



Renormalization group improvement and dynamical breaking of symmetry in a supersymmetric Chern–Simons-matter model

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Abstract

In this work, we investigate the consequences of the Renormalization Group Equation (RGE) in the determination of the effective superpotential and the study of Dynamical Symmetry Breaking (DSB) in an $\mathcal{N} = 1$ supersymmetric theory including an Abelian Chern–Simons superfield coupled to N scalar superfields in $(2 + 1)$ dimensional spacetime. The classical Lagrangian presents scale invariance, which is broken by radiative corrections to the effective superpotential. We calculate the effective superpotential up to two-loops by using the RGE and the beta functions and anomalous dimensions known in the literature. We then show how the RGE can be used to improve this calculation, by summing up properly defined series of leading logs (LL), next-to-leading logs (NLL) contributions, and so on... We conclude that even if the RGE improvement procedure can indeed be applied in a supersymmetric model, the effects of the consideration of the RGE are not so dramatic as it happens in the non-supersymmetric case.

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1. Introduction

Dynamical Symmetry Breaking (DSB) constitutes a very appealing scenario in field theory, where quantum corrections are entirely responsible for the appearance of nontrivial minima of the effective potential [1]. In the case of a classically scale invariant model, all mass scales are generated by these quantum corrections and are fixed as functions of the symmetry breaking scale. This scenario would be particularly interesting in the Standard Model, but earlier calculations pointed to a dead end: quantum corrections turned the effective potential unstable, rendering DSB impossible [2]. However, it has been shown that this conclusion, based on the effective potential calculated up to the one-loop level, could be modified substantially by using the Renormalization Group Equation (RGE) [3–10] to sum up infinite subsets of higher loop contributions to the effective potential. More than a quantitative correction over the one-loop result, this improvement lead to a new scenario, where DSB was operational [3,4]. More recent calculations were able to include corrections up to five loops in the effective potential [11,12], bringing the prediction for the Higgs mass relatively close to the experimental value indicated by the LHC (for other works regarding conformal symmetry in the Standard Model see for example [13,14]).

Besides being a viable ingredient to the Standard Model phenomenology, DSB also occurs in other contexts, such as lower dimensional theories. Particularly interesting are models involving the Chern–Simons (CS) term in $(2 + 1)$ spacetime dimensions [15]. The basic renormalization properties of such models have been studied for quite some time [16–22]. We shall be particularly interested in models with scale invariance at the classical level, that is, with a pure CS field coupled to massless scalars and fermions, with Yukawa quartic interactions and scalar ϕ^6 self-interactions. In these models, the one-loop corrections calculated using the dimensional reduction scheme [23] are rather trivial, since no singularities appear, and no DSB happens either; at the two-loop level, however, one finds renormalizable divergences. Also, the two-loops effective potential V_{eff} exhibits a nontrivial minimum, signaling the appearance of DSB. Due to the nontrivial β and γ functions at two-loop level, one may obtain an improvement in the calculation of V_{eff} by imposing the RGE

$$\left[\mu \frac{\partial}{\partial \mu} + \beta_x \frac{\partial}{\partial x} - \gamma_\phi \phi \frac{\partial}{\partial \phi} \right] V_{eff}(\phi; \mu, \alpha_i, L) = 0, \quad (1)$$

where x denotes collectively the coupling constants of the theory, μ is the mass scale introduced by the regularization, γ_ϕ is the anomalous dimension of scalar field ϕ ,

$$L = \ln \left[\frac{\phi^2}{\mu} \right], \quad (2)$$

and ϕ is the vacuum expectation value of ϕ . This improved effective potential was calculated in [24], and it was shown to imply in considerable changes in the properties of DSB in this model, thus providing another context where the consideration of the RGE is essential to a proper analysis of the phase structure of the model.

Our objective is to verify whether in supersymmetric models containing the CS field, the consideration of the RGE also induces considerable modifications in the scenario of DSB. Supersymmetric CS theories have been studied for quite some time [17,25–28], and have recently attracted much attention due to their relation to M2-branes [29]. The superconformal field theory describing multiple M2-branes is dual to the $D = 11$ Supergravity on $AdS_4 \times S^7 \sim [SO(2, 3)/SO(1, 3)] \times [SO(8)/SO(7)] \subset OSp(8|4)/[SO(1, 3) \times SO(7)]$, therefore the action for multiple M2-branes has $\mathcal{N} = 8$ supersymmetry. However, the on-shell degrees of freedom of

this theory are exhausted by bosons and physical fermions making its gauge sector to have no on-shell degrees of freedom. These requirements are satisfied by a Chern–Simons-matter theory called BLG theory [30–34], which describes two M2-branes. Relaxing the requirement of manifest $\mathcal{N} = 8$ supersymmetry, this approach can be generalized to a $\mathcal{N} = 6$ Chern–Simons-matter theory with the gauge group $U_k(N) \times U_{-k}(N)$ (k and $-k$ are CS levels) [35,36], which is expected to be enhanced to $\mathcal{N} = 8$ for $k = 1$ or $k = 2$ [37–39]. The quantization of such model was thoroughly studied in [40–48]. Also, detailed calculations of the effective superpotential within $\mathcal{N} = 2$ superfield theories in three dimension have been reported in [49,50].

The first part of our work is the computation of the effective superpotential of a generic supersymmetric CS theory coupled to matter superfields, up to two-loops. To this end, we use the RGE and the β and γ function calculated in [17,51], thus avoiding the direct computation of any supergraph. With this result in hand, we discuss how we can reorganize the expansion of the effective superpotential in terms of Leading Logs (LL), Next-to-Leading Logs (NLL) contributions, and so on, in a way that allows us to calculate coefficients arising from higher orders corrections, thus improving the two-loop evaluation of the effective superpotential. We are thus able to find an improved effective superpotential, which will be used to study DSB in our model. We will show that, contrary to what happens in the non-supersymmetric case [24], here the RGE improvement leads only to slight modifications in the DSB scenario.

In this work, we shall focus on calculations done in the superfield language [52,53], in which supersymmetry is manifest in all stages of the calculations. This paper is organized as follows: in Sec. 2, we present our model and calculate the effective superpotential with the knowledge of the renormalization group functions found in the literature, together with the RGE. Section 3 reviews the standard approach to RGE improvement of the effective potential in four dimensional models, and section 4 is devoted to adapt this procedure to the supersymmetric three-dimensional case. The resulting improved effective superpotential is used in Sec. 5 to study DSB in our model. Section 6 presents our conclusions and perspectives. Some explicit results and the Mathematica code used to obtain them is given as a Supplementary Material to this work.

2. Calculation of the effective superpotential

Our starting point is the classical action in $\mathcal{N} = 1$ superspace of a Chern–Simons superfield Γ_β coupled to N massless complex scalars superfields Φ_a , with a quartic self-interaction,

$$\mathcal{S} = \int d^5z \left\{ -\frac{1}{2} \Gamma^\alpha W_\alpha - \frac{1}{2} \overline{\nabla^\alpha \Phi_a} \nabla_\alpha \Phi_a + \frac{\lambda}{4} (\overline{\Phi_a} \Phi_a)^2 \right\}, \quad (3)$$

where $W^\alpha = \frac{1}{2} D^\beta D^\alpha \Gamma_\beta$ is the gauge superfield strength, $\nabla^\alpha = (D^\alpha - ig \Gamma^\alpha)$ is the gauge supercovariant derivative, and $a = 1, \dots, N$. We follow the basic conventions for three-dimensional supersymmetry found in [52].

The main object we shall be interested in studying is the three dimensional effective superpotential [54]. To define this object, we consider a shift in the N -th component of Φ_a ,

$$\Phi_N = \Phi_N^q + \sigma, \quad (4)$$

by the background (constant) superfield $\sigma = \sigma_1 - \theta^2 \sigma_2$. On general grounds, the effective superpotential can be cast as

$$\Gamma[\sigma] = \int d^5z K(\sigma) + \int d^5z F(\sigma, D_\alpha \sigma, D^2 \sigma). \quad (5)$$

As discussed in [27], the knowledge of K is sufficient for investigating the dynamical breaking of the gauge symmetry, and consequential generation of a mass scale in the model, while the study of a hypothetical supersymmetry breaking would involve also the calculation of F [55,56]. For simplicity, in this work we will restrict ourselves on the calculation of $K(\sigma)$, which we shall call the *effective superpotential* from now on.

The effective superpotential $K(\sigma)$ is particularly well suited for the approach we develop in this work, since we will be able to calculate it by using a simple ansatz, using the information already known from the literature regarding renormalization group functions for the model (3). The relevant results are given in [17], from which we extract the two-loop beta functions and anomalous dimension for the scalar superfield,

$$\beta_\lambda = c_3\lambda^3 + c_2\lambda^2 y + c_1\lambda y^2 + c_0 y^3, \tag{6a}$$

$$\beta_y = 0, \tag{6b}$$

$$\gamma_\phi = d_2\lambda^2 + d_0 y^2, \tag{6c}$$

in terms of the redefined gauge coupling constant

$$y = g^2. \tag{7}$$

The numerical coefficients present in (6) are given by

$$c_3 = \frac{3}{64\pi^2} (N + 2), \quad c_2 = \frac{1}{64\pi^2}, \quad c_1 = -\frac{2}{64\pi^2} (N + 2),$$

$$c_0 = -\frac{1}{64\pi^2} (N + 3), \quad d_2 = \frac{1}{4(64\pi^2)} (N + 1), \quad d_0 = -\frac{1}{4(64\pi^2)} (2N + 3). \tag{8}$$

These results are obtained from a two-loop computation of the divergent vertex functions of the theory, since at one loop no divergences appear, provided Feynman integrals are calculated by means of dimensional regularization.

We shall use for $K(\sigma)$ the ansatz

$$K(\sigma) = -\frac{1}{4}\sigma^4 S(L), \tag{9}$$

where

$$S(L) = A(y, \lambda) + B(y, \lambda)L + C(y, \lambda)L^2 + \dots, \tag{10}$$

and A, B, C, \dots are defined as series in powers of the coupling constants y and λ , and L is defined in (2). We will eventually adopt a shorthand notation where x will denote any of the two couplings in our model, so that a monomial like $y^n\lambda^m$ will be written as x^{m+n} . Comparison with the tree level action (3) shows us that

$$A(y, \lambda) = \lambda + \mathcal{O}(x^2), \tag{11}$$

but actually the value of $A(y, \lambda)$ will be fixed by the Coleman–Weinberg normalization of the effective superpotential,

$$\frac{1}{4!} \frac{d^4 K(\sigma)}{d^4 \sigma} = \frac{\lambda}{4}, \tag{12}$$

so only the L dependent pieces of $K(\sigma)$, involving B, C, \dots , have to be calculated.

The other ingredient we will need is the RGE given in (1). From Eq. (2) follows that $\partial_L = \frac{1}{2}\sigma\partial_\sigma = -\mu\partial_\mu$, and inserting (9) into (1), we obtain an alternative form for the RGE,

$$\left[-(1 + 2\gamma_\phi)\partial_L + \beta_\lambda\partial_\lambda - 4\gamma_\phi\right]S(L) = 0, \tag{13}$$

which will be used hereafter.

Inserting the ansatz (10) in (13), and separating the resulting expression by orders of L , we obtain a series of equations, of which we quote the first two:

$$-(1 + 2\gamma_\phi)B(y, \lambda) + \beta_\lambda\partial_\lambda A(y, \lambda) - 4\gamma_\phi A(y, \lambda) = 0, \tag{14}$$

and

$$-2(1 + 2\gamma_\phi)C(y, \lambda) + \beta_\lambda\partial_\lambda B(y, \lambda) - 4\gamma_\phi B(y, \lambda) = 0. \tag{15}$$

We now consider that all functions appearing in these equations are defined as a series in powers of the couplings x , writing Eq. (14) as

$$\begin{aligned} & -\left(B^{(1)} + B^{(2)} + B^{(3)} + \dots\right) - 2\left(\gamma_\phi^{(2)} + \gamma_\phi^{(3)} + \dots\right)\left(B^{(1)} + B^{(2)} + B^{(3)} + \dots\right) \\ & + \left(\beta_\lambda^{(3)} + \beta_\lambda^{(4)} + \dots\right)\left(\partial_\lambda A^{(1)} + \partial_\lambda A^{(2)} + \dots\right) \\ & - 4\left(\gamma_\phi^{(2)} + \gamma_\phi^{(3)} + \dots\right)\left(A^{(1)} + A^{(2)} + \dots\right) = 0, \end{aligned} \tag{16}$$

where the numbers in the superscripts denote the power of x of each term. Since all terms of the previous equation start at order x^3 , except the first, we conclude that $B^{(1)} = B^{(2)} = 0$, and obtain the relation

$$B^{(3)} = \beta_\lambda^{(3)} - 4\lambda\gamma_\phi^{(2)}, \tag{17}$$

after using Eq. (11). This last equation fixes the coefficients of $B^{(3)}$ in terms of the (known) coefficients of $\beta_\lambda^{(3)}$ and $\gamma_\phi^{(2)}$, in the following form,

$$B^{(3)} = b_3\lambda^3 + b_2\lambda^2y + b_1\lambda y^2 + b_0y^3, \tag{18}$$

where

$$b_0 = c_0; b_1 = c_1 - 4d_0; b_2 = c_2; b_3 = c_3 - 4d_2. \tag{19}$$

The corrections of the order x^3L we have found for S_{eff} could be obtained by a two-loop calculation of the effective superpotential, using supergraph methods. Since we do not know the coefficients of $\beta_\lambda^{(4)}$ and $\gamma_\phi^{(3)}$, which would appear from higher loop corrections, we cannot use Eq. (14) to calculate further coefficients of B or A , so this equation does not allow us to obtain information on higher-loops contributions to S_{eff} .

Now looking at Eq. (15) expanded in power of the couplings, one may conclude that $C(\lambda, y)$ starts at order x^5 , and obtain the relation,

$$C^{(5)} = \frac{1}{2}\beta^{(3)}\partial_\lambda B^{(3)} - 2\gamma^{(2)}B^{(3)}, \tag{20}$$

from which the coefficients of the form x^5L^2 of S_{eff} are calculated from known coefficients of the beta functions, anomalous dimension, and B^3 . The end result is as follows,

$$\begin{aligned}
 C^{(5)} = & \lambda^5 \left(\frac{3}{2} c_3 b_3 - 2d_2 b_3 \right) + \lambda^4 y \left(c_3 b_2 + \frac{3}{2} c_2 b_3 - 2d_2 b_2 \right) \\
 & + \lambda^3 y^2 \left(\frac{1}{2} c_3 b_1 + c_2 b_2 + \frac{3}{2} c_1 b_3 - 2d_0 b_3 - 2d_2 b_1 \right) \\
 & + \lambda^2 y^3 \left(\frac{1}{2} c_2 b_1 + c_1 b_2 + \frac{3}{2} c_0 b_3 - 2d_0 b_2 - 2d_2 b_0 \right) \\
 & + \lambda y^4 \left(\frac{1}{2} c_1 b_1 + c_0 b_2 - 2d_0 b_1 \right) + y^5 \left(\frac{1}{2} c_0 b_1 - 2d_0 b_0 \right). \tag{21}
 \end{aligned}$$

As a result, the RGE allows us to calculate terms of order $x^5 L^2$ which, in our model, would appear only in a four loops explicit evaluation of V_{eff} . Equation (15) does not provide us with order $x^6 L^2, x^7 L^2, \dots$ terms, however, since we do not have knowledge of higher orders coefficients of β_λ and γ_ϕ .

At this point, it is clear that one could go on calculating order $x^7 L^3, x^9 L^4, \dots$ terms from Eqs. (10) and (13), obtaining contributions to the effective superpotential arising from higher loop orders, based just on the information we have from the two loop calculation of β_λ and γ_ϕ . In section 4, we will give an explanation of this pattern of coefficients we are able to calculate, interpreting it as a leading logs summation of the effective superpotential.

3. RGE improvement and dynamical symmetry breaking: a short review of the four dimensional case

We now review the procedure for the RGE improvement of the effective potential calculation that was applied to the Standard Model in [3,4,57,12] and to the non-supersymmetric version of the model studied in this work in [24]. In doing so, further on we will be able to pinpoint the differences we find in the supersymmetric three dimensional case, still recognizing that the procedure outlined in the previous section is essentially the same used in these works.

Consider a scale invariant ϕ^4 model in four spacetime dimensions, coupled to other fermionic or gauge fields via a set of couplings denoted collectively by x . The effective potential $\mathcal{V}_{eff}(\phi; \mu, x, \mathcal{L})$ should satisfy the RGE

$$\left[\mu \frac{\partial}{\partial \mu} + \beta_x \frac{\partial}{\partial x} - \gamma_\phi \phi \frac{\partial}{\partial \phi} \right] \mathcal{V}_{eff}(\phi; \mu, x, \mathcal{L}) = 0, \tag{22}$$

where now

$$\mathcal{L} = \ln \left[\frac{\phi^2}{\mu^2} \right]. \tag{23}$$

As before, we can rewrite this in a more convenient fashion by defining

$$\mathcal{V}_{eff}(\phi; \mu, x, \mathcal{L}) = \phi^4 \mathcal{S}_{eff}(\mu, x, \mathcal{L}), \tag{24}$$

so that Eq. (24) implies

$$\left[- (2 + 2\gamma_\phi) \frac{\partial}{\partial \mathcal{L}} + \beta_x \frac{\partial}{\partial x} - 4\gamma_\phi \right] \mathcal{S}_{eff}(\mu, x, \mathcal{L}) = 0. \tag{25}$$

The central point of the general approach to RGE improvement discussed in the aforementioned references is to reorganize the contributions to $\mathcal{S}_{eff}(\mu, x, \mathcal{L})$ arising from different loop

orders according to the difference between the aggregate power of the couplings x and the logs \mathcal{L} , that is to say,

$$\mathcal{S}_{eff}(x, \mathcal{L}) = \mathcal{S}_{eff}^{LL}(x, \mathcal{L}) + \mathcal{S}_{eff}^{NLL}(x, \mathcal{L}) + \dots, \quad (26)$$

where \mathcal{S}_{eff}^{LL} contains the leading logs contributions,

$$\mathcal{S}_{eff}^{LL}(x, \mathcal{L}) = \sum_{n \geq 1} C_n^{LL} x^n \mathcal{L}^{n-1}, \quad (27)$$

\mathcal{S}_{eff}^{NLL} contains the next to leading logs terms,

$$\mathcal{S}_{eff}^{NLL}(x, \mathcal{L}) = \sum_{n \geq 2} C_n^{NLL} x^n \mathcal{L}^{n-2}, \quad (28)$$

and so on. Insertion of the ansatz (26) in the RGE (25) gives a set of coupled differential equations, of which we quote the first two,

$$\left[-2 \frac{\partial}{\partial \mathcal{L}} + \beta_x^{(2)} \frac{\partial}{\partial x} \right] \mathcal{S}_{eff}^{LL}(x, \mathcal{L}) = 0, \quad (29)$$

and

$$\left[-2 \frac{\partial}{\partial \mathcal{L}} + \beta_x^{(2)} \frac{\partial}{\partial x} \right] \mathcal{S}_{eff}^{NLL}(x, \mathcal{L}) + \left[\beta_x^{(3)} \frac{\partial}{\partial x} - 4\gamma_\phi^{(2)} \right] \mathcal{S}_{eff}^{LL}(x, \mathcal{L}) = 0. \quad (30)$$

Equation (29) results in a first order difference equation for C_n^{LL} , so the knowledge of the initial coefficient C_1^{LL} and the order x^2 contribution to the beta function from loop calculations, allows one to calculate all C_n^{LL} , therefore summing up all the leading logs contributions to the effective potential. This summation was the key to making the DSB scenario viable in the scale invariant Standard Model as shown in [3]. One does not need to stop at this point, however, since Eq. (30) can also be used to sum up the next to leading logs, after \mathcal{S}_{eff}^{LL} was calculated, provided one knows the first coefficient C_2^{NLL} of the series, as well as $\beta_x^{(3)}$ and $\gamma_\phi^{(2)}$. That means one can sum up sequentially several subseries of coefficients contributing to the effective potential, until exhausting the perturbative information encoded in β_x , γ_ϕ , and the V_{eff} calculated up to a certain loop order. This is a systematical procedure to extract the maximum amount of information concerning the effective potential from a perturbative calculation.

One important technical detail is that the renormalization group functions are usually calculated in the Minimal Subtraction (MS) renormalization scheme, and they need to be adapted to the procedure outlined in this section, as it was first pointed out in [58]. For simplicity, let us consider the case of a theory with a single coupling x . In the MS scheme, divergent integrals (in four spacetime dimension) appear with a factor

$$\tilde{\mathcal{L}} = \ln \left[\frac{x \phi^2}{2\mu^2} \right], \quad (31)$$

while in the so-called Coleman–Weinberg (CW) scheme, the effective potential depends on a log of the form (23). Both schemes can be related by a redefinition of the mass scale μ ,

$$\mu_{MS}^2 = f(x) \mu_{CW}^2, \quad (32)$$

which can be shown to imply in the following relation between the beta functions in both schemes,

$$\beta_{CW} = \beta_{MS} \left(1 - \frac{1}{2} \beta_{MS} \partial_x \ln f \right)^{-1}. \tag{33}$$

In four spacetime dimensions, divergences usually start at one loop, generating order x^2 contributions to β_{MS} , therefore,

$$\begin{aligned} \beta_{CW} &= \left(\beta_{MS}^{(2)} + \beta_{MS}^{(3)} + \dots \right) (1 + \mathcal{O}(x)) \\ &= \beta_{MS}^{(2)} + \mathcal{O}(x^3). \end{aligned} \tag{34}$$

The conclusion is that at one loop level, both beta functions can be used interchangeably, but if calculations are done at two loops or more, one has to adapt the MS functions to be used in the calculation of the CW effective potential. The same reasoning concerning the beta functions can be applied to the anomalous dimension, with similar conclusions.

To gain further insight into this problem, we present the following argument: in the MS and CW schemes, the effective potential would be calculated at one loop level in the forms

$$V_{MS} = \phi^4 \left(\tilde{A}(x) + \tilde{B}(x) \tilde{\mathcal{L}} \right), \tag{35}$$

and

$$V_{CW} = \phi^4 \left(A(x) + B(x) \mathcal{L} \right). \tag{36}$$

From Eqs. (23) and (31), we have

$$\tilde{\mathcal{L}} = \mathcal{L} + \ln \left[\frac{x}{2} \right], \tag{37}$$

and therefore one can rewrite V_{MS} in a form compatible with the CW scheme as follows,

$$V_{MS} = \phi^4 \left[\left(\tilde{A}(x) + \ln \frac{x}{2} \tilde{B}(x) \right) + \tilde{B}(x) \mathcal{L} \right]. \tag{38}$$

Since the value of $A(x)$ is immaterial in the CW potential, being fixed by the CW condition (12), we conclude that $\tilde{B}(x) = B(x)$ and that both V_{MS} and V_{CW} end up giving identical results at one loop. At two loops, however, V_{MS} contains a term of the form $\tilde{C}(x) \tilde{\mathcal{L}}^2$, so after employing (37), one would find a difference in the relevant term proportional to \mathcal{L} , meaning both potentials are not equivalent at two loops. The net result is that, at the two loop level, the RGE can be used to relate renormalization group functions and the effective potential in the CW and the MS scheme, but not interchangeably.

4. RGE improvement in the three dimensional supersymmetric case

Now we discuss how to adapt the procedure outlined in section 3 to our model. First of all, we consider the problem of interchangeability of MS and CW renormalization group functions when using the RGE to calculate the effective potential. In the supersymmetric three-dimensional model considered by us, divergences only start at two loops, and the beta functions start at order x^3 . This means that instead of Eq. (34) we have

$$\begin{aligned} \beta_{CW} &= \left(\beta_{MS}^{(3)} + \beta_{MS}^{(4)} + \dots \right) (1 + \mathcal{O}(x)) \\ &= \beta_{MS}^{(3)} + \mathcal{O}(x^4). \end{aligned} \tag{39}$$

Also, V_{eff} acquires a term proportional to \tilde{L}^2 only at loop orders greater than two. This means we are safe to use interchangeably functions calculated in the MS and the CW scheme, as we have done in section 2.

It is still not clear that the series of terms we calculated in the previous section, of orders $x^{2n+1}L^n$, have any relation to the leading logs summation described in section 3. Indeed, by repeating the steps outlined in the start of that section for our model, the fact that $\beta_\lambda^{(2)} = 0$ together with Eq. (29) would imply $C_n^{LL} = 0$ for $n > 1$, which in its turn would also trivialize Eq. (30). The conclusion would be that the RGE does not allow us to calculate any new contribution for the effective superpotential.

Actually, this apparent problem is a consequence of the particular pattern of divergences that appear in our model, whenever we use dimensional regularization to evaluate Feynman integrals. In four dimensional non-supersymmetric theories, divergences in general occur at any loop order n , the leading logs being of the order $x^{n+1}L^n$. In three dimensional supersymmetric models, divergences start only at two loops, and are of the order x^3L . At three loops, the only divergences arise from two loops subdiagrams, of the order x^4L . At four loops, we find again superficial divergent diagrams, of order x^5L^2 , while five loops diagrams contain at the most four and two loops divergent subdiagrams, of order x^6L^2 and x^6L . This pattern suggests that new superficial divergences appear only at even loops, and are of the order $x^{2n+1}L^n$, and these terms should be identified as “leading logs” in our case, despite the fact that the difference between the power of coupling constants and logs is not the same for all terms. Careful consideration of this divergence pattern suggests for supersymmetric three dimensional models the definition,

$$S_{eff}(x, L) = S_{eff}^{LL}(x, L) + S_{eff}^{NLL}(x, L) + \dots, \quad (40)$$

where leading logs contributions are of the form

$$S_{eff}^{LL}(x, L) = \sum_{n \geq 0} C_n^{NLL} x^{2n+1} L^n, \quad (41)$$

next to leading logs are given by

$$S_{eff}^{NLL}(x, L) = \sum_{n \geq 0} \left(C_n^{NLL} x^{2n+2} L^n + D_n^{NLL} x^{2n+3} L^n \right), \quad (42)$$

and so on. Inserting this ansatz into the RGE (13) gives us

$$\sum_{n \geq 0} \left[\left(-(n+1) C_{n+1}^{LL} + (2n+1) \beta^3 C_n^{LL} - 4\gamma_\phi^{(2)} C_n^{LL} \right) x^{2n+3} L^n + \mathcal{O}\left(x^{2n+5} L^n\right) \right] = 0, \quad (43)$$

which, very much like Eq. (29), provides a first order difference equation for C_n^{LL} , now involving the order x^3 terms in the beta function, as well as the order x^2 terms of the anomalous dimension. From this equation, the whole series of leading logs terms may be (in principle) summed up, and $S_{eff}^{LL}(x, L)$ determined from the two loop information we have at hand. Looking at other coefficients of the sum in Eq. (43) would provide equations for the calculation of next to leading log contributions, and so on. The result is that the leading logs summation procedure can be applied to three dimensional supersymmetric models, yet with nontrivial modifications, taking into account the peculiar divergence structure of such models.

To actually apply this technique to our model, one has to generalize the equations in last paragraph to the case of two couplings, which involves dealing with double sums of the form

$$S_{eff}^{LL}(\lambda, y, L) = \sum_{n \geq 0} \sum_{0 \leq \ell \leq 2n+1} C_{n, 2n+1-\ell}^{LL} \lambda^\ell y^{2n+1-\ell} L^n. \tag{44}$$

We do not quote here the algebraic details, but we developed a Mathematica code to calculate the coefficients C^{LL} up to an arbitrary (finite) order. With this code we could reproduce, as a consistency check, the result given in Eq. (21), as well as calculating corrections to S_{eff}^{LL} up to the order $x^{41} L^{20}$ in a few seconds. This result will be used, in the next section, to study the modifications introduced by the leading logs summation in the DSB in our model. The code as well as the explicit results are available as a Supplementary Material to this paper.

5. Dynamical breaking of symmetry

In this Section we study the dynamical breaking of the conformal symmetry that occurs in the present theory, based on the improved effective superpotential that was obtained in the previous sections, by summing up leading logs contributions up to the order $x^{41} L^{20}$. More explicitly, we consider,

$$K_{eff}^I(\sigma) = -\frac{1}{4}\sigma^4 \left[S_{eff}^{LL}(\lambda, y, L) + \rho \right], \tag{45}$$

ρ being a finite renormalization constant. The constant ρ is fixed using the CW normalization condition (12). Requiring that the $K_{eff}^I(\sigma)$ has a minimum at $\sigma^2 = \mu$ means that

$$\left. \frac{d}{d\sigma} K_{eff}^I(\sigma) \right|_{\sigma^2=\mu} = 0, \tag{46}$$

which can be used to determine the value of λ as a function of the free parameters y and N .

Upon explicit calculation, Eq. (46) turns out to be a polynomial equation in λ , and among its solutions we look for those which are real and positive, and correspond to a minimum of the potential, i.e.,

$$M_\Sigma = \left. \frac{d^2}{d\sigma^2} K_{eff}^I(\sigma) \right|_{\sigma^2=\mu} > 0. \tag{47}$$

This procedure was implemented in a *Mathematica* program, and we verified that it can be performed for any value of $y < 1$ and N . That means DSB is operational in this model for any reasonable value of its parameters. As an example: choosing $y = 0.1$ and $N = 1$, we found that $\lambda^I = 0.000023224294742$. To compare, by choosing the same values of y and N , but using the unimproved two-loop effective superpotential, including only corrections up to order $x^3 L$, we find $\lambda = 0.00002322078553849$. The difference between the two values being only of order 0.015%, we say that the improvement of the effective superpotential by means of the summation of the leading logs contributions provides only a small quantitative change on the parameters of the DSB. This is rather different from the scenario found in four dimensional models, or even the non-supersymmetric version of the same model considered by us, where the RGE improvement provided substantial qualitative changes in the phase structure of the DSB [24]. The incremental aspect of the improvement, in the present case, can also be seen by plotting both the improved and unimproved effective superpotentials as in Fig. 1, where only by choosing relatively high values of y and N we were able to get two graphs that do not superimpose.

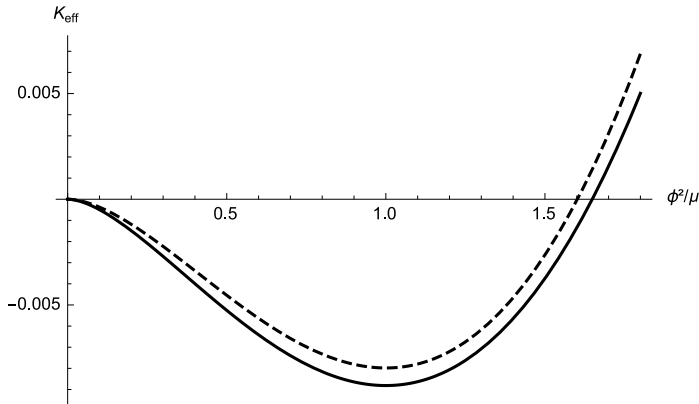


Fig. 1. Comparison of the unimproved (solid line) and improved (dashed line) effective superpotential, for $y = 0.8$ and $N = 90$.

6. Conclusions

The mechanism of symmetry breaking is central for the formulation of a consistent quantum field theory of the known elementary interactions, and the possibility that quantum corrections of a symmetric potential could alone induce such symmetry breaking is a rather interesting one, not only for its mathematical elegance, but also for physical reasons. Recently, for example, a mechanism of dynamical symmetry breaking in a scale-invariant version of the Standard Model is being discussed as a viable mechanism for generating a mass for the Higgs particle compatible with experimental observations. The idea of using the RGE to improve the calculation of the effective potential, summing up terms arising from higher loop orders organized as leading logarithms, next to leading logarithms, and so on, is central to this approach. We have shown how this program can be applied to a supersymmetric model in the superfield formalism, which is the main technical result of this paper.

We discussed an $\mathcal{N} = 1$ supersymmetric Abelian Chern–Simons model coupled to an arbitrary number of scalar and fermion superfields. Matter fields are assumed to be minimally coupled to the CS field, together with quartic self-interaction. The use of the renormalization group functions calculated in [17] together with the RGE allowed a calculation of the improved effective superpotential, that can be used to study DSB in our model. The end result was that DSB is operational for all reasonable values of the free parameters, and that the RGE improvement produces only a small quantitative change in the properties of the model.

In this particular model, therefore, the effects of the RGE improvement were not so dramatic as in its non-supersymmetric counterpart, however the question remains whether the same might happen in different models. It begs to say, however, that we do not expect this technique to be directly applicable to four dimensional supersymmetric models, for which non-renormalization theorems in general forbid DSB.

One final remark is in order. In this work, we used the superfield formalism for the evaluation of the effective superpotential and the study of the phase structure of the model in a manifestly supersymmetric way. One might wonder about the effective scalar potential V_{eff} , i.e., the effective potential of the scalar component of the constant background superfield σ . V_{eff} should be calculated from the full effective superpotential $\Gamma[\sigma]$ as described in [27]. In this case, one should be careful in isolating the contribution from the auxiliary field effective superpotential F

(see Eq. (5)), and also use the beta functions appropriate for the component fields (the superfield quartic coupling $\frac{\lambda}{4} (\overline{\Phi}_a \Phi_a)^2$ translates into a coupling $\frac{\lambda^2}{4} (\overline{\varphi}_a \varphi_a)^3$ in the component formalism, for example) for the RGE improvement. This approach would be natural if one were to consider an important aspect that was left out of this paper because of its technical complexity: the inclusion of the effects of the auxiliary field effective superpotential F , which would allow us to investigate the possibility of spontaneous breaking of supersymmetry. This would deserve a separate investigation. Since in the approximation we are considering there is no possibility of supersymmetry breaking, it is simpler to consider the $K(\sigma)$ effective superpotential as the central object of our study, as it was done in our work.

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Appendix A. Supplementary material

Supplementary material related to this article can be found online at <http://dx.doi.org/10.1016/j.nuclphysb.2016.04.015>.

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