

Revisiting the texture zero neutrino mass matrices

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 In the light of refined and large measurements of the reactor mixing angle θ_{13} , we have revisited the texture three- and two-zero neutrino mass matrices in the flavor basis. For Majorana neutrinos, it has been explicitly shown that all the texture three-zero mass matrices remain ruled out. Further, for both normal and inverted mass ordering, for the texture two-zero neutrino mass matrices one finds interesting constraints on the Dirac-like CP-violating phase δ and Majorana phases ρ and σ .

Subject Index B40

1. Introduction

The recent measurement of reactor mixing angle θ_{13} and its subsequent refinements [1–4], along with the precision measurement of the solar and atmospheric mixing angles θ_{12} and θ_{23} and the neutrino mass-squared differences, have given a new impetus to the neutrino oscillation phenomenology. The observation of a non-zero value of θ_{13} , on the one hand, restored the parallelism between quark and lepton mixing, while on the other hand it has triggered a great deal of interest in the exploration of CP violation in the leptonic sector. It has also brought into focus the issue of constraining the CP-violating phases of the mixing matrix from the textures of the mass matrices. This has further been strengthened by a very recent constraint on the sum of absolute neutrino masses provided by the Planck experiment [5]. In particular, efforts have been made to carry out fine-grained analysis regarding the compatibility of texture-specific mass matrices in the flavor [6–14] as well as non-flavor basis [15–19]. Specifically, in the flavor basis, wherein the charged lepton mass matrix is considered to be diagonal, a good number of attempts have been made to explore the compatibility of texture zero mass matrices for Majorana neutrinos with the neutrino oscillation data.

Present refinements in the data warrant a reconsideration of the compatibility of the texture zero neutrino mass matrices. Therefore, the purpose of the present paper is to reinvestigate all the possible structures of texture three- and two-zero neutrino mass matrices in the flavor basis. In particular, we have carried out an analysis, similar to the one carried out in Ref. [20], with an emphasis on obtaining useful constraints on the CP-violating phases. Specifically, for the viable possibilities of texture zero mass matrices, we have examined the implications of a relatively large and refined θ_{13} on the parameter space of the Dirac-like CP-violating phase δ and the Majorana phases ρ , σ .

The plan of the paper is as follows. To begin with, some essential details pertaining to the construction of the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix [21–24] from the mass matrices are presented in Sect. 2. Using the inputs given in Sect. 3, and keeping focus on the CP-violating phases ρ , σ , and δ , the results pertaining to the analyses of texture three-zero Majorana mass matrices are

presented in Sect. 4. For the texture two-zero case, results of the analysis for Majorana neutrino mass matrices are given in Sect. 5. Finally, Sect. 6 summarizes our conclusions.

2. Texture-specific mass matrices and construction of the PMNS matrix

Before proceeding further, we briefly underline the methodology relating the elements of the mass matrices to those of the mixing matrix. In the flavor basis, wherein the charged lepton mass matrix M_l is diagonal, the Majorana neutrino mass matrix M_ν can be expressed in terms of three neutrino masses m_1, m_2, m_3 and the flavor-mixing matrix V as

$$M_\nu = V \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} V^T. \quad (1)$$

The mixing matrix V can be written as $V = UP$, where U denotes the PMNS [21–24] neutrino mixing matrix consisting of three flavor mixing angles and one Dirac-like CP-violating phase, whereas the matrix P is a diagonal phase matrix, i.e., $P = \text{diag}(e^{i\rho}, e^{i\sigma}, 1)$, with ρ and σ being the two Majorana CP-violating phases. The neutrino mass matrix M_ν can then be rewritten as

$$M_\nu = \begin{pmatrix} M_{ee} & M_{e\mu} & M_{e\tau} \\ M_{e\mu} & M_{\mu\mu} & M_{\mu\tau} \\ M_{e\tau} & M_{\mu\tau} & M_{\tau\tau} \end{pmatrix} = U \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} U^T, \quad (2)$$

where $\lambda_1 = m_1 e^{2i\rho}$, $\lambda_2 = m_2 e^{2i\sigma}$, $\lambda_3 = m_3$.

For the calculations, we have adopted the parameterization of the mixing matrix U considered by Ref. [20], i.e.,

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -c_{12}s_{23}s_{13} - s_{12}c_{23}e^{-i\delta} & -s_{12}s_{23}s_{13} + c_{12}c_{23}e^{-i\delta} & s_{23}c_{13} \\ -c_{12}c_{23}s_{13} + s_{12}s_{23}e^{-i\delta} & -s_{12}c_{23}s_{13} - c_{12}s_{23}e^{-i\delta} & c_{23}c_{13} \end{pmatrix}, \quad (3)$$

where $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$ for $i, j = 1, 2, 3$ and δ is the CP-violating phase.

3. Inputs used in the present analysis

Before discussing the results of the analysis, we summarize the experimental information for various neutrino oscillation parameters. For both normal mass ordering (NO) and inverted mass ordering (IO), the best-fit values and the latest experimental constraints on neutrino parameters at 1σ , 2σ , and 3σ confidence levels (CLs), following Ref. [25], are given in Table 1.

4. Texture three-zero Majorana neutrino mass matrices

In the flavor basis, there are 20 possible texture three-zero patterns; the texture structures of these have been elaborated in Ref. [26], and it has been shown that they are all found to be incompatible with the neutrino oscillation data. Our present analysis, carried out with the current refined data, reinforces this earlier conclusion. To elaborate, we explicitly examine one of the possibilities and bring forward its incompatibility with the latest neutrino mixing data. In particular, we consider the

Table 1. 1σ , 2σ , and 3σ CL ranges of neutrino oscillation parameters. NO (IO) refers to normal (inverted) neutrino mass ordering.

Parameter	Best fit	1σ	2σ	3σ
δm^2 [10^{-5} eV ²]	7.60	7.42–7.79	7.26–7.99	7.11–8.18
$ \Delta m_{31}^2 $ [10^{-3} eV ²] (NO)	2.48	2.41–2.53	2.35–2.59	2.30–2.65
$ \Delta m_{31}^2 $ [10^{-3} eV ²] (IO)	2.38	2.32–2.43	2.26–2.48	2.20–2.54
θ_{12}	34.6°	33.6°–35.6°	32.7°–36.7°	31.8°–37.8°
θ_{23} (NO)	48.9°	41.7°–50.7°	40.0°–52.1°	38.8°–53.3°
θ_{23} (IO)	49.2°	46.9°–50.7°	41.3°–52.0°	39.4°–53.1°
θ_{13} (NO)	8.6°	8.4°–8.9°	8.2°–9.1°	7.9°–9.3°
θ_{13} (IO)	8.7°	8.5°–8.9°	8.2°–9.1°	8.0°–9.4°
δ (NO)	254°	182°–353°	0°–360°	0°–360°
δ (IO)	266°	210°–322°	0°–16° $\oplus$ 155°–360°	0°–360°

case wherein $M_{ee} = 0$, $M_{e\tau} = 0$, $M_{\mu\mu} = 0$, i.e.,

$$M_\nu = \begin{pmatrix} 0 & \times & 0 \\ \times & 0 & \times \\ 0 & \times & \times \end{pmatrix}. \quad (4)$$

The neutrino mass matrix M_ν can be diagonalized by using flavor mixing matrix U . For the present case, the constraints $M_{ee} = 0$ and $M_{e\tau} = 0$ yield the following expression for the ratios of the complex neutrino mass eigenvalues:

$$\frac{\lambda_1}{\lambda_3} = -\frac{s_{13}}{c_{13}^2} \left(\frac{s_{12}c_{23}}{c_{12}s_{23}} e^{i\delta} + s_{13} \right), \quad \frac{\lambda_2}{\lambda_3} = +\frac{s_{13}}{c_{13}^2} \left(\frac{c_{12}c_{23}}{s_{12}s_{23}} e^{i\delta} - s_{13} \right). \quad (5)$$

For $M_{\mu\mu} = 0$, we obtain

$$\lambda_3 \left(U_{\mu 1} U_{\mu 1} \cdot \frac{\lambda_1}{\lambda_3} + U_{\mu 2} U_{\mu 2} \cdot \frac{\lambda_2}{\lambda_3} + U_{\mu 3} U_{\mu 3} \right) = 0. \quad (6)$$

From the real and imaginary parts of the above equations, we obtain the following constraints on θ_{12} . From the real part, we get

$$\theta_{12} = \frac{1}{2} \text{acot} \left(\frac{s_{23}c_{23}^2 s_{13}^2 (2 + \cot 2\delta) - s_{23}^3 \cos 2\theta_{13}}{2c_{23}^2 s_{13} \cos \delta} \right), \quad (7)$$

whereas, from the imaginary part, we obtain

$$\theta_{12} = \frac{1}{2} \text{acot} \left(\frac{s_{13} \cos(2\delta) s_{23}}{2 \sin \delta} \right). \quad (8)$$

To check the compatibility of the above two solutions, in Fig. 1(a) and (b) we present the correlation plots between θ_{12} and θ_{23} for the constraints obtained from the real and imaginary parts respectively. These graphs have been obtained by giving full allowed variation to the phase δ and 3σ variation to θ_{13} . From Fig. 1(a), we find that the range obtained for θ_{12} here has no overlap with its experimental range, i.e., 31.8°–37.8°, whereas the plot given in Fig. 1(b) shows that θ_{12} found here includes its experimental range. For compatibility with the texture three-zero possibility considered here, both the graphs should include the experimental range of θ_{12} , therefore ruling out the present case. A similar analysis corresponding to the other possibilities of texture three-zero Majorana neutrino mass matrices yields the same result, therefore ruling these out.

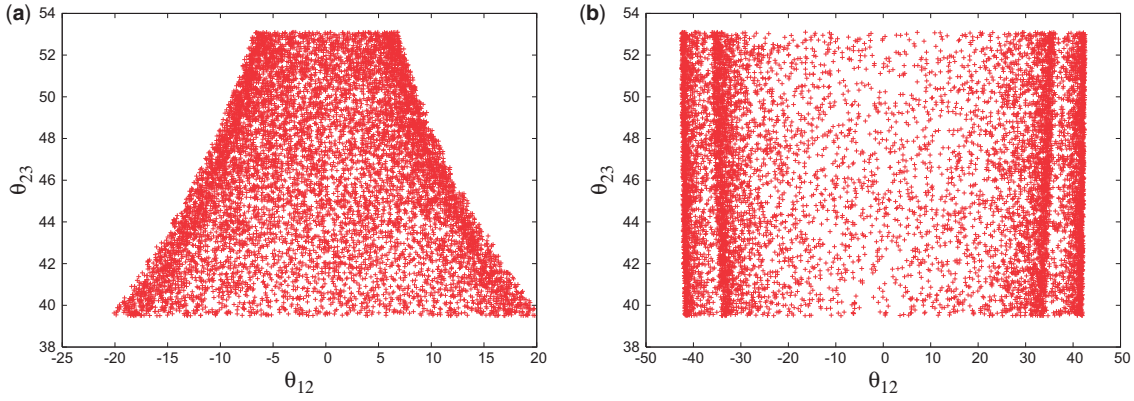


Fig. 1. Correlation plots for the (a) real and (b) imaginary parts of $M_{\mu\mu} = 0$ for three-zero texture with $M_{ee} = 0$, $M_{e\tau} = 0$, $M_{\mu\mu} = 0$, where the angles are measured in degrees.

5. Texture two-zero Majorana neutrino mass matrices

Coming to the case of texture two-zero Majorana neutrino mass matrices, it is well known that there are 15 possible structures of neutrino mass matrix. Adopting the classification scheme of Ref. [20], these can be divided into the six classes A, B, C, D, E, and F. For the purpose of examining whether a possible pattern of texture two-zero mass matrices is viable or not, one needs to examine the compatibility of the corresponding PMNS matrix constructed using them. To execute the present analysis, to begin with we vary the input neutrino oscillation parameters, i.e., the three mixing angles θ_{12} , θ_{23} , and θ_{13} , as well as the mass-squared differences δm^2 and Δm^2 , within their 3σ CL ranges as summarized in Table 1. In the absence of any experimental constraint on Dirac-like CP-violating phase δ , we give full variation of 0° to 360° to it. In order to obtain constraints on the Majorana phases ρ and σ , these can be expressed in terms of the mixing matrix elements:

$$\rho = \frac{1}{2} \arg \left(\frac{U_{a3}U_{b3}U_{l2}U_{m2} - U_{a2}U_{b2}U_{l3}U_{m3}}{U_{a2}U_{b2}U_{l1}U_{m1} - U_{a1}U_{b1}U_{l2}U_{m2}} \right) \quad (9)$$

and

$$\sigma = \frac{1}{2} \arg \left(\frac{U_{a1}U_{b1}U_{l3}U_{m3} - U_{a3}U_{b3}U_{l1}U_{m1}}{U_{a2}U_{b2}U_{l1}U_{m1} - U_{a1}U_{b1}U_{l2}U_{m2}} \right). \quad (10)$$

Coming to the results of the analysis, it may be mentioned that even with the present refinements of the mixing angle θ_{13} , out of the 15 possible texture two-zero Majorana neutrino mass matrices, the texture patterns $A_{1,2}$, $B_{1,2,3,4}$ and C remain viable for the normal mass ordering (NO). The possibilities $D_{1,2}$ have already been ruled out by Ref. [20] due to mixing angle θ_{12} being less than 38° ; the conclusions of our analysis in this regard are the same. Similarly, patterns $E_{1,2,3}$ and $F_{1,2,3}$ remain phenomenologically disfavored. In the following, for NO, we present results for the patterns of categories A, B, and C. In particular, we present the implications of the relatively large and refined θ_{13} on the parameter space of the three CP-violating phases δ , ρ , and σ .

5.1. Class A

This class consists of two possible texture two-zero matrices A_1 and A_2 . Interestingly, there exists a 2–3 permutation symmetry between these; therefore, the phenomenological implications of these are similar and thus we have discussed only pattern A_1 in detail. As a first step, to examine the

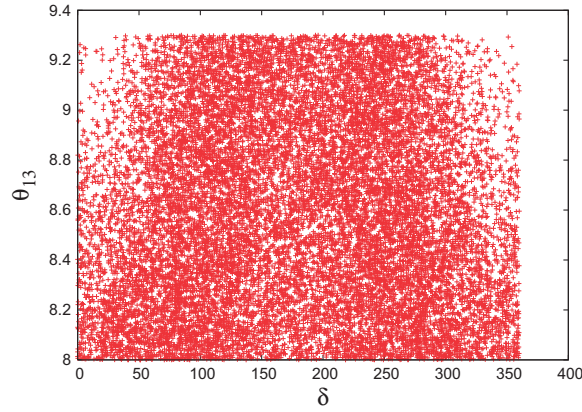


Fig. 2. Pattern A_1 : Mixing angle θ_{13} versus phase δ . All the parameters are in degrees.

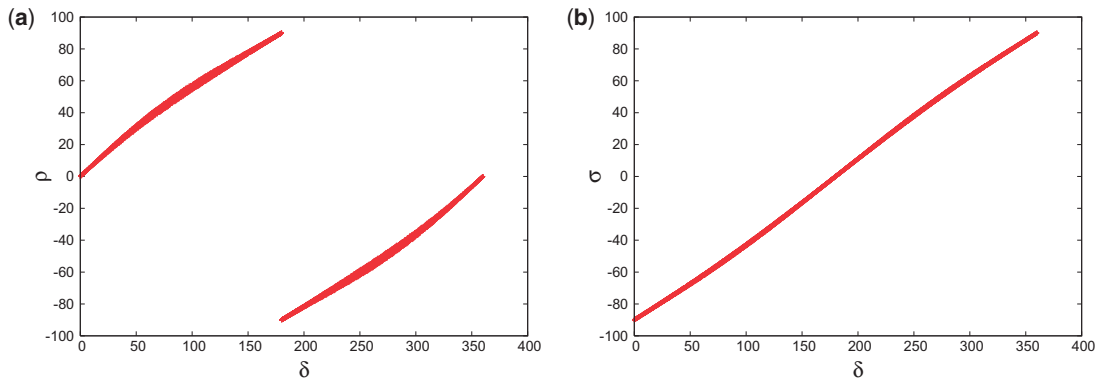


Fig. 3. Pattern A_1 : Correlation plots of phases ρ and σ versus δ . All the parameters are in degrees.

implications of non-zero and large θ_{13} on phase δ , in Fig. 2 we give a θ_{13} versus δ plot. From the graph, one finds that corresponding to the 3σ CL experimental range of angle θ_{13} , i.e. 7.9° to 9.3° , one does not obtain any constraint on phase δ , this being in agreement with the conclusions of Ref. [20].

As a next step, in Fig. 3(a) and (b) we show the correlation plots of Dirac-like CP-violating phase δ and Majorana phases ρ and σ . Again, we find that one is not able to obtain any useful constraints on these phases, in particular, both ρ and σ take values from -90° to 90° .

5.2. Class B

Class B consists of four possible texture two-zero matrices $B_{1,2,3,4}$. Similar to the matrices of class A, there exists a 2–3 permutation symmetry between the matrices B_1 and B_2 , as well as between B_3 and B_4 ; therefore, the phenomenological implications of these are similar and thus we only discuss patterns B_1 and B_3 in detail. In order to obtain constraints on Dirac-like CP-violating phase δ due to the refined measurement of mixing angle θ_{13} , in Fig. 4 we present a θ_{13} versus δ plot for patterns B_1 and B_3 . From the graph one finds that, unlike the case of matrices belonging to class A, the 3σ CL experimental range of angle θ_{13} considerably shrinks the parameter space of δ near 90° and 270° . Therefore, further refinements in the measurement of phase δ would have implications for the matrices considered here.

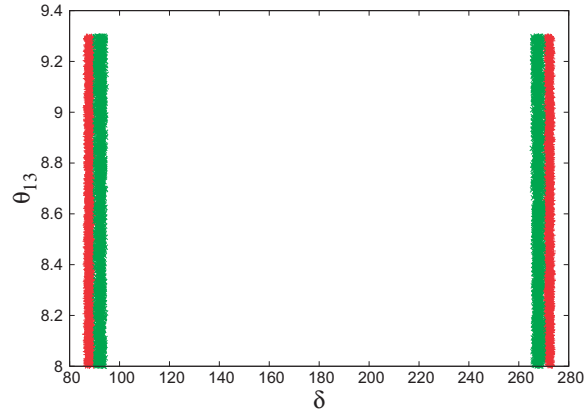


Fig. 4. Patterns B_1 (green) and B_3 (red): Mixing angle θ_{13} versus phase δ . All the parameters are in degrees.

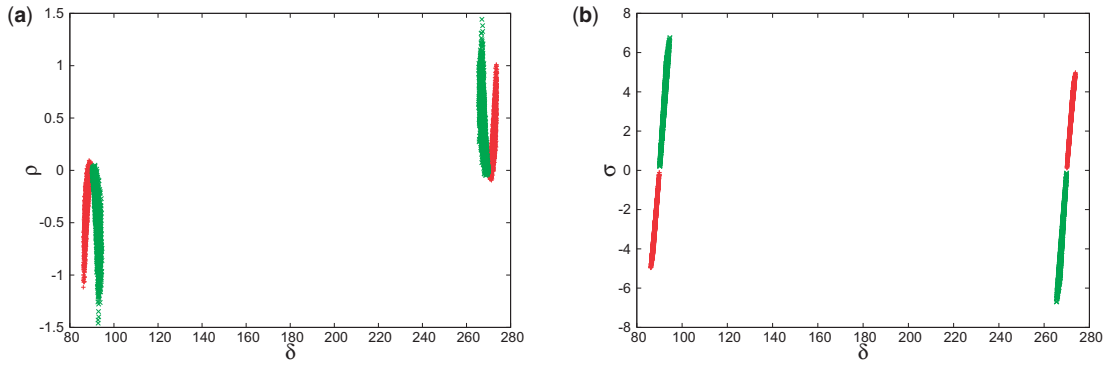


Fig. 5. Patterns B_1 (green) and B_3 (red): Correlation plots of phases ρ and σ versus δ . All the parameters are in degrees.

For the matrices B_1 and B_3 , the correlation plots of Dirac-like CP-violating phase δ and Majorana phases ρ and σ are shown in Fig. 5(a) and (b). The plots indicate that the Majorana phases ρ and σ now take quite small values; in particular, phase ρ lies between -1.5° and 1.5° , whereas σ takes values from -7° to 7° .

5.3. Class C

Similar to the earlier classes, for the matrix belonging to class C, as a first step, we have examined the implications of non-zero and large θ_{13} on phase δ ; in Fig. 6 we show θ_{13} versus δ . From the graph, one finds that corresponding to the 3σ CL experimental range of angle θ_{13} , the phase δ takes values in the ranges 0° – 60° , 120° – 230° , and 300° – 350° .

Figure 7(a) and (b) give the correlation plots of Dirac-like CP-violating phase δ and Majorana phases ρ and σ . None of these phases produce any useful constraint; i.e., they take values in the range -90° – 90° .

5.4. Comparing the results for normal and inverted mass ordering

After having discussed the implications of recent refinements of neutrino oscillation parameters on the Dirac-like CP-violating phase δ and the Majorana phases ρ and σ , we now briefly discuss and compare the results of the analysis for the normal (NO) and inverted (IO) mass orderings.

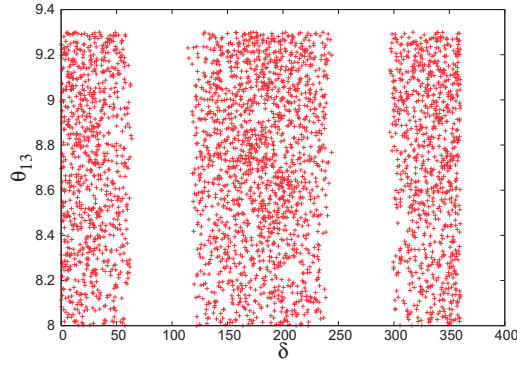


Fig. 6. Pattern C: Mixing angle θ_{13} versus phase δ . All the parameters are in degrees.

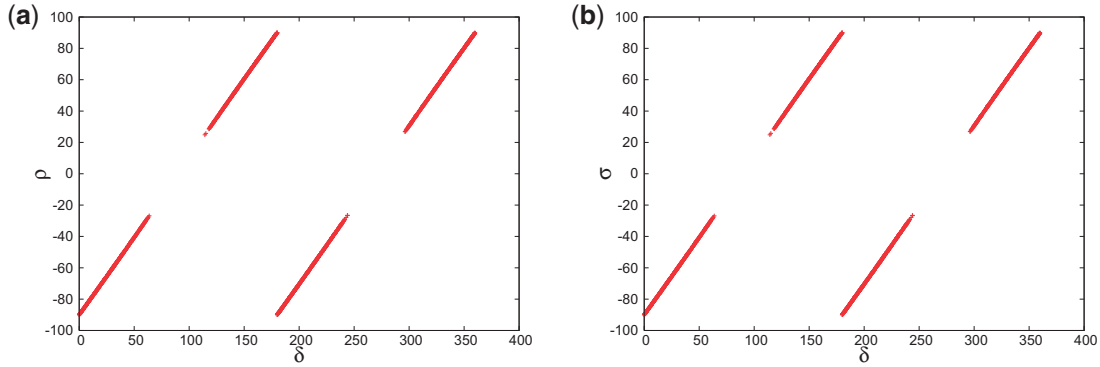


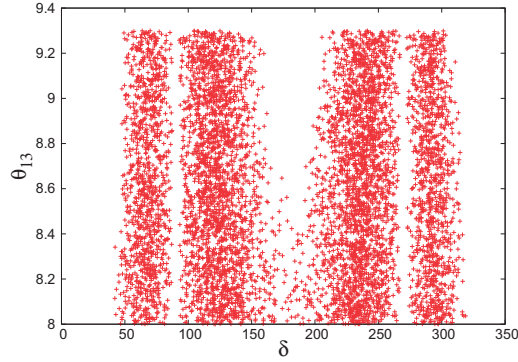
Fig. 7. Pattern C: Correlation plots of phases ρ and σ versus δ . All the parameters are in degrees.

To this end, in Table 2 we have summarized the allowed ranges of Dirac-like CP-violating phase δ , the Majorana phases ρ, σ , and three neutrino masses m_1, m_2, m_3 for the experimentally allowed classes. A few interesting points are as follows:

- As is obvious from Table 1, the matrices belonging to class A are ruled out for IO, whereas for NO, although both A_1 and A_2 are compatible, one is not able to obtain any useful constraints for either of the CP-violating phases.
- All the matrices belonging to class B are viable for both NO and IO. For both the mass orderings, phase δ takes values close to 90° and 270° . The Majorana phases ρ and σ are both quite small; in particular, for IO they take values smaller than the corresponding values for NO. For NO, these conclusions have already been shown graphically in Figs. 4 and 5(a), (b). The corresponding graphs pertaining to IO are similar and hence not shown here.
- Again, the matrix belonging to class C is viable for both NO and IO. Unlike the matrices of class B, one now finds significantly different constraints on CP-violating phase δ for NO and IO. In particular, for NO, δ takes values in the ranges 0° – 62.4° , 114° – 244° , and 294° – 360° . However, for IO the lower limit of δ becomes considerably higher; in particular, δ takes values in the ranges 40.5° – 86.8° , 93.2° – 266.5° , and 273.2° – 325° . Further, regarding the Majorana phases, for NO the phases σ and ρ lie between -90.0° and -27.3° and 27.3° and 90.0° . However, for IO, although phase σ remains unconstrained, the phase ρ gets considerably constrained to -18.4° – 18.2° .

Table 2. The allowed ranges of Dirac-like CP-violating phase δ , the Majorana phases ρ, σ , and three neutrino masses m_1, m_2, m_3 for the experimentally allowed classes. Masses are in eV.

Class	Normal mass ordering (NO)		Inverted mass ordering (IO)	
A_1	$\delta = 0^\circ\text{--}360^\circ$ $\rho = -90^\circ\text{--}90^\circ$ $\sigma = -90^\circ\text{--}90^\circ$	$m_1 = 0.0023\text{--}0.0113$ $m_2 = 0.0087\text{--}0.0144$ $m_3 = 0.0447\text{--}0.0565$	$\times$ $\times$ $\times$	$\times$ $\times$ $\times$
A_2	$\delta = 0^\circ\text{--}360^\circ$ $\rho = -90^\circ\text{--}90^\circ$ $\sigma = -90^\circ\text{--}90^\circ$	$m_1 = 0.0021\text{--}0.0113$ $m_2 = 0.0086\text{--}0.0144$ $m_3 = 0.0447\text{--}0.0565$	$\times$ $\times$ $\times$	$\times$ $\times$ $\times$
B_1	$\delta = 90.05^\circ\text{--}94.67^\circ \oplus 265.3^\circ\text{--}269.9^\circ$ $\rho = -1.8^\circ\text{--}0.05^\circ \oplus -0.008^\circ\text{--}1.5^\circ$ $\sigma = 0.07^\circ\text{--}7.2^\circ \oplus -7.4^\circ\text{--}0.1^\circ$	$m_1 = 0.0327\text{--}0.326$ $m_2 = 0.0349\text{--}0.327$ $m_3 = 0.0564\text{--}0.331$	$\delta = 89.5^\circ\text{--}91.03^\circ \oplus 268.8^\circ\text{--}270.4^\circ$ $\rho = 0.09^\circ\text{--}3.0^\circ \oplus -3.0^\circ\text{--}1.0^\circ$ $\sigma = -1.3^\circ\text{--}0.2^\circ \oplus -0.3^\circ\text{--}1.3^\circ$	$m_1 = 0.0540\text{--}0.278$ $m_2 = 0.0536\text{--}0.0811$ $m_3 = 0.0290\text{--}0.274$
B_2	$\delta = 83.18^\circ\text{--}89.7^\circ \oplus 270^\circ\text{--}277^\circ$ $\rho = -0.03^\circ\text{--}2.8^\circ \oplus -2.8^\circ\text{--}0.027^\circ$ $\sigma = -10.5^\circ\text{--}0.4^\circ \oplus 0.2^\circ\text{--}10.6^\circ$	$m_1 = 0.0276\text{--}0.15$ $m_2 = 0.0285\text{--}0.16$ $m_3 = 0.0522\text{--}0.13$	$\delta = 89^\circ\text{--}90.5^\circ \oplus 269.5^\circ\text{--}271^\circ$ $\rho = -2.6^\circ\text{--}0.07^\circ \oplus 0.1^\circ\text{--}2.3^\circ$ $\sigma = -0.2^\circ\text{--}1.3^\circ \oplus -1.2^\circ\text{--}0.2^\circ$	$m_1 = 0.0594\text{--}0.31$ $m_2 = 0.0599\text{--}0.31$ $m_3 = 0.0339\text{--}0.3$
B_3	$\delta = 86.2^\circ\text{--}89.9^\circ \oplus 270^\circ\text{--}273.9^\circ$ $\rho = -2.8^\circ\text{--}0.2^\circ \oplus -0.17^\circ\text{--}1.1^\circ$ $\sigma = -5.2^\circ\text{--}0.06^\circ \oplus 0.3^\circ\text{--}5.1^\circ$	$m_1 = 0.0350\text{--}0.326$ $m_2 = 0.0381\text{--}0.32$ $m_3 = 0.0588\text{--}0.32$	$\delta = 89.8^\circ\text{--}91.82^\circ \oplus 268.2^\circ\text{--}270.2^\circ$ $\rho = -2.8^\circ\text{--}0.2^\circ \oplus 0.06^\circ\text{--}2.8^\circ$ $\sigma = 0.12^\circ\text{--}3.4^\circ \oplus -3.4^\circ\text{--}0.05^\circ$	$m_1 = 0.054\text{--}0.17$ $m_2 = 0.055\text{--}0.17$ $m_3 = 0.0313\text{--}0.16$
B_4	$\delta = 90.1^\circ\text{--}95.9^\circ \oplus 263.9^\circ\text{--}269.9^\circ$ $\rho = -0.16^\circ\text{--}2.1^\circ \oplus -2.2^\circ\text{--}0.1^\circ$ $\sigma = 0.18^\circ\text{--}7.2^\circ \oplus -7.4^\circ\text{--}0.1^\circ$	$m_1 = 0.0298\text{--}0.22$ $m_2 = 0.0303\text{--}0.22$ $m_3 = 0.0545\text{--}0.22$	$\delta = 88.8^\circ\text{--}90^\circ \oplus 269.9^\circ\text{--}271^\circ$ $\rho = 0.04^\circ\text{--}2.3^\circ \oplus -2.0^\circ\text{--}1.8^\circ$ $\sigma = -2.3^\circ\text{--}0.09^\circ \oplus 0.04^\circ\text{--}2.1^\circ$	$m_1 = 0.0563\text{--}0.247$ $m_2 = 0.0565\text{--}0.32$ $m_3 = 0.0366\text{--}0.32$
C	$\delta = 0^\circ\text{--}62.4^\circ \oplus 114^\circ\text{--}244^\circ$ $\oplus 294^\circ\text{--}360^\circ$ $\rho = -90^\circ\text{--}27.3^\circ \oplus 27^\circ\text{--}90^\circ$ $\sigma = -90^\circ\text{--}27.3^\circ \oplus 27^\circ\text{--}90^\circ$	$m_1 = 0.135\text{--}0.32$ $m_2 = 0.130\text{--}0.32$ $m_3 = 0.137\text{--}0.32$	$\delta = 40.5^\circ\text{--}86.8^\circ \oplus 93.2^\circ\text{--}266.5^\circ$ $\oplus 273.2^\circ\text{--}325^\circ$ $\rho = -18.4^\circ\text{--}18.2^\circ$ $\sigma = -90^\circ\text{--}90^\circ$	$m_1 = 0.0482\text{--}0.32$ $m_2 = 0.0489\text{--}0.32$ $m_3 = 0.0197\text{--}0.32$

**Fig. 8.** Pattern C: Mixing angle θ_{13} versus phase δ for IO. All the parameters are in degrees.

For NO, these conclusions have already been shown graphically in Figs. 6 and 7(a), (b). For IO, the mixing angle θ_{13} versus phase δ graph is shown in Fig. 8. From the graph, one finds that corresponding to the 3σ CL experimental range of angle θ_{13} , i.e. 7.9° to 9.3° , one obtains a slight constraint on phase δ ; in particular, it largely lies in the range $40^\circ\text{--}325^\circ$.

Again, for IO, in Fig. 9(a) and (b) we show the correlation plots of Dirac-like CP-violating phase δ and Majorana phases ρ and σ . Interestingly, we find that although for phase σ we do not obtain any useful constraint, i.e., it takes values in the range $-90^\circ\text{--}90^\circ$, phase ρ is narrowed down

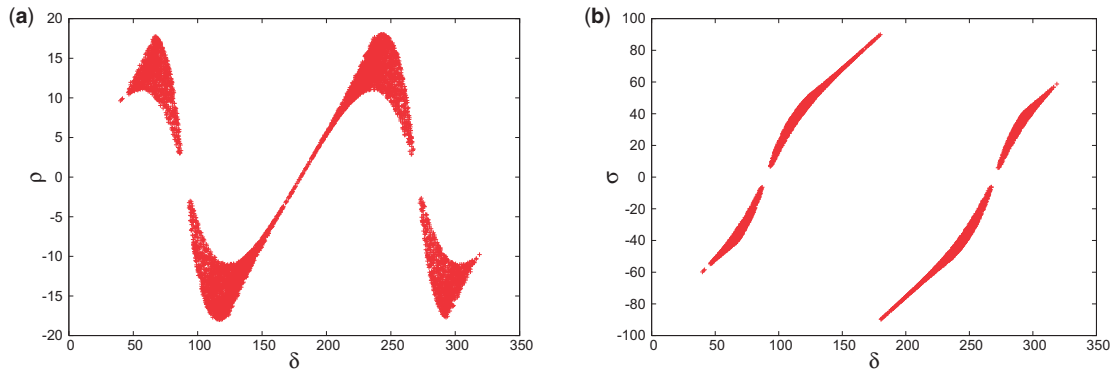


Fig. 9. Pattern C: Correlation plots of phases ρ and σ versus δ for IO. All the parameters are in degrees.

considerably, i.e., it lies in the range -18° – 18° . Also, one finds that phase ρ shows considerable dependence on the Dirac-like CP-violating phase δ .

6. Summary and conclusions

To summarize, in the light of recent refinements of mixing angle θ_{13} , we have reinvestigated all the possible texture structures of three-zero and two-zero neutrino mass matrices for both Majorana and Dirac neutrinos in the flavor basis. In particular, we have examined the implications of angle θ_{13} on the parameter space of three CP-violating phases: ρ , σ , and δ . For the case of texture three-zero mass matrices, confirming earlier results, we find that all possible structures remain ruled out even with the recent data. For a particular texture structure, this has been shown using correlation plots between angles θ_{12} and θ_{23} for the constraints obtained from the real and imaginary parts of ratios of complex neutrino mass eigenvalues. These graphs show that θ_{12} obtained here has no overlap with its experimental range, therefore ruling out the present texture-specific case.

For the case of texture two-zero mass matrices, out of the 15 possible structures, the matrices belonging to classes D, E, and F remain ruled out even with the latest data. For the classes A, B, and C, analysis has been carried out for both NO and IO. The analysis shows that the matrices of class A are ruled out for IO, whereas for NO one does not obtain any useful constraints on the CP-violating phases. For the class B matrices, all these are viable for both NO and IO. For both the mass orderings, phase δ takes values close to 90° and 270° ; however, the Majorana phases ρ and σ both acquire quite small values. The matrix belonging to class C is again viable for both NO and IO. For NO, phase δ has lower limit 0° , whereas for IO the lower limit is 40.5° . The Majorana phases also acquire different constraints for different mass orderings.

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