Hindawi Publishing Corporation
Advances in High Energy Physics
Volume 2014, Article ID 958137, 5 pages
http://dx.doi.org/10.1155/2014/958137

Research Article

Analysis of Scalar Field Cosmology with Phase Space Deformations

Sinuhe Perez-Payan,[1] M. Sabido,[1] E. Mena,[2] and C. Yee-Romero[3]

[1] *Departamento de Física de la Universidad de Guanajuato, Apartado Postal E-143, 37150 León, GTO, Mexico*
[2] *Centro Universitario de la Ciénega, Universidad de Guadalajara, Avenida Universidad 1115 Edificio B, 47820 Ocotlán, JAL, Mexico*
[3] *Departamento de Matemáticas, Facultad de Ciencias Universidad Autónoma de Baja California, Apartado Postal 1880, 22860 Ensenada, BC, Mexico*

Correspondence should be addressed to M. Sabido; msabido@fisica.ugto.mx

Received 19 December 2013; Revised 5 April 2014; Accepted 5 April 2014; Published 30 April 2014

Academic Editor: Chao-Qiang Geng

Phase space deformations on scalar field cosmology are studied. The deformation is introduced by modifying the symplectic structure of the minisuperspace variables. The effects of the deformation are studied in the "*C-frame*" and the "*NC-frame.*" In order to remove the ambiguities of working on different frames, a new principle is introduced. When we impose that both frames should be physically equivalent, we conclude that the only possibility for this model, is to have an effective cosmological constant $\Lambda_{\text{eff}} \geq 0$. Finally we bound the parameter space for θ and β.

1. Introduction

It is a common issue in cosmology to make use of scalar fields as the responsible agents of some of the most intriguing aspects of our universe (see [1–4] and references therein). We find that scalar fields are used as the inflaton which seeds the primordial perturbations for structure, as a cold dark matter candidate and as the dark energy component which seems to be driving the current accelerated expansion of the universe [5]. This current acceleration is probably the biggest conundrum in theoretical physics and is usually attributed to a small, nonvanishing Λ. Unfortunately there is no known mechanism that guarantees a positive near zero value for Λ in a stable or metastable vacuum.

The initial interest in noncommutative field theory [6–8] slowly but steadily permeated in the realm of gravity, from which several approaches to noncommutative gravity [9–18] were proposed. All of these formulations have shown that noncommutative theories of gravity are highly nonlinear. In order to study the effects of noncommutativity on different aspects of the universe, noncommutative cosmology was presented [19]. The authors noticed that the noncommutative deformations modify the noncommutative fields. Furthermore they worked in cosmology, where one imposes symmetry requirements upon the infinite-dimensional space of all possible three-dimensional geometries (superspace) to reduce it to a finite-dimensional minisuperspace, and conjectured that the effects of the full noncommutative theory of gravity should be reflected in the these minisuperspace variables. This was achieved by introducing the Moyal product of functions in the Wheeler-DeWitt equation, in the same manner as is done in noncommutative quantum mechanics. The analysis of the cosmological model was carried out at the quantum level; several works followed this idea [20–25]. Although the noncommutative deformations of the minisuperspace were originally analyzed at the quantum level, classical noncommutative formulations have been proposed. In [20], the authors considered classical noncommutative relations in the phase space for the Kantowski-Sachs cosmological model and established the classical noncommutative equations of motion. For scalar field cosmology, in [21–26] the classical minisuperspace is deformed and a scalar field is used as the matter component of the universe. The main idea of this classical noncommutativity is based on the assumption that modifying the Poisson brackets of the classical theory gives the noncommutative equations of motion [19–26]. These phase space deformations give rise to two generally different interpretations known as the "*C-frame*" and the "*NC-frame*," which in general are not physically equivalent [27].

The main purpose of this paper is to analyze the effects of more general phase space deformations in cosmology under a new principle, in order to eliminate the ambiguity of using the "*C*" or "*NC*" frame. To introduce the deformation we will follow the approach in [28] and we add a new principle, namely, that the physical parameter space for the deformation is given by the requirement that the physical descriptions in the *C* and *NC* frames are equivalent. These ideas will be applied to the model in [29] and analyze the permitted values of θ and β under this new principle. We conclude that for this model the only solution consistent with the principle is that of an effective cosmological constant that satisfies the bound $\Lambda_{\text{eff}} \geq 0$; we also determine the permitted values of θ and ω.

The paper is organized as follows. We start in Section 2 by introducing the deformed phase space model and its dynamics and leave the last section for discussion and concluding remarks.

2. The Model

Let us start with a homogeneous and isotropic universe with a flat Friedmann-Robertson-Walker metric:

$$ds^2 = -N^2(t)\,dt^2 + a^2(t)\left[dr^2 + r^2 d\Omega\right], \tag{1}$$

as usual $a(t)$ is the scale factor and $N(t)$ is the lapse function. We use the Einstein-Hilbert action and a scalar field ϕ as the matter content for the model. In units $8\pi G = 1$, the action takes the form

$$S = \int dt \left\{ -\frac{3a\dot{a}^2}{N} + a^3\left(\frac{\dot{\phi}^2}{2N} - N\Lambda\right)\right\}. \tag{2}$$

We make the following change of variables:

$$\begin{aligned} x &= m^{-1}a^{3/2}\sinh(m\phi), \\ y &= m^{-1}a^{3/2}\cosh(m\phi), \end{aligned} \tag{3}$$

where $m^{-1} = 2\sqrt{2/3}$. The Hamiltonian is

$$H_c = N\left(\frac{1}{2}P_x^2 + \frac{\omega^2}{2}x^2\right) - N\left(\frac{1}{2}P_y^2 + \frac{\omega^2}{2}y^2\right), \tag{4}$$

with $\omega^2 = -(3/4)\Lambda$. To find the dynamics we solve the equations of motion; for this model it can easily be integrated [29].

To construct the deformed model, we usually follow the canonical quantum cosmology approach, where, after canonical quantization, one formally obtains the Wheeler-deWitt equation. An alternative to study quantum mechanical effects is to introduce deformations to the phase space of the system. The approach is an equivalent path to quantization and is part of a complete and consistent type of quantization known as deformation quantization [30]. Our interest is in cosmology and cosmological models can be constructed in the minisuperspace. Following the previous discussion we can assume that studying cosmological models in deformed phase space could be interpreted as studying quantum effects to cosmological solutions [28]. In the deformed phase space approach, the deformation is introduced by the Moyal brackets to get a deformed Poisson algebra. To construct a deformed Poisson algebra we will follow the approach in [28, 29]. We start with the following transformation on the classical phase space variables $\{x, y, P_x, P_y\}$, which satisfy the usual Poisson algebra:

$$\begin{aligned} \hat{x} &= x + \frac{\theta}{2}P_y, \qquad & \hat{y} &= y - \frac{\theta}{2}P_x, \\ \hat{P}_x &= P_x - \frac{\beta}{2}y, \qquad & \hat{P}_y &= P_y + \frac{\beta}{2}x. \end{aligned} \tag{5}$$

These new variables satisfy a deformed algebra:

$$\begin{aligned} \{\hat{y}, \hat{x}\} = \theta, \qquad \{\hat{x}, \hat{P}_x\} = \{\hat{y}, \hat{P}_y\} = 1 + \sigma, \\ \{\hat{P}_y, \hat{P}_x\} = \beta, \end{aligned} \tag{6}$$

where $\sigma = \theta\beta/4$. Furthermore, as in [28], we assume that the deformed variables satisfy the same relations as their commutative counterpart equation (3).

Now that we construct the deformed theory, first, we start with a Hamiltonian which is formally analogous to (4) but constructed with the variables that obey the modified algebra equation (6):

$$H = \frac{1}{2}\left[\left(P_x^2 - P_y^2\right) - \omega_1^2\left(xP_y + yP_x\right) + \omega_2^2\left(x^2 - y^2\right)\right], \tag{7}$$

where we have used the change of variables equation (5) and the following definitions:

$$\omega_1^2 = \frac{\beta - \omega^2\theta}{1 - \omega^2\theta^2/4}, \qquad \omega_2^2 = \frac{\omega^2 - \beta^2/4}{1 - \omega^2\theta^2/4}. \tag{8}$$

Written in terms of the original variables the Hamiltonian explicitly has the effects of the phase space deformation. These effects are encoded by the parameters θ and β. In [29], the late time behaviour of this model was studied. From this formulation two different physical theories arise, one that considers the variables x and y and a different theory based on $\hat{x}$ and $\hat{y}$. The first theory is interpreted as a "commutative" theory with a modified interaction; this theory is referred to as being realised in the "*C-frame*" [27]. The second theory, which privileges the variables $\hat{x}$ and $\hat{y}$, is a theory with "noncommutative" variables but with the standard interaction and is referred to as realised in the "*NC-frame*." One usually privileges one of the frames or assumes that the differences between them are negligible, but in some cases there can be dramatic differences in the physics on each frame. Although the physical implications between the frames can be startlingly different [27], there are cases when the predictions are very similar. In those instances, one can take advantage of the formulation in the "*C-frame*" and have very clear interpretation of the deformation.

In the "*C-frame*" our deformed model has a very nice interpretation that of a ghost oscillator in the presence of constant magnetic field and allows us to write the effects of the noncommutative deformation as minimal coupling on the Hamiltonian and write the Hamiltonian in terms of the magnetic B-field [29].

To obtain the dynamics for the model, we derive the equations of motion from the Hamiltonian equation (7). The solutions for the variables $x(t)$ and $y(t)$ in the "*C-frame*" are

$$\begin{aligned} x(t) &= \eta_0 e^{-(\omega_1^2/2)t}\cosh\left(\omega' t + \delta_1\right) \\ &\quad - \zeta_0 e^{(\omega_1^2/2)t}\cosh\left(\omega' t + \delta_2\right), \\ y(t) &= \eta_0 e^{-(\omega_1^2/2)t}\cosh\left(\omega' t + \delta_1\right) \\ &\quad + \zeta_0 e^{(\omega_1^2/2)t}\cosh\left(\omega' t + \delta_2\right), \end{aligned} \tag{9}$$

where $\omega' = \sqrt{(\beta^2 - 4\omega^2)/(4 - \omega^2\theta^2)}$. For $\omega'^2 < 0$, the hyperbolic functions are replaced by harmonic functions. There is a different solution for $\beta = 2\omega$; the solutions in the "*C-frame*" are

$$\begin{aligned} x(t) &= (a + bt)\, e^{-(\omega_1^2/2)t} + (c + dt)\, e^{(\omega_1^2/2)t}, \\ y(t) &= (a + bt)\, e^{-(\omega_1^2/2)t} - (c + dt)\, e^{(\omega_1^2/2)t}. \end{aligned} \tag{10}$$

To compute the volume of the universe in the "*C-frame*" we use (9) and (3):

$$a^3(t) = V_0 \cosh^2\left(\omega' t\right), \tag{11}$$

where we have taken $\delta_1 = \delta_2 = 0$. For the case $\omega'^2 < 0$, the hyperbolic function is replaced by a harmonic function. For the case $\beta = 2\omega$, the volume is given by

$$a^3(t) = V_0 + At + Bt^2, \tag{12}$$

where V_0, A, and B are constructed from the integration constants.

To find the dynamics in the "*NC-frame*" we start from the "*C-frame*" solutions and use (5); we get for the volume

$$\hat{a}^3(t) = \begin{cases} \widehat{V_0}\left[\cosh^2\left(\omega' t\right) - \dfrac{\omega'^2\theta^2}{\left(2 - \omega_1^2\theta\right)^2}\sinh^2\left(\omega' t\right)\right] & \text{for } \omega'^2 > 0; \\ \widehat{V_0} + Bt + Ct^2 & \text{for } \omega'^2 = 0; \\ \widehat{V_0}\left[\cos^2\left(\left|\omega'\right| t\right) - \dfrac{\left|\omega'\right|^2\theta^2}{\left(2 - \omega_1^2\theta\right)^2}\sin^2\left(\left|\omega'\right| t\right)\right] & \text{for } \omega'^2 < 0, \end{cases} \tag{13}$$

where $\widehat{V_0}$ are the initial volume in the "*NC-frame*." We can see that for $\theta = 0$ the description in the two frames is the same. In the next section we will analyse under what conditions we can have an accelerating universe.

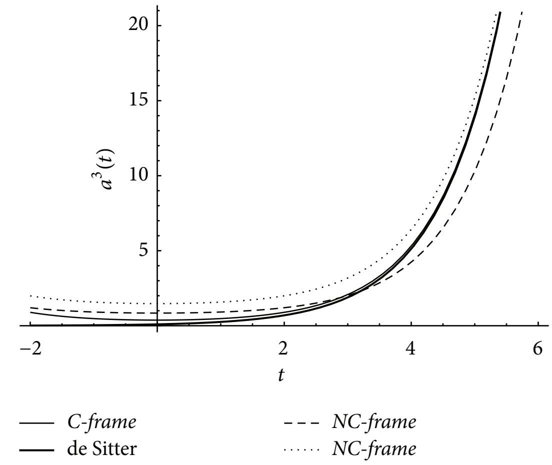

FIGURE 1: Dynamics of the deformed phase space model for $\omega'^2 > 0$. In all the plots of the deformed model, $\delta_2 = \delta_1 = 0$. The thin line and dashed lines are for $\eta_0 = \zeta_0 = 1$, $\beta = 1$, $\omega = 0$, and $\theta = 5$; the dotted line is for $\eta_0 = 5/8$, $\zeta_0 = 4$, $\theta = 4\sqrt{6/5}$, $\omega = 1$, and $\beta = 1/\sqrt{5}$. Finally the thick line corresponds to the volume of the de Sitter space time. We can see that for large values of t the behaviour is exponential.

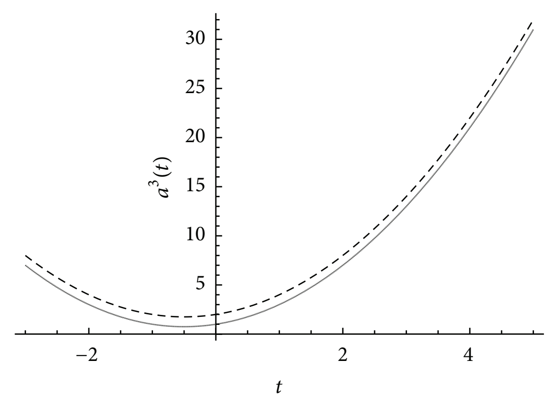

FIGURE 2: Dynamics of the deformed phase space model for the case $\omega'^2 = 0$ or equivalently $\beta = 2\omega$. We can see that the behaviour is the same in the "*C-frame*" and in the "*NC-frame*."

3. Discussion

As already discussed in the previous section, the phase space deformation yields two physically inequivalent descriptions. If we are to consider phase space deformations as physically valid, then the existence of these inequivalent descriptions is unappealing. To solve this issue we will impose the following principle: "Deformed phase space models are only valid when the "NC" and "C" descriptions are equivalent."

This means that we only use those solutions that have the same behaviour, this will restrict the permitted values of the deformation parameters β and θ. We can see in Figures 1 and 2 that the volume has the same behaviour in both frames. In Figure 1, the thin plot ("*C-frame*") and the dashed plot ("*NC-frame*") are plotted with the same values for the deformation parameters and initial conditions; the dotted line plot ("*NC-frame*") corresponds to different values of the parameters; this

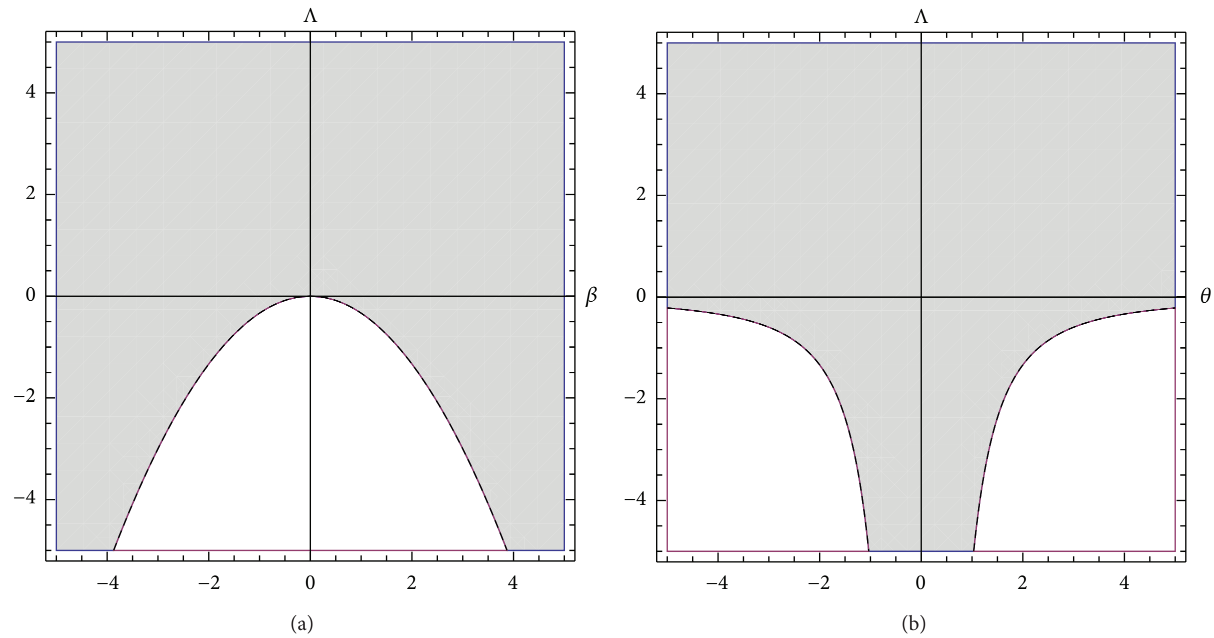

Figure 3: The grey area of the first plot corresponds to $f > 0$. The grey area in the second plot corresponds to $g > 0$. The white region in both plots corresponds to the negative values of f and g.

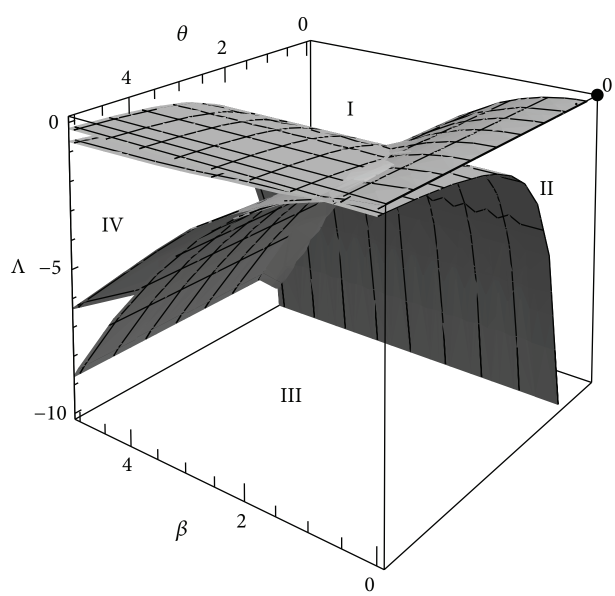

Figure 4: The parameter space where the Λ_{eff} is positive corresponds to the regions I and III of the plot. In this plot the origin 0 is in the upper right corner.

is done to show that the same asymptotic behaviour results in both frames.

By comparing the volume in the two frames and considering the restriction the only cases that are consistent with our condition are $\omega'^2 \geq 0$, the first two cases in (13). The last one is discarded because the volume in the "*NC-frame*" will be negative. Considering the result in [29], where it was shown that for $\omega'^2 > 0$ an effective cosmological constant exists and is given by

$$\Lambda_{\text{eff}} = \frac{1}{3}\left(\frac{\beta^2 + 3\Lambda}{1 + (3/16)\,\Lambda\theta^2}\right). \tag{14}$$

We conclude that "*under phase-space transformations and the imposed restriction, the effective cosmological constant is positive*."

To determine the space parameters where the deformed model is valid, we have the following conditions:

(i) $\Lambda_{\text{eff}} > 0$;

(ii) $f(\beta,\Lambda) = \beta^2 + 3\Lambda$ and $g(\theta,\Lambda) = 1 + (3/16)\Lambda\theta^2$ have the same sign;

(iii) from the "*NC-frame*" there is the condition, $h(\beta,\theta,\Lambda) = \omega'^2\theta^2/(2-\omega_1^2\theta)^2 < 1$.

These conditions can be seen in the plots in Figure 3. The grey areas correspond to positive f and g and the white area to negative values for these functions. Therefore to meet the conditions we must take values that are simultaneously either in the grey area or white area. Adding the last condition limits the deformation for the model to regions I and III of Figure 4. We can see that even if Λ is negative, it is possible to have a positive effective cosmological constant and a small value for the deformation parameters θ and β.

In this paper we have proposed that in order to have a viable theory under phase space deformations there are two steps:

(1) use the deformed algebra equation (6);

(2) impose the condition: "*Deformed phase space models are only valid when the "NC" and "C" descriptions are equivalent*."

For the toy model we have studied we analysed the possible solutions under the minisuperspace deformation. We constructed the space of allowed values for the deformation parameters. Finally we have shown that imposing the new principle of the equivalence of the "*C*" and "*NC*" on the deformed phase space model the only valid solution is for $\Lambda_{\text{eff}} \geq 0$.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

This work is supported by CONACYT Grants 84798 and 135023. This work is part of the PROMEP research network "Gravitación y Física Matemática."

References

[1] E. J. Copeland, M. Sami, and S. Tsujikawa, "Dynamics of dark energy," *International Journal of Modern Physics D*, vol. 15, no. 11, pp. 1753–1935, 2006.

[2] B. Ratra and P. J. E. Peebles, "Cosmological consequences of a rolling homogeneous scalar field," *Physical Review D*, vol. 37, no. 12, pp. 3406–3427, 1988.

[3] L. P. Chimento and A. S. Jakubi, "Scalar field cosmologies with perfect fluid in Robertson-Walker metric," *International Journal of Modern Physics D*, vol. 5, no. 1, pp. 71–84, 1996.

[4] A. R. Liddle and L. A. Urena-Lopez, "Inflation, dark matter, and dark energy in the string landscape," *Physical Review Letters*, vol. 97, Article ID 161301, 2006.

[5] K. Bamba, S. Capozziello, S. Nojiri, and S. D. Odintsov, "Dark energy cosmology: the equivalent description via different theoretical models and cosmography tests," *Astrophysics and Space Science*, vol. 342, pp. 155–228, 2012.

[6] N. Seiberg and E. Witten, "String theory and noncommutative geometry," *Journal of High Energy Physics*, vol. 3, no. 9, article 032, 1999.

[7] A. Connes, M. R. Douglas, and A. Schwarz, "Noncommutative geometry and Matrix theory," *Journal of High Energy Physics*, vol. 2, article 003, 1998.

[8] M. R. Douglas and N. A. Nekrasov, "Noncommutative field theory," *Reviews of Modern Physics*, vol. 73, no. 4, pp. 977–1029, 2001.

[9] H. Garcia-Compean, O. Obregon, C. Ramirez, and M. Sabido, "Noncommutative self-dual gravity," *Physical Review D*, vol. 68, Article ID 044015, 2003.

[10] H. Garcia-Compean, O. Obregon, C. Ramirez, and M. Sabido, "Noncommutative topological theories of gravity," *Physical Review D*, vol. 68, Article ID 045010, 2003.

[11] P. Aschieri, M. Dimitrijević, F. Meyer, and J. Wess, "Noncommutative geometry and gravity," *Classical and Quantum Gravity*, vol. 23, no. 6, pp. 1883–1911, 2006.

[12] X. Calmet and A. Kobakhidze, "Noncommutative general relativity," *Physical Review D: Particles, Fields, Gravitation and Cosmology*, vol. 72, no. 4, Article ID 045010, pp. 1–5, 2005.

[13] L. Alvarez-Gaume, F. Meyer, and M. A. Vazquez-Mozo, "Comments on noncommutative gravity," *Nuclear Physics B*, vol. 753, pp. 92–117, 2006.

[14] S. Estrada-Jimenez, H. Garcia-Compean, O. Obregon, and C. Ramirez, "Twisted covariant noncommutative self-dual gravity," *Physical Review D*, vol. 78, Article ID 124008, 2008.

[15] J. Lukierski, H. Ruegg, A. Nowicki, and V. N. Tolstoy, "q-deformation of Poincaré algebra," *Physics Letters B: Nuclear, Elementary Particle and High-Energy Physics*, vol. 264, no. 3-4, pp. 331–338, 1991.

[16] S. Majid and H. Ruegg, "Bicrossproduct structure of κ-Poincare group and non-commutative geometry," *Physics Letters B: Nuclear, Elementary Particle and High-Energy Physics*, vol. 334, no. 3-4, pp. 348–354, 1994.

[17] T. Ohl and A. Schenkel, "Cosmological and black hole spacetimes in twisted noncommutative gravity," *Journal of High Energy Physics*, vol. 2009, no. 10, article 052, 2009.

[18] K. S. Gupta, E. Harikumar, T. Juric, S. Meljanac, and A. Samsarov, "Effects ofnoncommutativity on the black hole entropy," http://arxiv.org/abs/1312.5100 .

[19] H. Garcia-Compean, O. Obregon, and C. Ramirez, "Noncommutative quantum cosmology," *Physical Review Letters*, vol. 88, Article ID 161301, 2002.

[20] G. D. Barbosa and N. Pinto-Neto, "Noncommutative geometry and cosmology," *Physical Review D: Particles, Fields, Gravitation and Cosmology*, vol. 70, no. 10, Article ID 103512, 2004.

[21] L. O. Pimentel and C. Mora, "Noncommutative quantum cosmology," *General Relativity and Gravitation*, vol. 37, no. 4, pp. 817–821, 2005.

[22] L. O. Pimentel and O. Obregón, "Non commuting quantum cosmology with scalar matter," *General Relativity and Gravitation*, vol. 38, no. 4, pp. 553–560, 2006.

[23] M. Aguero, J. A. S. Aguilar, C. Ortiz, M. Sabido, and J. Socorro, "Noncommutative bianchi type II quantum cosmology," *International Journal of Theoretical Physics*, vol. 46, no. 11, pp. 2928–2934, 2007.

[24] W. Guzmán, C. Ortiz, M. Sabido, J. Socorro, and M. A. Agüero, "Non-commutative Bianchi quantum cosmology," *International Journal of Modern Physics D*, vol. 16, no. 10, pp. 1625–1632, 2007.

[25] B. Vakili, N. Khosravi, and H. R. Sepangi, "Bianchi spacetimes in noncommutative phase space," *Classical and Quantum Gravity*, vol. 24, no. 4, pp. 931–949, 2007.

[26] W. Guzman, M. Sabido, and J. Socorro, "Noncommutativity and scalar field cosmology," *Physical Review D*, vol. 76, Article ID 087302, 2007.

[27] G. D. Barbosa, "Noncommutative conformally coupled scalar field cosmology and its commutative counterpart," *Physical Review D*, vol. 71, Article ID 063511, 2005.

[28] B. Vakili, P. Pedram, and S. Jalalzadeh, "Late time acceleration in a deformed phase space model of dilaton cosmology," *Physics Letters B: Nuclear, Elementary Particle and High-Energy Physics*, vol. 687, no. 2-3, pp. 119–123, 2010.

[29] S. Perez-Payan, M. Sabido, and C. Yee-Romero, "Effects of deformed phase space on scalar field cosmology," *Physical Review D*, vol. 88, Article ID 027503, 2013.

[30] R. Cordero, H. Garcia-Compean, and F. J. Turrubiates, "Deformation quantization of cosmological models," *Physical Review D*, vol. 83, Article ID 125030, 2011.