Potential of octant degeneracy resolution in JUNO*

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Abstract: This work extends the idea of using a cyclotron-based antineutrino source for purposes of neutrino physics. Long baseline experiments suffer from degeneracies and correlations between Θ_{23}, $\delta_{\rm CP}$ and the mass hierarchy. However, the combination of a superconducting cyclotron and a big liquid scintillator detector like JUNO in a medium baseline experiment, which does not depend on the mass hierarchy, may allow to determine whether the position of the mixing angle Θ_{23} is in the lower octant or the upper octant. Such an experiment would improve the precision of the Θ_{23} measurement to a degree which depends on the CP-phase.

Keywords: neutrino oscillations, octant, mixing angle, JUNO

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1 Problem of octant degeneracy

In the framework of 3-flavor neutrino mixing through Pontecorvo-Maki-Nakagawa-Sakata [1] unitary mixing matrix:

$$U_{\rm PMNS}=\begin{pmatrix} U_{\rm e1} & U_{\rm e2} & U_{\rm e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}, \tag{1}$$

$\sin^2(\Theta_{23})$ and $\cos^2(\Theta_{23})$ can be expressed in the standard parametrization as:

$$\sin^2(\Theta_{23})=\frac{|U_{\mu 3}|^2}{1-|U_{\rm e3}|^2},\ \cos^2(\Theta_{23})=\frac{|U_{\tau 3}|^2}{1-|U_{\rm e3}|^2}.$$

It is clear that if Θ_{23}=45°, then mixing between ν_μ and ν_τ becomes maximal. This would indicate symmetry between the $\nu_e\to\nu_\mu$ and $\nu_e\to\nu_\tau$ oscillation processes. The octant problem refers to the degeneracy between Θ_{23} and $\pi/2-\Theta_{23}$, when the mixing angle enters in the oscillation probability as a term within $\sin(2\Theta_{23})$. However, the degeneracy between the lower octant (LO) and the upper octant (UO) can be eliminated if a measurement is sensitive to terms with $\sin(\Theta_{23})$ or $\cos(\Theta_{23})$. Until recently, there was a quite large uncertainty in the measurements of $\sin^2(\Theta_{23})$: $\sin^2(\Theta_{23})=0.35-0.65$ (90%C.L.) for normal hierarchy (NH), and $\sin^2(\Theta_{23})=0.34-0.67$ (90%C.L.) for inverted hierarchy (IH), from the combined analysis of the MINOS experiment [2]. T2K reported the best fit value of $\sin^2(\Theta_{23})=0.532$ (NH) and $\sin^2(\Theta_{23})=0.534$ (IH) with smaller uncertainty and consistent with hypothesis of maximal mixing [3]. Recent data from the NOνA experiment favors Θ_{23} in either LO or UO, and disfavors maximal mixing at 0.8σ significance [4].

Since the leading approximation of oscillation probability for reactor experiments does not depend on the mixing angle Θ_{23}, the current scientific program of JUNO [5] will not allow for a solution to the problem of octant degeneracy. However, precise measurements of $\bar{\nu}_e$ appearance from $\bar{\nu}_\mu$ disappearance could provide a possibility to partially resolve this degeneracy.

2 Methodology of the numerical analysis

2.1 Proposal of the experimental setup

The full description of our proposal is presented in [6], which is based on the DAEδALUS experiment project [7]. It is worthwhile to summarize the main as-

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pects of this proposal. We suggest using the appearance channel for electron antineutrinos from muon antineutrinos. In the framework of standard three neutrino mixing theory the oscillation probability without matter effect can be expressed as [8]:

$$P(\bar{\nu}_\mu \to \bar{\nu}_e) = \sin^2\theta_{23}\sin^2 2\theta_{13}\sin^2\Delta_{31} + \cos^2\theta_{23}\sin^2 2\theta_{12}\sin^2\Delta_{21} + \sin 2\theta_{13}\sin 2\theta_{23}\sin 2\theta_{12}\sin\Delta_{31}\sin\Delta_{21}\cdot\cos(\Delta_{31}-\delta_{\rm CP}), \tag{2}$$

where $\Delta_{ij} = \Delta m^2_{ij}\cdot L/(4E_\nu)$; Δm^2_{ij} – the neutrino mass squared difference; L – the distance between source and detector; E_ν – neutrino energy; $\delta_{\rm CP}$ – Dirac phase of CP violation. The source of $\bar{\nu}_\mu$ is a three-body decay of μ^+ from decay at rest of stopped π^+, which are produced by a superconducting cyclotron [9]. The contribution to electron antineutrino spectrum is around 10^{-4} from π^-, which are created together with π^+ [7]. Two cyclotrons (near and far) are located at distances of 1.5 km and 20 km, respectively. The power of the near cyclotron is 1 MW. It is needed as a flux monitor. There are two options for the power of the far cyclotron: 5 MW and 10 MW. We are planning to use JUNO as a liquid scintillator detector, which has a total mass of 20 kt. The expected exposure time of the experiment is 10 years. NH is assumed, because at a distance of 20 km the experiment is insensitive to mass hierarchy.

The estimated IBD-event spectrum as a function of energy is depicted in Fig. 1. It is clear that the neutrino rate increases with mixing angle Θ_{23}.

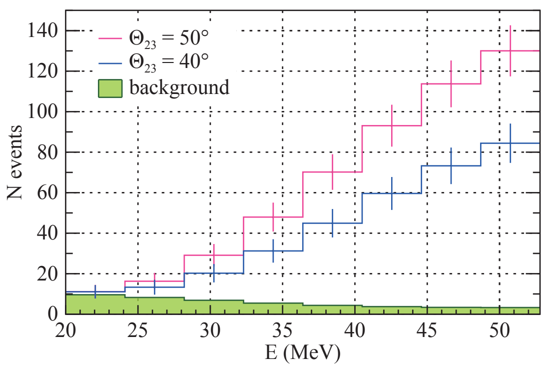

Fig. 1. (color online) The shape of the IBD-event spectrum as a function of energy for two values of Θ_{23} (we assume a power of 10 MW of the far cyclotron, 200 kt·year exposure time, $\delta_{\rm CP} = -\pi/2$). The green area shows the background.

2.2 Statistical evaluation of MC simulations

Event rate analysis is based on statistical treatment of the expected IBD signal rate inside the detector. Initial muon antineutrinos have a continuous spectrum with an endpoint of 52.8 MeV. In order to exclude a significant part of the atmospheric background, we chose an energy window between 20 and 52.8 MeV. However, this is not sufficient to disregard the background completely.

The current statistical analysis is devided in two parts. The first part concerns the sensitivity to octant degeneracy; the second part is about the precise measurement of Θ_{23}.

2.2.1 Sensitivity to discovery of true octant

We follow the so-called classical method of calculating a confidence level. This method is based on the calculation of a $\Delta\chi^2$ function, which, as Wilks's theorem predicts [10], should follow a chi-square distribution. The number of degrees of freedom can be calculated as the difference between the degrees of freedom of initial chi-square functions. Usually, this number is equal to the number of estimating parameters. In our case, there is only one parameter – Θ_{23}.

A χ^2 distribution with one degree of freedom has the same distribution as the square of a single normally distributed variable [11]. Therefore, standard Gaussian confidence levels 1σ (68.3%), 2σ (95.4%), 3σ (99.7%) etc. correspond to values of χ^2: 1, 4, 9 etc.

In general, the sensitivity to octant degeneracy can be calculated by minimization of a $\Delta\chi^2$ function, which is given by:

$$\Delta\chi^2 = |\chi^2_{\rm min}(90^\circ - \Theta_{23}) - \chi^2_{\rm min}(\Theta_{23})|, \tag{3}$$

where "min" means that both chi-square functions $\chi^2(90^\circ - \Theta_{23})$ and $\chi^2(\Theta_{23})$ have to be minimized in their parameter spaces; Θ_{23} is a scanning parameter, which is fixed for each iteration of an MC cycle. In our case, the chi-square function has only one minimum, which is close to the test-true value of Θ_{23}. In the opposite octant this function always increases. Consequently we need to redefine the $\Delta\chi^2$ function as:

$$\Delta\chi^2 = |\chi^2_{\rm min}(45^\circ) - \chi^2_{\rm min}(\Theta_{23})|, \tag{4}$$

where 45° corresponds to a border between two octants.

We use the chi-square function presented in [12, 13].

$$\chi^2(\Theta_{23}) = \chi^2_{\rm pull} + \chi^2_{\rm prior}, \tag{5}$$

where the pull-term includes Poisson statistics, and takes into account the background and flux normalization. Additional Gaussian penalties are also added.

$$\chi^2_{\rm pull} = 2\sum_{i=1}^{N_b}\left[\mu_i - n_i + n_i\cdot\ln\frac{n_i}{\mu_i}\right] + \frac{s^2}{\sigma_s^2} + \frac{b^2}{\sigma_b^2}. \tag{6}$$

Here, N_b – is the total number of bins in the histogram; μ_i – predicted counts in the i-th bin; n_i – observed counts in the i-th bin; s and b – so-called nuisance parameters for signal and background, respectively; σ_s and σ_b – systematic errors for signal and background counts. μ_i is given by :

$$\mu_i = N^i_s\cdot(1+s) + N^i_{bkg}\cdot(1+b),$$

where N^i_s and N^i_{bkg} are the number of counts in the i-th bin for signal and background, respectively. The prior-term in equation (5) corresponds to uncertainties of oscillation parameters and can be written as:

$$\chi^2_{\text{prior}} = \sum_{j=1}^{N_p} \frac{(\eta_j - \eta_j^o)^2}{(\delta\eta_j)^2}, \tag{7}$$

where N_p – is the number of oscillation parameters; η_j – j-th oscillation parameter; η_j^o – best fit value of η_j; $\delta\eta_j$ – one sigma error of η_j^o.

2.2.2 The accuracy of Θ_{23} measurement

The estimation of the accuracy of measurement for the current best fit value of Θ_{23} can be obtained by minimizing the chi-square function (5) in the whole parameter space. It should be emphasized that from recent experimental data the best fit value of Θ_{23} is split between LO and UO [14]. Consequently, we use two values of Θ_{23} in the precision calculations.

Further, we give a set of oscillation parameters and their uncertainties taken from PDG in Table 1.

Table 1 The list of oscillation parameters and their uncertainties from PDG [14]. Most are used in the prior-term of the chi-square function in our calculations, except the parameter of interest – Θ_{23}. The normal hierarchy is assumed.

η_j	$\Delta m^2_{21}\cdot 10^{-5}/\text{eV}^2$	$\Delta m^2_{32}\cdot 10^{-3}/\text{eV}^2$	$\sin^2(\Theta_{12})$	$\sin^2(\Theta_{23})$	$\sin^2(\Theta_{13})\cdot 10^{-2}$
η_j^o	7.53	2.51	0.307	0.597(UO) 0.417(LO)	2.12
$\delta\eta_j$	0.18	0.05	0.013	0.026	0.08

2.2.3 Monte-Carlo simulations

The expected electron antineutrino event spectra at a distance of 20 km were simulated using the Monte-Carlo method including oscillations. The energy resolution of the JUNO detector is 3% per MeV. The beam power of the far cyclotron is 5 or 10 MW with systematic flux uncertainty σ_s=2%, which includes the uncertainties of shape and normalization. We treat neutral current events (NC) as background. The initial estimation gives 439 NC events for an exposure time of 200 kt·year with a duty factor of 33%. Using the technique from [15], which is based on the signal coincidence and pulse shape discrimination, this background can be significantly reduced, to 33 NC events. Adding also fast neutron and charge current atmospheric events, the total background equals 45 events. This number is used in simulations with systematic uncertainty σ_b=5%.

To investigate the sensitive region of octant degeneracy, 1k MC "fake" experiments were calculated for each sample with particular fixed values of δ_{CP}. We did not apply any constraints to the parameter Θ_{23}. Both parts of $\Delta\chi^2$ in equation (3) were minimized using the ROOT package Minuit [16, 17]. Finally, the sensitivity region was calculated as defined in section 2.2.1.

In order to evaluate the potential of JUNO to accurately measure the mixing angle Θ_{23}, 5k MC "fake" experiments were simulated for each sample with a particular fixed value of δ_{CP}. The chi-square function (5) was minimized in the entire parameter space. A histogram was then filled with the extracted values of Θ_{23}. The shape of the histogram is Gaussian, since we assumed that all parameter uncertainties have Gaussian distribution. The 1σ error of Θ_{23} was obtained as a standard deviation of the aforementioned histogram. This procedure was repeated for the whole range of CP-phases, from $-\pi$ to π.

3 Results

Experimental sensitivity to octant degeneracy is depicted in Fig. 2. The yellow area shows the 68.3% confidence interval, within which the experiment is insensitive to octant degeneracy. The green area shows the insensitive region with confidence level 99.7%.

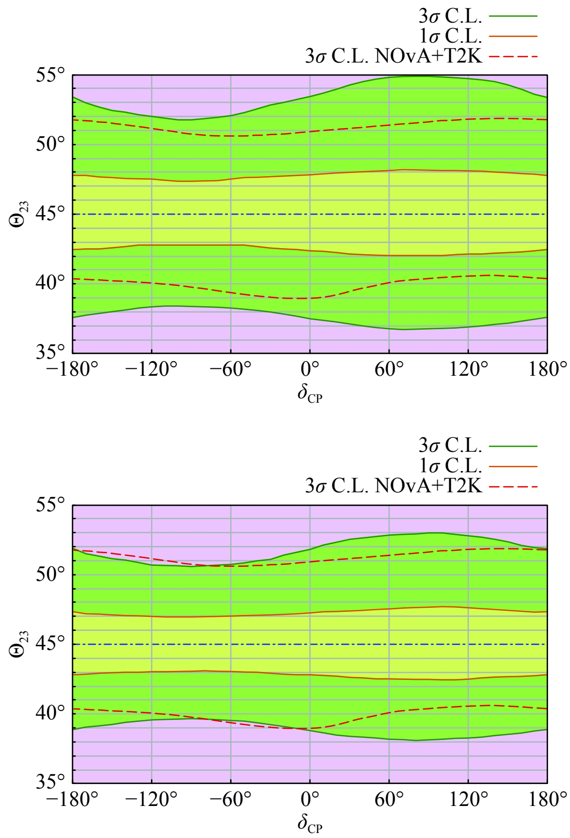

Fig. 2. (color online) The sensitive area for determining the octant as a function of δ_{CP} assuming an exposure time of 200 kt·year. The top panel corresponds to a 5 MW source, the bottom panel –10 MW. The yellow area corresponds to insensitivity with 68.3% C.L. The green area corresponds to insensitivity with 99.7% C.L. The pink area is sensitive to the octant with a significance of more than 3σ. Dashed red lines show 99.7% C.L for the combined analysis of T2K and NOνA presented in [18].

In the pink area the octant can be determined with a significance of more than 3σ. As can be seen, the sensitivity to octant is better for negative values of δ_{CP}. For these values, a 5 MW cyclotron can distinguish the octant if the mixing angle Θ_{23} is outside the range 38.5°-52.9°. A 10 MW cyclotron can measure the octant if Θ_{23} is outside 39.7°-50.8°. Therefore, higher statistics leads to an improvement of the sensitivity. The result for the 10 MW case is slightly worse than the expected result from the combined analysis of T2K+NOνA.

Figure 3 gives a quantitative estimation of the uncertainty for two possible values of Θ_{23} as a function of δ_{CP}. The top row corresponds to $\sin^2(\Theta_{23}) = 0.597$ and the bottom row to $\sin^2(\Theta_{23}) = 0.417$.

The wave behavior of curves in Fig. 3 can be explained by the maximum of the probability function (2) for $\delta_{CP} = \pi/2$ and the minimum for $\delta_{CP} = -\pi/2$. As can be seen in Fig. 3, the main uncertainty comes from oscillation parameters. Our estimation shows that the dominant uncertainty comes from the mixing angle Θ_{13}. The influence of the background is quite small, especially for higher statistics with a 10 MW source. Statistically, the improvement of the results is possible only with a 10 MW source. However, in reality only the LO values can improve the result in the case of a negative CP-phase.

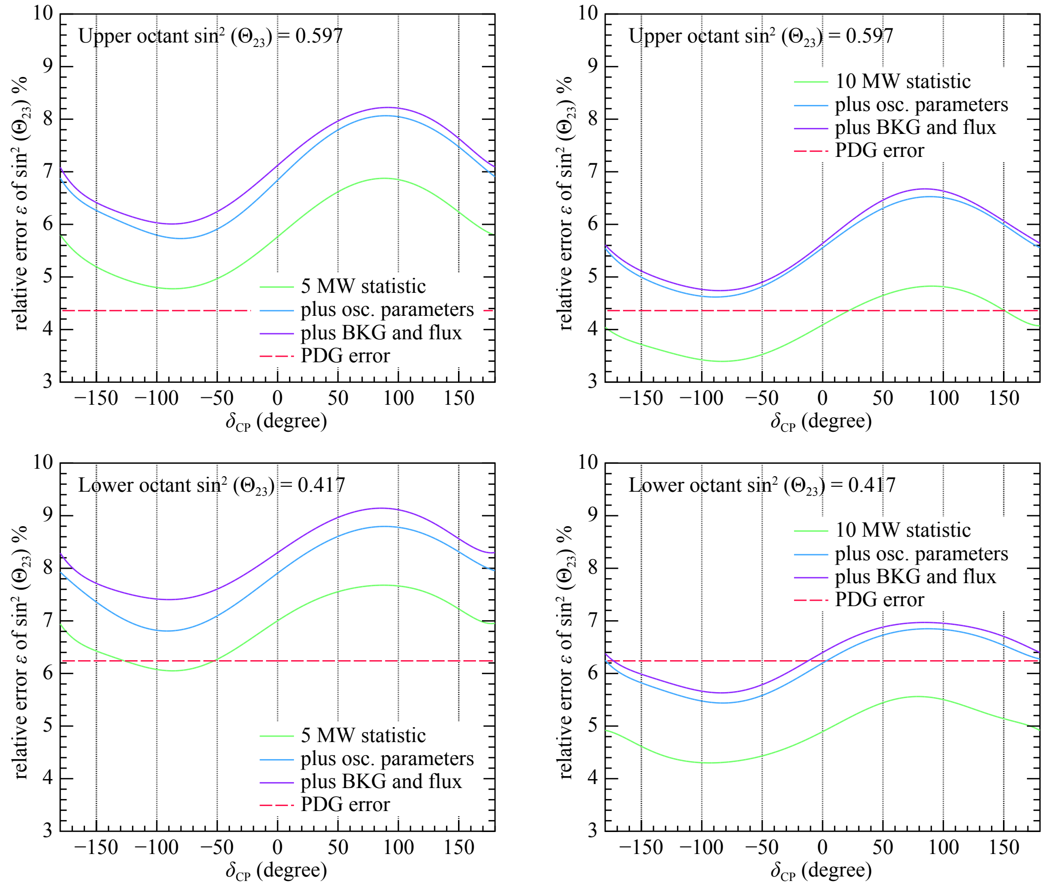

Fig. 3. (color online) Measurement accuracy of the mixing angle Θ_{23}. The top row is for $\sin^2(\Theta_{23}) = 0.597$ and the bottom for $\sin^2(\Theta_{23}) = 0.417$. Two values of the power of the far cyclotron are assumed. Dashed red lines correspond to the current value of the relative error $\sin^2(\Theta_{23})$ from PDG, where $\varepsilon(\sin^2(\Theta_{23}) = 0.417)$=6.24% and $\varepsilon(\sin^2(\Theta_{23}) = 0.597)$=4.36%.

4 Conclusions

The present work demonstrates another application of superconducting cyclotrons for measurements in neutrino physics. The transition channel $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ allows to explore not only the problem of CP violation, but at the same time to realize a precise measurement of Θ_{23} and partially resolve the octant degeneracy.

It was shown that the distinction between LO and UO is comparable to the combined analysis of T2K and NOνA , especially for negative values of δ_{CP}. Regarding the measurement precision of Θ_{23}, the current best fit value can be improved only in the case of a 10 MW source, especially if the mixing angle is in LO. There are two main difficulties with precision measurements: uncertainties in the oscillation parameters and small statistics. The problem of statistics can be alleviated by using a small water detector for monitoring neutrino flux instead of the near cyclotron. This allows to use the far cyclotron in a continuous mode, as proposed for the TNT2K experiment [19].

The combination of JUNO and superconducting cyclotrons could be a good alternative to conventional beam experiments. It would allow the measurement of Θ_{23} and δ_{CP} in the current scientific program without affecting JUNO's main goals.

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