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Research Article

Quark-Antiquark Potential from a Deformed AdS/QCD

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In this work we calculate the static limit of the energy for a quark-antiquark pair from the Nambu-Goto action using a holographic approach with a deformed AdS space, with warp factor $\exp\{(\kappa z)^n/n\}$. From this energy we derive the Cornell potential for the quark-antiquark interaction. We also find a range of values for our parameters which fits exactly the Cornell potential parameters. In particular, setting the zero energy of the Cornell potential at 0.33 fermi, we find that $\kappa = 0.56$ GeV and $n = 1.3$.

1. Introduction

The quark-antiquark potential has been a very useful tool for the investigation of strong interactions and quark confinement. This potential can be used, for example, to analyze the transition between the confined and deconfined phases of matter (see, for instance, [1]).

Recently, efforts have been made to obtain the quark-antiquark potential [2–11] using the well-known AdS-CFT correspondence. For another approach using effective string theory, see, for instance, [12]. This correspondence was originally formulated as a mapping of correlation functions of a superconformal $\mathcal{N} = 4$ Yang-Mills theory defined on the boundary of the AdS space and a string theory living in its bulk. It works in such a way that a strongly coupled regime on the boundary theory is mapped into a weakly coupled one in the bulk [13–17].

However, since the original formulation of the correspondence is based on a conformal field theory, which has no characteristic scale, the confining behavior of the potential is not contemplated once confinement implies a typical length scale.

In order to describe both the confining and nonconfining behaviors, it becomes necessary to break the conformal invariance of the theory. There are various ways of doing so but we mention just two of them: the hardwall [18–24] and the softwall [25–28] models which break conformal invariance introducing a cutoff in the action. Inspired by [6], here we break the conformal invariance modifying the background metric instead of the bulk action. So the metric is given by

$$ds^2 = g_{mn} dX^m dX^n = \frac{R^2}{z^2} h(z) \left(dx^i dx_i + dz^2 \right), \qquad (1)$$

where R is the AdS radius, $m, n = 0, 1, 2, 3, z$, where z is the holographic coordinate while x^i with $i = 0, 1, 2, 3$ represents an Euclidean space in four dimensions. The warp factor that we consider here in this work is given by

$$h(z) = \exp\left\{ \frac{1}{n} (\kappa z)^n \right\}, \qquad (2)$$

in which κ has dimensions of inverse length and n is a dimensionless number. We will keep these constants arbitrary until Section 3, where we relate our results to phenomenology of the quark-antiquark potential. Note that, if we restrict $n = 2$, we reobtain the results of [6].

The main goal of this work is to calculate the energy configuration for a quark-antiquark pair from the Nambu-Goto action using a holographic approach within the deformed metric (1) with the warp factor given by (2). From this energy we will obtain the Cornell potential [29–33] (for excellent reviews of the Cornell potential see [34, 35]):

$$V(\ell) = -\frac{\xi}{\ell} + \frac{\ell}{a^2} + C, \qquad (3)$$

and also find a range of values for the parameters κ and n which describe $h(z)$ in order to fit this potential.

This work is organised as follows. In Section 2, using the warp factor $\exp\{(\kappa z)^n/n\}$, we compute the separation and the energy of the quark-antiquark pair using the Wilson loop from the AdS/CFT correspondence. In Section 3, we discuss the matching of our parameters κ and n to fit the Cornell potential. Finally, in Section 4, we present our comments and conclusions. We also include an appendix where we give some details of the calculation of the energy and the separation distance of the string.

2. The Wilson Loop and the Quark Potential

The starting point of our calculations involves the Wilson loop. For convenience we choose one circuit corresponding to a rectangular spacetime loop with temporal extension T and spatial extension ℓ in the association with the area of the string worldsheet that lives in the AdS space, whose boundary is just the flat spacetime in 4 dimensions where the loop is defined [2, 3].

So, following this prescription, we just have to calculate the Nambu-Goto action of a string with the endpoints (identified as the quark and antiquark) fixed at $z = 0$, assuming a "U-shape" equilibrium configuration in the bulk of deformed AdS.

Assuming also that the string configuration is, by hypothesis, static, i.e., it moves in the interior of the deformed AdS without change in its shape, one can show that the interquark separation and energy for the type of metric (1) are, respectively, given by (see the appendix for details)

$$\ell = 2\int_0^{z_0} \frac{z^2}{z_0^2}\frac{h(z_0)}{h(z)} \frac{1}{\sqrt{1-(h(z_0)/h(z))^2 (z^4/z_0^4)}} dz, \qquad (4)$$

$$E = \frac{1}{\pi\alpha'}\int_0^{z_0} \frac{R^2}{z^2} h(z) \frac{1}{\sqrt{1-(h(z_0)/h(z))^2 (z^4/z_0^4)}} dz. \qquad (5)$$

Note that z_0 is the minimum of the z coordinate and corresponds to the bottom of the U-shape curve.

The form of (4) and (5) is very convenient because it makes explicit that the expressions of energy and separation distance depend only on the warp factor chosen for the metric and the value of z_0.

It is useful to rewrite the integrals (4) and (5) in terms of a dimensionless variable. If we define $v := z/z_0$, the integrals become

$$\ell = 2z_0 \int_0^1 \frac{h(1)}{h(v)} v^2 \left[1 - \left(\frac{h(1)}{h(v)}\right)^2 v^4\right]^{-1/2} dv. \qquad (6)$$

$$E = \frac{R^2}{\pi\alpha'}\frac{1}{z_0}\int_0^1 h(v)\, v^{-2} \left[1 - \left(\frac{h(1)}{h(v)}\right)^2 v^4\right]^{-1/2} dv, \qquad (7)$$

which makes explicit the dimensions of ℓ and E since the integrals are now dimensionless, and where we identify $h(v) \equiv h(z)$. Note also that the ratio $R^2/\pi\alpha'$ is dimensionless.

Now we introduce the dimensionless parameter $\lambda := (\kappa z_0)^n$ such that (6) and (7) become

$$\ell = 2\frac{\lambda^{1/n}}{\kappa}\int_0^1 v^2 e^{(\lambda/n)(1-v^n)} \left(1 - e^{(2\lambda/n)(1-v^n)} v^4\right)^{-1/2} dv, \qquad (8)$$

$$E = \frac{R^2}{\pi\alpha'}\frac{\kappa}{\lambda^{1/n}}\int_0^1 e^{(\lambda/n)v^n} v^{-2} \left(1 - e^{(2\lambda/n)(1-v^n)} v^4\right)^{-1/2} dv, \qquad (9)$$

where κ has the dimension of energy. Let us analyze the above expressions when $\lambda \approx 0$ and $\lambda \approx 2$, which are the interesting physical limits since for $\lambda \longrightarrow 0$ one has $\ell \longrightarrow 0$, while for $\lambda \longrightarrow 2$ one has $\ell \longrightarrow \infty$, as we are going to discuss below.

2.1. Calculation of ℓ

2.1.1. λ Close to Zero. If we express the integrand in (8) as a power series in λ centered at zero, to first order in λ, and integrate it, we obtain

$$I(\lambda, n) = -\frac{\sqrt{\pi}}{2n}\left(\frac{\Gamma(3/4)(\lambda-2n)}{\Gamma(1/4)} - \frac{\lambda\Gamma((n+3)/4)}{\Gamma((n+1)/4)}\right), \qquad (10)$$

where the above result is valid only if $n > -3$; otherwise the integral does not converge.

Substituting this result in (8) and grouping terms proportional to λ, one finds

$$\ell = \frac{1}{\rho_0}\frac{\lambda^{1/n}}{\kappa}\left\{1 - \frac{\lambda}{2n}\left[1 - F(n)\pi\rho_0\right]\right\} + \mathcal{O}(\lambda^2); \quad (\lambda \approx 0). \qquad (11)$$

Here we have defined the dimensionless number $1/\rho_0 := (2\pi)^{3/2}/\Gamma^2(1/4)$ and function $F(n) := (2/\sqrt{\pi})(\Gamma((3+n)/4)/\Gamma((1+n)/4))$.

2.1.2. λ Close to 2. If we repeat the procedure of last subsection for λ now centered at 2, we will not be able to achieve an analytic expression for the integral. We note however that the integral of (8) is dominated by $v \sim 1$. We thus expand the integrand around $v = 1$ to first order and integrate it, obtaining the following.

$$\begin{aligned} I(\lambda, n) &= \left(\frac{1}{\sqrt{\lambda(2\lambda+n-9)+10}}\right) \\ &\cdot \left[-\log(-2(\lambda-2)[\lambda(2\lambda+n-9)+10]) + 2\right. \\ &\cdot \log\left(\lambda(2\lambda+n-9)\right. \\ &+ \sqrt{[\lambda(2\lambda+n-11)+14][\lambda(2\lambda+n-9)+10]} \\ &\left.\left.+ 10\right)\right] \end{aligned} \qquad (12)$$

As the first logarithm of (12) diverges when $\lambda = 2$, one would expand again around $\lambda = 2$ up to first order. However, since terms of order $\mathcal{O}(1)$ in the expansion will not contribute to the functional form of the Cornell potential and we are extracting just the leading behavior of (8) for $\lambda \approx 2$, we can safely neglect contributions of order $\mathcal{O}(\lambda)$ in the aforementioned expansions, obtaining

$$I(\lambda, n) = -\frac{1}{\sqrt{2n}} \log(2-\lambda) + \mathcal{O}(1), \tag{13}$$

which, due to (8), leads to

$$\ell = \frac{2^{1/n}}{\kappa}\left[-\sqrt{\frac{2}{n}}\log(2-\lambda) + \mathcal{O}(1)\right]; \quad (\lambda \approx 2). \tag{14}$$

As mentioned above, the limit $\lambda \longrightarrow 2$ implies $\ell \longrightarrow \infty$.

2.2. Calculation of the Energy. Before we calculate the integral in (9), let us point out that it diverges as $1/v^2$ when $v \longrightarrow 0$. This becomes clear if one analyzes the series expansion of the integrand in λ close to 0 and 2.

So, we choose the renormalization of (9) as

$$\begin{aligned} E_{\text{Ren.}} = \frac{R^2}{\pi\alpha'}\frac{\kappa}{\lambda^{1/n}}\Bigg\{&-1 \\ &+ \int_0^1 e^{(\lambda/n)v^n} v^{-2}\left[\left(1 - e^{(2\lambda/n)(1-v^n)}v^4\right)^{-1/2} - 1\right] dv\Bigg\}, \end{aligned} \tag{15}$$

such that this energy expression is finite and now we can analyze again the limits of λ close to 0 and 2.

2.2.1. λ Close to Zero. Expanding the integrand in (15) with respect to λ, centered at zero, we find

$$\begin{aligned} I(\lambda, n) = 1 &- \frac{1}{2\rho_0} \\ &+ \left[\frac{\sqrt{\pi}(n+1)\Gamma((n-1)/4)}{8n\Gamma((n+1)/4)} - \frac{1}{4n}\frac{1}{\rho_0}\right]\lambda, \end{aligned} \tag{16}$$

so that the renormalized energy is

$$\begin{aligned} E_{\text{Ren.}} = &-\frac{R^2}{\pi\alpha'}\frac{1}{2\rho_0} \\ &\cdot \frac{\kappa}{\lambda^{1/n}}\left\{1 + \frac{\lambda}{2n}\left[1 - G(n)\pi\rho_0\right] + \mathcal{O}\left(\lambda^2\right)\right\}, \end{aligned} \tag{17}$$

where we defined the dimensionless function $G(n) = (n+1)\Gamma((n-1)/4)/2\sqrt{\pi}\Gamma((n+1)/4)$.

Writing the prefactor $\kappa/\lambda^{1/n}$ as a function of ℓ (c.f. (11)), substituting in (17), and keeping only linear terms in λ, we get

$$E_{\text{Ren.}} = \frac{R^2}{\pi\alpha'}\left\{-\frac{\xi_0}{\ell} + \frac{\lambda}{4\ell}\left[\frac{G(n)-F(n)}{\rho_0 n}\right] + \mathcal{O}\left(\lambda^2\right)\right\}, \tag{18}$$

where we defined the dimensionless number $\xi_0 \coloneqq 1/(2\rho_0^2)$. Using (11) we can rewrite $\lambda \approx 0$ in terms of ρ_0 and κ:

$$\lambda \approx (\kappa\ell\rho_0)^n\left[1 + \frac{\lambda}{2}\left(1 - F(n)\pi\rho_0\right)\right]. \tag{19}$$

Substituting this result in (18) and keeping in mind that $\lambda \approx 0$ is equivalent to the regime of short distances, one can safely disregard terms proportional to ℓ^{2n-1} in comparison with the terms proportional to ℓ^{n-1}. Then, we obtain

$$E_{\text{Ren.}} = \frac{R^2}{\pi\alpha'}\left\{-\frac{\xi_0}{\ell} + \sigma_0(n)\ell^{n-1} + \mathcal{O}\left(\ell^{2n-1}\right)\right\}, \tag{20}$$

where we defined the function $\sigma_0(n) \coloneqq (1/4)\kappa^n\rho_0^{n-1}[(G(n) - F(n))/n]$, with dimensions of (energy)^n.

2.2.2. λ Close to 2. In this section we are going to calculate the renormalized energy for λ close to 2. Repeating the procedure employed in Section 2.1.2, i.e., rewriting all the integrand in (15) inside the square root

$$I(\lambda, v, n) = \left[v^4 e^{-2\lambda v^n/n}\left(1 - v^4 e^{2\lambda(1-v^n)/n}\right)\right]^{-1/2} - v^{-2}, \tag{21}$$

and expanding this integrand with respect to v centered at 1, to second order we find

$$\begin{aligned} I(\lambda, v, n) = &\ 2(2-\lambda)(1-v)e^{-2\lambda/n} \\ &- e^{-2\lambda/n}\left(6\lambda^2 - \lambda n - 23\lambda + 22\right)(1-v)^2 \\ &+ 3(1-v)^2 + 2(1-v). \end{aligned} \tag{22}$$

For the above expression to be real, the first two terms must be positive and the last one must be negative which implies, respectively, that $\lambda < 2$ and $(n+23)/12 - (1/12)\sqrt{n^2+46n+1} < \lambda < (1/12)\sqrt{n^2+46n+1} + (n+23)/12$. Now, integrating (22), one has

$$\begin{aligned} I(\lambda, n) = &-3 - \frac{\log(4-2\lambda)}{\sqrt{e^{-2\lambda/n}\left(-6\lambda^2 + \lambda(n+23) - 22\right)}} \\ &+ \frac{2\log\left[\sqrt{\lambda(-6\lambda+n+21) - 18} + \sqrt{\lambda(-6\lambda+n+23) - 22}\right]}{\sqrt{e^{-2\lambda/n}\left[-6\lambda^2 + \lambda(n+23) - 22\right]}}. \end{aligned} \tag{23}$$

Keeping only terms in lowest order of λ and substituting $\lambda = 2$ in the denominator of above expression, we get from (15)

$$\begin{aligned} E_{\text{Ren.}} &= \frac{R^2}{\pi\alpha'}\frac{\kappa}{2^{1/n}}\left\{-\frac{e^{2/n}\log(2-\lambda)}{\sqrt{2n}} + \mathcal{O}(1)\right\} \\ &= \frac{R^2}{\pi\alpha'}\left[\sigma(n)\ell + \mathcal{O}(1)\right], \end{aligned} \tag{24}$$

where we have used the relation between ℓ and λ given by (14) and defined $\sigma(n) \coloneqq (1/2)\kappa^2(e/2)^{2/n}$.

3. Phenomenology

Summarizing the results of the last section, the renormalized energies (20) and (24) in terms of the separation ℓ are given by

$$E^{\lambda\approx 0}_{\text{Ren.}} = \frac{R^2}{\pi\alpha'}\left\{-\frac{\xi_0}{\ell} + \sigma_0(n)\ell^{n-1} + \mathcal{O}\left(\ell^{2n-1}\right)\right\}, \tag{25}$$

$$E^{\lambda\approx 2}_{\text{Ren.}} = \frac{R^2}{\pi\alpha'}\left[\sigma(n)\ell + \mathcal{O}(1)\right], \tag{26}$$

with

$$\xi_0 := \frac{1}{2\rho_0^2};$$
$$\frac{1}{\rho_0} := \frac{(2\pi)^{3/2}}{\Gamma^2(1/4)}; \quad (27)$$
$$\sigma(n) := \frac{1}{2}\kappa^2\left(\frac{e}{2}\right)^{2/n}.$$

The precise definition of $\sigma_0(n)$, given after (20), will not be needed here since in this section we are going to disregard the term proportional to ℓ^{n-1} in comparison with the term of order ℓ^{-1}, once $n > 0$ and in (25) $\ell \approx 0$.

Now we are going to fit the constants of our model with the phenomenological constants of the Cornell potential (3) with $\xi = 0.52$ and $a = 2.34\text{GeV}^{-1}$ [29–33] (for excellent reviews of the Cornell potential see [34, 35]).

First of all, we fix the dimensionless ratio $R^2/\pi\alpha'$ from the slope of the linear potential at long distances, where the stringy picture is more reliable. Since this regime is equivalent to $\lambda \approx 2$, we compare (3) with (26), which leads to the condition $1/a^2 = R^2\sigma(n)/\pi\alpha'$ and therefore

$$\frac{R^2}{\pi\alpha'} = \frac{2}{(a\kappa)^2} \times \left(\frac{2}{e}\right)^{2/n}. \quad (28)$$

Next, we compare the expression (25) with (3), finding $R^2/\pi\alpha' = \xi/\xi_0$, so that eliminating the ratio $R^2/\pi\alpha'$, one obtains

$$\xi = \frac{1}{(2.34\kappa)^2} \times \left(\frac{2}{e}\right)^{2/n} \frac{1}{\rho_0^2}. \quad (29)$$

The above equation can be solved graphically for given values of κ: we present some of these solutions in Figure 1, for the interval $(0.55 \leq \kappa \leq 0.70)$ GeV.

With the values of parameter κ and its corresponding values of n we can investigate the energy associated with the quark-antiquark pair through numerical calculations. In Figure 2 we plot the quark-antiquark potential $E_{Ren.}$ in terms of the quark separation ℓ, for some values of κ.

If we fix the constant C in the Cornell potential (3) to be zero, we can obtain a phenomenological constraint such that $V(\ell) = 0$ occurs for $\ell \approx 0.33$ fermi. Then, for our warp factor such behavior is achieved for $\kappa = 0.56$ GeV and $n = 1.3$, which corresponds approximately to the red dashed line in Figure 2.

Note also that in Figure 2, for the linear confining behavior all the curves shown present the same slope. This is not a universal property of the deformation we have considered but rather is a choice to fit the Cornell potential parameters.

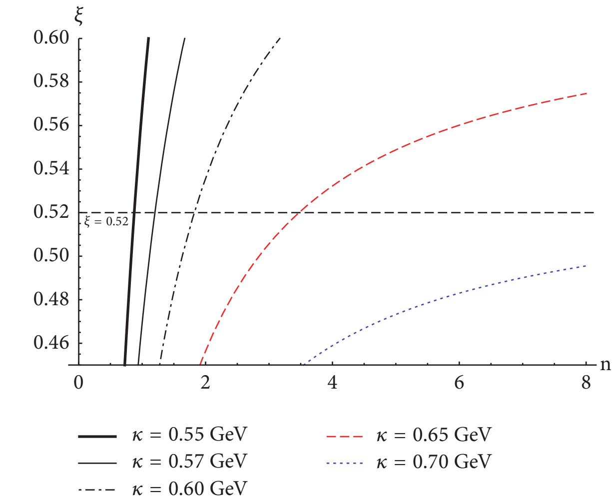

FIGURE 1: Equation (29) solved graphically: The curves are plots of (29) for some values of κ with $a = 2.34$ GeV^{-1}. The horizontal dashed line represents the phenomenological desired value of the parameter ξ, i.e., $\xi = 0.52$ to fit the Cornell potential.

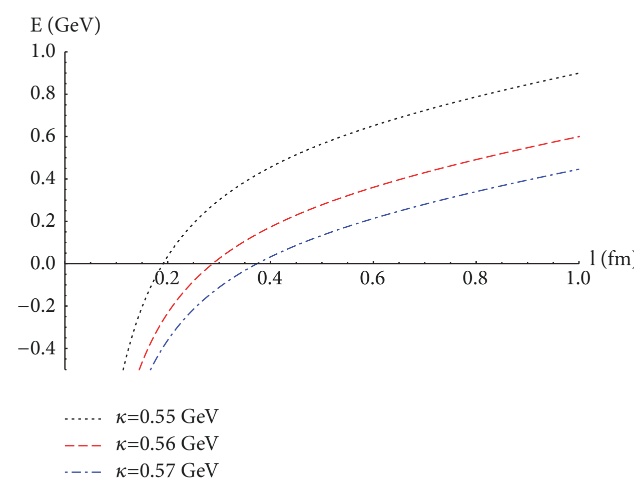

FIGURE 2: $E_{Ren.}$ against ℓ obtained directly from (8) and (15) through numerical integration, for three particular values of κ: 0.55 GeV, 0.56 GeV, 0.57 GeV and their respective approximate values of n: 1.2, 1.3, 1.4. These curves correspond to possible matches with the Cornell potential. The values n come from Figure 1 for each curve corresponding to a given κ.

4. Concluding Remarks

In this work we have calculated the energy corresponding to a given separation between a quark-antiquark pair from the Nambu-Goto action using a deformed AdS space as a background. The choice of the deformed AdS space is based on the introduction of an exponential factor given by $h(z) = \exp\{(\kappa z)^n/n\}$, (2). We have also shown that this configuration energy has the shape of a Cornell potential. In order to fit the Cornell potential parameters we can choose a variety of possibilities for the pair (κ, n). In Figure 1, we have shown some of these possibilities, and in Figure 2, we presented some profiles for the Cornell potential. Note that in Figure 2 we observe the transition from a confining to a nonconfining regime around $\ell \sim 0.3$ fm. Specifically, for the choice $\kappa = 0.56$ GeV and $n = 1.3$, we matched the Cornell potential with the condition $C = 0$, as represented by the red dashed line.

Another interesting feature of our model is the universal nonconfining behavior for $\ell \approx 0$, already pointed out by [2]. In the context of our model, this universal behavior for short distances is due to the fact that $\ell \approx 0$ is equivalent to $\lambda \approx 0$ (c.f. Section 2.1.1) which means that $h(z) \longrightarrow 1$, hence we recover the geometry of pure AdS space, and therefore we must obtain the nonconfining term due to the conformal symmetry of the background space.

Also one can see that our deformation of the AdS, which is a UV deformation, is the fact that it only affects the large distance physics. This modification is encapsulated by the coefficients of the linear term in (26), which become dependent on the deformation $h(z)$, where κ and n are the parameters that control the deformation. It is interesting that the confining behavior is maintained despite the choice of κ and n which is actually an explicit manifestation of the criterion discussed by [3].

Appendix

We start this appendix following [3], defining a metric given by

$$ds^2 = -G_{00}(s)\,dt^2 + G_{x_\parallel x_\parallel} dx_\parallel^2 + G_{ss} ds^2 + G_{x_T x_T} dx_T^2, \quad (A.1)$$

and the Nambu-Goto action:

$$S = \frac{1}{2\pi\alpha'} \int d\sigma d\tau \sqrt{\det\left[\partial_\alpha X^M \partial_\beta X^N G_{MN}\right]}. \quad (A.2)$$

Choosing the gauge $\sigma = x$ and $\tau = t$ and integrating with respect to t, one gets

$$S = \frac{T}{2\pi\alpha'} \int dx \sqrt{G_{00}(s(x))\,G_{x_\parallel x_\parallel}(s(x)) + G_{00}(s(x))\,G_{ss}(s(x))\,(\partial_x s)^2}. \quad (A.3)$$

Here T is the temporal extension of the Wilson loop.

Then, we define [3]

$$f^2(s(x)) = G_{00}(s(x))\,G_{x_\parallel x_\parallel}(s(x)) \quad (A.4)$$

$$g^2(s(x)) = G_{00}(s(x))\,G_{ss}(s(x)) \quad (A.5)$$

so that we are left with the integral

$$S = \frac{T}{2\pi\alpha'} \int dx \sqrt{f^2(s(x)) + g^2(s(x))\,(\partial_x s)^2}. \quad (A.6)$$

Using the differential equation for the geodesic of the string in its equilibrium configuration, we get that the separation of the endpoints (or, in our perspective, the quark and antiquark distance) is given by

$$\begin{aligned} l &= \int dx = \int \left(\frac{ds}{dx}\right)^{-1} ds \\ &= 2\int_{s_0}^{s_1} \frac{g(s)}{f(s)} \frac{f(s_0)}{\sqrt{f^2(s) - f^2(s_0)}} ds, \end{aligned} \quad (A.7)$$

where s_0 and s_1 are, respectively, de equilibrium position of the bottom of the string and the position of its endpoints.

Since the action has dimensions of energy × time, the energy of the configuration associated with the string will be given by (A.6)

$$E = \frac{1}{\pi\alpha'} \int_{s_0}^{s_1} \frac{g(s)}{f(s)} \frac{f^2(s)}{\sqrt{f^2(s) - f^2(s_0)}} ds. \quad (A.8)$$

Performing the change of variable $s = R^2/z$, where R is the AdS radius, we have $z_0 = R^2/s_0$, $z_1 = R^2/s_1$, and $ds = -(R^2/z^2)dz$. We take the limit $s_1 \longrightarrow \infty$, which means that $z_1 \longrightarrow 0$ and we can rewrite (A.7) and (A.8) as

$$l = 2\int_0^{z_0} \frac{g(z)}{f(z)} \frac{f(z_0)}{\sqrt{f^2(z) - f^2(z_0)}} \frac{R^2}{z^2} dz \quad (A.9)$$

$$E = \frac{1}{\pi\alpha'} \int_0^{z_0} \frac{g(z)}{f(z)} \frac{f^2(z)}{\sqrt{f^2(z) - f^2(z_0)}} \frac{R^2}{z^2} dz. \quad (A.10)$$

Using metric (1) we have $f(z) = h(z)(R^2/z^2)$ and $g(s) = h(z)$. Then, from (A.9) and (A.10), one gets (4) and (5).

Data Availability

We declare that all data used in the article is available upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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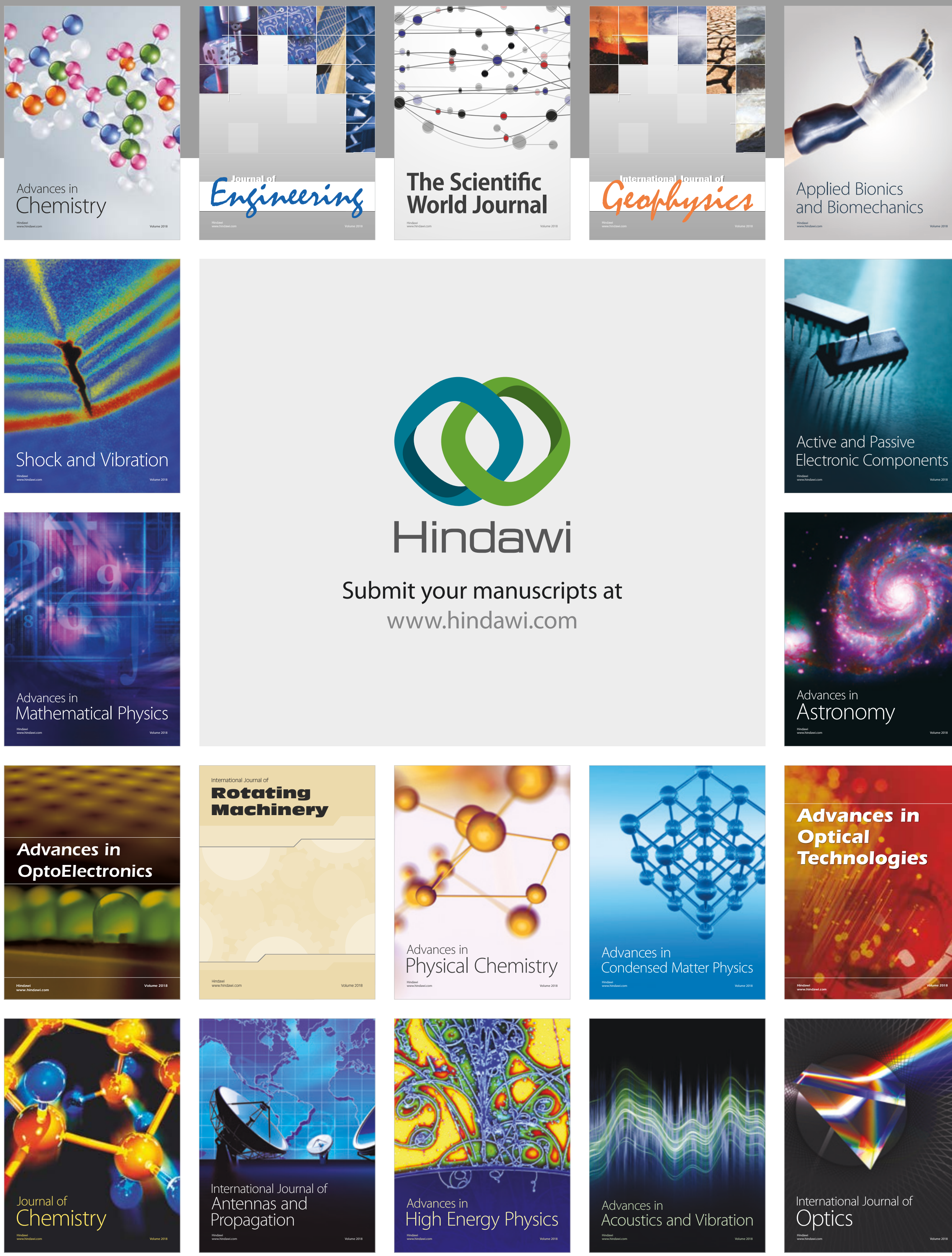

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