



M2-branes on a constant flux background[☆]

M.P. Garcia del Moral^{a,*}, C. Las Heras^a, P. Leon^a, J.M. Pena^{a,b}, A. Restuccia^{a,*}

^a Departamento de Física, Universidad de Antofagasta, Aptdo 02800, Chile

^b Departamento de Física, Facultad de Ciencias, Universidad Central de Venezuela, A.P. 47270, Caracas 1041-A, Venezuela

ARTICLE INFO

Article history:

Received 28 June 2019

Received in revised form 3 September 2019

Accepted 5 September 2019

Available online 9 September 2019

Editor: M. Cvetič

Keywords:

Supermembrane

Fluxes

Backgrounds

Supergravity

ABSTRACT

We describe a compactified Supermembrane, or M2-brane, with 2-form fluxes generated by constant three-forms that are turned on a 2-torus of the target space $M_9 \times T^2$. We compare this theory with the one describing a 11D M2-brane formulated on $M_9 \times T^2$ target space subject to an irreducible wrapping condition. We show that the flux generated by the bosonic 3-form under consideration is in a one to one correspondence to the irreducible wrapping condition. After a canonical transformation both Hamiltonians are exactly the same up to a constant shift in one particular case. Consequently both of them, share the same spectral properties. We conclude that the Hamiltonian of the M2-brane with 2-form target space fluxes on a torus has a purely discrete spectrum with eigenvalues of finite multiplicity and it can be considered to describe a new sector of the microscopic degrees of freedom of M-theory. We also show that the total membrane momentum in the direction associated to the flux condition acquires a quantized contribution in correspondence to the flux units that have been turned on.

© 2019 The Authors. Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>). Funded by SCOAP³.

1. Introduction

Supermembranes, also called M2-branes, are $2 + 1$ supersymmetric extended objects that evolve in an eleven dimensional target space [1]. They are described by a nonlinear interacting field theory invariant under global supersymmetry, local diffeomorphisms and local kappa symmetry [2]. Supermembranes are part of the building blocks of M-theory and they are sources of 11D supergravity [1,3], a relation that was emphasized in [4]. Moreover, in [5] it is shown that the supermembrane emerges as an exact solution of supergravity field equations. A remarkable property of supermembrane theory is that all the five string theories at least at kinematical level and by double dimensional reduction can be obtained from it [7–10]. Recently it has been shown that M2-brane toroidally compactified is U-dual invariant [11,12].

The Supermembrane theory was originally expected to describe the microscopic degrees of freedom of M-theory however when formulated on 11D Minkowski background [2], it was rigorously proved in the context of matrix model regularization that it has continuous spectrum from $[0, \infty)$ [13]. This property led to the community to re-interpret this theory as a second quantized theory [14] and the compactification by itself does not change this

behavior [15]. The string-like spikes at zero cost energy survive in the classical potential [16]. Interestingly, the semiclassical quantization of the toroidally compactified supermembrane theory however exhibits a purely discrete spectrum as it was proven in [17]. There are just few cases described in the literature -up to our knowledge- in which the complete formulation of supermembrane theory exhibits discreteness of the spectrum: A first case the so called *supermembrane with central charges* [20] that is irreducibly wrapped [21] around a 2-torus whose full-fledged Hamiltonian has also a purely discrete spectrum as rigorously shown in [18,19]. The case considered in [17], also has a discrete spectrum, since it contains a particular realization of the central charge condition. This condition that has been extended to other compactifications -that adds a topological condition on the wrapping, see for example [22–24], and a second type, corresponding the supermembrane on a pp-wave background [25,26] whose matrix model regularization corresponds to the BMN matrix model [27] and whose properties of discreteness were proven on [19].

Since M-theory is a candidate for unification theory, at least a sector of the theory will be described in terms of the Supermembrane theory degrees of freedom. Consequently it becomes increasingly clear the need to obtain the M2-brane theory formulated on more general backgrounds. Previous formulation of the M2-brane in the Light Cone Gauge (L.C.G) on arbitrary curved backgrounds in the formalism of the superspace to second order in Grassmann variables was done in [28]. In this paper we analyze the quantum properties of a simple but nontrivial case of a M2-brane on

[☆] This work started to be written in a Visiting Research (August–September 2018) at Departamento de Física, Universidad de Los Andes, Bogotá, Colombia.

* Corresponding authors.

E-mail address: maria.garciadelmoral@uantof.cl (M.P. Garcia del Moral).

a $M_9 \times T^2$ background with particular target space 2-form fluxes generated by the presence of non vanishing components of a constant form C_{\pm} . These backgrounds are consistent with supergravity. Indeed, backgrounds with a constant C_3 were considered by the authors [5], corresponding to a M2-brane acting as a source, see also [6]. The analysis that we do includes backgrounds corresponding to the asymptotic limit of those considered in [29]. The paper is organized as follows: In section 2 we obtain the Hamiltonian and constraints of the M2-brane on $M_9 \times T^2$ with target space fluxes. In section 3 we compare this theory with the M2-brane theory irreducibly wrapped around the 2-torus. Based on this result we characterize its spectrum. In section 4 we establish the relation between the two formulations and based on this result we characterize its spectral properties. In section 5 we present our conclusions.

2. M2-brane subject to 2-form fluxes induced by constant C_3

In this section we will analyze the supermembrane theory in the L.C.G. formulated on $M_9 \times T^2$ background with 2-form fluxes induced by the presence of constant bosonic 3-form gauge fields $C_{\mu\nu\lambda}$. The supersymmetric action of the M2-brane on a generic 11D noncompact background was found by [1].

$$S = T \int d^3\xi \left[-\frac{1}{2} \sqrt{-g} g^{uv} \Pi_u^{\hat{m}} \Pi_v^{\hat{n}} \eta_{\hat{m}\hat{n}} + \frac{1}{2} \sqrt{-g} - \frac{1}{6} \varepsilon^{uvw} \Pi_u^{\hat{A}} \Pi_v^{\hat{B}} \Pi_w^{\hat{C}} C_{\hat{A}\hat{B}\hat{C}} \right], \quad (1)$$

where, $g_{uv} = \Pi_u^{\hat{m}} \Pi_v^{\hat{n}} \eta_{\hat{m}\hat{n}}$ with $u, v, w, = 0, 1, 2$ the worldvolume indices. (Here $\hat{A}, \hat{B}, \hat{C}$, are tangent superspace indices; $\hat{m}, \hat{n}, \hat{l}$, bosonic tangent space indices and $\hat{a}, \hat{b}, \hat{c}$, fermionic tangent space indices.) Pullback of the supervielbein $E_M^{\hat{A}}$ to the worldvolume is given by $\Pi_u^{\hat{A}} = \frac{\partial Z^{\hat{A}}}{\partial \xi^u} E_M^{\hat{A}}$ where M is superspace index. In our study we will focus on flat metric $G_{\mu\nu} = \eta_{\mu\nu}$ backgrounds but in the presence of some constant components of the three-form:

$$\begin{aligned} \Pi_v^{\hat{m}} &= \partial_v X^\mu \delta_\mu^{\hat{m}} + \bar{\theta} \Gamma^{\hat{m}} \partial_v \theta, & \Pi_v^{\hat{a}} &= \partial_v \theta^{\hat{a}}, \\ C_{\mu\alpha\beta} &= (\bar{\theta} \Gamma_{\mu\nu})_{(\alpha} (\bar{\theta} \Gamma^{\nu})_{\beta)}, & C_{\alpha\beta\gamma} &= (\bar{\theta} \Gamma_{\mu\nu})_{(\alpha} (\bar{\theta} \Gamma^{\mu})_{\beta} (\bar{\theta} \Gamma^{\nu})_{\gamma)}, \\ C_{\mu\nu\rho} &= \text{const}, & C_{\mu\nu\alpha} &= (\bar{\theta} \Gamma_{\mu\nu})_{\alpha}, \end{aligned} \quad (2)$$

The embedding coordinates in the superspace formalism are $(X^\mu(\xi), \theta^\alpha(\xi))$ with ξ^u the worldvolume coordinates and where μ, ν, λ and α, β , are bosonic and fermionic target space indices, respectively. They are scalars under reparametrizations on ξ . In this background the action of the supermembrane takes the following form:

$$S = -T \int d^3\xi \left\{ \sqrt{-g} + \varepsilon^{uvw} \bar{\theta} \Gamma_{\mu\nu} \partial_w \theta \left[\frac{1}{2} \partial_u X^\mu (\partial_v X^\nu + \bar{\theta} \Gamma^\nu \partial_v \theta) + \frac{1}{6} \bar{\theta} \Gamma^\mu \partial_u \theta \bar{\theta} \Gamma^\nu \partial_v \theta \right] + \frac{1}{6} \varepsilon^{uvw} \partial_u X^\mu \partial_v X^\nu \partial_w X^\rho C_{\rho\nu\mu} \right\}. \quad (3)$$

This background is consistent with supergravity in 11D dimensions and in a particular case it corresponds to the asymptotic limit of a supergravity solution generated by an M2-brane acting as a source [5,29]. The line element in this case is given by

$$ds^2 = (1 + \frac{k}{r^6})^{-\frac{2}{3}} dx^{\bar{\mu}} dx^{\bar{\nu}} \eta_{\bar{\mu}\bar{\nu}} + (1 + \frac{k}{r^6})^{-\frac{1}{3}} dy^{\bar{m}} dy^{\bar{n}} \delta_{\bar{m}\bar{n}}, \quad (4)$$

where $\bar{\mu} = 0, 1, 2$, $\bar{m} = 3, \dots, 10$ and $r = \sqrt{y^{\bar{m}} y^{\bar{m}}}$ is the radial isotropic coordinate in the transverse space. On the other hand, the ansatz for the 3-form produces:

$$C_{\bar{\mu}\bar{\nu}\bar{\sigma}} = \epsilon_{\bar{\mu}\bar{\nu}\bar{\sigma}} (1 + \frac{k}{r^6})^{-1}, \quad (5)$$

with the other components set to zero. When $r \rightarrow \infty$, the metric (4) goes to Minkowski metric and (5) is constant. We may now formulate the supermembrane action on a $M_9 \times T^2$ target space with constant gauge field $C_{\mu\nu\sigma}$ closely following the definitions of [15]. We start the L.C.G. fixing by taking, $X^+(\xi) = X^+(0) + \tau$, where τ is the time coordinate on the worldvolume, so that $\partial_u X^+ = \delta_{u\tau}$, and $\Gamma^+ \theta = 0$. Now the supersymmetric action formulated in this partial gauge fixing is

$$S = T \int d^3\xi \{ -\sqrt{\bar{g}} \Delta - \varepsilon^{rs} \partial_r X^a \bar{\theta} \Gamma^- \Gamma_a \partial_s \theta + C_+ + \partial_\tau X^- C_- + \partial_\tau X^a C_a + C_{+-} \} \quad (6)$$

with¹

$$\begin{aligned} C_a &= -\varepsilon^{rs} \partial_r X^- \partial_s X^b C_{-ab} + \frac{1}{2} \varepsilon^{rs} \partial_r X^b \partial_s X^c C_{abc}, \\ C_{\pm} &= \frac{1}{2} \varepsilon^{rs} \partial_r X^a \partial_s X^b C_{\pm ab}, \quad C_{+-} = \varepsilon^{rs} \partial_r X^- \partial_s X^a C_{+-a}, \end{aligned} \quad (7)$$

where σ^r , $r = 1, 2$ are the spacelike coordinates of the base manifold $\Sigma \times R$, being Σ a torus. It is possible to fix the variation of some components of the 3-form by virtue of its gauge invariance. In particular it is possible to fix $C_{+-a} = 0$ and $C_{-ab} = \text{constant}$. The Hamiltonian takes the form

$$H = T \int d^2\sigma \left\{ \frac{1}{(P_- - C_-)} \left[\frac{1}{2} (P_a - C_a)^2 + \frac{1}{4} (\varepsilon^{rs} \partial_r X^a \partial_s X^b)^2 \right] + \varepsilon^{rs} \bar{\theta} \Gamma^- \Gamma_a \partial_s \theta \partial_r X^a - C_+ \right\} \quad (8)$$

subject to the primary constraints

$$P_a \partial_r X^a + P_- \partial_r X^- + \bar{S} \partial_r \theta \approx 0, \quad S + (P_- - C_-) \Gamma^- \theta \approx 0, \quad (9)$$

with

$$\begin{aligned} P_a &= \sqrt{\frac{\bar{g}}{\Delta}} (\partial_\tau X_a - u_r \bar{g}^{rs} \partial_s X_a) + C_a, \\ P_- &= \sqrt{\frac{\bar{g}}{\Delta}} + C_-, \quad S = -\sqrt{\frac{\bar{g}}{\Delta}} \Gamma^- \theta. \end{aligned} \quad (10)$$

The contribution of C_+ to the Hamiltonian is a boundary term that does not necessarily vanish since there exists a compact sector of target space. X^- appears explicitly through C_a in the Hamiltonian. On the other side X^- can be solved from the constraint in terms of the physical degrees of freedom. However non-locality is introduced in the procedure. In order to achieve a local canonical polynomial reduction of the Hamiltonian one may perform the following transformation

$$P_a \rightarrow \hat{P}_a \equiv P_a - C_a, \quad P_- \rightarrow \hat{P}_- \equiv P_- - C_-, \quad S \rightarrow \hat{S} \equiv S, \quad (11)$$

keeping invariant the rest of the canonical variables. These preserve all the Poisson brackets, as it is a canonical transformation on the phase space. In fact, the kinetic terms remain invariant under (11),

¹ In order to be self-contained we include the definitions of [2]: $\Delta = -g_{00} + u_r \bar{g}^{rs} u_s$ being $\bar{g}^{rs} g_{st} = \delta^r_t$ and $g \equiv \det g = -\Delta \bar{g}$ (with $\varepsilon^{0rs} = \varepsilon^{rs}$). $\bar{g}_{rs} \equiv g_{rs} - \partial_r X^a \partial_s X^b \delta_{ab}$; $u_r \equiv g_{0r} = \partial_r X^- + \partial_0 X^a \partial_r X^b \delta_{ab} + \bar{\theta} \Gamma^- \partial_r \theta$; $\bar{g}_{00} = 2\partial_0 X^- + \partial_0 X^a \partial_0 X^b \delta_{ab} + 2\bar{\theta} \Gamma^- \partial_0 \theta$.

$$\int_{\Sigma} (P_a \dot{X}^a + P_- \dot{X}^- + \bar{S} \dot{\theta}) = \int_{\Sigma} (\hat{P}_a \dot{\hat{X}}^a + \hat{P}_- \dot{\hat{X}}^- + \hat{S} \dot{\hat{\theta}}), \quad (12)$$

and the new constraints are

$$P_a \partial_r X^a + P_- \partial_r X^- + \bar{S} \partial_r \theta = \hat{P}_a \partial_r \hat{X}^a + \hat{P}_- \partial_r \hat{X}^- + \hat{S} \partial_r \hat{\theta} \approx 0 \quad (13)$$

$$\chi \approx S + (P_- - C_-) \Gamma^- \theta = \hat{S} + \hat{P}_- \Gamma^- \hat{\theta} \approx 0 \quad (14)$$

Now we may use the residual gauge symmetry generated by the constraints to impose the gauge fixing condition $\hat{P}_- = \hat{P}_-^0 \sqrt{w}$, where \sqrt{w} is a time independent scalar density and \hat{P}_-^0 a zero mode defined as in [2]. We may then eliminate (\hat{X}^-, \hat{P}_-) as canonical variables and obtain a formulation solely in terms of (\hat{X}^a, \hat{P}_a) . The remaining constraint after the partial gauge fixing corresponds, as usual, to the area preserving ones:

$$d(\hat{P}_a d\hat{X}^a + \bar{\theta} \Gamma^- d\hat{\theta}) = 0, \quad \int_{\Sigma} d(\hat{P}_a d\hat{X}^a + \bar{\theta} \Gamma^- d\hat{\theta}) = 0, \quad (15)$$

where the first constraint is the local integrability condition which must be satisfied in order to have a solution for \hat{X}^- . The second integral constraint, is the condition that the periods $d\hat{X}^-$ are trivial and hence $d\hat{X}^-$ is an exact one form. We then have the following hamiltonian density for a membrane on a general background field, after the elimination of the conjugate pairs (\hat{X}^-, \hat{P}_-) and (\hat{X}^+, \hat{P}_+) ,

$$\hat{\mathcal{H}}_{total} = \hat{\mathcal{H}} - C_+ = \frac{1}{\hat{P}_-} \left[\frac{1}{2} \hat{P}_a \hat{P}^a + \frac{1}{4} (\varepsilon^{rs} \partial_r \hat{X}^a \partial_s \hat{X}^b)^2 \right] + \varepsilon^{rs} \bar{\theta} \Gamma^- \Gamma_a \partial_s \hat{\theta} \partial_r \hat{X}^a - C_+ \quad (16)$$

Since we are considering a toroidal compactification of the target space $M_9 \times T^2$, the bosonic components decompose in the compact and non-compact sector. The even embedding maps \hat{X}^a associated to the compact sector which always appear as closed one-forms in the action. They decompose into an exact one-form, plus a harmonic one-form. The latter may have nontrivial periods on the basis of homology of the compact base Riemann surface Σ . The odd embedding maps $\hat{\theta}^a$ we assume to be single valued on the base manifold. The gauge fixing procedure is consistent under this compactification since the local gauge transformations only involve the exact part of \hat{X}^a . The Hamiltonian of the compactified theory is the following one:

$$\begin{aligned} \int_{\Sigma} d^2 \sigma \hat{\mathcal{H}}_{total} &= \int_{\Sigma} d^2 \sigma \left\{ \frac{\sqrt{w}}{\hat{P}_-^0} \left[\frac{1}{2} \left(\frac{P_m}{\sqrt{w}} \right)^2 + \frac{1}{2} \left(\frac{P_i}{\sqrt{w}} \right)^2 \right. \right. \\ &\quad \left. \left. + \frac{1}{4} \{X^i, X^j\}^2 + \frac{1}{2} \{X^i, X^m\}^2 + \frac{1}{4} \{X^m, X^n\}^2 \right] \right. \\ &\quad \left. + \sqrt{w} [\bar{\theta} \Gamma^- \Gamma_m \{X^m, \theta\} + \bar{\theta} \Gamma^- \Gamma_i \{X^i, \theta\}] \right\} - C_+, \end{aligned} \quad (17)$$

where the index m denotes the maps from the base Σ to M_9 and $i, j = 1, 2$ the map from Σ to T^2 . Generically, $dX^i = M_j^i d\hat{X}^j + dA^i$, M_j^i are integers in order to have a map to cycles. $d\hat{X}^j$ is a normalized basis of harmonic one-forms and dA^i are exact one-form components. There are no further requirements on M_j^i . We notice the differences between the above Hamiltonian and the one on a Minkowski M_{11} target space where no harmonic contribution is present. We will give the complete expression of the Hamiltonian in terms of $d\hat{X}^i, A^i, dX^m$ in the next section. This Hamiltonian is

subject to the local and global constraints associated to the Area Preserving Diffeomorphisms

$$d(P_i dX^i + P_m dX^m + \bar{\theta} \Gamma^- d\theta) = 0,$$

$$\oint_{C_s} d(P_i dX^i + P_m dX^m + \bar{\theta} \Gamma^- d\theta) = 0. \quad (18)$$

In the compactified case, in contrast to the noncompact one, the last term in (17) for constant bosonic 3-form is a total derivative of a multivalued function (due to the harmonic contribution). Therefore its integral is not necessarily zero.

Classically the dynamics of this Hamiltonian contains string-like spikes which render the quantum spectrum of the theory continuous. The main point to study in this paper is the behavior of the theory when we add a flux quantization condition on the 3-form. Given the target space $M_9 \times T^2$ a flux condition on it corresponds to a closed two form F_2 whose integral on the compact sector is an integer number. This flux condition is equivalent to the existence of an $U(1)$ principle bundle over T^2 and of a 1-form connection on it whose curvature is F_2 :

$$\int_{T^2} F_2 = k \in \mathbb{Z} \setminus \{0\}. \quad (19)$$

In this paper we consider the closed two-forms generated by C_+ or C_- . We will define them in the following sections. We are interested in the quantum properties of the supermembrane on a target space with nontrivial C_{\pm} , under a flux condition generated by them. In order to perform this analysis we are going to compare the Hamiltonian (17) subject to a flux condition with the Hamiltonian of the M2-brane irreducibly wrapped on a target space $M_9 \times T^2$.

3. The M2-brane with irreducible wrapping

The M2-brane with irreducible wrapping is defined as follows. The embedding maps satisfy winding conditions over the non-trivial 1-cycles of the 2-torus with $\oint_{C_j} dX^i = M_j^i$, M_j^i are winding numbers integers and C_s the homological basis of the 2-torus. The embedding maps associated to the wrapping on T_2 satisfy the following topological condition [21]

$$\int_{\Sigma} dX^i \wedge dX^j = \epsilon^{ij} n A, \quad n \in \mathbb{Z} \setminus \{0\}, \quad (20)$$

with A denoting the area of the 2-torus (which we can be normalized to 1) and n an integer that is chosen to be different from zero. This condition is a quantization condition and ensures that the harmonic modes appear in a nontrivial way in the expression of X^i . This condition is related with the existence of a central charge in the SUSY algebra and for this reason this sector of the Supermembrane has been denoted as *Supermembrane with central charges*. Indeed, it implies that the supermembrane is a calibrated submanifold [22]. Other studies analyzing the M2-brane on holomorphic curves was considered in [30] and [31]. From a geometrical point of view, irreducibility condition ensures the existence of a non-trivial $U(1)$ principal bundle over the worldvolume of the supermembrane, characterized by the integer n associated to its first Chern class. A particular n fixes and restricts the allowed class of principal fiber bundle where it can be formulated. The canonical connections are $U(1)$ monopoles expressed in terms of the embedding maps (which are minimal immersions) of the supermembrane

in the compactified space. Indeed, this corresponds to have a non-trivial 2-form flux over the supermembrane world-volume

$$\int_{\Sigma} F_2 = n \in \mathbb{Z}/\{0\}. \quad (21)$$

See [21] for further details. The Hamiltonian of a supermembrane wrapped on a 2-torus subject to (20) found in [20] is the following one

$$\begin{aligned} H^{Irr} = & \int_{\Sigma} d^2\sigma \sqrt{w} \left[\frac{1}{2} \left(\frac{P_m}{\sqrt{w}} \right)^2 + \frac{1}{2} \left(\frac{P_i}{\sqrt{w}} \right)^2 + \frac{1}{4} \{X^m, X^m\}^2 \right. \\ & + \frac{1}{2} (\mathcal{D}_i X^m)^2 + \frac{1}{4} (\mathcal{F}_{ij})^2 \Big] \\ & + \int_{\Sigma} d^2\sigma \sqrt{w} \left[\Lambda \left(\mathcal{D}_i \left(\frac{P_i}{\sqrt{w}} \right) + \left\{ X^m, \frac{P_m}{\sqrt{w}} \right\} \right) \right] + (n^2 Area_{T^2}^2) \\ & + \int_{\Sigma} d^2\sigma \sqrt{w} \left[-\bar{\theta} \Gamma_- \Gamma_i \mathcal{D}_i \theta - \bar{\theta} \Gamma_- \Gamma_m \{X^m, \theta\} \right. \\ & \left. + \Lambda \{ \bar{\theta} \Gamma_-, \theta \} \right], \end{aligned} \quad (22)$$

where there is a symplectic covariant derivative and symplectic curvature defined

$$\mathcal{D}_i X^m = D_i X^m + \{A_i, X^m\}, \quad \mathcal{F}_{ij} = D_i A_j - D_j A_i + \{A_i, A_j\}, \quad (23)$$

with D_i a covariant derivative defined in terms of the moduli of the torus, the winding numbers M_j^i and the harmonic one-forms, see [11]. The symplectic connection transforming under area preserving diffeomorphisms given by $\delta_\epsilon A = \mathcal{D}\epsilon$. In [32], the authors showed that this hamiltonian classically does not contain string-like configurations. At a quantum level it has the remarkable property of having a supersymmetric discrete spectrum with finite multiplicity, [18,19] in distinction with the supermembrane compactified on a torus without this restriction (20) which has continuous spectrum from $[0, \infty)$ [13,15]. The irreducible wrapping condition is a flux condition over the worldvolume that generalizes the Dirac monopole construction to Riemann surfaces of arbitrary genus ≥ 1 [33]. The theory defined in this way is a restriction of the supermembrane theory. All configurations must satisfy, in addition, the global constraint. The constraint (20) does not change the local symmetries of the supermembrane theory since it is topological condition. In particular, the invariance under area preserving diffeomorphisms is preserved.

4. Relation between both formulations

We start by considering a flux condition generated by C_{\pm} on the base manifold Σ . C_{\pm} is a density with the dimensions of the membrane momentum in the directions associated with coordinates X^{\pm} . It is defined on Σ , we then consider its associated 2-form

$$\begin{aligned} C_{\pm} d\sigma^1 \wedge d\sigma^2 &= \frac{1}{2} \frac{\partial X^a}{\partial \sigma^r} \frac{\partial X^b}{\partial \sigma^s} C_{\pm ab} d\sigma^r \wedge d\sigma^s \\ &= \frac{1}{2} C_{\pm ab} dX^a(\sigma, \tau) \wedge dX^b(\sigma, \tau) \end{aligned} \quad (24)$$

Under the assumption that $C_{\pm ab}$ is constant, it is a closed two-form on Σ . We impose the flux condition

$$\int_{\Sigma} F_2 = \int_{\Sigma} C_{\pm} d\sigma^1 \wedge d\sigma^2 = k_{\pm} \in \mathbb{Z}/\{0\}. \quad (25)$$

The maps from $\Sigma \rightarrow M_9 \times T^2$ decompose into maps from $\Sigma \rightarrow M_9$ and the ones from $\Sigma \rightarrow T^2$. The former are labeled with an index m and the latter with index i, j . As we have stated the closed one-forms dX^i can always be expressed in terms of the harmonic part $M_j^i d\tilde{X}^j$ and its exact part. In the flux condition the exact one-forms cancel and we are only left with the harmonic sector. We then have for the flux condition

$$\int_{\Sigma} F_2 = \int_{\Sigma} \frac{1}{2} C_{\pm ij} M_k^i M_l^j d\tilde{X}^k \wedge d\tilde{X}^l = k_{\pm} \in \mathbb{Z}/\{0\}, \quad (26)$$

The normalized basis of harmonic one-forms on Σ , $d\tilde{X}^i$, satisfy $\int_{C_i} d\tilde{X}^j = \delta_i^j$, with C_i , being the homology basis on Σ . Using the bilinear Riemann relations we get $\int_{\Sigma} d\tilde{X}^1 \wedge d\tilde{X}^2 = 1$ It is convenient to define the density \sqrt{w} introduced in the gauge fixing procedure as $\sqrt{w} = \frac{\partial \tilde{X}^1}{\partial \sigma^r} \frac{\partial \tilde{X}^2}{\partial \sigma^s} \epsilon^{rs}$ it is a regular density on Σ , we then have $\int_{\Sigma} \sqrt{w} d\sigma^1 \wedge d\sigma^2 = 1$. We can now change variables from (σ^1, σ^2) on Σ to local coordinates $(\tilde{X}^1, \tilde{X}^2)$ on the 2-torus T^2 , the map is defined by $\tilde{X}^i = \tilde{X}^i(\sigma^1, \sigma^2)$. In fact, the Jacobian of the change of variables is \sqrt{w} which is nonzero on Σ . C_{\pm} defines then a flux condition on T^2 ,

$$\int_{\Sigma} F_2 = c_{\pm} \det M \int_{T^2} d\tilde{X}^1 \wedge d\tilde{X}^2 = \int_{T^2} \frac{1}{2} C_{\pm ij} M_k^i M_l^j d\tilde{X}^k \wedge d\tilde{X}^l = \int_{T^2} \tilde{F}_2, \quad (27)$$

where we have expressed $C_{\pm ij} = c_{\pm} \epsilon_{ij}$, and denoted $\det M$ the determinant of the matrix M_j^i . There is then a one to one correspondence between the flux condition generated by C_{\pm} on Σ and on T^2 . The main point is that $\det M$ must be nonzero, that is the condition of irreducible wrapping must be satisfied. There is then a one to one correspondence between the supermembrane with irreducible wrapping and the supermembrane on a background with a flux condition on T^2 generated by C_{\pm} . In fact, given the latter it implies that the supermembrane has an irreducible wrapping. Conversely given a supermembrane with irreducible wrapping there always exists a three form with a flux condition compatible with the nontrivial wrapping. Moreover, if there is no flux, (27) is equal to zero, then supermembranes with reducible wrapping are admissible in the configurations space and the spectrum is consequently continuous from zero to infinity.

Furthermore the Hamiltonian of both theories differ at most in a constant, arising from the C_{+} term in the Hamiltonian (17), hence the spectrum of the supermembrane with fluxes generated by C_{\pm} has also discrete spectrum with finite multiplicity, a remarkable property. The effect of the C_{\pm} background produces a discrete shift in some components of the momentum of the supermembrane, and in the Hamiltonian density. Comparing with the original configuration variables (X^a, P_a) and considering the total momentum of the supermembrane, we have

$$P_-^0 = \int_{\Sigma} P_- d\sigma^1 \wedge d\sigma^2 = \int_{\Sigma} (\hat{P}_- + C_-) d\sigma^1 \wedge d\sigma^2 = \hat{P}_-^0 + k_-, \quad (28)$$

$$\begin{aligned} P_+^0 &= \int_{\Sigma} P_+ d\sigma^1 \wedge d\sigma^2 = \int_{\Sigma} (\hat{P}_+ + C_+) d\sigma^1 \wedge d\sigma^2 \\ &= \int_{\Sigma} \hat{\mathcal{H}} d\sigma^1 \wedge d\sigma^2 + k_+, \end{aligned} \quad (29)$$

$$\begin{aligned}
P_a^0 &= \int_{\Sigma} P_a d\sigma^1 \wedge d\sigma^2 = \int_{\Sigma} (\hat{P}_a + C_a) d\sigma^1 \wedge d\sigma^2 \\
&= \int_{\Sigma} \hat{P}_a d\sigma^1 \wedge d\sigma^2,
\end{aligned} \tag{30}$$

where we have used (27) and $\int_{\Sigma} C_a d\sigma^1 \wedge d\sigma^2 = 0$. $\hat{\mathcal{H}}$ is the Hamiltonian density of the supermembrane with fluxes (analogously with irreducible wrapping). We conclude that the interaction of the supermembrane with the background we have considered has rendered a quantized change of the membrane momentum on the directions associated with the coordinate X^{\pm} compared to the case when the C_{\pm} fluxes are set off.

5. Discussion and Conclusions

In this paper we study the effects of fluxes in the quantum properties of the M2-brane. We discuss the Hamiltonian formulation of the M2-brane compactified on a torus with 2-form fluxes induced by the 3-form C_{\pm} . We study the effect of the fluxes on the quantum properties of the theory. We establish an equivalence relation between a theory that contains 2-form fluxes induced by the C_{\pm} on the target space ‘M2-brane with fluxes’ and the supermembrane satisfying a topological condition over the worldvolume associated to an ‘irreducible wrapping’ where no reference to the 3-form background is present. When we consider the M2-brane on a C_{\pm} background and the target space is noncompact or even compactified on the 2-torus times Minkowski but no fluxes are present, the spectrum of the theory is continuous. Classically it can be understood from the fact that it contains -as in the uncompactified case-, string-like spikes that can be attached to the spectrum without any cost of energy. The case we analyze corresponds to have 2-form fluxes on the target space induced by the C_{\pm} . In this case the spectral behavior of the theory changes drastically: its mass spectrum becomes discrete. The flux backreacts on the worldvolume generating a induced flux on the worldvolume associated to the presence of fixed nontrivial $U(1)$ fiber bundle whose first chern class is k . It acts as a new constraint on the Hamiltonian and it is associated to the existence of a nontrivial central charge condition. The Hamiltonian becomes the Hamiltonian of the Supermembrane theory irreducible wrapped on a flat torus, shifted by a constant term proportional to k . An immediate consequence of this equivalence between both actions is the fact that a supermembrane formulated on a Minkowski background in the presence of a three form C_{\pm} toroidally compactified with an induced 2-form flux condition has a purely discrete spectrum with eigenvalues of finite multiplicity at quantum level. It represents a new sector of M2-brane with this property. The membrane momentum becomes shifted by the flux units in the directions of the X^+ or X^- coordinates corresponding to flux C_+ or C_- respectively. This property had also been observed at the level of matrix theory, see [34]. An old question posed by the authors in [35] was the relation between the matrix model on a noncommutative torus and its M-theory origin in terms of a M2-brane in the presence of a quantization condition over a constant C_- [28]. We show that this last theory corresponds exactly to the supermembrane with central charges. The results shown in the paper suggest the interest to generalize the precise relation between fluxes and quantization properties of M2-brane in order to describe new sectors of the microscopic degrees of freedom of M-theory.

Acknowledgements

A.R. and M.P.G.M. are partially supported by Projects FONDECYT 1161192 (Chile). C.L.H. Y P.L. are supported by the Project

ANT1756, ANT1855 and ANT1856 of the MINEDUC-Universidad de Antofagasta. J.M.P. is grateful to MINEDUC-Universidad de Antofagasta for kind hospitality and financial support during part of the realization of this work supported by MINEDUC-UA project, code Projects ANT1755 and ANT1855. Also, J.M.P. want to thank to Professor P. Bargueno for his hospitality at Uniandes (Bogotá, Colombia) during a visiting research from August to September of 2018 where part of this work was also done.

References

- [1] E. Bergshoeff, E. Sezgin, P.K. Townsend, Supermembranes and eleven-dimensional supergravity, *Phys. Lett. B* 189 (1987) 75–78 [69 (1987)].
- [2] B. de Wit, J. Hoppe, H. Nicolai, On the quantum mechanics of supermembranes, *Nucl. Phys. B* 305 (4) (1988) 545–581.
- [3] E. Bergshoeff, E. Sezgin, P.K. Townsend, Properties of the eleven-dimensional supermembrane theory, *Ann. Phys.* 185 (2) (1988) 330–368.
- [4] B.A. Ovrut, D. Waldram, Membranes and three-form supergravity, *Nucl. Phys. B* 506 (1997) 236–266, arXiv:hep-th/9704045.
- [5] M.J. Duff, K.S. Stelle, Multi-membrane solutions of $d = 11$ supergravity, *Phys. Lett. B* 253 (1) (1991) 113–118.
- [6] V. Balasubramanian, D. Kastor, J.H. Traschen, K.Z. Win, The spin of the M2-brane and spin-spin interactions via probe techniques, *Phys. Rev. D* 59 (1999) 084007, arXiv:hep-th/9811037.
- [7] F. Aldabe, A.L. Larsen, The $SO(32)$ heterotic and type IIB membranes, *J. High Energy Phys.* 11 (1998) 012, arXiv:hep-th/9811234.
- [8] F. Aldabe, Heterotic and type I strings from twisted supermembranes, *Nucl. Phys. B* 473 (1996) 63–73, arXiv:hep-th/9603183.
- [9] J.H. Schwarz, An $SL(2, \mathbb{Z})$ multiplet of type IIB superstrings, *Phys. Lett. B* 360 (1995) 13–18, Erratum: *Phys. Lett. B* 364 (1995) 252, arXiv:hep-th/9508143.
- [10] M.J. Duff, P.S. Howe, T. Inami, K.S. Stelle, Superstrings in $d=10$ from supermembranes in $d=11$, *Phys. Lett. B* 191 (1) (1987) 70–74.
- [11] M.P. Garcia del Moral, J.M. Pena, A. Restuccia, Supermembrane origin of type II gauged supergravities in 9D, *J. High Energy Phys.* 09 (2012) 063, arXiv:1203.2767 [hep-th].
- [12] M.P. Garcia del Moral, J.M. Pena, A. Restuccia, T-duality invariance of the supermembrane, 2012, *Int. J. Geom. Methods Mod. Phys.* 10 (2013) 1360010, arXiv:1211.2434 [hep-th].
- [13] B. de Wit, M. Luscher, H. Nicolai, The supermembrane is unstable, *Nucl. Phys. B* 320 (1989) 135–159.
- [14] T. Banks, W. Fischler, S.H. Shenker, Leonard Susskind, M theory as a matrix model: a conjecture, *Phys. Rev. D* 55 (1997) 5112–5128 [435 (1996)] arXiv:hep-th/9610043.
- [15] B. de Wit, K. Peeters, J. Plefka, Supermembranes with winding, *Phys. Lett. B* 409 (1997) 117–123, arXiv:hep-th/9705225.
- [16] M.P. Garcia del Moral, A. Restuccia, Spectrum of a noncommutative formulation of the $D = 11$ supermembrane with winding, *Phys. Rev. D* 66 (2002) 045023.
- [17] M.J. Duff, T. Inami, C.N. Pope, E. Sezgin, K.S. Stelle, Semiclassical quantization of the supermembrane, *Nucl. Phys. B* 297 (1998) 515–538.
- [18] L. Boulton, M.P. Garcia del Moral, A. Restuccia, Discreteness of the spectrum of the compactified $d=11$ supermembrane with nontrivial winding, *Nucl. Phys. B* 671 (2003) 343–358, arXiv:hep-th/0211047.
- [19] L. Boulton, M.P. Garcia del Moral, A. Restuccia, Spectral properties in supersymmetric matrix models, *Nucl. Phys. B* 856 (2012) 716–747, arXiv:1011.4791 [hep-th].
- [20] I. Martin, J. Ovalle, A. Restuccia, Compactified $D = 11$ supermembranes and symplectic noncommutative gauge theories, *Phys. Rev. D* 64 (2001) 046001, arXiv:hep-th/0101236.
- [21] I. Martin, A. Restuccia, Rafael S. Torrealba, On the stability of compactified $D = 11$ supermembranes, *Nucl. Phys. B* 521 (1998) 117–128, arXiv:hep-th/9706090.
- [22] J. Bellorin, A. Restuccia, $D=11$ supermembrane wrapped on calibrated submanifolds, *Nucl. Phys. B* 737 (2006) 190–208, arXiv:hep-th/0510259.
- [23] M.P. Garcia del Moral, J.M. Pena, A. Restuccia, $N=1$ 4D supermembrane from 11D, *J. High Energy Phys.* 07 (2008) 039, arXiv:0709.4632 [hep-th].
- [24] A. Belhaj, M.P. Garcia del Moral, A. Restuccia, A. Segui, J.P. Vairo, The supermembrane with central charges on a G2 manifold, *J. Phys. A* 42 (2009) 325201, arXiv:0803.1827 [hep-th].
- [25] K. Sugiyama, K. Yoshida, Supermembrane on the PP wave background, *Nucl. Phys. B* 644 (2002) 113–127, arXiv:hep-th/0206070.
- [26] K. Dasgupta, M.M. Sheikh-Jabbari, M. Van Raamsdonk, Matrix perturbation theory for M theory on a PP wave, *J. High Energy Phys.* 05 (2002) 056, arXiv:hep-th/0205185.
- [27] D.E. Berenstein, J.M. Maldacena, H.S. Nastase, Strings in flat space and pp waves from $N=4$ super Yang-Mills, *J. High Energy Phys.* 04 (2002) 013, arXiv:hep-th/0202021.
- [28] B. de Wit, K. Peeters, J. Plefka, Superspace geometry for supermembrane backgrounds, *Nucl. Phys. B* 532 (1998) 99–123, arXiv:hep-th/9803209.

- [29] K.S. Stelle, Lectures on supergravity p-branes, in: High Energy Physics and Cosmology. Proceedings, Summer School, Trieste, Italy, June 10–July 26, 1996, 1996, pp. 287–339, arXiv:hep-th/9701088.
- [30] T.Z. Husain, M2-branes wrapped on holomorphic curves, J. High Energy Phys. 12 (2003) 037, arXiv:hep-th/0211030.
- [31] J.P. Gauntlett, N. Kim, S. Pakis, D. Waldram, Membranes wrapped on holomorphic curves, Phys. Rev. D 65 (2002) 026003, arXiv:hep-th/0105250.
- [32] M.P. Garcia del Moral, A. Restuccia, Spectrum of a noncommutative formulation of the $D = 11$ supermembrane with winding, Phys. Rev. D 66 (2002) 045023, arXiv:hep-th/0103261.
- [33] I. Martin, A. Restuccia, Magnetic monopoles over topologically nontrivial Riemann surfaces, Lett. Math. Phys. 39 (1997) 379–391, arXiv:hep-th/9603035.
- [34] C.S. Chu, P.M. Ho, M. Li, Matrix theory in a constant C field background, Nucl. Phys. B 574 (2000) 275–287, arXiv:hep-th/9911153.
- [35] A. Connes, M.R. Douglas, A.S. Schwarz, Noncommutative geometry and matrix theory: compactification on tori, J. High Energy Phys. 02 (1998) 003, arXiv:hep-th/9711162.