Hindawi Publishing Corporation
Advances in High Energy Physics
Volume 2014, Article ID 390101, 7 pages
http://dx.doi.org/10.1155/2014/390101

Research Article

Slowly Rotating Black Holes with Nonlinear Electrodynamics

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Received 13 May 2014; Accepted 15 July 2014; Published 20 August 2014

Academic Editor: Rong-Gen Cai

We study charged slowly rotating black hole with a nonlinear electrodynamics (NED) in the presence of cosmological constant. Starting from the static solutions of Einstein-NED gravity as seed solutions, we use the angular momentum as the perturbative parameter to obtain slowly rotating black holes. We perform the perturbations up to the linear order for black holes in 4 dimensions. These solutions are asymptotically AdS and their horizon has spherical topology. We calculate the physical properties of these black holes and study their dependence on the rotation parameter a as well as the nonlinearity parameter β. In the limit $\beta \to \infty$, the solution describes slowly rotating AdS type black holes.

1. Introduction

The Reissner-Nordström (RN) solution is the static solution of the Einstein-Maxwell gravity. These solutions are asymptotically flat and their horizon has spherical topology. This black hole can be characterized by its mass and electric charge and the geometry of the RN black hole is singular at the origin of the radial coordinate. So, people were searching to construct a good regular alternative for it. For several decades a great deal of attention has been focused on some gravitating NED as regular generalizations of the RN solution of the Einstein-Maxwell gravity, in which the most popular example is the gravitating Born-Infeld (BI) theory [1–4]. On the other hand, theories such as the theory of black objects, with a logarithmic electrodynamic Lagrangian coupled to gravity, have been studied in an attempt to remove some of the singularities associated with a charged black hole [5]. The divergence of the energy-momentum tensor was successfully removed, although the spacetime still exhibited a curvature singularity, albeit of a weaker variety. Some other NED models supporting asymptotically Schwarzschild-like solutions have been also considered in the literature [5–7]. One can look for regular solutions with nonlinear electromagnetic fields of the Born-Infeld type [8–12].

On the other hand, exact solutions for charged static black holes with NED have been studied by many authors (see, e.g., [5, 11, 13–18]). This showed that the presence of the NED has important consequences for the black hole properties which motivate us to investigate charged black holes with NED, for instance, various limitations of the linear electrodynamics [19, 20], clarification of the self-interaction of virtual electron-positron pairs [21–23], and description of radiation propagation inside specific materials [24–27]. Moreover, from astrophysical point of view, one finds the effects of NED become indeed quite important in superstrongly magnetized compact objects, such as pulsars, and particular neutron stars (some examples include the so-called magnetars and strange quark magnetars) [28–30]. Also, NED modifies in a fundamental basis the concept of gravitational redshift and its dependency of any background magnetic field as compared to the well-established method introduced by standard general relativity. In addition, it was recently shown that NED objects can remove both of the big bang and black hole singularities [31–36]. Furthermore, it was shown that the nonlinearity may change the geometric properties of the black hole horizon(s). For example, unlike charged black holes in Einstein-Maxwell gravity, there is a new situation of black hole horizon in the presence of nonlinear electrodynamics (see [11] for more details). Finally we should note that although NED theories have complicated calculation, there is no objection against these theories and they may lead to new consequences in the future, as Plebanski in 1968 said: [37]: *"If in recent times the interest in NED cannot be said to be very popular, it is not due to the fact that one could rise some serious objections against this theory. It is simply rather difficult in its mathematical formulation, what causes that it is very unlikely to derive some concrete results in closed form."*

However, generalizing these static black holes to the rotating solutions is not easy due to the complexity of the equations. This forces us to restrict ourselves to the limit of slow rotation. Slowly charged rotating black hole solutions in higher dimensions with a single rotation parameter have been investigated by many authors [38–46]. Using perturbation theory, they have introduced the rotation parameter as a perturbative parameter and solved the equations of motion up to the linear order of the angular momentum parameter. Here, we use the same approach. We started from the static charged black holes with NED in antide Sitter (AdS) spacetime [11] and then considered the effect of adding a small angular momentum to the solutions. We discarded any terms involving a^2 or higher powers in a, where a is the rotation parameter. Finally we study the physical properties of these black holes. In particular, we have shown that the perturbative parameter a and the nonlinearity parameter β do not change the value of gyromagnetic ratio of these slowly rotating black holes.

The rest of the paper is organized as follows. In the next section, we give a brief review of the Einstein-Λ field equations in the presence of NED. In Section 3, we present 4-dimensional slowly rotating charged black hole solutions in the presence of two new classes of NED. Then, we obtain mass, electric charge, temperature, entropy, angular momentum, and gyromagnetic ratio of the solutions. We finish our paper with some concluding remarks.

2. Basic Field Equations

The Lagrangian of Einstein gravity in the presence of negative cosmological constant, Λ, with a NED is

$$£ = R - 2\Lambda + \mathscr{L}_{\mathrm{NED}}, \tag{1}$$

where R is the Ricci scalar and $\mathscr{L}_{\mathrm{NLED}}$ can be selected as a one of two new classes of NED [10–12], namely, Exponential form of NED (ENED) and Logarithmic form of NED (LNED), whose Lagrangians are

$$L(\mathscr{F}) = \begin{cases} \beta^2\left(\exp\left(-\dfrac{\mathscr{F}}{\beta^2}\right) - 1\right), & \text{ENED}, \\ -8\beta^2 \ln\left(1 + \dfrac{\mathscr{F}}{8\beta^2}\right), & \text{LNED}, \end{cases} \tag{2}$$

where β is the nonlinearity parameter and $\mathscr{F} = F_{\mu\nu}F^{\mu\nu}$ is Maxwell invariant. It is easy to show that in the limit $\beta \to \infty$ (weak field limit), the mentioned $L(\mathscr{F})$'s reduce to the Lagrangian of the standard Maxwell field

$$L(\mathscr{F}) \longrightarrow -\mathscr{F} + O\left(\mathscr{F}^2\right). \tag{3}$$

Using the Euler-Lagrange equation, we can obtain gravitational field equations as well as electromagnetic ones with the following explicit form:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{1}{2}g_{\mu\nu}L(\mathscr{F}) - 2L_{\mathscr{F}}F_{\mu\lambda}F_{\nu}^{\ \lambda}, \tag{4}$$

$$\partial_\mu\left(\sqrt{-g}L_{\mathscr{F}}F^{\mu\nu}\right) = 0, \tag{5}$$

where $L_{\mathscr{F}} = dL(\mathscr{F})/d\mathscr{F}$.

3. 4-Dimensional Slowly Rotating Charged Black Holes

Here, we look for the slowly rotating nonlinear charged black hole solutions. Inspection of the 4-dimensional Kerr and Kerr-Newman spacetimes shows that the only term in the metric that changes to the first order of the angular momentum parameter a is $g_{t\phi}$. In other words, using series expansion for slowly rotating Kerr-Newman spacetime, one obtains

$$\begin{aligned} ds^2 = & -G(r)\,dt^2 + \frac{dr^2}{G(r)} \\ & + 2aF(r)\,K(\theta)\,dt\,d\phi + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \end{aligned} \tag{6}$$

where the functions $G(r)$, $F(r)$, and $K(\theta)$ are known functions in Einstein-Maxwell gravity and a is a parameter associated with its angular momentum. In this paper, we take into account metric (6), to solve (4) and (5) to first order in the angular momentum parameter a. Using metric (6), one finds that the consistent one-form gauge potential is

$$A = h(r)\left[dt + aK(\theta)\,d\phi\right]. \tag{7}$$

After some algebraic calculations, we find that (5) leads to two independent differential equations. One of these equations is free of metric function with the following explicit form:

$$e_1 = \begin{cases} E'(r) + \dfrac{2E(r)}{r}\dfrac{\beta^2}{4E^2(r) + \beta^2} = 0, & \text{ENEF}, \\ E'(r) - \dfrac{2E(r)}{r}\dfrac{E^2(r) - 4\beta^2}{E^2(r) + 4\beta^2} = 0, & \text{LNEF} \end{cases} \tag{8}$$

and the following solution:

$$E(r) = \begin{cases} \dfrac{q}{r^2}\exp\left[\dfrac{-L_W}{2}\right], & \text{ENEF}, \\ \dfrac{2q}{r^2(\Gamma + 1)}, & \text{LNEF}, \end{cases} \tag{9}$$

where $E(r) = h'(r)$, prime denotes the first derivative with respect to r, $\Gamma = \sqrt{1 + (q^2/(\beta^2 r^4))}$, and q is an integration constant which is the electric charge of the black holes. In addition, $L_W = \mathrm{Lambert}W(4q^2/(\beta^2 r^4))$ which satisfies

$\mathrm{Lambert}W(x)\exp[\mathrm{Lambert}W(x)] = x$. (for more details, see [47, 48]). Now, we expand the function $E(r)$ for large r ($r \gg 1$) to investigate the asymptotic behavior of the electromagnetic field

$$E(r)|_{\mathrm{Larger}} = \frac{q}{r^2} - \frac{\chi q^3}{4\beta^2 r^6} + O\left(\frac{q^5}{\beta^4 r^{10}}\right), \tag{10}$$

where $\chi = 1$ and 8 for LNED and ENED, respectively. This equation shows that for large values of r, the dominant term of the nonlinear electromagnetic field is the same as that in 4-dimensional RN black holes.

Another independent differential equation of (5) depends on $h(r)$, $h'(r)$, $G(r)$, $F(r)$, and $K(\theta)$. In order to obtain consistent unknown functions, one should fix $K(\theta) = \cos(\theta)$. After simplification, one finds that second differential equation of (5) leads to

$$e_2 = \begin{cases} E'(r) + \dfrac{E(r)}{rG(r)} \times \dfrac{\beta^2\left\{r\left[G'(r)+F'(r)\right]-2F(r)\right\}}{4E^2(r)+\beta^2} = 0, & \text{ENEF,} \\ E'(r) - \dfrac{E(r)}{rG(r)} \times \dfrac{\left(E^2(r)-4\beta^2\right)\left\{r\left[G'(r)+F'(r)\right]-2F(r)\right\}}{E^2(r)+4\beta^2} = 0, & \text{LNEF.} \end{cases} \tag{11}$$

Having the electromagnetic field at hand, we are in a position to solve the gravitational field equation (4). After some cumbersome manipulation, one can show that the nonzero independent components of the field equation (4) for ENED and LNED branches are, respectively,

$$\begin{aligned} e_3 &= rG' + G - 1 + \Lambda r^2 + \frac{4\beta^2 r^2}{\chi}\left(\Delta + 2\omega\Delta_\omega\right) = 0, \\ e_4 &= rG'' + 2G' + 2\Lambda r + \frac{8r\beta^2\Delta}{\chi} = 0, \\ e_5 &= r^2 GF'' - 2F(G-1) - rFe_4 + 4r^2 GE^2\Delta_\omega = 0, \end{aligned} \tag{12}$$

with

$$\Delta = \begin{cases} 1 - e^{\omega}, & \text{ENEF,} \\ \ln(1-\omega), & \text{LNEF,} \end{cases} \tag{13}$$

where $\Delta_\omega = -d\Delta/d\omega$ and $\omega = \chi E^2/4\beta^2$.

Now, we are looking for the solutions which satisfy (12) (and also (11)) with the second equation of electromagnetic field (11), simultaneously. It leads to

$$G(r) = 1 - \frac{2M}{r} - \frac{\Lambda r^2}{3} + \begin{cases} -\dfrac{\beta^2 r^2}{6} + \dfrac{\beta q}{r}\left(\displaystyle\int \frac{dr}{\sqrt{L_W}} - \int \sqrt{L_W}\,dr\right), & \text{ENEF,} \\ \dfrac{8\beta^2 r^2}{3} - \dfrac{4q^2}{r}\displaystyle\int \frac{dr}{r^2(\Gamma-1)} - \dfrac{4\beta^2}{r}\int r^2 \ln\left(\frac{2}{\Gamma+1}\right)dr, & \text{LNEF,} \end{cases} \tag{14}$$

$$F(r) = -G(r), \tag{15}$$

where M is the integration constant which is related to the ADM (Arnowitt-Deser-Misner) mass of the black hole. One may note that in Maxwell limit, $\beta \to \infty$, these slowly rotating solutions reduce to the 4-dimensional Kerr-Newmann black holes with slow rotation.

At this point, it is worthwhile to investigate the causal structure and physical properties of these solutions. One can show that the Kretschmann scalar $R_{\mu\nu\lambda\kappa}R^{\mu\nu\lambda\kappa}$ diverge at $r = 0$ and it is finite for $r \neq 0$ and goes to $24/l^2$ as $r \to \infty$. So we find that there is an essential singularity at $r = 0$. Now, we look for the existence of horizons. The horizons, if any exist, are given by the zeros of the function $G(r) = (g_{rr})^{-1}$. Let us denote the largest positive root of $G(r) = 0$ by r_+. Regardless of naked singularity, the black hole covers with two horizons or an extreme horizon for ordinary β. Furthermore, considering (14), one can find that for small values of the nonlinearity parameter the Schwarzschild black hole with a nonextreme horizon may be recovered. In other words, one can show that for $r \to 0$, the function $G(r)$ may be positive or negative for $\beta > \beta_c$ or $\beta < \beta_c$, respectively. The new situation appears for $\beta < \beta_c$, in which the black holes have one nonextreme horizon with positive temperature as it happens for Schwarzschild black holes (see [11] for more details).

Moreover, we can obtain some information about causal structure by considering the temperature of the black hole. By using the definition of hawking temperature on the outer horizon r_+ which may be obtained through the definition of surface gravity

$$T_+ = \frac{1}{2\pi}\sqrt{-\frac{1}{2}\left(\nabla_\mu \chi_\nu\right)\left(\nabla^\mu \chi^\nu\right)}, \tag{16}$$

where the Killing vector χ is the null generator of the event horizon. After some calculations, we find

$$T_+ = \frac{G'(r)}{4\pi} + O\left(a^2\right) = \frac{1-\Lambda r_+^2}{4\pi r_+} + \begin{cases} \dfrac{\beta q\left(1-L_{W_+}\right)}{4\pi r_+ \sqrt{L_{W_+}}} - \dfrac{\beta^2 r_+}{8\pi}, & \text{ENEF} \\ \dfrac{q^2\left(2-\Gamma_+\right)}{\pi r_+^3 \Gamma_+\left(\Gamma_+-1\right)} + \dfrac{\beta^2 r_+}{\pi}\left(\ln\left(\dfrac{\Gamma_+^2-1}{2}\right) - \dfrac{2}{\Gamma_+}\right), & \text{LNEF} \end{cases} + O\left(a^2\right). \tag{17}$$

Now, we are in a position to investigate the effects of the nonlinearity parameter on the geometry of the solutions. It is interesting to mention that for the small values of the nonlinearity parameter, a new situation appears. Expanding the function $G(r)$ for large and small values of β, one can obtain

$$\begin{aligned} G(r) = {} & 1 - \frac{2M}{r} - \frac{\Lambda r^2}{3} + \frac{q^2}{r^2} - \frac{q^4}{5\beta^2 r^6} \\ & + O\left(\frac{q^6}{\beta^4 r^{10}}\right), \quad \text{for large } \beta \text{ or large } r. \end{aligned} \tag{18}$$

Equations (18) shows that for large values of the nonlinearity parameter one can recover the RN black hole, as it should be. In addition, this equation shows that the asymptotic behavior of the solutions is AdS.

In what follows we investigate the other conserved and thermodynamics quantities. The entropy of the black hole typically satisfies the so-called area law which states that the entropy of the black hole is a quarter of the event horizon area [49–55]. This near universal law applies to almost all kinds of black holes in Einstein gravity. Since the area of the event horizon does not change up to the linear order of the rotating parameter a, we can easily show that the entropy of the black hole on the outer event horizon r_+ can be written as

$$S = \pi r_+^2. \tag{19}$$

Next, we calculate the angular momentum, the electrical charge, and the gyromagnetic ratio of these rotating nonlinear charged black holes which appear in the limit of slow rotation parameter. The angular momentum of the black hole can be calculated through the use of the quasilocal formalism of the Brown and York [56]. According to the quasilocal formalism, the quantities can be constructed from the information that exists on the boundary of a gravitating system alone. Such quasilocal quantities will represent information about the spacetime contained within the system boundary, just like the Gauss's law. In our case the stress-energy tensor can be written as

$$T^{ab} = \frac{1}{8\pi}\left[\Theta^{ab} - \Theta\gamma^{ab}\right] \tag{20}$$

which is obtained by variation of the Lagrangian (1) with respect to γ_{ab}. To compute the angular momentum of the spacetime, one should choose a spacelike surface $\mathscr{B}$ in $\partial\mathscr{M}$ with metric σ_{ij} and write the boundary metric in ADM form

$$\gamma_{ab}dx^a dx^a = -N^2 dt^2 + \sigma_{ij}\left(d\varphi^i + V^i dt\right)\left(d\varphi^j + V^j dt\right), \tag{21}$$

where the coordinates φ^i are the angular variables parameterizing the hypersurface of constant r around the origin and N and V^i are the lapse and shift functions, respectively. When there is a Killing vector field ξ on the boundary, then the quasilocal conserved quantities associated with the stress-energy tensors of (20) can be written as

$$Q(\xi) = \int_{\mathscr{B}} d^{n-1}x\sqrt{\sigma}T_{ab}n^a\xi^b, \tag{22}$$

where σ is the determinant of the metric σ_{ij} and ξ and n^a are the Killing vector field and the unit normal vector on the boundary $\mathscr{B}$, respectively. For boundaries with timelike ($\xi = \partial/\partial t$) and rotational ($\varsigma = \partial/\partial\varphi$) Killing vector fields, one obtains the quasilocal mass and angular momentum

$$\mathscr{M} = \int_{\mathscr{B}} d^{n-1}x\sqrt{\sigma}T_{ab}n^a\xi^b = M, \tag{23}$$

$$J = \int_{\mathscr{B}} d^{n-1}x\sqrt{\sigma}T_{ab}n^a\varsigma^b = Ma, \tag{24}$$

provided the surface $\mathscr{B}$ contains the orbits of ς. For $a = 0$, the angular momentum vanishes, and therefore a is the rotational parameter of the nonlinear charged black hole. The quasilocal mass presented here is the same as ADM mass calculated by Abbott and Deser [57]. Combining (23) with (24) we get

$$J = \mathscr{M}a. \tag{25}$$

At last, we calculate the gyromagnetic ratio of these rotating nonlinear charged black holes. One of the important subjects about the 4-dimensional charged black hole in the Einstein gravity is that it can be assigned a gyromagnetic ratio $g = 2$ just like the electron in Dirac theory. Here we want to know how does the value of the gyromagnetic ratio change for slowly rotating nonlinear charged black holes in four dimensions. The magnetic dipole moment for this slowly rotating black hole is

$$\mu = qa. \tag{26}$$

Therefore, the gyromagnetic ratio is given by

$$g = \frac{2\mu\mathcal{M}}{qJ} = 2, \tag{27}$$

which is the gyromagnetic ratio of the 4-dimensional Kerr-Newman black holes. Since both of the angular momenta and the magnetic dipole momenta of these black holes first appear at the linear order in rotation parameter a, we have led to the conclusion that the value of the gyromagnetic ratio remains $g = 2$. Also, we find that the nonlinearity of electromagnetic field does not change the gyromagnetic ratio of the slowly rotating black hole.

4. Summary and Conclusion

In this paper, we have found a new stationary solution to gravity with NED in the presence of negative cosmological constant. This solution generalizes the corresponding static solution of [11] to include a small amount of angular momentum. Our strategy for obtaining these solutions was based on the perturbative method, where we solved the equations of motion up to the linear order in the perturbative parameter, which we chose proportional to the angular momentum. More precisely, we started from the nonrotating black hole solutions in 4 dimensions with NED [11] and considered the effect of adding a small angular momentum to the solutions. In the limit $\beta \rightarrow \infty$, the obtained solution reduces to the standard Einstein-Maxwell slowly rotating AdS black holes as expected. We calculated the mass M, electric charge Q, temperature T, entropy S, angular momentum J, and gyromagnetic ratio g which appear up to the linear order of the angular momentum parameter a. Interestingly enough, we found that β does not modify the value of the gyromagnetic ratio g of the slowly rotating black holes.

In closing, we recall that this paper only considers the slow rotation approximation. One can consider second order of the angular momentum parameter to obtain smooth and precise solutions. Arbitrarily fast rotating black hole solutions with a NED are more complicated, and we have not tackled this difficult problem here. In addition, one may think about nonsingular black hole solutions of these NED theories. Let us finally mention that we have only studied four-dimensional slowly rotating black hole solutions which can be extended to D-dimensional nonlinear black hole solutions. This extension will appear in a forthcoming publication.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

S. H. Hendi wishes to thank Shiraz University Research Council. M. Allahverdizadeh is supported by a FCT grant. This work has been supported financially by Center for Excellence in Astronomy & Astrophysics of Iran (CEAAI-RIAAM).

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