal{K}_2^{(0)}.\end{aligned}\quad (3.18)$$

Here, the diagram (a) vanishes while the diagrams (b) and (c) are given by $C_1^{\text{EFT}}(\mu_{\text{IR}})$ and $C_2^{\text{EFT}}(\mu_{\text{IR}})$, respectively:

$$C_1^{\text{EFT}}(\mu_{\text{IR}}) = -C_2^{\text{EFT}}(\mu_{\text{IR}}) \equiv \frac{1}{16\pi^2} \frac{1}{2} \left(\frac{2}{\epsilon'} + 1 - \ln \frac{\mu_{\text{IR}}^2}{\mu^2} \right). \quad (3.19)$$

These functions correspond to the coefficients defined in Eq. (B.17) in the limit: p^2 vanishes. The effective Kähler potentials up to the one-loop level are described as

$$\begin{aligned}\mathcal{K}_1^{\text{eff}} &= \mathcal{K}_1^{(1):\text{eff}} + \mathcal{K}_1^{(0)} + \left(Z_{C_1} Z_U^{\text{EFT}1/2} Z_Q^{\text{EFT}1/2} Z_D^{\text{EFT}1/2} Z_L^{\text{EFT}1/2} - 1 \right) \mathcal{K}_1^{(0)}, \\ \mathcal{K}_2^{\text{eff}} &= \mathcal{K}_2^{(1):\text{eff}} + \mathcal{K}_2^{(0)} + \left(Z_{C_2} Z_E^{\text{EFT}1/2} Z_U^{\text{EFT}1/2} Z_Q^{\text{EFT}1/2} - 1 \right) \mathcal{K}_2^{(0)}.\end{aligned}\quad (3.20)$$

The logarithmic divergences are absorbed by the counter terms of $\mathcal{C}_A$, and then we have

$$\begin{aligned}Z_{C_1} &= 1 - \frac{2}{16\pi^2\epsilon'} \left(\frac{11}{30}g_1^2 + \frac{3}{2}g_2^2 + \frac{4}{3}g_3^2 \right), \\ Z_{C_2} &= 1 - \frac{2}{16\pi^2\epsilon'} \left(\frac{23}{30}g_1^2 + \frac{3}{2}g_2^2 + \frac{4}{3}g_3^2 \right).\end{aligned}\quad (3.21)$$

These are consistent with the results of Ref. [33]. In the next section, we determine the threshold corrections for the wave functions and the Wilson coefficients of the dimension-six baryon-number violating operators by matching the full and effective theories.

4. Threshold corrections of the dimension-six operators

In the previous section, we have shown the radiative corrections to two-, three-, and four-point vertex functions in the SUSY $SU(5)$ GUTs and we have shown also the radiative corrections to the Wilson coefficients of the dimension-six operators in the EFT. Now, we determine the threshold corrections by matching the amplitudes in the EFT and those in the full theory.

First, let us discuss the threshold corrections to the two-point functions for matter superfields. As we have seen in Eq. (3.4), the one-loop two-point functions are divided into two parts: one is linear to $f(M_X^2)$ and the other is linear to $f(\mu_{\text{IR}}^2)$. The latter is the contribution from the MSSM gauge interactions, and the former is the contribution from the broken gauge interaction in $SU(5)$. On the other hand, the two-point functions in the EFT at the GUT scale have the form;

$$\Gamma_{\Phi}^{2\text{-pt};\text{eff}} = (1 - \lambda_{\Phi}) \left(1 + b_{\Phi} f(\mu_{\text{IR}}^2) \right) \Gamma_{\Phi 0}^{2\text{-pt}}. \quad (4.1)$$

Here, the chiral superfield in the EFT is given by $(1 - \lambda_{\Phi}/2)\Phi$ (Φ is in the full theory). λ_{Φ} is determined so as to match the two-point function in the EFT and that in the full theory:

$$\lambda_{\Phi}(\mu) = \frac{g_5^2}{16\pi^2} \hat{\lambda}_{\Phi} f(M_X^2), \quad (4.2)$$

where $(\hat{\lambda}_Q, \hat{\lambda}_U, \hat{\lambda}_D, \hat{\lambda}_L, \hat{\lambda}_E) = (3, 4, 2, 3, 6)$ is defined.

Next, we determine the threshold corrections for the baryon-number violating dimension-six operators. The two-point functions of the matter superfields in the full theory and the EFT are matched above, and we have determined the threshold corrections to the renormalizable kinetic terms. For a matter superfield Φ , the renormalizable kinetic term has the form $(1 - \lambda_{\Phi})\Phi^{\dagger}\Phi$ in the EFT. The finite corrections to the two-point functions in the EFT appear in the correction to the Wilson coefficients of higher-dimensional operators. The Wilson coefficients of higher-dimensional operators themselves also include the finite corrections. Thus, we redefine the effective Kähler potentials $\mathcal{K}_I^{\text{eff}}$ ($I = 1, 2$) as the ones with threshold corrections up to the one-loop level as follows:

$$\begin{aligned} \mathcal{K}_1^{\text{eff}} &= \mathcal{K}_1^{(1);\text{eff}} + \left(1 - \lambda_1 - \frac{1}{2}(\lambda_U + \lambda_Q + \lambda_D + \lambda_L) \right) \mathcal{K}_1^{(0)} \\ &\quad + \left(Z_{C_1} Z_U^{\text{EFT}^{1/2}} Z_Q^{\text{EFT}^{1/2}} Z_D^{\text{EFT}^{1/2}} Z_L^{\text{EFT}^{1/2}} - 1 \right) \mathcal{K}_1^{(0)}, \\ \mathcal{K}_2^{\text{eff}} &= \mathcal{K}_2^{(1);\text{eff}} + \left(1 - \lambda_2 - \frac{1}{2}(\lambda_E + \lambda_U + 2\lambda_Q) \right) \mathcal{K}_2^{(0)} \\ &\quad + \left(Z_{C_2} Z_E^{\text{EFT}^{1/2}} Z_U^{\text{EFT}^{1/2}} Z_Q^{\text{EFT}} - 1 \right) \mathcal{K}_2^{(0)}. \end{aligned} \quad (4.3)$$

λ_1 and λ_2 are the threshold corrections to the Wilson coefficients for the baryon-number violating operators.

In the full theory (the SUSY $SU(5)$ GUTs), we have computed the effective Kähler potential for the dimension-six operators at the one-loop level,

$$\begin{aligned} \mathcal{K}_1^{\text{full}} &= -\frac{1}{2} \frac{1}{M_X^2 + \Sigma(0)} C_{V_2} C_{V_1} \mathcal{O}^{(1)} + \mathcal{K}_1^{(\text{Box})}, \\ \mathcal{K}_2^{\text{full}} &= -\frac{1}{2} \frac{1}{M_X^2 + \Sigma(0)} C_{V_2} C_{V_3} \mathcal{O}^{(2)} + \mathcal{K}_2^{(\text{Box})}. \end{aligned} \quad (4.4)$$

The first terms include the vacuum polarization of the X boson $\Sigma(0)$ and the one-loop effective couplings $\mathcal{C}_{V_1}$, $\mathcal{C}_{V_2}$, and $\mathcal{C}_{V_3}$ which are defined in Eq. (3.14).

There are IR divergences in $\mathcal{K}_I^{\text{full}}$ and $\mathcal{K}_I^{\text{eff}}$ ($I = 1, 2$), which are represented by μ_{IR} . The divergences are absorbed by the operators $\mathcal{O}^{(I)}$.⁵ Then, we divide the effective Kähler potentials into the coefficients $\mathcal{C}_I$ and the renormalized operator $\mathcal{O}_r^{(I)}$:

$$\mathcal{K}_I^{\text{full}} = \mathcal{C}_I^{\text{full}} \mathcal{O}_r^{(I)}, \quad \mathcal{K}_I^{\text{eff}} = \mathcal{C}_I^{\text{eff}} \mathcal{O}_r^{(I)}. \quad (4.5)$$

The one-loop coefficients in the full theory are given by

$$\begin{aligned} \frac{\mathcal{C}_1^{(1):\text{full}}}{\mathcal{C}_1^{(0)}} &= \frac{M_X^2}{M_X^2 + \Sigma(0)} - \frac{g_5^2}{16\pi^2} \left[6 + 6 \left(1 - \ln \frac{M_X^2}{\mu^2} \right) - \frac{16}{5} \ln \frac{M_X^2}{\mu^2} \right], \\ \frac{\mathcal{C}_2^{(1):\text{full}}}{\mathcal{C}_2^{(0)}} &= \frac{M_X^2}{M_X^2 + \Sigma(0)} - \frac{g_5^2}{16\pi^2} \left[\frac{32}{5} + 8 \left(1 - \ln \frac{M_X^2}{\mu^2} \right) - \frac{18}{5} \ln \frac{M_X^2}{\mu^2} \right], \end{aligned} \quad (4.6)$$

where $\mathcal{C}_1^{(0)}$ and $\mathcal{C}_2^{(0)}$ are the tree-level ones: $\mathcal{C}_1^{(0)} = \mathcal{C}_2^{(0)} = -g_5^2/M_X^2$. In the EFT, the coefficients are

$$\begin{aligned} \frac{\mathcal{C}_1^{(1):\text{eff}}}{\mathcal{C}_1^{(0)}} &= 1 - \lambda_1 - \frac{6g_5^2}{16\pi^2} \left(1 - \ln \frac{M_X^2}{\mu^2} \right) - \frac{14}{5} \frac{g_5^2}{16\pi^2}, \\ \frac{\mathcal{C}_2^{(1):\text{eff}}}{\mathcal{C}_2^{(0)}} &= 1 - \lambda_2 - \frac{8g_5^2}{16\pi^2} \left(1 - \ln \frac{M_X^2}{\mu^2} \right) - \frac{14}{5} \frac{g_5^2}{16\pi^2}. \end{aligned} \quad (4.7)$$

We assume that the matching scale is $\mu = M_{\text{GUT}} (\simeq M_X)$, where the unification $g_1 = g_2 = g_3 = g_5$ is achieved. By comparing the amplitudes obtained in the full and effective theories, we determine the threshold corrections to the Wilson coefficients of dimension-six operators λ_1 and λ_2 at the one-loop level:

$$\begin{aligned} \lambda_1 &= \frac{\Sigma(0)}{M_X^2 + \Sigma(0)} + \frac{g_5^2}{16\pi^2} \frac{16}{5} \left(1 - \ln \frac{M_X^2}{\mu^2} \right), \\ \lambda_2 &= \frac{\Sigma(0)}{M_X^2 + \Sigma(0)} + \frac{g_5^2}{16\pi^2} \frac{18}{5} \left(1 - \ln \frac{M_X^2}{\mu^2} \right). \end{aligned} \quad (4.8)$$

We find that the corrections to the wave function for the matter field and the vertex of the X boson are canceled with each other as expected from the Ward identity and that the threshold corrections come from the corrections to the vacuum polarization and the box-like contributions.

Now, we give numerical results of the short-range renormalization factor including threshold corrections in the minimal SUSY $SU(5)$ GUT and its vector-like extension. In the minimal SUSY $SU(5)$ GUT, the X multiplet, the color-triplet Higgs multiplets, and the adjoint Higgs multiplet acquire heavy mass through the VEV of the adjoint Higgs multiplet. First, we set the masses of the GUT particles to be degenerate in mass 2.0×10^{16} GeV since they are model-dependent parameters. The dependence of the threshold correction on the GUT scale mass spectrum is

⁵ Since the IR divergent terms from the box-like diagrams are proportional to $\ln M_X^2/\mu_{\text{IR}}^2$, we divide this into $\ln M_X^2/\mu^2 + \ln \mu^2/\mu_{\text{IR}}^2$ where μ denotes the renormalization scale, and then the IR divergent terms are absorbed by the operators.

shown later. The threshold corrections in the minimal SUSY $SU(5)$ GUT are divided into two parts: the one comes from the vacuum polarization of the X boson as

$$\lambda_1|_{\text{vac.}} = \lambda_2|_{\text{vac.}} = \frac{\Sigma(0)}{M_X^2 + \Sigma(0)} = 1.26 \times 10^{-2}, \quad (4.9)$$

another one comes from the box-type diagram:

$$\begin{aligned} \lambda_1|_{\text{vert.}} &= \frac{g_5^2}{16\pi^2} \frac{16}{5} \left(1 - \ln \frac{M_X^2}{\mu^2} \right) = 1.05 \times 10^{-2}, \\ \lambda_2|_{\text{vert.}} &= \frac{g_5^2}{16\pi^2} \frac{18}{5} \left(1 - \ln \frac{M_X^2}{\mu^2} \right) = 1.18 \times 10^{-2}. \end{aligned} \quad (4.10)$$

Then, by combining these contribution we obtain the numerical values of threshold corrections as

$$\lambda_1(M_{\text{GUT}}) = 2.31 \times 10^{-2}, \quad \lambda_2(M_{\text{GUT}}) = 2.44 \times 10^{-2}, \quad (4.11)$$

where we assume that all sparticle masses are set to be $M_{\text{SUSY}} = 1$ TeV. We set the renormalization scale at which we match the amplitudes in the full theory and the EFT to $M_{\text{GUT}} = 2.0 \times 10^{16}$ GeV.

The short-range renormalization factors of Wilson coefficients of the dimension-six operators which include two-loop RGEs and threshold corrections are defined as:

$$A_S^{(I)} \equiv (1 - \lambda_I) \frac{C_I(M_{\text{SUSY}})}{C_I(M_{\text{GUT}})}, \quad (4.12)$$

where $C_I(\mu)$ are the Wilson coefficients of the dimension-six operators at renormalization scale μ , which do not include threshold corrections at GUT scale. These numerical factors are obtained by using the RGEs at the two-loop level⁶

$$A_S^{(1)} = 1.927, \quad A_S^{(2)} = 2.015. \quad (4.13)$$

We have also evaluated the short-range renormalization factor to the partial decay rate ($p \rightarrow e^+ + \pi^0$). We define the ratio of the short-range renormalization factor with and without the threshold correction to the Wilson coefficients of the dimension-six operators as

$$R \equiv \frac{A_S^{(1)2} + (1 + |V_{ud}|^2) A_S^{(2)2} \Big|_{\text{w}}}{A_S^{(1)2} + (1 + |V_{ud}|^2) A_S^{(2)2} \Big|_{\text{w/o}}}, \quad (4.14)$$

where the denominator and numerator correspond to the short-range enhancement factor of the nucleon decay rate without and with threshold corrections, respectively. V_{ud} denotes $(1, 1)$ component of the CKM matrix. We obtain $R = 0.952$ in the minimal SUSY $SU(5)$ GUT, that is, there is about 5% suppression compared with the short-range renormalization factor without threshold corrections.

⁶ The RGEs for the gauge and Yukawa coupling constants and the Wilson coefficients for the baryon-number violating dimension-six operators are summarized in [Appendix C](#).

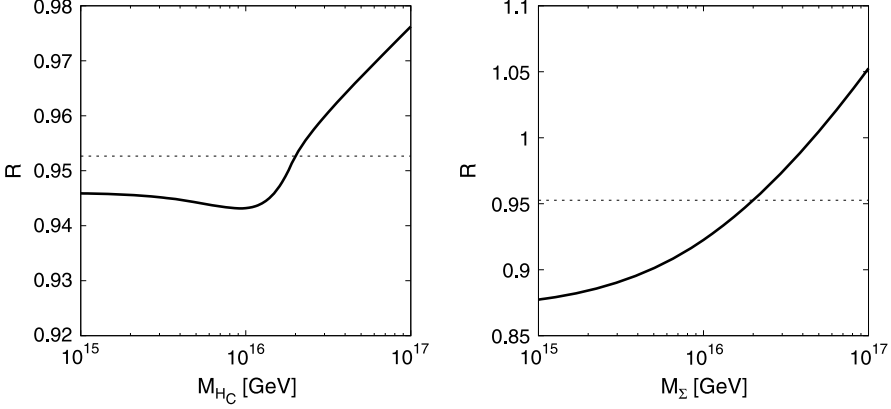


Fig. 6. GUT-scale particle mass dependence on renormalization factor of proton decay. Left panel shows M_{H_C} dependence for $M_{\Sigma} = M_X = 2 \times 10^{16}$ GeV. Right panel shows M_{Σ} dependence for $M_{H_C} = M_X = 2 \times 10^{16}$ GeV. Dotted line shows the degenerate mass case in each panels.

We note that the mass relation between M_{Σ} and $M_{\Sigma_{24}}$ is $M_{\Sigma} = 5M_{\Sigma_{24}}$ in the minimal SUSY $SU(5)$ GUT. When we adopt this mass relation and we assume the masses of the GUT particles are set to be $M_X = M_{\Sigma} = 2.0 \times 10^{16}$ GeV, we have

$$A_S^{(1)} = 1.916, \quad A_S^{(2)} = 2.004, \quad (4.15)$$

and then, we obtain $R = 0.942$.

In Fig. 6, we describe the heavy mass dependence on the ratio of the short-range renormalization factor in the minimal SUSY $SU(5)$ GUT. Here, we set the mass of the component fields of the adjoint Higgs multiplet to be degenerate in M_{Σ} , that is, we set $M_{\Sigma_{24}} = M_{\Sigma}$, for simplicity. The left panel of Fig. 6 shows the color-triplet Higgs mass (M_{H_C}) dependence of the ratio with the fixed adjoint Higgs mass $M_{\Sigma} = 2.0 \times 10^{16}$ GeV. The right panel of Fig. 6 shows the adjoint Higgs mass (M_{Σ}) dependence of the ratio with the fixed color-triplet Higgs mass $M_{H_C} = 2.0 \times 10^{16}$ GeV. Since, in a large M_{H_C} region, the vacuum polarization behaves as $\Sigma \sim \frac{1}{2}M_X^2(\frac{1}{2} - \ln M_{H_C}^2/\mu^2)$, the decay rate of proton is slightly enhanced in this region.

In the SUSY $SU(5)$ GUT with light vector-like matter scenario, the threshold corrections to the Wilson coefficients of the dimension-six operators are enhanced since the unified gauge coupling becomes large. This large unified coupling leads to the large renormalization effect to the Wilson coefficients of the dimension-six operators.

In Fig. 7, we show the ratio of the short-range renormalization factors in the vector-like matter scenario. The horizontal line and the vertical line present the mass scale of the vector-like matters and the ratio of the short-range renormalization effect, respectively. The solid lines correspond to the case that the number of $\mathbf{5} + \bar{\mathbf{5}}$ vector-like matters is set to be $n_5 = 1, \dots, 4$ from top to bottom without $\mathbf{10} + \bar{\mathbf{10}}$ vector-like matter. In this estimation, we assume the masses of the heavy multiplets and the GUT scale are set to be 2.0×10^{16} GeV. If the mass (number) of the vector-like superfields is sufficiently light (large), the unified gauge coupling at the GUT scale becomes larger, and then the threshold correction makes the proton lifetime longer.

In Fig. 8, we show the partial proton lifetime ($p \rightarrow \pi^0 + e^+$) in the minimal SUSY $SU(5)$ and its vector-like extension. In this evaluation, we assume the masses of the GUT spectrum are set to be the same mass (2.0×10^{16} GeV), especially the X -boson mass is set to be $M_X = 2.0 \times$

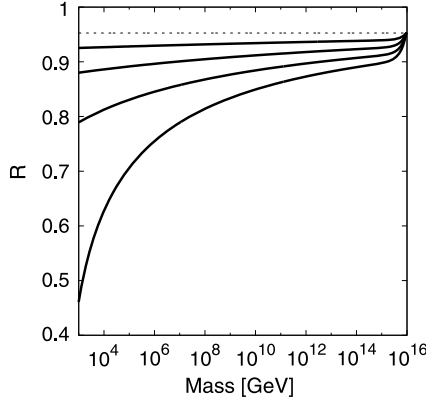


Fig. 7. Ratio of short-range renormalization effects with and without threshold effect in the minimal SUSY $SU(5)$ GUT with light vector-like matters. We take $n_5 = 1, \dots, 4$ in solid lines from top to bottom. The case of the minimal SUSY $SU(5)$ with no light vector-like matter is shown in dotted line.

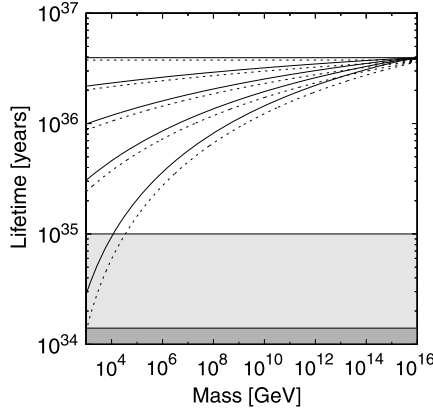


Fig. 8. Partial proton lifetime ($p \rightarrow \pi^0 + e^+$) in vector-like extension scenario. In solid (dotted) lines, we take $n_5 = 0, 1, \dots, 4$ with (without) threshold corrections at GUT scale. Deep gray (gray) region corresponds to experimental excluded region by Super-Kamiokande (the future sensitivity by the Hyper-Kamiokande).

10^{16} GeV. We use the two-loop RGEs of the Wilson coefficients of the dimension-six operators as short-distance [25,34] and as long-distance [24]. We also use the hadron matrix elements evaluated with the lattice calculation [35]. The deep gray region is corresponding to the present lower bound on this decay mode by the Super-Kamiokande ($\tau(p \rightarrow \pi^0 + e^+) > 1.4 \times 10^{34}$ years). The gray region, on the other hand, corresponds to the future sensitivity on this decay mode by the Hyper-Kamiokande ($\tau(p \rightarrow \pi^0 + e^+) > 1.0 \times 10^{35}$ years). Due to the extra fields, the lifetime is suppressed since the unified coupling becomes large at GUT scale. However, the threshold correction lifts the partial proton lifetime up.

5. Conclusion and discussion

In this study, we have derived the threshold corrections to the Wilson coefficients which cause proton decay ($p \rightarrow \pi^0 + e^+$) at the GUT scale in SUSY $SU(5)$ GUTs. We find that the threshold

correction makes the proton decay rate suppressed about 5% in the minimal SUSY $SU(5)$ GUT. Furthermore, we also have investigated the threshold effect on the partial proton decay rate in the extended SUSY $SU(5)$ GUT with additional vector-like pairs, motivated by the achievement of the 126 GeV Higgs boson. In these models, we find that the decay rate is $O(10)\%$ suppressed due to the large unified gauge coupling.

In our study, we neglect the threshold corrections induced by the Yukawa interactions, because the Yukawa interactions involving light quarks and leptons are negligibly small at the GUT scale. Similarly, we do not estimate the threshold correction at the scale where superparticles are decoupled. In this work, we have concentrated on the effect of vector-like matters at the GUT scale. In order to complete the evaluation of two-loop level corrections, we should include the one-loop threshold correction at the SUSY scale. We will calculate these corrections on another occasion.

There exists the additional loop suppression in the next-to-next-to leading order (NNLO) calculations such as three-loop RGEs and two-loop threshold corrections. The loop factor at the GUT scale, that is $g_5^2(M_{\text{GUT}})/16\pi^2$, becomes 3.3×10^{-3} to 1.5×10^{-2} corresponding to the number of vector-like matters being $N_5 = 0$ to $N_5 = 4$. Thus, the NNLO calculations should be much smaller than the uncertainty of the matrix elements derived by using lattice QCD simulation as discussed below.

The matrix elements relevant to nucleon decay have been evaluated with the lattice QCD and they have 30% uncertainty at present [35]. In this work, we have revealed that the threshold correction at the GUT scale can be comparable to the uncertainty in the vector-like matter extended models. On the other hand, the correction in the minimal SUSY $SU(5)$ GUT is small in comparison with the uncertainty of the matrix elements. We expect that the uncertainty would be reduced in the future.

Finally, we note the application of our work to the other SUSY GUTs. We only have investigated the threshold effects in the minimal SUSY $SU(5)$ GUT and the extra vector-like matter extensions in this paper. When, however, we apply our formulae for the extension of the SUSY $SU(5)$ GUTs, for instance the missing-partner model [36], we only have to evaluate additional contributions to the vacuum polarization for the X boson. That is remaining as one of our future work.

Acknowledgements

This work is supported by Grant-in-Aid for Scientific research from the Ministry of Education, Science, Sports, and Culture (MEXT), Japan, No. 24340047 (for J.H.) and No. 23104011 (for J.H. and Y.O.). The work of J.H. is also supported by World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan.

Appendix A. Decomposition of $SU(5)$ interactions

A.1. Interactions of vector superfields

In super-Yang–Mills theories, the renormalizable Lagrangian is written as

$$\mathcal{L}_{\text{SYM}} = \frac{1}{8g^2} \text{tr} \int d^2\theta \mathcal{W}^\alpha \mathcal{W}_\alpha + \text{h.c.}, \quad (\text{A.1})$$

where the field strength chiral superfield is given in Eq. (2.5). The Lagrangian is expanded in the vector superfield V as

$$\begin{aligned}
& \frac{1}{8g^2} \text{tr} \int d^2\theta \mathcal{W}^\alpha \mathcal{W}_\alpha \\
&= -\frac{1}{8} \text{tr} \int d^4\theta \left[-V \mathcal{D}^\alpha \bar{\mathcal{D}}^2 \mathcal{D}_\alpha V + 2g V \mathcal{D}^\alpha \bar{\mathcal{D}}^2 [V, \mathcal{D}_\alpha V] \right. \\
&\quad \left. + g^2 [V, \mathcal{D}^\alpha V] \bar{\mathcal{D}}^2 [V, \mathcal{D}_\alpha V] + \frac{4g^2}{3} \mathcal{D}_\alpha V \bar{\mathcal{D}}^2 [V, [V, \mathcal{D}_\alpha V]] + \dots \right]. \quad (\text{A.2})
\end{aligned}$$

The decomposition of the $SU(5)$ vector superfield $\mathcal{V}_3$ is given by Eq. (2.6). As mentioned in text, we denote $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$ vector superfields in the MSSM with G , W , and B . The kinetic terms of the vector superfields in the $SU(5)$ GUTs are given into the following form;

$$\begin{aligned}
\mathcal{L}_{VV} &= 2 \text{tr} \int d^4\theta G \square P_T G + 2 \text{tr} \int d^4\theta W \square P_T W + \int d^4\theta B \square P_T B \\
&\quad + 2 \int d^4\theta X^\dagger \square P_T X, \quad (\text{A.3})
\end{aligned}$$

where X denotes the massive vector superfield associated with the broken $SU(5)$ generators. Here, $P_T (\equiv \mathcal{D}^\alpha \bar{\mathcal{D}}^2 \mathcal{D}_\alpha / (8\square))$ is the projection operator to the transverse mode ($P_T^2 = P_T$).

From the second term of Eq. (A.2), the three-point interaction terms between X and MSSM vector superfields are obtained as

$$\mathcal{L}_{X\text{-3pt}} = \int d^4\theta \left[\delta_r^s (T^a)^\beta_\alpha (\mathcal{K}_{XG}^a)^{r\alpha}_{s\beta} + (t^a)_r^s \delta_\alpha^\beta (\mathcal{K}_{XW}^a)^{r\alpha}_{s\beta} + \frac{5}{2\sqrt{15}} \delta_r^s \delta_\alpha^\beta (\mathcal{K}_{XB}^a)^{r\alpha}_{s\beta} \right], \quad (\text{A.4})$$

where

$$\begin{aligned}
(\mathcal{K}_{XV}^a)^{r\alpha}_{s\beta} &\equiv \frac{g_5}{4} \left[X_\beta^r (\bar{\mathcal{D}}^2 \mathcal{D} X^\dagger)_s^\alpha \mathcal{D} V^a + V^a (\bar{\mathcal{D}}^2 \mathcal{D} X)_\beta^r (\mathcal{D} X^\dagger)_s^\alpha + X_s^{\dagger\alpha} (\bar{\mathcal{D}}^2 \mathcal{D} V^a) \mathcal{D} X_\beta^r \right. \\
&\quad \left. - X_\beta^r (\bar{\mathcal{D}}^2 \mathcal{D} V^a) (\mathcal{D} X^\dagger)_s^\alpha - V^a (\bar{\mathcal{D}}^2 \mathcal{D} X^\dagger)_s^\alpha \mathcal{D} X_\beta^r - X_s^{\dagger\alpha} (\bar{\mathcal{D}}^2 \mathcal{D} X)_\beta^r \mathcal{D} V^a \right]. \quad (\text{A.5})
\end{aligned}$$

Here, spinor indices are contracted like $^\alpha_\alpha$ or $_{\dot{\alpha}}^{\dot{\alpha}}$. The four-point self interaction of X is given as

$$\begin{aligned}
\mathcal{L}_{X\text{-4pt}} &= -\frac{g_5^2}{48} \int d^4\theta \left[(\bar{\mathcal{D}}^2 \mathcal{D} X^\dagger_r) (X_\beta^r X_s^{\dagger\alpha} (\mathcal{D} X)_\alpha^s - 2X_\beta^r (\mathcal{D} X^\dagger)_s^\beta X_\alpha^s + (\mathcal{D} X)_\beta^r X_s^{\dagger\beta} X_\alpha^s) \right. \\
&\quad \left. + (\bar{\mathcal{D}}^2 \mathcal{D} X_\alpha^r) (X_s^{\dagger\alpha} X_\beta^s (\mathcal{D} X^\dagger)_r^\beta - 2X_s^{\dagger\alpha} (\mathcal{D} X)_\beta^s X_r^{\dagger\beta} + (\mathcal{D} X^\dagger)_s^\alpha X_\beta^s X_r^{\dagger\beta}) \right]. \quad (\text{A.6})
\end{aligned}$$

A.2. Vector–ghost interactions

The Lagrangian for the massless Fadeev–Popov ghost chiral superfields, which are denoted by b and c , are given as

$$\mathcal{L}_{\text{FP}} = 2 \text{tr} \int d^4\theta (b + b^\dagger) \mathcal{L}_g V \left[(c + c^\dagger) + \coth(\mathcal{L}_g V) (c - c^\dagger) \right], \quad (\text{A.7})$$

where $\mathcal{L}_A B$ is the Lie derivative ($\mathcal{L}_A B \equiv [A, B]$). Therefore, the kinetic terms for ghost fields in the $SU(5)$ GUTs are obtained as

$$\begin{aligned}
\mathcal{L}_{\text{ghost}} &= 2 \int d^4\theta \left[\text{tr}(b_3^\dagger c_3 - b_3 c_3^\dagger) + \text{tr}(b_2^\dagger c_2 - b_2 c_2^\dagger) \right] + \int d^4\theta (b_1^\dagger c_1 - b_1 c_1^\dagger) \\
&\quad + \int d^4\theta \left[(b_X^\dagger c_X - b_X c_X^\dagger) + (b_{X^\dagger}^\dagger c_{X^\dagger} - b_{X^\dagger} c_{X^\dagger}^\dagger) \right], \quad (\text{A.8})
\end{aligned}$$

where the ghost multiplets are decomposed in a similar way to the gauge multiplets as

$$b = \begin{pmatrix} b_3 - \frac{2}{\sqrt{30}}b_1 & \frac{1}{\sqrt{2}}b_X \\ \frac{1}{\sqrt{2}}b_{X^\dagger} & b_2 + \frac{3}{\sqrt{30}}b_1 \end{pmatrix}, \quad c = \begin{pmatrix} c_3 - \frac{2}{\sqrt{30}}c_1 & \frac{1}{\sqrt{2}}c_X \\ \frac{1}{\sqrt{2}}c_{X^\dagger} & c_2 + \frac{3}{\sqrt{30}}c_1 \end{pmatrix}. \quad (\text{A.9})$$

After spontaneously breaking of the GUT group by the adjoint Higgs chiral superfield, there exist kinetic mixing terms between X and the Nambu–Goldstone chiral superfields $\Sigma_{(3,2)}$ and $\Sigma_{(3^*,2)}$. By using the supersymmetric R_ξ -gauge [37], we remove the kinetic mixing terms, and we find the mass terms for the ghost chiral superfields [37] as:

$$\mathcal{L}_{\text{ghost mass}} = \int d^4\theta \left[(b_X + b_X^\dagger) \frac{M_X^2}{\xi \square} (c_X - c_X^\dagger) + (b_{X^\dagger} + b_{X^\dagger}^\dagger) \frac{M_X^2}{\xi \square} (c_{X^\dagger} - c_{X^\dagger}^\dagger) \right]. \quad (\text{A.10})$$

We note that the terms such as $b_X c_X$ and $b_X^\dagger c_X^\dagger$ vanish by the superspace integral since these are chiral (or antichiral) superfields. Then, the propagator for massive ghost superfields is modified as

$$\Delta_{bc} = \frac{i}{k^2} \delta^4(\theta_1 - \theta_2) \rightarrow \frac{i}{k^2} \frac{1}{1 - \frac{M_X^2}{\xi k^2}} \delta^4(\theta_1 - \theta_2). \quad (\text{A.11})$$

In the evaluation of the self energy of X , we need interaction terms for X and the massive ghosts. In general, three-point and four-point interaction terms of ghost superfields and vector superfields are obtained from Eq. (A.7) as follows,

$$\mathcal{L}_{bVc} = \text{tr} \int d^4\theta \left\{ 2g(b + b^\dagger) [V, (c + c^\dagger)] + \frac{2g^2}{3} (b + b^\dagger) [V, [V, (c - c^\dagger)]] + \mathcal{O}(V^3) \right\}. \quad (\text{A.12})$$

Then, the interaction terms between X and the ghosts are given by:

$$\mathcal{L}_{bXc} = \int d^4\theta \left[\delta_r^s (T^a)_\alpha^\beta (\mathcal{K}_{bcG}^a)^{r\alpha}_{s\beta} - (t^a)_r^s \delta_\alpha^\beta (\mathcal{K}_{bcW}^a)^{r\alpha}_{s\beta} - \frac{5}{2\sqrt{15}} \delta_r^s \delta_\alpha^\beta (\mathcal{K}_{bcB})^{r\alpha}_{s\beta} \right], \quad (\text{A.13})$$

and

$$\begin{aligned} \mathcal{L}_{bX^2c} = & -\frac{g_5^2}{6} \int d^4\theta (\delta_{\alpha\gamma}^{\beta\delta} \delta_{su}^{tr} + \delta_{\alpha\gamma}^{\delta\beta} \delta_{su}^{rt}) X_r^\dagger{}^\alpha X_\beta^s \\ & \times \left[(b_{X^\dagger})_t^\gamma (c_{X^\dagger}^\dagger)_\delta^u - (b_X^\dagger)_t^\gamma (c_X)_\delta^u - (b_{X^\dagger}^\dagger)_\delta^u (c_{X^\dagger})_t^\gamma + (b_X)_\delta^u (c_X^\dagger)_t^\gamma \right]. \end{aligned} \quad (\text{A.14})$$

Here, we define $\delta_{\alpha\gamma}^{\beta\delta} \equiv \delta_\alpha^\beta \delta_\gamma^\delta$ and $\delta_{su}^{tr} \equiv \delta_s^t \delta_u^r$. In the three-point interactions, we define the term $(\mathcal{K}_{bcV}^a)^{r\alpha}_{s\beta}$ as:

$$\begin{aligned} (\mathcal{K}_{bcV}^a)^{r\alpha}_{s\beta} \equiv & ((b_X + b_{X^\dagger})_\beta^r X_s^{\dagger\alpha} - X_\beta^r (b_{X^\dagger} + b_X)_s^\alpha) (c_V + c_V^\dagger)^a \\ & + (b_V + b_V^\dagger)^a (X_\beta^r (c_{X^\dagger} + c_X)_s^\alpha - (c_X + c_{X^\dagger})_\beta^r X_s^{\dagger\alpha}). \end{aligned} \quad (\text{A.15})$$

A.3. Gauge interactions of matter superfields

Now, we summarize the gauge interactions of the matter and Higgs multiplets in SUSY $SU(5)$ GUTs. The renormalizable Kähler potential in the $SU(5)$ GUTs is given as:

$$\begin{aligned}\mathcal{K} = & \Phi^{\dagger A} (e^{-2g_5 \mathcal{V}_5})_A^B \Phi_B + \Psi_{AB}^{\dagger} (e^{2g_5 \mathcal{V}_5})_C^A (e^{2g_5 \mathcal{V}_5})_D^B \Psi^{CD} \\ & + 2\Sigma_B^{\dagger A} (e^{-2g_5 \mathcal{V}_5})_A^C (e^{2g_5 \mathcal{V}_5})_D^B \Sigma_C^D + H_5^{\dagger A} (e^{-2g_5 \mathcal{V}_5})_A^B H_{\bar{5}B} \\ & + H_{\bar{5}A}^{\dagger} (e^{2g_5 \mathcal{V}_5})_B^A H_5^B.\end{aligned}\quad (\text{A.16})$$

The three-point gauge interaction of the $\bar{5}$ representation matter field Φ is given as

$$\begin{aligned}\mathcal{K}_{\Phi^{\dagger}V\Phi} = & -g_5 D^{C\dagger} \left(2G - \frac{2}{\sqrt{15}} B \right) D^C + g_5 L^{\dagger} \left(2W - \frac{3}{\sqrt{15}} B \right) L \\ & - \sqrt{2} g_5 \left[D^{C\dagger} (X \cdot L) + \text{h.c.} \right].\end{aligned}\quad (\text{A.17})$$

For the four-point vertices, we only use the interactions which include only one X ,

$$\mathcal{K}_{\Phi^{\dagger}V^2\Phi} \ni \sqrt{2} g_5^2 \left(D^{C\dagger} G (X \cdot L) + \frac{1}{\sqrt{60}} D^{C\dagger} B (X \cdot L) + D^{C\dagger} (WX \cdot L) \right) + \text{h.c.} \quad (\text{A.18})$$

Here, $(A \cdot B) \equiv \epsilon_{rs} A^r B^s$. We also obtain the relevant gauge interactions from the **10** representation matter field Ψ ,

$$\begin{aligned}\mathcal{K}_{\Psi^{\dagger}V\Psi} = & -g_5 U^{C\dagger} \left(2G + \frac{4}{\sqrt{15}} B \right) U^C + g_5 Q^{\dagger} \left(2G + 2W + \frac{1}{\sqrt{15}} B \right) Q \\ & + g_5 \frac{6}{\sqrt{15}} E^{C\dagger} B E^C + \sqrt{2} g_5 \left[[Q^{\dagger} X U^C] - (Q^{\dagger} \cdot X^{\dagger}) E^C + \text{h.c.} \right],\end{aligned}\quad (\text{A.19})$$

$$\begin{aligned}\mathcal{K}_{\Psi^{\dagger}V^2\Psi} \ni & \sqrt{2} g_5^2 \left[[(GQ^{\dagger}) X U^C] - [Q^{\dagger} X (GU^C)] - \frac{3}{\sqrt{60}} B [Q^{\dagger} X U^C] + [(WQ^{\dagger}) X U^C] \right. \\ & \left. + E^{C\dagger} \left((X \cdot GQ) + (X \cdot WQ) + \frac{7}{\sqrt{60}} (X \cdot BQ) \right) \right] + \text{h.c.},\end{aligned}\quad (\text{A.20})$$

where $[ABC] \equiv \epsilon^{\alpha\beta\gamma} A_{\alpha} B_{\beta} C_{\gamma}$ or $\epsilon_{\alpha\beta\gamma} A^{\alpha} B^{\beta} C^{\gamma}$.

There are also the three- and four-point interactions with Higgs multiplets of X . One of those comes from the interaction of the anti-fundamental Higgs superfield $\bar{H} = (H_{\bar{C}}, H_d)$,

$$\begin{aligned}\mathcal{K}_{\bar{H}^{\dagger}X\bar{H}} = & -\sqrt{2} g_5 \left[H_{\bar{C}}^{\dagger} (X \cdot H_d) + \text{h.c.} \right] \\ & + g_5^2 \left[\sqrt{2} \left(H_{\bar{C}}^{\dagger} G (X \cdot H_d) + \frac{1}{2\sqrt{15}} H_{\bar{C}}^{\dagger} B (X \cdot H_d) + H_{\bar{C}}^{\dagger} (WX \cdot H_d) \right) + \text{h.c.} \right. \\ & \left. + H_{\bar{C}}^{\dagger\alpha} X_{\alpha}^r X_r^{\dagger\beta} H_{\bar{C}\beta} + (X^{\dagger} \cdot H_d^{\dagger}) (X \cdot H_d) \right].\end{aligned}\quad (\text{A.21})$$

Another one comes from the fundamental Higgs superfield $H = (H_C, H_u)$,

$$\begin{aligned}\mathcal{K}_{H^{\dagger}XH} = & \sqrt{2} g_5 \left[H_u^{\dagger} X H_C + \text{h.c.} \right] \\ & + g_5^2 \left[\sqrt{2} \left(H_u^{\dagger} X G H_C + \frac{1}{2\sqrt{15}} H_u^{\dagger} X B H_C + H_u^{\dagger} W X H_C \right) + \text{h.c.} \right. \\ & \left. + H_{C\alpha}^{\dagger} X_{\alpha}^r X_r^{\dagger\beta} H_C^{\beta} + H_{ur}^{\dagger} X_{\alpha}^r X_s^{\dagger\alpha} H_u^s \right].\end{aligned}\quad (\text{A.22})$$

The adjoint Higgs superfield is decomposed as

$$\Sigma = \begin{pmatrix} \Sigma_8 - \frac{2}{\sqrt{60}}\Sigma_{24} & \frac{1}{\sqrt{2}}\Sigma_{(3,2)} \\ \frac{1}{\sqrt{2}}\Sigma_{(3^*,2)} & \Sigma_3 + \frac{3}{\sqrt{60}}\Sigma_{24} \end{pmatrix}. \quad (\text{A.23})$$

In our calculation, we need the interaction terms with the adjoint Higgs superfield of X ,

$$\begin{aligned} \mathcal{K}_{\Sigma^\dagger X \Sigma} = & -2g_5 \left[\Sigma_{(3^*,2)}^\dagger X \Sigma_8 - \Sigma_8^\dagger X \Sigma_{(3,2)} \right] - 2g_5 \left[\Sigma_3^\dagger X \Sigma_{(3,2)} - \Sigma_{(3^*,2)}^\dagger X \Sigma_3 \right] \\ & - \frac{5}{\sqrt{15}} g_5 \left[\Sigma_{(3^*,2)}^\dagger X \Sigma_{24} - \Sigma_{24}^\dagger X \Sigma_{(3,2)} \right] + \text{h.c.}, \end{aligned} \quad (\text{A.24})$$

$$\begin{aligned} \mathcal{K}_{\Sigma^\dagger X^\dagger X \Sigma} = & g_5^2 \left\{ 2X^\dagger (\Sigma_8^\dagger \Sigma_8 + \Sigma_8 \Sigma_8^\dagger) X + 2X^\dagger (\Sigma_3^\dagger \Sigma_3 + \Sigma_3 \Sigma_3^\dagger) X + \frac{5}{3} X^\dagger \Sigma_{24}^\dagger \Sigma_{24} X \right. \\ & + \frac{10}{\sqrt{15}} (\Sigma_{24}^\dagger X^\dagger X \Sigma_8 - \Sigma_{24}^\dagger X^\dagger X \Sigma_3 + \text{h.c.}) \\ & - 2 \left(2\Sigma_8^\dagger X^\dagger X \Sigma_3 + (\Sigma_{(3^*,2)}^\dagger)_r^\alpha X_\alpha^s X_\beta^r (\Sigma_{(3,2)})_s^\beta + \text{h.c.} \right) \\ & \left. + (\delta_{st}^{ru} \delta_{\delta\beta}^{\alpha\gamma} + \delta_{ts}^{ru} \delta_{\beta\delta}^{\alpha\gamma}) X_\gamma^\dagger X_u^\dagger \left((\Sigma_{(3,2)}^\dagger)_\alpha^s (\Sigma_{(3,2)})_r^\beta + (\Sigma_{(3^*,2)}^\dagger)_r^\beta (\Sigma_{(3^*,2)})_\alpha^s \right) \right\}. \end{aligned} \quad (\text{A.25})$$

After symmetry breaking of GUT, there exist the three-point interaction terms between MSSM vector superfields, Nambu–Goldstone multiplet, and X with VEV v_Σ of the adjoint Higgs multiplet.

$$\begin{aligned} \mathcal{K}_{v_\Sigma X} = & -10g_5^2 v_\Sigma \left\{ \Sigma_{(3,2)} \left[GX - WX + \frac{5}{\sqrt{30}} BX \right] \right. \\ & \left. + \Sigma_{(3^*,2)} \left[GX^\dagger - WX^\dagger + \frac{5}{\sqrt{30}} BX^\dagger \right] \right\} + \text{h.c.} \end{aligned} \quad (\text{A.26})$$

Appendix B. Radiative corrections at one-loop

In this appendix, we give the explicit formulae of the loop integrals in terms of supergraphs. All the external momenta of the chiral (antichiral) superfields are set to be p , and the masses of the MSSM vector superfields are set to be μ_{IR} in order to regularize the IR divergence. For simplicity, we set all coupling constants to be 1 through this appendix. For the corrections to the three-point vertex functions and the box-like corrections, the loop integrals in text are the coefficients of Kähler potentials in the limit that the external momenta p^2 vanishes.

B.1. Radiative corrections to two-point functions for matter superfields

The correction to the self energy of the chiral and antichiral matter superfields in the first generation is induced by the gauge interactions. The one-loop contribution is given as

$$\begin{aligned} i\Gamma_\Phi = & i^2 \int d^4\theta_1 d^4\theta_2 \int \frac{d^D l}{(2\pi)^D} \frac{-i}{2(l^2 - M^2)} \frac{i}{(l+p)^2} \frac{1}{16} (\mathcal{D}_2^2 \delta_{21} \overleftarrow{\mathcal{D}}_1^2) \delta_{12} \Phi(p, \theta_1) \Phi^\dagger(p, \theta_2) \\ = & -\frac{1}{2} \int \frac{d^D l}{(2\pi)^D} \frac{1}{l^2 - M^2} \frac{1}{(l+p)^2} \int d^4\theta \Phi^\dagger(p, \theta) \Phi(p, \theta), \end{aligned} \quad (\text{B.1})$$

where p is external momentum and M is the mass for the internal vector superfield. δ_{ij} denotes the δ -function for the Grassmann valuable, $\delta_{ij} \equiv (\delta_i - \delta_j)^2 (\bar{\delta}_i - \bar{\delta}_j)^2$. The renormalized one-loop two-point function of matter superfields in the $SU(5)$ GUTs are given as

$$\Gamma_\Phi = -\frac{g_5^2}{8\pi^2} \left[(c_5^\Phi - \sum_{n=1}^3 c_n^\Phi) f(M_X^2) + \sum_{n=1}^3 c_n^\Phi f(\mu_{\text{IR}}^2) \right] \int d^4\theta \Phi^\dagger \Phi, \quad (\text{B.2})$$

where function $f(M^2)$ is defined in Eq. (3.6). c_5^Φ and c_n^Φ ($n = 3, 2, 1$) are the quadratic Casimir defined in text. In the MSSM, we also obtain

$$\Gamma_\Phi^{\text{EFT}} = -\frac{g_5^2}{8\pi^2} \sum_{n=1}^3 c_n^\Phi f(\mu_{\text{IR}}^2) \int d^4\theta \Phi^\dagger \Phi. \quad (\text{B.3})$$

B.2. Radiative corrections to two-point function for vector superfield

Three diagrams in Fig. 1 contribute to the radiative corrections to two-point functions from the (massive) chiral superfields. The corrections from the diagram (a) in Fig. 1 are

$$i\Gamma_{XX}^{(a)} = -i^2 \int d^4\theta X_r^{\dagger\alpha}(-p, \theta) \int \frac{d^D l}{(2\pi)^D} \frac{l^2 - \frac{1}{2} l_\mu \sigma_{\alpha\dot{\alpha}}^\mu \mathcal{D}^\alpha \bar{\mathcal{D}}^{\dot{\alpha}} + \frac{1}{16} \mathcal{D}^2 \bar{\mathcal{D}}^2}{(l^2 - M_1^2)[(l+p)^2 - M_2^2]} X_\alpha^r(p, \theta), \quad (\text{B.4})$$

where M_1 and M_2 are the masses of the chiral superfields in the loop diagram. After picking the transverse mode and regularizing the UV divergence, we obtain the finite correction to the two-point function as follows:

$$\Gamma_{XX}^{(a)} = \frac{1}{16\pi^2} B(p^2, M_1^2, M_2^2) \int d^4\theta X_r^{\dagger\alpha}(-p, \theta) P_T X_\alpha^r(p, \theta) + (\text{longitudinal mode}), \quad (\text{B.5})$$

where the loop function is defined in Eq. (3.11). The massive chiral superfields also have the non-zero contribution from the diagram (b) in Fig. 1,

$$\Gamma_{XX}^{(b)} = -\frac{M^2}{16\pi^2} \left(1 - \ln \frac{M^2}{\mu^2} \right) \int d^4\theta X_r^{\dagger\alpha}(-p, \theta) P_T X_\alpha^r(p, \theta), \quad (\text{B.6})$$

where M is for the masses of chiral superfields running in the internal line. The third contribution (the diagram (c) in Fig. 1) comes from the vertex which includes the VEV of the adjoint Higgs superfield,

$$\Gamma_{XX}^{(c)} = \frac{1}{16\pi^2} A(p^2, M_1^2, M_2^2) \int d^4\theta X_r^{\dagger\alpha}(-p, \theta) P_T X_\alpha^r(p, \theta). \quad (\text{B.7})$$

Here, the loop function A is also defined in Eq. (3.11).

B.3. Radiative corrections to three-point vertices

The one-loop diagrams for the three-point vertex correction are shown in Fig. 3. In our momentum assignment, the momentum of the X boson is $q = 0$. The one-loop vertex correction induced by the diagram in Fig. 3(a) is given as

$$i\Gamma_1^{(v)}(p; M) = i^3 \int d^4\theta_1 d^4\theta_2 d^4\theta_3 \int \frac{d^D l}{(2\pi)^D} \frac{i}{(l+p)^2} \frac{i}{(l+p)^2} \frac{-i}{2(l^2 - M^2)} \\ \times \frac{1}{16} (\overline{\mathcal{D}}_2^2 \delta_{23} \overleftarrow{\mathcal{D}}_3^2) \frac{1}{16} (\overline{\mathcal{D}}_3^2 \delta_{31} \overleftarrow{\mathcal{D}}_1^2) \delta_{12} \Phi(\theta_1) \Phi^\dagger(\theta_2) V(\theta_3). \quad (\text{B.8})$$

By integrating by part and also using the $\mathcal{D}$ algebra, we always decompose the vertex correction into the effective Kähler terms $\mathcal{K}$ and the auxiliary terms which vanish as $\mathcal{D}_\alpha \Phi, \overline{\mathcal{D}}_{\dot{\alpha}} \Phi^\dagger = 0$. The effective Kähler term induced by the diagram Fig. 3(a) has the following form:

$$i\mathcal{K}_1^{(v)}(p; M) = \frac{1}{2} \int \frac{d^D l}{(2\pi)^D} \frac{1}{[(l+p)^2]^2} \frac{1}{l^2 - M^2} (l+2p)^2 \Phi \Phi^\dagger V, \quad (\text{B.9})$$

where we remove the Grassmann valuables in the effective Kähler term, for simplicity.

Next we show the effective Kähler term described in Fig. 3(b) and (c). In our momentum assignment, the diagrams both of Fig. 3(b) and (c) give the same expression, and we find the one-loop vertex correction and the effective Kähler term as

$$i\Gamma_2^{(v)}(p; M) = i^2 \int d^4\theta_1 d^4\theta_2 \int \frac{d^D l}{(2\pi)^D} \frac{i}{(l+p)^2} \frac{-i}{2(l^2 - M^2)} \\ \times \frac{1}{16} \overline{\mathcal{D}}_2^2 \delta_{21} \overleftarrow{\mathcal{D}}_1^2 \delta_{12} \Phi(\theta_1) \Phi^\dagger(\theta_2) V(\theta_2), \\ i\mathcal{K}_2^{(v)}(p; M) = -\frac{1}{2} \int \frac{d^D l}{(2\pi)^D} \frac{1}{(l+p)^2} \frac{1}{l^2 - M^2} \Phi \Phi^\dagger V. \quad (\text{B.10})$$

The diagrams (d) and (e) in Fig. 3 include the three-point vertices of vector superfields. After carrying out the superspace integral, the vertex corrections from the diagrams Fig. 3(d) and (e) are obtained as

$$i\Gamma_3^{(v)}(p; M) = -4 \int \frac{d^D l}{(2\pi)^D} \frac{1}{l^2 - \mu_{\text{IR}}^2} \frac{1}{(l+p)^2} \frac{(l+p)^2 + p^2}{l^2 - M^2} \int d^4\theta \Phi^\dagger \Phi V, \\ i\Gamma_4^{(v)}(p; M) = 8 \int \frac{d^D l}{(2\pi)^D} \frac{1}{l^2 - M^2} \frac{1}{l^2 - \mu_{\text{IR}}^2} \int d^4\theta \Phi^\dagger \Phi V. \quad (\text{B.11})$$

Since they do not include the auxiliary terms, $\Gamma_n^{(v)}(p; M)$ ($n = 3, 4$) is just the Kähler term $\int d^4\theta \mathcal{K}_n^{(v)}(p; M)$ ($n = 3, 4$).

The contribution from a diagram (f) in Fig. 3 is zero as mentioned in the text.

B.4. Box-like corrections

Now we show the effective Kähler terms from the box-like diagrams presented in Fig. 4. These diagrams include one massless and one massive vector superfields. The correction from the box diagram (Fig. 4(a)) is given as:

$$i\Gamma_{\text{box}}(p; M) \\ \equiv p^2 \int \frac{d^D q}{(2\pi)^D} \frac{1}{q^2 - M^2} \frac{1}{(q+p)^2} \frac{1}{q^2 - \mu_{\text{IR}}^2} \frac{1}{(q-p)^2} \int d^4\theta \Phi_1^\dagger \Phi_2 \Phi_3^\dagger \Phi_4. \quad (\text{B.12})$$

Here, we do not write the external momenta of external superfields for simplicity since we set them to be the same momentum p . As mentioned above, we set the mass of massless vector

superfields to be μ_{IR} as IR regularization. $\Gamma_{\text{box}}(p; M)$ vanishes at the point with $p^2 = 0$, as mentioned in the text.

The contribution of the crossing-box diagram (Fig. 4(b)) is given by

$$\begin{aligned} i\Gamma_{\text{cross}}(p; M) &= \frac{1}{4} \int \frac{d^D q}{(2\pi)^D} \frac{1}{q^2 - M^2} \frac{1}{[(q-p)^2]^2} \frac{1}{q^2 - \mu_{\text{IR}}^2} \int d^4 \theta \left[(q-p)^2 \Phi_1^\dagger \Phi_2 \Phi_3^\dagger \Phi_4 \right. \\ &\quad \left. + \frac{1}{2} (q-p)_\mu (\bar{\sigma}^\mu \bar{\mathcal{D}} \mathcal{D}) \left(\Phi_1^\dagger \Phi_4 \right) \Phi_2 \Phi_3^\dagger + \frac{1}{16} \bar{\mathcal{D}}^2 \mathcal{D}^2 \left(\Phi_1^\dagger \Phi_4 \right) \Phi_3^\dagger \Phi_2 \right]. \end{aligned} \quad (\text{B.13})$$

Here, we define the mnemonic symbol $(\bar{\sigma}^\mu \bar{\mathcal{D}} \mathcal{D}) \equiv (\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} \bar{\mathcal{D}}_{\dot{\alpha}} \mathcal{D}_\alpha$. This correction has the auxiliary terms. The corresponding Kähler term is given by removing the auxiliary terms as

$$i\mathcal{K}_{\text{cross}}(p; M) = \frac{1}{4} \int \frac{d^D q}{(2\pi)^D} \frac{(q-2p)^2}{(q^2 - M^2)[(q-p)^2]^2 (q^2 - \mu_{\text{IR}}^2)} \Phi_1^\dagger \Phi_2 \Phi_3^\dagger \Phi_4. \quad (\text{B.14})$$

Finally, we show the contribution from the triangle diagram in Fig. 4(c). The correction from the triangle diagram is obtained as follows:

$$i\Gamma_{\text{triangle}}(p; M) \equiv -\frac{1}{4} \int \frac{d^D q}{(2\pi)^D} \frac{1}{q^2 - M^2} \frac{1}{(q+p)^2} \frac{1}{q^2 - \mu_{\text{IR}}^2} \int d^4 \theta \Phi_1^\dagger \Phi_2 \Phi_3^\dagger \Phi_4. \quad (\text{B.15})$$

Since auxiliary terms are not included in the radiative corrections Γ_{box} and Γ_{triangle} , the corresponding Kähler terms are just written by these corrections as $\Gamma_n = \int d^4 \theta \mathcal{K}_n$ ($n = \text{box}, \text{triangle}$).

The diagram in Fig. 4(d) vanishes as mentioned in the text.

B.5. One-loop corrections in EFT

In the last of this appendix, we show the radiative corrections in EFT presented in Fig. 5. We obtain the one-loop effective vertex functions Γ_1^{EFT} , Γ_2^{EFT} , and Γ_3^{EFT} which correspond to the diagram Fig. 5(b), (c), and (a), respectively, as follows:

$$\begin{aligned} i\Gamma_1^{\text{EFT}}(p; \mu_{\text{IR}}) &= \frac{1}{2} \int \frac{d^D q}{(2\pi)^D} \frac{1}{[(q+p)^2]^2} \frac{1}{q^2 - \mu_{\text{IR}}^2} \int d^4 \theta \left[(q+p)^2 \Phi_1^\dagger \Phi_2 \Phi_3^\dagger \Phi_4 \right. \\ &\quad \left. + \frac{1}{2} (q+p)_\mu (\bar{\sigma}^\mu \bar{\mathcal{D}} \mathcal{D}) \left(\Phi_1^\dagger \Phi_2 \right) \Phi_3^\dagger \Phi_4 + \frac{1}{16} \bar{\mathcal{D}}^2 \mathcal{D}^2 \left(\Phi_1^\dagger \Phi_2 \right) \Phi_3^\dagger \Phi_4 \right], \\ i\Gamma_2^{\text{EFT}}(p; \mu_{\text{IR}}) &= -\frac{1}{2} \int \frac{d^D q}{(2\pi)^D} \frac{1}{q^2 - \mu_{\text{IR}}^2} \frac{1}{(q+p)^2} \int d^4 \theta \Phi_1^\dagger \Phi_2 \Phi_3^\dagger \Phi_4, \\ i\Gamma_3^{\text{EFT}}(p; \mu_{\text{IR}}) &= \frac{1}{2} \int \frac{d^D q}{(2\pi)^D} \frac{1}{[(q+p)^2]^2} \frac{1}{q^2 - \mu_{\text{IR}}^2} (2p)^2 \int d^4 \theta \Phi_1^\dagger \Phi_2 \Phi_3^\dagger \Phi_4. \end{aligned} \quad (\text{B.16})$$

The momentum assignment is the same as in calculation of the box-like diagrams. The corresponding Kähler terms are given by removing the auxiliary terms as

$$\begin{aligned} i\mathcal{K}_1^{\text{EFT}}(p; \mu_{\text{IR}}) &= \frac{1}{2} \int \frac{d^D q}{(2\pi)^D} \frac{(q+2p)^2}{[(q+p)^2]^2 (q^2 - \mu_{\text{IR}}^2)} \Phi_1^\dagger \Phi_2 \Phi_3^\dagger \Phi_4, \\ i\mathcal{K}_2^{\text{EFT}}(p; \mu_{\text{IR}}) &= -\frac{1}{2} \int \frac{d^D q}{(2\pi)^D} \frac{1}{q^2 - \mu_{\text{IR}}^2} \frac{1}{(q+p)^2} \Phi_1^\dagger \Phi_2 \Phi_3^\dagger \Phi_4, \end{aligned}$$

$$i\mathcal{K}_3^{\text{EFT}}(p; \mu_{\text{IR}}) = 2p^2 \int \frac{d^D q}{(2\pi)^D} \frac{1}{[(q+p)^2]^2} \frac{1}{q^2 - \mu_{\text{IR}}^2} \Phi_1^\dagger \Phi_2 \Phi_3^\dagger \Phi_4. \quad (\text{B.17})$$

Here, we skip over the ways in which we obtain the effective vertex functions from the diagrams since these structures are similar as mentioned above. $i\mathcal{K}_3^{\text{EFT}}(p; \mu_{\text{IR}})$ vanishes when $p^2 = 0$ is set, as mentioned in the text.

Appendix C. Renormalization group equations

C.1. Gauge couplings and Yukawa couplings

In our analysis, we have used the RGEs at the two-loop level. The RGEs for the gauge coupling constants are as follows [38,39]:

$$\frac{dg_i}{d \ln \mu} = \frac{g_i}{16\pi^2} \left[b_i g_i^2 + \frac{1}{16\pi^2} \left(\sum_j b_{ij} g_i^2 g_j^2 - \sum_{j=U,D,E} a_{ij} g_i^2 \text{tr}[\mathbf{Y}_j \mathbf{Y}_j^\dagger] \right) \right]. \quad (\text{C.1})$$

Here, $\mathbf{Y}_U = \mathbf{U}$, $\mathbf{Y}_D = \mathbf{D}$, and $\mathbf{Y}_E = \mathbf{E}$ are the Yukawa coupling matrices. The coefficients in the SM are given as:

$$b_{ij} = \begin{pmatrix} \frac{199}{50} & \frac{27}{10} & \frac{44}{5} \\ \frac{9}{10} & \frac{35}{6} & 12 \\ \frac{11}{10} & \frac{9}{2} & -26 \end{pmatrix}, \quad b_i = \left(\frac{41}{10}, -\frac{19}{6}, -7 \right), \quad a_{ij} = \begin{pmatrix} \frac{17}{10} & \frac{1}{2} & \frac{3}{2} \\ \frac{3}{2} & \frac{3}{2} & \frac{1}{2} \\ 2 & 2 & 0 \end{pmatrix}. \quad (\text{C.2})$$

The one-loop RGEs for the Yukawa coupling matrices are given as.⁷

$$\begin{aligned} \frac{d\mathbf{U}}{d \ln \mu} &= \frac{1}{16\pi^2} \left[-\sum_i c_i^{\text{SM}} g_i^2 + \frac{3}{2} \mathbf{U} \mathbf{U}^\dagger - \frac{3}{2} \mathbf{D} \mathbf{D}^\dagger + Y_2(S) \right] \mathbf{U}, \\ \frac{d\mathbf{D}}{d \ln \mu} &= \frac{1}{16\pi^2} \left[-\sum_i c_i'^{\text{SM}} g_i^2 + \frac{3}{2} \mathbf{D} \mathbf{D}^\dagger - \frac{3}{2} \mathbf{U} \mathbf{U}^\dagger + Y_2(S) \right] \mathbf{D}, \\ \frac{d\mathbf{E}}{d \ln \mu} &= \frac{1}{16\pi^2} \left[-\sum_i c_i''^{\text{SM}} g_i^2 + \frac{3}{2} \mathbf{E} \mathbf{E}^\dagger + Y_2(S) \right] \mathbf{E}, \end{aligned} \quad (\text{C.3})$$

where

$$c_i^{\text{SM}} = \left(\frac{17}{20}, \frac{9}{4}, 8 \right), \quad c_i'^{\text{SM}} = \left(\frac{1}{4}, \frac{9}{4}, 8 \right), \quad c_i''^{\text{SM}} = \left(\frac{9}{4}, \frac{9}{4}, 0 \right), \quad (\text{C.4})$$

and

$$Y_2(S) = \text{tr} \left[3\mathbf{U} \mathbf{U}^\dagger + 3\mathbf{D} \mathbf{D}^\dagger + \mathbf{E} \mathbf{E}^\dagger \right]. \quad (\text{C.5})$$

The coefficients of the RGEs for the gauge coupling constants in the MSSM are obtained as

⁷ In our calculation, we need the RGEs for the gauge couplings at the two-loop level. It is sufficient to take into account the RGEs for the Yukawa couplings at the one-loop level since the Yukawa couplings appear in the two-loop-level RGEs for the gauge couplings.

$$b_{ij} = \begin{pmatrix} \frac{199}{25} & \frac{27}{5} & \frac{88}{5} \\ \frac{9}{5} & 25 & 24 \\ \frac{11}{5} & 9 & 14 \end{pmatrix}, \quad b_i = \left(\frac{33}{5}, 1, -3 \right), \quad a_{ij} = \begin{pmatrix} \frac{26}{5} & \frac{14}{5} & \frac{18}{5} \\ 6 & 6 & 2 \\ 4 & 4 & 0 \end{pmatrix}. \quad (\text{C.6})$$

The one-loop RGEs for the Yukawa matrices in the MSSM are given as

$$\begin{aligned} \frac{d\tilde{\mathbf{U}}}{d\ln\mu} &= \frac{1}{16\pi^2} \left[-\sum_i c_i^{\text{MSSM}} g_i^2 + 3\tilde{\mathbf{U}}\tilde{\mathbf{U}}^\dagger + \tilde{\mathbf{D}}\tilde{\mathbf{D}}^\dagger + \text{tr}(3\tilde{\mathbf{U}}\tilde{\mathbf{U}}^\dagger) \right] \tilde{\mathbf{U}}, \\ \frac{d\tilde{\mathbf{D}}}{d\ln\mu} &= \frac{1}{16\pi^2} \left[-\sum_i c_i^{\text{MSSM}} g_i^2 + 3\tilde{\mathbf{D}}\tilde{\mathbf{D}}^\dagger + \tilde{\mathbf{U}}\tilde{\mathbf{U}}^\dagger + \text{tr}(3\tilde{\mathbf{D}}\tilde{\mathbf{D}}^\dagger + \tilde{\mathbf{E}}\tilde{\mathbf{E}}^\dagger) \right] \tilde{\mathbf{D}}, \\ \frac{d\tilde{\mathbf{E}}}{d\ln\mu} &= \frac{1}{16\pi^2} \left[-\sum_i c_i^{\text{MSSM}} g_i^2 + 3\tilde{\mathbf{E}}\tilde{\mathbf{E}}^\dagger + \text{tr}(3\tilde{\mathbf{D}}\tilde{\mathbf{D}}^\dagger + \tilde{\mathbf{E}}\tilde{\mathbf{E}}^\dagger) \right] \tilde{\mathbf{E}}, \end{aligned} \quad (\text{C.7})$$

where

$$c_i^{\text{MSSM}} = \left(\frac{13}{15}, 3, \frac{16}{3} \right), \quad c_i^{\text{MSSM}} = \left(\frac{7}{15}, 3, \frac{16}{3} \right), \quad c_i^{\text{MSSM}} = \left(\frac{9}{5}, 3, 0 \right). \quad (\text{C.8})$$

The boundary conditions for the Yukawa coupling constants at the SUSY breaking scale (M_S) are

$$\tilde{U}(M_S) = \frac{1}{\sin\beta} U(M_S), \quad \tilde{Y}_j(M_S) = \frac{1}{\cos\beta} Y_j(M_S) \quad (j = D, E), \quad (\text{C.9})$$

where $\tan\beta$ is the ratio of vacuum expectation values in the MSSM.

When the vector-like matters are introduced in the MSSM, the RGEs for the gauge coupling constants are modified as

$$b_i \rightarrow b_i + \delta b_i, \quad b_{ij} \rightarrow b_{ij} + \delta b_{ij}, \quad (\text{C.10})$$

where b_i and b_{ij} are the coefficients of the one-loop and two-loop RGEs in the MSSM, respectively. δb_i and δb_{ij} are given by [40]:

$$\begin{aligned} \delta b_i &= (n_5 + 3n_{10}, n_5 + 3n_{10}, n_5 + 3n_{10}), \\ \delta b_{ij} &= \begin{pmatrix} \frac{7}{15}n_5 + \frac{23}{5}n_{10} & \frac{9}{5}n_5 + \frac{3}{5}n_{10} & \frac{32}{15}n_5 + \frac{48}{5}n_{10} \\ \frac{3}{5}n_5 + \frac{1}{5}n_{10} & 7n_5 + 21n_{10} & 16n_{10} \\ \frac{4}{15}n_5 + \frac{6}{5}n_{10} & 6n_{10} & \frac{34}{3}n_5 + 34n_{10} \end{pmatrix}, \end{aligned} \quad (\text{C.11})$$

where n_5 and n_{10} denote the number of $\mathbf{5} + \bar{\mathbf{5}}$ and $\mathbf{10} + \bar{\mathbf{10}}$ vector-like matter superfields, respectively.

C.2. Wilson coefficients of $D = 6$ Baryon-number violating operators

In Ref. [25], they have derived the two-loop RGEs for the Wilson coefficients of the following dimension-six baryon-number violating operators in the SUSY invariant theories,

$$\mathcal{L}_{D=6} = \sum_{I=1}^2 \mathcal{C}^{(I)} \mathcal{O}^{(I)}, \quad (\text{C.12})$$

where

$$\begin{aligned}\mathcal{O}^{(1)} &= \epsilon_{\alpha\beta\gamma}\epsilon_{rs} \int d^4\theta U^{C\dagger\alpha} D^{C\dagger\beta} e^{-\frac{2}{3}g_Y B} (e^{2g_3 G} Q^r)^\gamma L^s, \\ \mathcal{O}^{(2)} &= \epsilon_{\alpha\beta\gamma}\epsilon_{rs} \int d^4\theta E^{C\dagger} (e^{-2g_3 G} U^{C\dagger})^\alpha e^{\frac{2}{3}g_Y B} Q^{r\beta} Q^{s\gamma}.\end{aligned}\quad (\text{C.13})$$

The RGEs for the Wilson coefficients are given as

$$\mu \frac{d\mathcal{C}^{(I)}}{d\mu} = \frac{\mathcal{C}^{(I)}}{16\pi^2} \left[\sum_i \alpha_i^{(I)} g_i^2 + \frac{1}{16\pi^2} \sum_{i,j} \alpha_{ij}^{(I)} g_i^2 g_j^2 \right], \quad (\text{C.14})$$

where $i = 1, \dots, 3$, and the coefficients are given as

$$\alpha_i^{(1)} = \left(-\frac{11}{15}, -3, -\frac{8}{3} \right), \quad \alpha_i^{(2)} = \left(-\frac{23}{15}, -3, -\frac{8}{3} \right), \quad (\text{C.15})$$

$$\begin{aligned}\alpha_{ij}^{(1)} &= \begin{pmatrix} \frac{113}{150} + b_1 & \frac{3}{5} & \frac{34}{15} \\ \frac{3}{5} & \frac{9}{2} + 3b_2 & 6 \\ \frac{34}{15} & 6 & \frac{64}{3} + 8b_3 \end{pmatrix}, \\ \alpha_{ij}^{(2)} &= \begin{pmatrix} \frac{91}{50} + \frac{9}{5}b_1 & \frac{1}{5} & \frac{38}{15} \\ \frac{1}{5} & \frac{9}{2} + 3b_2 & 10 \\ \frac{38}{15} & 10 & \frac{64}{3} + 8b_3 \end{pmatrix}.\end{aligned}\quad (\text{C.16})$$

Here, b_i ($i = 1-3$) is given in Eq. (C.2).

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