



# Diluting the inflationary axion fluctuation by a stronger QCD in the early Universe



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## ABSTRACT

We propose a new mechanism to suppress the axion isocurvature perturbation, while producing the right amount of axion dark matter, within the framework of supersymmetric axion models with the axion scale induced by supersymmetry breaking. The mechanism involves an intermediate phase transition to generate the Higgs  $\mu$ -parameter, before which the weak scale is comparable to the axion scale and the resulting stronger QCD yields an axion mass heavier than the Hubble scale over a certain period. Combined with that the Hubble-induced axion scale during the primordial inflation is well above the intermediate axion scale at present, the stronger QCD in the early Universe suppresses the axion fluctuation to be small enough even when the inflationary Hubble scale saturates the current upper bound, while generating an axion misalignment angle of order unity.

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The non-observation of the neutron EDM requires the CP violating QCD angle to be as tiny as  $|\bar{\theta}| < 10^{-10}$ , causing the strong CP problem. An appealing solution of this puzzle is to introduce a spontaneously broken global Peccei–Quinn (PQ) symmetry [1]. Then  $\bar{\theta}$  corresponds to the vacuum value of the associated Nambu–Goldstone boson, the axion, which is determined to be vanishing by the low energy QCD dynamics [2].

An interesting consequence of this solution is that axions can explain the dark matter in our Universe. Yet, the prospect for axion dark matter depends on the cosmological history of the PQ phase transition. A possible scenario is that the spontaneous PQ breaking occurs after the primordial inflation is over. In such a case, the model is constrained to have the domain-wall number  $N_{\text{DW}} = 1$ , where  $N_{\text{DW}}$  corresponds to the integer-valued  $U(1)_{\text{PQ}} \times SU(3)_c \times SU(3)_c$  anomaly coefficient. Then axions are produced mainly by the annihilations of axionic strings and domain-walls, which would result in the right amount of axion dark matter for the axion scale  $f_a \sim 5 \times 10^{10}$  GeV [3]. However it appears to be difficult to realize this scenario within the framework of a fundamental theory such as string theory, since it requires a PQ symmetry with  $N_{\text{DW}} = 1$ , as

well as a restored PQ phase until some moment after the primordial inflation.

Another scenario which we will focus on in this paper is that  $U(1)_{\text{PQ}}$  is spontaneously broken during the primordial inflation and never restored afterwards. Then the model is not subject to the condition  $N_{\text{DW}} = 1$ , but is constrained by the axion isocurvature perturbation [4–6]. For instance, from the observed CMB power spectrum, one finds [7],

$$\left(\frac{\delta T}{T}\right)_{\text{iso}} \simeq \frac{4}{5} \left(\frac{\Omega_a}{\Omega_{\text{DM}}}\right) \frac{\delta\theta}{\theta_{\text{mis}}} < 3.8 \times 10^{-6}, \quad (1)$$

where  $\theta_{\text{mis}}$  and  $\delta\theta$  denote the average misalignment angle and the angle fluctuation, respectively, for the axion field right before the conventional QCD phase transition when  $m_a(t_{\text{QCD}}) \approx H(t_{\text{QCD}})$  with a temperature  $T(t_{\text{QCD}}) \sim 1$  GeV. The relic axion density is given by

$$\frac{\Omega_a}{\Omega_{\text{DM}}} \simeq 1.7 \theta_{\text{mis}}^2 \left(\frac{f_a(t_0)}{10^{12} \text{ GeV}}\right)^{1.19}, \quad (2)$$

with  $\Omega_{\text{DM}} \approx 0.24$  being the total dark matter fraction. Here we have assumed that  $|\delta\theta| \ll |\theta_{\text{mis}}|$  and there is no significant evolution of  $f_a$  from  $t_{\text{QCD}}$  to the present time  $t_0$  so that  $f_a(t_{\text{QCD}}) \approx f_a(t_0)$ . In inflationary cosmology, the primordial quantum fluctuation of the axion field results in

$$\delta\theta \equiv \delta\theta(t_{\text{QCD}}) = \gamma \delta\theta(t_1) = \gamma \frac{H(t_1)}{2\pi f_a(t_1)}, \quad (3)$$

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where  $f_a(t_I)$  and  $H(t_I)$  denote the axion scale and the Hubble parameter, respectively, during the primordial inflation epoch  $t_I$ , and the factor  $\gamma$  is introduced to take into account the evolution of  $\delta\theta$  from  $t_I$  to  $t_{\text{QCD}}$ . Note that the inflationary Hubble scale  $H(t_I)$  is bounded by the tensor-to-scalar ratio of the CMB perturbation as

$$r \simeq 0.16 \left( \frac{H(t_I)}{10^{14} \text{GeV}} \right)^2 < 0.11, \quad (4)$$

and the weak gravity conjecture [8] suggests that generic axion scales are bounded as

$$f_a \lesssim \mathcal{O} \left( \frac{g^2}{8\pi^2} M_{\text{Pl}} \right), \quad (5)$$

where  $M_{\text{Pl}} \simeq 2.4 \times 10^{18}$  GeV is the reduced Planck mass.

To discuss the implication of the isocurvature constraint (1), one needs to specify the cosmological evolution of the axion scale after the primordial inflation is over. If  $f_a(t_I) \sim f_a(t_0)$  as has been assumed in most of the previous studies, it requires that either  $H(t_I)$  is smaller than its upper bound  $\sim 10^{14}$  GeV by at least five orders of magnitude, so that the CMB tensor mode is too small to be observable, or  $\delta\theta$  should experience a large suppression after the primordial inflation, which appears to be difficult to be implemented.

The above observation suggests a more attractive scenario realizing  $f_a(t_I) \gg f_a(t_0)$  [9] in a natural manner. Indeed supersymmetric axion models offer a natural scheme to realize such a scenario, generating the axion scale through the competition between the tachyonic SUSY breaking mass term and a supersymmetric, but Planck-scale-suppressed higher dimensional term in the scalar potential [10–13]. One then finds

$$\begin{aligned} f_a(t_0) &\sim \sqrt{m_{\text{SUSY}} M_{\text{Pl}}}, \\ f_a(t_I) &\sim \sqrt{H(t_I) M_{\text{Pl}}}, \end{aligned} \quad (6)$$

which explains elegantly the origin of an intermediate axion scale at present, while giving a Hubble-induced inflationary axion scale well above the present axion scale, if the supersymmetry (SUSY) breaking mass  $m_{\text{SUSY}}$  at present is around TeV scale. Furthermore, this type of axion models can be successfully embedded into string theory. Specifically, they can be identified as a low energy limit of string models involving an anomalous  $U(1)_A$  gauge symmetry with vanishing Fayet–Iliopoulos term [12,14]. In such string models, the  $U(1)_A$  gauge boson is decoupled from the low energy world by receiving a heavy mass  $M_A \sim g^2 M_{\text{Pl}}/8\pi^2$  through the Stückelberg mechanism, while leaving the global part of  $U(1)_A$  as an unbroken PQ symmetry in the supersymmetric limit. Once SUSY breaking is introduced properly, in both the present Universe and the inflationary early Universe, the residual PQ symmetry can be spontaneously broken to generate the axion scales as (6).

In this paper, we discuss a novel mechanism to suppress the axion isocurvature perturbation, while producing the right amount of axion dark matter, within the framework of supersymmetric axion models with the axion scales given by (6). The isocurvature constraint (1) and the relic axion density (2) suggest that for  $H(t_I)$  near the current upper bound  $\sim 10^{14}$  GeV, the allowed amount of axion dark matter is maximal when  $f_a(t_0) \sim 10^{11}$ – $10^{13}$  GeV, while  $f_a(t_I)$  nearly saturates the weak gravity bound (5), e.g.  $f_a(t_I) \sim 10^{16}$ – $10^{17}$  GeV. Interestingly, the axion scales generated by SUSY breaking as (6) automatically realize such pattern if  $m_{\text{SUSY}}$  is around TeV scale. More specifically, for the case

$$f_a(t_0)/f_a(t_I) \approx \sqrt{m_{\text{SUSY}}/H(t_I)}, \quad (7)$$

the isocurvature bound (1) reads off

$$\frac{H(t_I)}{10^{14} \text{GeV}} < \left( \frac{0.08}{\gamma} \right)^2 \left( \frac{\Omega_{\text{DM}}}{\Omega_a} \right) \left( \frac{f_a(t_0)}{10^{12} \text{GeV}} \right)^{0.8} \left( \frac{1 \text{TeV}}{m_{\text{SUSY}}} \right),$$

when combined with (2). This implies that a high scale inflation scenario with  $H(t_I) \sim 10^{13}$ – $10^{14}$  GeV, which would give an observable tensor-to-scalar ratio  $r = \mathcal{O}(0.1\text{--}0.01)$  in the CMB perturbation, can be compatible with the axion dark matter  $\Omega_a = \Omega_{\text{DM}}$ , if the axion field fluctuation experiences just a mild suppression after  $t_I$ , e.g.  $\gamma = \mathcal{O}(0.1\text{--}0.01)$  in (3).

To suppress  $\delta\theta$  through its cosmological evolution, one needs a period with  $m_a(t) > H(t)$  well before  $t_{\text{QCD}}$ . On the other hand, usually this is not easy to be realized because the axion mass should be generated mostly by the QCD anomaly in order for the strong CP problem solved by the PQ mechanism. (See Refs. [15–18] for an alternative possibility.) In the following, we propose a simple scheme to achieve such a cosmological period by having a phase of stronger QCD in the early Universe. Although the simplest model that realizes our scenario suffers from a new domain-wall problem, we can address it by adding PQ-charged superfields such that the discrete symmetry of the model is broken before inflation.

Our scheme is based on a phase transition at  $t = t_\mu \gg t_I$ , which will be called the  $\mu$ -transition in the following as it generates the Higgs  $\mu$ -parameter through the superpotential term [19],

$$\mu(X) H_u H_d \equiv \frac{\kappa_1 X^2 H_u H_d}{M_{\text{Pl}}}, \quad (8)$$

where  $X$  is a PQ-charged gauge-singlet superfield. Specifically,

$$X(t \leq t_\mu) = 0, \quad X(t > t_\mu) \sim \sqrt{m_{\text{SUSY}} M_{\text{Pl}}}, \quad (9)$$

so that

$$\mu(t \leq t_\mu) = 0, \quad \mu(t > t_\mu) \sim m_{\text{SUSY}}. \quad (10)$$

With this transition, the weak scale experiences an unusual evolution in a way that the weak scale before the  $\mu$ -transition is comparable to the axion scale (6), as will be discussed below.

To proceed, let us discuss first the key features of the scheme, and later present an explicit model to realize the whole ingredients. Including the Hubble-induced contribution, the mass of the  $D$ -flat Higgs direction  $H_u H_d$  is generically given by

$$m_\phi^2 = c_\phi H^2 + \xi_\phi m_{\text{SUSY}}^2 + 2|\mu|^2 \quad (\phi^2 \equiv H_u H_d), \quad (11)$$

where  $c_\phi$  and  $\xi_\phi$  are model-dependent parameters of order unity. In our scheme, both  $c_\phi$  and  $\xi_\phi$  are assumed to be negative, so  $m_\phi^2 < 0$  before the  $\mu$ -transition. Then  $\phi = \sqrt{H_u H_d}$  is stabilized by the competition between the tachyonic  $m_\phi^2 |\phi|^2$  and a supersymmetric term of  $\mathcal{O}(|\phi|^6/M_{\text{Pl}}^2)$  in the scalar potential, which results in

$$\begin{aligned} f_a(t_I) &\sim \phi(t_I) \sim \sqrt{H(t_I) M_{\text{Pl}}}, \\ f_a(t_\mu) &\sim \phi(t_\mu) \sim \sqrt{m_{\text{SUSY}} M_{\text{Pl}}}. \end{aligned} \quad (12)$$

On the other hand, after the  $\mu$ -transition,  $m_\phi^2 > 0$  due to  $\mu \sim m_{\text{SUSY}}$ . The resulting weak scale and axion scale at present are given by

$$\begin{aligned} \phi(t_0) &= \mathcal{O}(100) \text{ GeV}, \\ f_a(t_0) &\sim X(t_0) \sim \sqrt{m_{\text{SUSY}} M_{\text{Pl}}}. \end{aligned} \quad (13)$$

A simple consequence of the above evolution of  $H_u H_d$  is that the weak scale is comparable to the axion scale before the  $\mu$ -transition:

$$\tilde{\phi} \equiv \phi(t \leq t_\mu) \sim \tilde{f}_a \equiv f_a(t \leq t_\mu). \quad (14)$$

This results in a higher QCD scale, i.e. a stronger QCD, and therefore a heavier axion mass which might be even bigger than the Hubble scale for a certain period. Let us estimate the QCD scale  $\tilde{\Lambda}_{\text{QCD}}$  before the  $\mu$ -transition, which is defined as the scale where the 1-loop QCD coupling blows up, as well as the resulting axion mass  $\tilde{m}_a$ . For the case with  $\tilde{\Lambda}_{\text{QCD}} < \tilde{m}_{\tilde{g}}(\tilde{m}_{\tilde{g}}) < 10^{-5}\tilde{\phi}$ , where  $\tilde{m}_{\tilde{g}}$  denotes the gluino mass before the  $\mu$ -transition, we find

$$\tilde{\Lambda}_{\text{QCD}} \approx 23 \text{ TeV} \left( \frac{\tilde{m}_{\tilde{g}}}{30 \text{ TeV}} \right)^{2/11} \times \left( \frac{\tan \beta}{10} \right)^{3/11} \left( \frac{\tilde{\phi}}{10^{12} \text{ GeV}} \right)^{6/11}, \quad (15)$$

where  $\tan \beta = \langle H_u \rangle / \langle H_d \rangle$  at present, and  $\tilde{m}_{\tilde{g}}/\tilde{g}_3^2(\tilde{m}_{\tilde{g}}) \simeq m_{\tilde{g}}/g_3^2(m_{\tilde{g}})$  for the gluino mass  $m_{\tilde{g}}$  at present. Here we assume that  $g_3^2(M_{\text{GUT}}) = \tilde{g}_3^2(M_{\text{GUT}})$  and  $y_q(M_{\text{GUT}}) = \tilde{y}_q(M_{\text{GUT}})$  for the QCD coupling and the quark Yukawa couplings. When the temperature  $T \lesssim \tilde{\Lambda}_{\text{QCD}}$ , the axion mass during the period of stronger QCD is estimated to be

$$\tilde{m}_a \approx \tilde{\Lambda}_{\text{QCD}}^2 / \tilde{f}_a. \quad (16)$$

On the other hand, if  $\tilde{m}_{\tilde{g}}(\tilde{\Lambda}_{\text{np}}) < \tilde{\Lambda}_{\text{QCD}} < 10^{-5}\tilde{\phi}$ , the resulting QCD scale is estimated as

$$\tilde{\Lambda}_{\text{QCD}} \approx 21 \text{ TeV} \left( \frac{\tan \beta}{10} \right)^{1/3} \left( \frac{\tilde{\phi}}{10^{12} \text{ GeV}} \right)^{2/3}, \quad (17)$$

with the axion mass

$$\tilde{m}_a \approx \tilde{m}_{\tilde{g}}^{1/2} \tilde{\Lambda}_{\text{QCD}}^{3/2} / \tilde{f}_a. \quad (18)$$

Here  $\tilde{\Lambda}_{\text{np}}$  denotes the scale where the stronger QCD becomes non-perturbative, i.e. around  $\tilde{g}_3^2 = 8\pi^2/N_c$  with  $N_c = 3$ . Note that the axion potential for the axion mass (18) can be obtained by a single insertion of the SUSY breaking spurion  $\tilde{m}_{\tilde{g}}\theta^2$  to the nonperturbative superpotential  $W_{\text{np}} \sim \tilde{\Lambda}_{\text{QCD}}^3$  induced by the gluino condensation.

If the stronger QCD scale  $\tilde{\Lambda}_{\text{QCD}}$  is high enough, there could be a period with  $\tilde{m}_a(t) > H(t)$  well before the conventional QCD phase transition. As is well known, in such a period the axion field experiences a damped oscillation, with an amplitude  $\bar{a}$  (averaged over each oscillation period) evolving as

$$\bar{a} \propto R^{-3/2}(t), \quad (19)$$

where  $R(t)$  is the scale factor of the expanding Universe. Then the spatially averaged vacuum value of the axion field is settled down at the minimum of the axion potential induced by the stronger QCD, while the axion angle fluctuation is diluted according to

$$\delta\theta = \gamma \frac{H(t_i)}{2\pi f_a(t_i)} \approx \left( \frac{T(t_\mu)}{T(t_i)} \right)^{3/2} \frac{H(t_i)}{2\pi f_a(t_i)}, \quad (20)$$

where  $t = t_i$  denotes the moment when the damped axion oscillation begins, and  $t = t_\mu$  is the moment when it is over. Note that, after the  $\mu$ -transition, the weak scale and the QCD scale quickly roll down to the present values, so the axion mass becomes negligible compared to  $H(t)$  until  $t \sim t_{\text{QCD}}$  when the Universe undergoes the conventional QCD phase transition. Also, the minimum of the axion potential induced by the stronger QCD is generically different from the minimum of the axion potential at present. As a result, our scheme generates an axion misalignment angle of order unity:

$$\theta_{\text{mis}} \equiv \left\langle \frac{a(t_\mu)}{f_a(t_\mu)} \right\rangle - \left\langle \frac{a(t_0)}{f_a(t_0)} \right\rangle = \mathcal{O}(1), \quad (21)$$

together with an intermediate axion scale at present, so gives rise to  $\Omega_a = \Omega_{\text{DM}}$  in a natural way.

In our case, the damped axion oscillation induced by the stronger QCD begins at a temperature  $T(t_i) \sim \tilde{\Lambda}_{\text{QCD}}$  as  $\tilde{m}_a$  is highly suppressed by thermal effects for  $T \gg \tilde{\Lambda}_{\text{QCD}}$ . On the other hand, the scalar field  $X$  generating  $\mu$  through (8) is trapped at the origin by thermal effects until the Universe cools down to a temperature  $T(t_\mu) \sim m_{\text{SUSY}}$ . In fact, our scheme involves a variety of dimensionless parameters which affect the naive estimate of the involved scales. We find that there is a large fraction of the natural parameter region where the axion mass

$$\tilde{m}_a \approx 0.4 \text{ MeV} \left( \frac{\tilde{f}_a}{10^{12} \text{ GeV}} \right)^{-1} \left( \frac{\tilde{\Lambda}_{\text{QCD}}}{20 \text{ TeV}} \right)^2 \quad (22)$$

is larger than the Hubble scale

$$H(t_\mu) \simeq 0.2 \text{ MeV} \left( \frac{\sqrt{V_0}}{1 \text{ TeV} \times 10^{12} \text{ GeV}} \right), \quad (23)$$

over the period  $t_i \lesssim t \lesssim t_\mu$  with a temperature ratio:

$$T(t_\mu)/T(t_i) = \mathcal{O}(10^{-1}-10^{-2}). \quad (24)$$

Then the resulting  $\delta\theta$  given by (20) can be small enough to satisfy the isocurvature bound (1) even when  $H(t_i)$  saturates its upper bound  $\sim 10^{14}$  GeV. Note that during  $t_i \lesssim t \lesssim t_\mu$ ,

$$\phi(t) - \phi(t_0) \sim X(t) - X(t_0) \sim \sqrt{m_{\text{SUSY}} M_{\text{Pl}}},$$

so the corresponding vacuum energy density  $V_0 = \mathcal{O}(m_{\text{SUSY}}^2 f_a^2(t_0))$ . This means that in this period the Universe is dominated by the vacuum energy density with the Hubble scale given by (23), which is often called the thermal inflation [20].

It should be stressed that in our scheme the axion isocurvature perturbation is suppressed by two steps. The first suppression is due to  $f_a(t_0)/f_a(t_i) \sim \sqrt{m_{\text{SUSY}}/H(t_i)} \ll 1$ , and the second is due to the stronger QCD dynamics before the  $\mu$ -transition, yielding a further suppression by  $\gamma \sim (m_{\text{SUSY}}/\tilde{\Lambda}_{\text{QCD}})^{3/2}$ . To illustrate the result, we depict in Fig. 1 the upper bound on the inflationary Hubble scale  $H(t_i)$  resulting from the isocurvature constraint (1) for  $\Omega_a = \Omega_{\text{DM}}$ . To make a comparison, we depict the results for three distinct cases: (i) the conventional scenario of  $f_a(t_i) = f_a(t_0)$  without a stronger QCD, (ii) a scheme with  $f_a(t_i)/f_a(t_0) \sim \sqrt{H(t_i)/m_{\text{SUSY}}}$ , but without a stronger QCD, (iii) our scheme with  $f_a(t_i)/f_a(t_0) \sim \sqrt{H(t_i)/m_{\text{SUSY}}}$  and a stronger QCD before the  $\mu$ -transition.

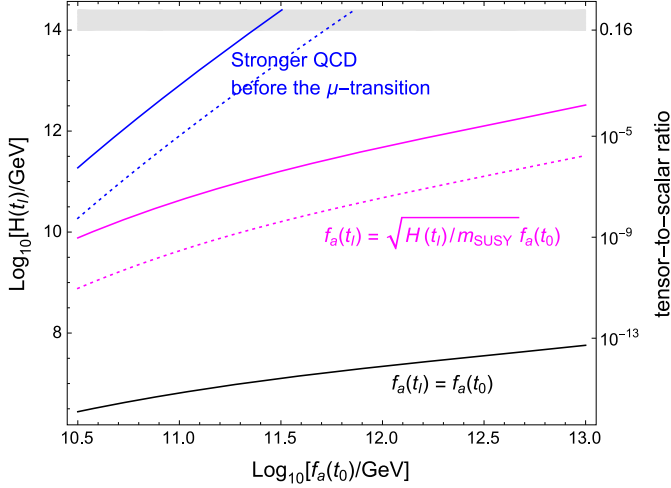
Let us now present an explicit model implementing the mechanisms discussed above. As a simple example, we consider a model with the following superpotential,

$$W = (\text{MSSM Yukawa terms}) + \lambda Y \Phi \Phi^c + \frac{\kappa_1 X^2 H_u H_d}{M_{\text{Pl}}} + \frac{\kappa_2 X Y^3}{M_{\text{Pl}}} + \frac{\kappa_3 (H_u H_d) (L H_u)}{M_{\text{Pl}}}, \quad (25)$$

where  $X$  and  $Y$  are PQ-charged gauge singlets responsible for the  $\mu$ -transition,  $L$  is the MSSM lepton doublet, and  $\Phi + \Phi^c$  are  $U(1)_Y$ -charged exotic matter fields introduced to give a thermal mass to  $Y$ . Then the scalar potential for the  $\mu$ -transition is given by

$$V_1 = m_X^2 |X|^2 + m_Y^2 |Y|^2 + \left( \frac{\kappa_2 A_2}{M_{\text{Pl}}} X Y^3 + \text{h.c.} \right) + \frac{|\kappa_2|^2}{M_{\text{Pl}}^2} (|Y|^6 + 9|X|^2 |Y|^4), \quad (26)$$

where



**Fig. 1.** Upper bound on the inflationary Hubble scale consistent with the axion dark matter,  $\Omega_a = \Omega_{\text{DM}}$ . Here we have taken  $m_{\tilde{g}} = 3$  TeV,  $\tan\beta = 10$ , and  $T(t_\mu) = 1$  TeV. The shaded region is excluded by the Planck results. The black solid line is the constraint in the conventional scenario with  $f_a(t_1) = f_a(t_0)$ . The magenta lines are for the scenario with  $f_a(t_1)/f_a(t_0) = \sqrt{H(t_1)/m_{\text{SUSY}}}$ , but without a stronger QCD. The blue lines are for our scheme which leads to a further suppression of  $\delta\theta$  by the stronger QCD. The SUSY breaking mass has been taken  $m_{\text{SUSY}} = 1$  TeV for the solid lines and 10 TeV for the dotted lines. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

$$\begin{aligned} m_X^2 &= c_X H^2 + \xi_X m_{\text{SUSY}}^2 + 4|\mu_X|^2, \\ m_Y^2 &= c_Y H^2 + \xi_Y m_{\text{SUSY}}^2 + \alpha_Y T^2, \end{aligned} \quad (27)$$

for  $\mu_X = \kappa_1 H_u H_d / M_{\text{Pl}}$ . Here  $c_{X,Y} H^2$  are the Hubble-induced masses,  $\xi_{X,Y} m_{\text{SUSY}}^2$  are the SUSY breaking masses at zero temperature, and  $\alpha_Y T^2$  is the thermal mass of  $Y$  induced by the coupling  $\lambda Y \Phi \Phi^c$ , which is of  $\mathcal{O}(|\lambda|^2 T^2)$  for  $|\lambda Y| < T$ .

For simplicity, we will assume that all the dimensionless parameters appearing in the superpotential and the SUSY breaking scalar masses are of order unity. However it should be noted that these parameters can have a variation of  $\mathcal{O}(0.1-10)$  easily. In particular, the superpotential parameters  $\kappa_n$  can have a much wider variation without invoking fine-tuning. This gives us a rather large room to get an enough suppression of the axion angle fluctuation  $\delta\theta$  through a stronger QCD before the  $\mu$ -transition. At any rate, assuming that  $c_{X,Y} > 0$ ,  $\xi_X > 0$  and  $\xi_Y < 0$ , the scalar potential (26) indeed yields the desired  $\mu$ -transition as

$$\begin{aligned} X = Y = 0 & \quad \text{at } t \leq t_\mu, \\ X \sim Y \sim \sqrt{m_{\text{SUSY}} M_{\text{Pl}}} & \quad \text{at } t > t_\mu, \end{aligned} \quad (28)$$

with  $T(t_\mu) \sim m_{\text{SUSY}}$ .<sup>1</sup>

Now the Higgs and slepton fields can have a nontrivial evolution along the following flat direction:

$$\begin{aligned} H_d^T &= (\phi_d, 0), \quad L^T = (\phi_\ell, 0), \\ H_u^T &= (0, \sqrt{|\phi_d|^2 + |\phi_\ell|^2}), \end{aligned} \quad (29)$$

<sup>1</sup> In our example (25), the cosmological evolution of  $X$  and  $Y$  can cause a domain-wall problem associated with the discrete symmetry  $\Gamma$  generated by  $X \rightarrow -X$  and  $Y \rightarrow e^{i\pi/3} Y$ . This problem can be avoided by extending the model to make  $\Gamma$  not restored. For instance, one can introduce additional PQ-charged fields  $Z$  and  $\varphi$  with  $Z\varphi^2 + Y\varphi^3/M_{\text{Pl}}$  in the superpotential. If  $\varphi$  has a tachyonic SUSY breaking mass ( $\xi_\varphi < 0$ ), as well as a tachyonic Hubble-induced mass ( $c_\varphi < 0$ ), the discrete symmetry of the model, which is now involving  $\varphi^3 \rightarrow e^{-i\pi/3}\varphi^3$ , is never restored in the early Universe, and the model is free from the domain-wall problem, while implementing our mechanism. It is also possible to construct a model in which the  $\mu$ -transition is triggered by a PQ singlet in a way not having any discrete symmetry.

which satisfies the  $F$  and  $D$  flat conditions. The relevant terms of the scalar potential of  $\phi_{d,\ell}$  are given by

$$\begin{aligned} V_2 &= \sum m_i^2 |\phi_i|^2 + \sqrt{\sum |\phi_i|^2} (B\mu\phi_d + \text{h.c.}) \\ &+ \left( \sum |\phi_i|^2 \right) \left( \frac{\kappa_3 A_3 \phi_d \phi_\ell}{M_{\text{Pl}}} + \text{h.c.} \right) \\ &+ \sqrt{\sum |\phi_i|^2} \left( \frac{\mu^* \kappa_3 \phi_\ell}{M_{\text{Pl}}} (3|\phi_d|^2 + |\phi_\ell|^2) + \text{h.c.} \right) \\ &+ \frac{|\kappa_3|^2}{M_{\text{Pl}}^2} \left( \sum |\phi_i|^2 \right) (|\phi_d|^4 + 6|\phi_d \phi_\ell|^2 + |\phi_\ell|^4), \end{aligned} \quad (30)$$

for  $\mu = \kappa_1 X^2 / M_{\text{Pl}}$ , where

$$\begin{aligned} m_{\phi_d}^2 &= c_d H^2 + \xi_d m_{\text{SUSY}}^2 + 2|\mu|^2, \\ m_{\phi_\ell}^2 &= c_\ell H^2 + \xi_\ell m_{\text{SUSY}}^2 + |\mu|^2. \end{aligned} \quad (31)$$

Again, assuming  $c_{d,\ell} < 0$  and  $\xi_{d,\ell} < 0$ , but  $m_{\phi_{d,\ell}}^2(t_0) > 0$  due to  $\mu(t_0) \sim m_{\text{SUSY}}$ , the above scalar potential yields

$$\begin{aligned} f_a(t_1) &\sim \phi_{d,\ell}(t_1) \sim \sqrt{H(t_1) M_{\text{Pl}}}, \\ f_a(t_\mu) &\sim \phi_{d,\ell}(t_\mu) \sim \sqrt{m_{\text{SUSY}} M_{\text{Pl}}}, \end{aligned} \quad (32)$$

and

$$\begin{aligned} \phi_d(t_0) &= \mathcal{O}(100) \text{ GeV}, \quad \phi_\ell(t_0) = 0, \\ f_a(t_0) &\sim X(t_0) \sim Y(t_0) \sim \sqrt{m_{\text{SUSY}} M_{\text{Pl}}}. \end{aligned} \quad (33)$$

To summarize, under a reasonably plausible assumption on the SUSY breaking during the primordial inflation and in the present Universe, the model with the superpotential (25) can successfully realize the desired cosmological evolution of the three relevant scales: the axion scale, the weak scale, and the QCD scale as given by (28), (32) and (33). Being generated by SUSY breaking, an inflationary axion scale  $f_a(t_1) \sim \sqrt{H(t_1)/m_{\text{SUSY}}} f_a(t_0)$  is determined to be well above the present axion scale  $f_a(t_0) \sim \sqrt{m_{\text{SUSY}} M_{\text{Pl}}}$ , and a stronger QCD in the early Universe is realized to yield an enough suppression of the axion angle fluctuation even when  $H(t_1)$  saturates its upper bound. We note that the minimum of the axion potential induced by the stronger QCD depends on  $\arg(\kappa_3 A_3)$ , but not on  $\arg(B\mu)$ , while the minimum of the axion potential at present depends on  $\arg(B\mu)$ , but not on  $\arg(\kappa_3 A_3)$ . As a result, the stronger QCD generates an axion misalignment angle  $\theta_{\text{mis}} = \mathcal{O}(1)$ , so that the axion dark matter with  $\Omega_a = \Omega_{\text{DM}}$  arises naturally in our scheme.

There is a remaining issue which should be addressed to complete our scheme. As we have noticed, the  $\mu$ -transition is foregone by a late-time thermal inflation. This suggests that the scheme should be accompanied by a late-time baryogenesis operating after the  $\mu$ -transition. In fact, the model of (25) offers an elegant mechanism to generate the baryon asymmetry through the rolling flat direction  $LH_u$  [21]. More detailed cosmology of our scheme, including the leptogenesis by rolling  $LH_u$ , will be discussed elsewhere [22].

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