



Gravitational Casimir–Polder effect



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ABSTRACT

The interaction due to quantum gravitational vacuum fluctuations between a gravitationally polarizable object modelled as a two-level system and a gravitational boundary is investigated. This quantum gravitational interaction is found to be position-dependent, which induces a force in close analogy to the Casimir–Polder force in the electromagnetic case. For a Dirichlet boundary, the quantum gravitational potential for the polarizable object in its ground-state is shown to behave like z^{-5} in the near zone, and z^{-6} in the far zone, where z is the distance to the boundary. For a concrete example, where a Bose–Einstein condensate is taken as a gravitationally polarizable object, the relative correction to the radius of the BEC caused by fluctuating quantum gravitational waves in vacuum is found to be of order 10^{-21} . Although the correction is far too small to observe in comparison with its electromagnetic counterpart, it is nevertheless of the order of the gravitational strain caused by a recently detected black hole merger on the arms of the LIGO.

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1. Introduction

Gravitational waves, which are ripples of spacetime, are natural consequences of the theory of general relativity established by Einstein one hundred years ago [1]. Since their amplitudes are extraordinarily small, gravitational waves have never been detected directly until the recent breakthrough made by the Laser Interferometer Gravitational-wave Observatory (LIGO) and Virgo collaborations [2] in the culmination of a decades-long quest. What LIGO detected are actually the classical effects of gravitational waves on the length differences between the arms which were revealed by the laser interferometry. Naturally, one may wonder what effects are if gravitational waves are quantized and whether they can be detectable. Although strictly speaking an ultimate answer of these questions requires a full theory of quantum gravity which is still elusive and quantum gravitational effects are generally expected near the Planck scale, which is far from the energy scales accessible so far, one can still do something using general relativity as a consistent effective field theory at low energies [3–7]. In the present paper, we are interested in yet another low energy

quantum gravitational effect, i.e., quantum gravitational interaction between a gravitationally polarizable object and a boundary.

A fundamental difference between a quantum theory and a classical one is the quantum fluctuations in vacuum necessitated by the uncertainty principle. Vacuum fluctuations, although seem fictional, may cause observational effects. A well-known example is the Casimir force between two neutral conducting plates in vacuum [8]. Classically, no force other than the universal gravitation would be expected between the two plates because of the absence of external fields. However, quantum mechanically, the fluctuating quantum electromagnetic field modes in vacuum are modified due to the presence of reflecting boundaries, and a force is thus induced. One can also replace one of the plates with a neutral polarizable object in the above configuration, and the resulting force is usually referred to as the Casimir–Polder force [9]. The Casimir and Casimir–Polder effects have played an important role in our understanding of the quantization of electromagnetic fields [10,11]. One naturally expects that, if gravity has a quantum nature, it should also generate Casimir-like forces. Here, we plan to calculate the gravitational Casimir–Polder force between a gravitationally polarizable object and a gravitational medium. Let us note that, the quantum corrections to classical gravitational forces between two polarizable objects from the induced quadrupole moments due to two graviton exchange have recently been studied in Ref. [7], and later in Refs. [12] and [13] in different approaches.

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2. The basic formalism

We model a gravitationally polarizable object as a harmonic oscillator. For simplicity, we treat it as a two-level system, which is taken as an open quantum system in interaction with a bath of quantum fluctuating gravitational fields in vacuum. The total Hamiltonian takes the form

$$H = H_A + H_B + H_I. \quad (1)$$

Here H_A is the Hamiltonian of the two-level system

$$H_A = \hbar\omega_0 S_z, \quad (2)$$

where $S_z = \frac{1}{2}(|+\rangle\langle+| - |-\rangle\langle-|)$, and $|+\rangle$ and $|-\rangle$ denote the excited and the ground states, respectively. H_B is the Hamiltonian of the gravitational field, whose explicit expression is not needed here. H_I denotes the quadrupolar gravitational interaction Hamiltonian, which can be written as

$$H_I = -\frac{1}{2} \sum_{ij} Q_{ij} E_{ij}, \quad (3)$$

where Q_{ij} is the quadrupole moment of the object and $E_{ij} = -\nabla_i \nabla_j \phi$ with ϕ being the gravitational potential. In Newtonian theory, E_{ij} determines the tidal gravitational acceleration between two nearby test particles, while in general relativity, the similar role is played by the Weyl tensor, i.e. $E_{ij} = -c^2 C_{0i0j}$ ¹ [15]. Here E_{ij} and $B_{ij} = \frac{1}{2} c^2 \epsilon_{imn} C^{mn}_{0j}$ are the gravito-electric and gravito-magnetic tensors which satisfy the linearized Einstein field equations written in a form in analogy to the Maxwell equations known as gravitoelectromagnetism [15–21]. Note that the gravito-electric field E_{ij} here is supposed to be quantized.

Initially, the whole system is described by $\rho_{\text{tot}} = \rho(0) \otimes \rho_B$, in which $\rho(0)$ is the initial reduced density matrix of the object, and ρ_B characterizes the state of the environment. The time evolution of the whole system in the frame of the object follows the quantum Liouville equation

$$\frac{\partial \rho_{\text{tot}}(\tau)}{\partial \tau} = -\frac{i}{\hbar} [H(\tau), \rho_{\text{tot}}(\tau)]. \quad (4)$$

The dynamics of the reduced system can be obtained by tracing over the degrees of freedom of the field, and in the limit of weak-coupling, the reduced density matrix is found to satisfy the master equation in the interaction picture [22–24]

$$\frac{d}{d\tau} \rho(\tau) = -\frac{i}{\hbar} [H_{LS}, \rho(\tau)] + \mathcal{D}(\rho(\tau)). \quad (5)$$

We observe from Eq. (5) that the contribution of the vacuum gravitational fields can be separated into two parts. The first part

$$H_{LS} = \hbar \sum_{\omega} \sum_{ijkl} S_{ijkl}(\omega) A_{ij}^{\dagger}(\omega) A_{kl}(\omega), \quad (6)$$

is unitary, where $A_{ij}(\omega) = -\frac{1}{2} \sum_{\nu'=\omega} \Pi(\nu) Q_{ij} \Pi(\nu')$, with $\Pi(\nu)$ denoting the projection onto the eigenspace belonging to the eigenvalue ν of the Hamiltonian H_S . The function $S_{ijkl}(\omega)$ can be written as

$$S_{ijkl}(\omega) = \frac{i}{2} \mathcal{G}_{ijkl}(\omega) - i \Gamma_{ijkl}(\omega), \quad (7)$$

where $\mathcal{G}_{ijkl}(\omega)$ is the Fourier transform of the field correlation function $\langle E_{ij}(s) E_{kl}(0) \rangle$,

$$\mathcal{G}_{ijkl}(\omega) = \frac{1}{\hbar^2} \int_{-\infty}^{\infty} ds e^{i\omega s} \langle E_{ij}(s) E_{kl}(0) \rangle, \quad (8)$$

and $\Gamma_{ijkl}(\omega)$ is the one-side Fourier transform

$$\Gamma_{ijkl}(\omega) = \frac{1}{\hbar^2} \int_0^{\infty} ds e^{i\omega s} \langle E_{ij}(s) E_{kl}(0) \rangle. \quad (9)$$

Then it can be shown, with the help of

$$\frac{1}{x \mp i\epsilon} = \mathcal{P} \frac{1}{x} \pm i\pi \delta(x), \quad (10)$$

that

$$S_{ijkl}(\omega) = -\frac{\mathcal{P}}{2\pi} \int_{-\infty}^{\infty} \frac{\mathcal{G}_{ijkl}(\lambda)}{\lambda - \omega} d\lambda, \quad (11)$$

where $\mathcal{P}$ means the principal value. The unitary part H_{LS} is usually referred to as the Lamb shift Hamiltonian, which arises from the object's quadrupolar interaction with vacuum fluctuations of the gravitational fields and leads to an energy shift of the object. This part is our main focus in the present paper, and in the following we will show that this part becomes position-dependent when a boundary is present, which induces a force in close analogy to the Casimir–Polder interaction in the electromagnetic case. The second part

$$\mathcal{D}(\rho) = \sum_{\omega} \sum_{ijkl} \mathcal{G}_{ijkl}(\omega) \left(A_{kl}(\omega) \rho A_{ij}^{\dagger}(\omega) - \frac{1}{2} \{ A_{ij}^{\dagger}(\omega) A_{kl}(\omega), \rho \} \right), \quad (12)$$

is the dissipator of the master equation, which is nonunitary and represents the decoherence and dissipation due to environment.

3. The quantum gravitational interaction

Now, we calculate the quantum gravitational interaction between a static gravitationally polarizable two-level system whose trajectory can be described as

$$t(\tau) = \tau, \quad x(\tau) = y(\tau) = 0, \quad z(\tau) = z, \quad (13)$$

and a boundary located at $z = 0$. As has been discussed above, one needs to calculate the Lamb shift Hamiltonian (6) which is related to the correlation functions of the gravito-electric field $\langle E_{ij}(s) E_{kl}(0) \rangle$. Consequently, the wave equation in a gravitational medium is needed to fix the boundary conditions, cf. Ref. [25]. In the present paper, we apply the Dirichlet boundary condition for simplicity, which models an ideal reflecting boundary for transverse gravito-electric field modes. If we expand the space-time metric as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, where $\eta_{\mu\nu} = \{-1, 1, 1, 1\}$ is the Minkowski metric, and work in the transverse traceless (TT) gauge, the gravito-electric field tensor $E_{ij} = \frac{1}{2} \ddot{h}_{ij}$, where a dot denotes derivative with respect to t . The Wightman function for gravitons in the TT gauge reads [26]

$$\begin{aligned} \langle h_{ij}(x) h_{kl}(x') \rangle &= \frac{32\pi G \hbar^2}{c^4} (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk} - \delta_{ij} \delta_{kl} + D_{ijkl}) \langle 0 | \phi(x) \phi(x') | 0 \rangle, \end{aligned} \quad (14)$$

where

¹ We follow the sign convention of Misner, Thorne, and Wheeler [14].

$$D_{ijkl} = \left(\frac{\partial_i \partial'_j}{\nabla^2} \delta_{kl} + \frac{\partial_k \partial'_l}{\nabla^2} \delta_{ij} - \frac{\partial_i \partial'_k}{\nabla^2} \delta_{jl} - \frac{\partial_i \partial'_l}{\nabla^2} \delta_{jk} \right. \\ \left. - \frac{\partial_j \partial'_l}{\nabla^2} \delta_{ik} - \frac{\partial_j \partial'_k}{\nabla^2} \delta_{il} + \frac{\partial_i \partial'_j \partial_k \partial'_l}{\nabla^4} \right), \quad (15)$$

and $\langle 0|\phi(x)\phi(x')|0\rangle$ is the scalar field two-point function. Here ∇^{-2} in Eq. (15) should be understood in the sense of a Green's function, and when working in momentum space its effect is to bring in a factor of k^{-2} . The scalar field two-point function can then be written as the sum of a free space term and a term due to the presence of the boundary with the help of the method of images as

$$\langle 0|\phi(x)\phi(x')|0\rangle \\ = -\frac{c}{4\pi^2\hbar} \frac{1}{(ct - ct' - i\epsilon)^2 - (x - x')^2 - (y - y')^2 - (z - z')^2} \\ + \frac{c}{4\pi^2\hbar} \frac{1}{(ct - ct' - i\epsilon)^2 - (x - x')^2 - (y - y')^2 - (z + z')^2}. \quad (16)$$

Since we are interested in the energy shift caused by the boundary, in the following we consider the boundary-dependent terms only.

For a two-level system, the summation over ω in Eq. (6) contains two terms only, i.e. $\omega = \pm\omega_0$. As a result, Eq. (6) can be written explicitly as

$$H_{LS} = \frac{\hbar}{4} \sum_{ijkl} S_{ijkl}(\omega_0) |+\rangle \langle +| Q_{ij} |-\rangle \langle -| Q_{kl} |+\rangle \langle +| \\ + \frac{\hbar}{4} \sum_{ijkl} S_{ijkl}(-\omega_0) |-\rangle \langle -| Q_{ij} |+\rangle \langle +| Q_{kl} |-\rangle \langle -|. \quad (17)$$

Therefore, the energy-level shifts of the ground state and excited state are

$$\delta\mathcal{E}_- = \frac{\hbar}{4} \sum_{ijkl} S_{ijkl}(-\omega_0) \langle -| Q_{ij} |+\rangle \langle +| Q_{kl} |-\rangle, \quad (18)$$

$$\delta\mathcal{E}_+ = \frac{\hbar}{4} \sum_{ijkl} S_{ijkl}(\omega_0) \langle +| Q_{ij} |-\rangle \langle -| Q_{kl} |+\rangle, \quad (19)$$

respectively. Since the external environment is in its vacuum state, we focus on the energy shift of the ground-state, which can be calculated as

$$\delta\mathcal{E}_- = \frac{G}{z^5} \sum_{ijkl} Q_{ij} Q_{kl}^* f_{ijkl}(\omega_0, z). \quad (20)$$

Here and after we use $Q_{ij} = \langle -| Q_{ij} |+\rangle$, $Q_{ij}^* = \langle +| Q_{ij} |-\rangle$, and $|Q_{ij}|^2 = Q_{ij} Q_{ij}^*$ for brevity, and

$$f_{1111}(\omega_0, z) = \frac{\omega_0 z}{64\pi c} \int_0^\infty du \frac{16u^4 + 16u^3 + 20u^2 + 18u + 9}{u^2 + \omega_0^2 z^2/c^2} e^{-2u}, \quad (21)$$

$$f_{3333}(\omega_0, z) = \frac{\omega_0 z}{8\pi c} \int_0^\infty du \frac{4u^2 + 6u + 3}{u^2 + \omega_0^2 z^2/c^2} e^{-2u}, \quad (22)$$

$$f_{1122}(\omega_0, z) = -\frac{\omega_0 z}{64\pi c} \int_0^\infty du \frac{(2u+1)(8u^3 + 4u^2 - 3)}{u^2 + \omega_0^2 z^2/c^2} e^{-2u}, \quad (23)$$

$$f_{1133}(\omega_0, z) = -\frac{\omega_0 z}{16\pi c} \int_0^\infty du \frac{4u^2 + 6u + 3}{u^2 + \omega_0^2 z^2/c^2} e^{-2u}, \quad (24)$$

$$f_{1212}(\omega_0, z) = \frac{\omega_0 z}{64\pi c} \int_0^\infty du \frac{16u^4 + 16u^3 + 12u^2 + 6u + 3}{u^2 + \omega_0^2 z^2/c^2} e^{-2u}, \quad (25)$$

$$f_{1313}(\omega_0, z) = -\frac{\omega_0 z}{16\pi c} \int_0^\infty du \frac{4u^3 + 6u^2 + 6u + 3}{u^2 + \omega_0^2 z^2/c^2} e^{-2u}, \quad (26)$$

$$f_{1111}(\omega_0, z) = f_{2222}(\omega_0, z), \quad f_{1122}(\omega_0, z) = f_{2211}(\omega_0, z), \quad (27)$$

$$f_{1212}(\omega_0, z) = f_{2121}(\omega_0, z), \quad (28)$$

$$f_{1133}(\omega_0, z) = f_{3311}(\omega_0, z) = f_{2233}(\omega_0, z) = f_{3322}(\omega_0, z) \quad (29)$$

with other components being zero.

In analogy to electrodynamics, we define a gravitational polarizability tensor α_{ij} such that $\alpha_{ij} \equiv |Q_{ij}|^2/\hbar\omega_0$. In the near regime, i.e. when the distance between the object and the reflecting surface is much less than the transition wavelength ($\omega_0 z/c \ll 1$), the energy shift takes the form

$$\delta\mathcal{E}_- = \frac{3\hbar\omega_0 G}{128z^5} (2\alpha_{11} + 2\alpha_{22} + 17\alpha_{33} + 2\alpha_{12} - 8\alpha_{13} - 8\alpha_{23}), \quad (30)$$

in which the symmetric and traceless properties of the quadrupole tensor have been taken into account. This shows that the energy shift decays with distance as z^{-5} , which can be understood as a gravitational quadrupole–quadrupole interaction between the object and its image. In the long-distance regime, i.e. $\omega_0 z/c \gg 1$, we have

$$\delta\mathcal{E}_- = \frac{3\hbar G c}{4\pi z^6} (\alpha_{11} + \alpha_{22} + \alpha_{33} + \alpha_{12} - \alpha_{13} - \alpha_{23}). \quad (31)$$

In this regime, the energy shift decreases with distance as z^{-6} , and the factor c appears as a result of retardation.

It has been found in Ref. [7] that the quantum gravitational potential of a couple of polarizable objects are proportional to z^{-10} and z^{-11} respectively in the near and far regimes [7]. The difference in the power law can be understood by dimensional analysis. Here our results are proportional to α_{ij} , while those in Ref. [7] are proportional to α_{ij}^2 . Dimensionally $[\alpha_{ij}] = L^5/G$, then it is reasonable that there is a difference $\propto z^{-5}$ in the dependence of z between the two results.

For a concrete example, we consider a Bose–Einstein condensate (BEC), which, we note, is actually a multilevel system in a harmonic trap as a gravitationally polarizable object for an order-of-magnitude estimate of the effect we just computed. The BEC will be stretched and squeezed when a gravitational wave passes by, and a gravitational quadrupole will be induced. The gravitational polarizability of the BEC can be calculated in a harmonic oscillator model with the help of the geodesic deviation equation [18], and it is of the order of MR^2/ω_0^2 , where M is the mass of the BEC and R is the radius which can be characterized by the harmonic oscillator length $\sqrt{\hbar/m\omega_0}$ when the interatomic interactions are neglected [27], with m being the mass of a single atom in the BEC, and ω_0 the center-of-mass oscillating frequency which is the same with the trap frequency in the absence of perturbations. The BEC is in interaction with quantum vacuum gravitational fluctuations modified by the presence of a boundary, and a quantum gravitational potential V_{surf} is generated, which will cause a relative shift to the center-of-mass oscillating frequency as [28,29]

$$\gamma \equiv \frac{\omega_0 - \omega}{\omega_0} \simeq -\frac{1}{2M\omega_0^2} \frac{\partial^2}{\partial z^2} V_{\text{surf}}. \quad (32)$$

If the frequency of a harmonic oscillator is suddenly changed from ω_0 to ω at some position $z = z_0$, its amplitude will be changed from R to R' , and the variables can be related to each other on equalling the kinetic energy of the oscillator at $z = z_0$ as [18]

$$R'^2 = R^2 + \frac{\omega_0^2 - \omega^2}{\omega^2} (R^2 - z_0^2). \quad (33)$$

Plugging Eq. (32) and $\langle z_0^2 \rangle = R^2/2$ into Eq. (33), we have

$$\frac{R' - R}{R} \simeq \frac{1}{2} \gamma. \quad (34)$$

That is, due to the presence of V_{surf} , both the oscillating frequency and the radius of the BEC are modified, which are both of the order of γ . For a BEC composed of $N = 10^6$ ^{87}Rb atoms trapped with $\omega_0 \sim 10^2$ Hz, the typical size $R \sim 1$ μm . We assume that the center of mass of the BEC is located at a distance $z \simeq R \sim 1$ μm to the boundary, then $\gamma \sim \frac{\hbar G R^2}{z^2 \omega_0^3} \sim 10^{-21}$. Here let us note that the relative oscillating frequency shift of a trapped BEC has been utilized to detect the electromagnetic Casimir–Polder force, where $\gamma \sim 10^{-2} - 10^{-4}$ in the retarded and thermal regimes [28,29]. In the electromagnetic case, V_{surf} is calculated as a summation of the Casimir–Polder potentials of individual atoms, while in the gravitational case here, it is calculated by considering the BEC as a whole system. The relative correction is exceedingly small as compared to that in the electromagnetic case, so, far too small to observe. It is interesting to note however that the relative correction of the radius of the BEC $\gamma \sim 10^{-21}$ caused by the quantum gravitational interaction is of the same order of the gravitational strain caused by a black hole merger on the arms of the LIGO [2]. Noteworthy, the relative correction here is caused by fluctuating quantum gravitational waves in vacuum whereas the correction observed by LIGO was caused by classical gravitational waves produced by a binary black hole merger. We must point out that an experimental verification of this quantum gravitational effect would be a much greater challenge than that of the classical one detected by LIGO, even if we could find matter that would reflect gravitational waves significantly, since the radius of the BEC is a lot more difficult to be measured precisely.

Now a few comments are in order for our assumption of a plane that perfectly reflects gravitational waves. Let us first note that the absorption of gravitational waves, both the classical absorption, and the absorption of gravitons by the analog of the photoelectric effect and by phonon excitation, has been analyzed in Ref. [30], and it has been argued that the construction of a conductor that could absorb or reflect gravitational waves effectively would be impossible. At the same time, however, Press studied the possibility of constructing mirrors for gravitational waves with elastic and viscous materials, and found that the phase velocity of sound in such a material has to be greater than the speed of light [31]. These materials are found to be possible but unstable [32–34]. If somehow there were such materials in our Universe, then the propagation of certain gravitational waves through them might be noticeably affected. Besides, there have been interesting suggestions that the interaction between gravitational waves and quantum fluids (e.g. superconductors, superfluids, quantum Hall fluids, and Bose–Einstein condensates) might be significantly enhanced compared with ordinary matter (see Ref. [35] for a review), and even some work has been done on possible implications of these suggestions [36]. Finally, it is interesting to revisit the quantum effects we just studied using a more realistic model of a gravitational boundary rather than a perfect reflector, which, though a much more complicated issue, is currently under investigation.

4. Conclusion

In summary, we have studied the interaction caused by fluctuating quantum vacuum gravitational fields between a polarizable object modelled as a gravitational two-level system and a boundary. The position-dependent potential induces a force in close analogy to the Casimir–Polder force. We have worked out the explicit analytical expressions of the quantum gravitational interaction potential for a Dirichlet boundary, which decreases with distance as z^{-5} in the short-distance regime, and z^{-6} in the long-distance regime. Taking a Bose–Einstein condensate as a gravitationally polarizable object, we have found that the relative correction to the radius caused by fluctuating quantum gravitational waves in vacuum is of the order 10^{-21} .

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