



Cosmology from Newton–Chern–Simons gravity

S. Lepe^c, G. Rubio^b, P. Salgado^{a,b,*}

^a *Facultad de Ciencias, Universidad Arturo Prat de Iquique, Av. Arturo Prat Chacón 2120, Iquique, Chile*

^b *Departamento de Física, Universidad de Concepción, Casilla 160-C, Concepción, Chile*

^c *Instituto de Física, Pontificia Universidad Católica de Valparaíso, Avenida Brasil 2950, Valparaíso, Chile*

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Abstract

We study a five-dimensional non-relativistic gravity theory whose action is composed of a gravitational sector and a sector of matter where the gravitational sector is given by the so called Newton–Chern–Simons gravity and where the matter sector is described by a perfect fluid. At time to do cosmology, the obtained field equations shows a close analogy with the projectable version of the Hořava–Lifshitz theory in $(3 + 1)$ -dimensions. Solutions and their asymptotic limits are found. In particular a phantom solution with a future singularity reminiscent of a Little Big Rip future singularity is obtained.

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1. Introduction

In Ref. [1] was studied a five-dimensional Einstein–Chern–Simons gravity whose action $S = S_g + S_M$ is composed of a gravitational sector and a sector of matter, where the gravitational sector is given by a particular Chern–Simons gravity action [2] instead of the Einstein–Hilbert action and where the matter sector is given by the so called perfect fluid.

The corresponding Chern–Simons Lagrangian of Ref. [2] is a Lagrangian for the so called \mathfrak{B} algebra whose generators $\{J_{ab}, P_a, Z_{ab}, Z_a\}$ satisfy the commutation relation given by in the first equation of Ref. [1]. This Lagrangian can be constructed from the one-form gauge connection

* Corresponding author.

E-mail addresses: samuel.lepe@pucv.cl (S. Lepe), gurubio@udec.cl (G. Rubio), pasalgad@udec.cl (P. Salgado).

$$A = \frac{1}{2}\omega^{ab} J_{ab} + \frac{1}{l}e^a P_a + \frac{1}{2}k^{ab} Z_{ab} + \frac{1}{l}h^a Z_a, \quad (1)$$

and the two-form curvature

$$F = \frac{1}{2}R^{ab} J_{ab} + \frac{1}{l}T^a P_a + \frac{1}{2}K^{ab} Z_{ab} + \frac{1}{l}H^a Z_a, \quad (2)$$

where $T^a = D_\omega e^a$, $R^{ab} = d\omega^{ab} + \omega^a_c \omega^{cb}$, $H^a = D_\omega h^a + k^a_b e^b$, $K^{ab} = D_\omega k^{ab} + \frac{1}{l^2}e^a e^b$, are the corresponding ‘‘curvatures’’. In fact, using the extended Cartan’s homotopy formula [3,4], and integrating by parts, we find that the five-dimensional Chern–Simons lagrangian for the \mathfrak{B} algebra is given by [1]

$$L_{\text{ChS}}^{(5)} = \alpha_1 l^2 \epsilon_{abcde} R^{ab} R^{cd} e^e + \alpha_3 \epsilon_{abcde} \left(\frac{2}{3} R^{ab} e^c e^d e^e + 2l^2 k^{ab} R^{cd} T^e + l^2 R^{ab} R^{cd} h^e \right) + dB_{\text{EChS}}^{(4)}, \quad (3)$$

where the surface term $B_{\text{EChS}}^{(4)}$, given by

$$B_{\text{EChS}}^{(4)} = \alpha_1 l^2 \epsilon_{abcde} e^a \omega^{bc} \left(\frac{2}{3} d\omega^{de} + \frac{1}{2} \omega^d_f \omega^{fe} \right) + \alpha_3 \epsilon_{abcde} \left[l^2 \left(h^a \omega^{bc} + k^{ab} e^c \right) \left(\frac{2}{3} d\omega^{de} + \frac{1}{2} \omega^d_f \omega^{fe} \right) + l^2 k^{ab} \omega^{cd} \left(\frac{2}{3} de^e + \frac{1}{2} \omega^d_f e^e \right) + \frac{1}{6} e^a e^b e^c \omega^{de} \right]. \quad (4)$$

In the above mentioned reference [1] and also in Ref. [5] was shown that:

(i) the field equations can be obtained from the Lagrangian $L = L_{\text{ChS}}^{(5)} + \kappa L_M$, where $L_M = L_M(e^a, h^a, \omega^{ab})$ is the matter Lagrangian and κ is a coupling constant related to the effective Newton’s constant. In fact, the variation of the lagrangian (3) w.r.t. the dynamical fields vielbein e^a , spin connection ω^{ab} , h^a and k^{ab} , leads to the following field equations

$$\epsilon_{abcde} R^{ab} e^c e^d = 4\kappa_5 \left(\frac{\delta L_M}{\delta e^e} + \alpha \frac{\delta L_M}{\delta h^e} \right), \quad (5)$$

$$\frac{\delta L_M}{\delta h^e} = \frac{l^2}{8\kappa_5} \epsilon_{abcde} R^{ab} R^{cd}, \quad (6)$$

$$\epsilon_{abcde} R^{cd} D_\omega h^e = 0, \quad (7)$$

where we have imposed the conditions $T^a = 0$, $k^{ab} = 0$ and $\delta L_M / \delta \omega^{ab} = 0$ and where $\kappa_5 = \kappa / 8\alpha_3$ and $\alpha = -\alpha_1 / \alpha_3$. Note that the equation (5) is analogous to Einstein’s equation, where $\delta L_M / \delta h^a$ correspond to the energy-momentum tensor for the field h^a .

In the case where the equations (5)-(7) satisfy the cosmological principle and the ordinary matter is negligible compared to the dark energy, we find that corresponding the FLRW equations take the form

$$6 \left(\frac{\dot{a}^2 + k}{a^2} \right) = \kappa_5 \alpha \rho^{(h)}, \quad (8)$$

$$3 \left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}^2 + k}{a^2} \right) \right] = -\kappa_5 \alpha p^{(h)}, \quad (9)$$

$$\frac{3l^2}{\kappa_5} \left(\frac{\dot{a}^2 + k}{a^2} \right)^2 = \rho^{(h)}, \tag{10}$$

$$\frac{3l^2}{\kappa_5} \frac{\ddot{a}}{a} \left(\frac{\dot{a}^2 + k}{a^2} \right) = -p^{(h)}, \tag{11}$$

$$\left(\frac{\dot{a}^2 + k}{a^2} \right) \left[(h - h(0)) \frac{\dot{a}}{a} + \dot{h} \right] = 0. \tag{12}$$

These field equations were completely resolved in reference [1] for the age of dark energy, where was found that the field h^a has a similar behavior of a cosmological constant.

(ii) The equations (8)-(12) have solutions that describe an accelerated expansion for the three possible cosmological models of the universe. Namely, spherical expansion ($k = 1$), flat expansion ($k = 0$) and hyperbolic expansion ($k = -1$) when the constant α is greater than zero. This mean that the FRW–Einstein–Chern–Simons field equations have as a of their solutions an universe in accelerated expansion. This result allows us to conjecture that these solutions are compatible with the era of Dark Energy and that the energy-momentum tensor for the field h^a corresponds to a form of positive cosmological constant.

In summary in Refs. [1,5] were studied the implications of replacing in the action $S = S_g + S_M$ the Einstein–Hilbert action by the Einstein–Chern–Simons action on the cosmological evolution for a Friedmann–Lemaître–Robertson–Walker metric (FLRW). In the case that the matter action S_M is the action for a perfect fluid, was found that the FRW–Einstein–Chern–Simons field equations have solutions that describe an accelerated expansion for the three possible cosmological models of the universe.

On the other hand, in Ref. [6] was found that the non-relativistic limit of Einstein–Chern–Simons gravity action is given by the so called Newton–Chern–Simons gravity action. This action is invariant under the so called non-relativistic algebra $\mathcal{G}\mathfrak{B}_5$, which can be obtained as the non-relativistic limit of the generalized Poincaré algebra \mathfrak{B}_5 .

One of the purpose of this article is to find a non-relativistic limit of the results found in references [1,5], i.e., some cosmological solutions for the field equations which can be obtained from the Newton–Chern–Simons action studied in Ref. [6].

This paper is organized as follows: In Section 2 we obtain the field equations for the Lagrangian $L = L_{\text{ChS}}^{(5)} + \kappa L_M$, where $L_{\text{ChS}}^{(5)}$ is the Newton–Chern–Simons Lagrangian and L_M is the corresponding matter Lagrangian. These field equations correspond to the non-relativistic limit of the field equations studied in Refs. [1,5]. In Section 3 we find the field equations for a Newton–Chern–Simons cosmology. In Section 4 it is shown that the Newton–Chern–Simons cosmology is a sort of analogue of the projectable version of the Hořava–Lifshitz theory in (3 + 1)-dimensions, although one of the terms is not present. Solutions and their asymptotic limits are found, which show interesting properties. In particular a phantom solution with a future singularity reminiscent of a Little Big Rip future singularity is obtained. Finally, a brief revision of the adiabaticity in the cosmic evolution is made.

2. Newton–Chern–Simons gravity

In this section we will make a brief review of the so-called Newton–Chern–Simons gravity. The non-relativistic algebra $\mathcal{G}\mathfrak{B}_5$ has the following commutation relation [6] (see also [7]),

$$\begin{aligned} [J_{ij}, J_{kl}] &= \eta_{kj} J_{il} + \eta_{lj} J_{ki} - \eta_{ki} J_{jl} - \eta_{li} J_{kj}, \\ [J_{ij}, K_k] &= \eta_{jk} K_i - \eta_{ik} K_j, \quad [K_i, P_j] = -\delta_{ij} M, \end{aligned}$$

$$\begin{aligned}
[J_{ij}, P_k] &= \eta_{jk} P_i - \eta_{ik} P_j, \quad [K_i, H] = -P_i, \\
[J_{ij}, Z_{kl}] &= \eta_{kj} Z_{il} + \eta_{lj} Z_{ki} - \eta_{ki} Z_{jl} - \eta_{li} Z_{kj}, \\
[J_{ij}, Z_{k0}] &= \eta_{jk} Z_{i0} - \eta_{ik} Z_{j0}, \quad [K_i, Z_j] = -\delta_{ij} N, \\
[Z_{ij}, K_k] &= \eta_{jk} Z_{i0} - \eta_{ik} Z_{j0}, \quad [K_i, Z_0] = -Z_i, \\
[J_{ij}, Z_k] &= \eta_{jk} Z_i - \eta_{ik} Z_j, \quad [Z_{i0}, P_j] = -\delta_{ij} N, \\
[Z_{ij}, P_k] &= \eta_{jk} Z_i - \eta_{ik} Z_j, \quad [Z_{i0}, H] = -Z_i, \\
[P_i, H] &= Z_{i0}.
\end{aligned} \tag{13}$$

The one-form gauge connection A valued in the \mathcal{GB}_5 algebra is given by

$$\begin{aligned}
A &= \frac{c}{l} \tau H + \frac{1}{l} e^i P_i + \frac{c}{l} \hat{\tau} Z_0 + \frac{1}{l} h^i Z_i + \frac{1}{cl} m M + \frac{1}{cl} n N \\
&\quad + \frac{1}{c} \omega^i K_i + \frac{1}{c} k^i Z_{i0} + \frac{1}{2} \omega^{ij} J_{ij} + \frac{1}{2} k^{ij} Z_{ij},
\end{aligned} \tag{14}$$

where l and c are parameters of dimensions of length and velocity respectively. The corresponding 2-form curvature $F = dA + AA$ is then given by [6]

$$\begin{aligned}
F &= \frac{c}{l} R(H)H + \frac{1}{l} R^i(P_i)P_i + \frac{c}{l} R(Z_0)Z_0 + \frac{1}{l} R^i(Z_i)Z_i + \frac{1}{cl} R(M)M \\
&\quad + \frac{1}{cl} R(N)N + \frac{1}{c} R^i(K_i)K_i + \frac{1}{c} R^i(Z_{i0})Z_{i0} + \frac{1}{2} R^{ij}(J_{ij})J_{ij} + \frac{1}{2} R^{ij}(Z_{ij})Z_{ij},
\end{aligned} \tag{15}$$

where

$$\begin{aligned}
R(H) &= d\tau, \quad R^i(P_i) = T^i - \omega^i \tau, \\
R(Z_0) &= d\hat{\tau}, \quad R(M) = dm - \omega^i e_i, \\
R^i(Z_i) &= Dh^i - \omega^i \hat{\tau} - k^i \tau + k^i_j e^j, \\
R(N) &= dn - \omega^i h_i - k^i e_i, \\
R^i(Z_{i0}) &= Dk^i + v^2 e^i \tau + k^i_j \omega^j, \quad R^i(K_i) = D\omega^i, \\
R^{ij}(J_{ij}) &= R^{ij}, \quad R^{ij}(Z_{ij}) = Dk^{ij},
\end{aligned} \tag{16}$$

with $v = c/l$, $T^i = de^i + \omega^{ij} e_j$ and $R^{ij} = d\omega^{ij} + \omega^i_k \omega^{kj}$.

From the gauge connection transformation for A , $\delta A = d\Lambda + [A, \Lambda]$, with

$$\begin{aligned}
\Lambda &= \frac{v}{l} \zeta^0 H + \frac{1}{l} \zeta^i P_i + \frac{v}{l} \rho^0 Z_0 + \frac{1}{l} \rho^i Z_i + \frac{1}{vl} \sigma M + \frac{1}{vl} \gamma N \\
&\quad + \frac{1}{v} \lambda^i K_i + \frac{1}{v} \chi^i Z_{i0} + \frac{1}{2} \lambda^{ij} J_{ij} + \frac{1}{2} \chi^{ij} Z_{ij},
\end{aligned} \tag{17}$$

it is direct to find the variations of the different gauge fields [6]

$$\begin{aligned}
\delta \tau &= d\zeta^0, \quad \delta e^i = D\zeta^i - \omega^i \zeta^0 - \lambda^{ij} e_j + \tau \lambda^i, \\
\delta h^i &= D\rho^i - \omega^i \rho^0 - \lambda^{ij} h_j + h^0 \lambda^i + k^{ij} \zeta_j - k^i \zeta^0 - \chi^{ij} e_j + \tau \chi^i, \\
\delta m &= d\sigma - \omega^i \zeta_i + e^i \lambda_i, \quad \delta \omega^i = D\lambda^i - \lambda^{ij} \omega_j, \\
\delta n &= d\gamma - k^i \zeta_i + h^i \lambda_i - \omega^i \rho_i + e^i \chi_i, \quad \delta h^0 = d\rho^0, \\
\delta k^i &= D\chi^i - \lambda^{ij} k_j - \chi^{ij} \omega_j + k^{ij} \lambda_j + e^i \zeta^0 - \zeta^i \tau, \quad \delta \omega^{ij} = D\lambda^{ij}, \\
\delta k^{ij} &= D\chi^{ij} + k^i_k \lambda^{kj} + k^j_k \lambda^{ik},
\end{aligned} \tag{18}$$

where the derivative D is covariant with respect to the J -transformations.

From (18) we can see that only the gauge fields e_μ^i , τ_μ , m_μ , h_μ^i , h_μ^0 and n_μ transform under P and H transformations. These are the fields that should remain independent, while the remaining fields will be dependent upon the aforementioned fields. This can be achieved with the following constraints

$$\begin{aligned} R(H) &= d\tau = 0, \quad R^i(P_i) = T^i - \omega^i \tau = 0, \\ R(M) &= dm - \omega^i e_i = 0, \quad R(Z_0) = dh^0 = 0, \\ R^i(Z_i) &= Dh^i - \omega^i h^0 - k^i \tau + k_j^i e^j = 0, \\ R(N) &= dn - \omega^i h_i - k^i e_i = 0. \end{aligned} \tag{19}$$

Using the subspaces separation method introduced in Ref. [4], was found that, except for surface terms, the so called Newton–Chern–Simons lagrangian is given by

$$\begin{aligned} L_{\text{NRChS}} &= \alpha_1 \varepsilon_{ijkl} \left(-2R^{ij} T^k \omega^l - \frac{4}{3} R^{ij} \omega^k \omega^l \tau + 2R^{ij} D\omega^k e^l - R^{ij} R^{kl} m \right) \\ &\quad + \alpha_3 \varepsilon_{ijkl} \left(\frac{4}{3} v^2 R^{ij} e^k e^l \tau - 2R^{ij} Dh^k \omega^l - \frac{4}{3} R^{ij} k^k \omega^l \tau - \frac{4}{3} R^{ij} \omega^k \omega^l \hat{\tau} \right. \\ &\quad + 2R^{ij} D\omega^k h^l - \frac{4}{3} Dk^{ij} T^k \omega^l - Dk^{ij} \omega^k \omega^l \tau - R^{ij} k^{kl} dm - \frac{2}{3} R^{ij} k^{kl} e^m \omega_m \\ &\quad - \frac{2}{3} R^{ij} \omega_m k^{ml} m - \frac{4}{3} k^{ij} T^k D\omega^l - k^{ij} D\omega^k \omega^l \tau - 2R^{ij} T^k k^l - \frac{4}{3} R^{ij} \omega^k k^l \tau \\ &\quad \left. + \frac{2}{3} R^{ij} k^{km} \omega_m e^l + \frac{2}{3} \omega_m^i k^{jm} D\omega^k e^l - R^{ij} R^{kl} n - 2R^{ij} \omega^{km} k_m e^l \right), \end{aligned} \tag{20}$$

where ν , α_1 , α_3 are parameters of the theory and κ is a constant (for detail see [1,5,6]).

In the next section we will consider obtaining the equations of motion associated with the action whose Lagrangian is given by the eq. (20).

3. Newton–Chern–Simons field equations

In presence of matter, the complete Lagrangian of the theory is

$$L = \kappa L_M + L_{\text{NRChS}} \tag{21}$$

where L_{NRChS} is the Newton–Chern–Simons lagrangian given in (20) and L_M is the corresponding matter Lagrangian.

The field equations obtained from the action (21) are given by

$$\varepsilon_{ijkl} \left(-\frac{4}{3} \alpha_1 R^{ij} \omega^k \omega^l + \frac{4}{3} \alpha_3 v^2 R^{ij} e^k e^l \right) = \kappa \frac{\delta L_M}{\delta \tau}, \tag{22}$$

$$\frac{4}{3} \alpha_3 \varepsilon_{ijkl} R^{ij} \omega^k \omega^l = -\kappa \frac{\delta L_M}{\delta \hat{\tau}}, \tag{23}$$

$$2\varepsilon_{ijkl} \left(\alpha_1 R^{ij} D\omega^k - \frac{4}{3} \alpha_3 v^2 R^{ij} e^k \tau \right) = \kappa \frac{\delta L_M}{\delta e^l}, \tag{24}$$

$$2\alpha_3 \varepsilon_{ijkl} R^{ij} D\omega^k = \kappa \frac{\delta L_M}{\delta h^l}, \tag{25}$$

$$a_1 \varepsilon_{ijkl} R^{ij} R^{kl} = -\kappa \frac{\delta L_M}{\delta m}, \quad (26)$$

$$a_3 \varepsilon_{ijkl} R^{ij} R^{kl} = -\kappa \frac{\delta L_M}{\delta n}, \quad (27)$$

$$4\varepsilon_{ijkl} \left(\frac{2}{3} \alpha_1 R^{ij} \omega^k \tau - \alpha_1 R^{ij} T^k + \frac{2}{3} \alpha_3 R^{ij} \omega^k \hat{\tau} - \alpha_3 R^{ij} Dh^k \right) = \kappa \frac{\delta L_M}{\delta \omega^l}, \quad (28)$$

$$\begin{aligned} \varepsilon_{ijkl} \left(-2\alpha_1 R^{km} e_m \omega^l - 4\alpha_1 T^k D\omega^l - \frac{4}{3} \alpha_1 \omega^k \omega^l d\tau - \frac{8}{3} \alpha_1 D\omega^k \omega^l \tau + 2\alpha_1 R^{km} \omega_m e^l \right. \\ \left. - 2\alpha_1 R^{kl} dm + \frac{8}{3} v^2 \alpha_3 T^k e^l \tau + \frac{4}{3} \alpha_3 e^k e^l d\tau - 2\alpha_3 R^{km} h_m \omega^l - 4\alpha_3 Dh^k D\omega^l \right. \\ \left. - \frac{4}{3} \alpha_3 \omega^k \omega^l d\hat{\tau} - \frac{8}{3} \alpha_3 D\omega^k \omega^l \hat{\tau} + 2\alpha_3 R^{km} \omega_m h^l - \alpha_3 R^{kl} dn \right) = \kappa \frac{\delta L_M}{\delta \omega^{ij}}, \end{aligned} \quad (29)$$

where we have imposed the $k^{ij} = k^i = 0$ conditions, and used

$$T_i = * \left(\frac{\delta L_M}{\delta e^i} \right), \quad T_0 = * \left(\frac{\delta L_M}{\delta \tau} \right).$$

From (19) and Bianchi identities we find

$$DT^i = D\omega^i \tau, \quad R^{ij} e_j = D\omega^i \tau, \quad R_{[\lambda\mu}{}^{ij} e_{\nu]j} = (D_{[\lambda} \omega_{\mu]})^i \tau_{\nu}, \quad (30)$$

$$e_{[\lambda i} (D_{\mu} \omega_{\nu]})^i = 0. \quad (31)$$

For simplicity we will assume that the torsion vanishes. In this case $\delta L_M / \delta \omega^{ij} = 0$. Using the constraint (19) we find that (28) takes the form

$$\begin{aligned} 4\varepsilon_{ijkl} \left(\frac{\alpha_1}{3} R^{ij} \omega^k \tau + \frac{\alpha_3}{3} R^{ij} \omega^k \hat{\tau} \right) = 0, \\ \hat{\tau} = -\frac{\alpha}{3} \tau. \end{aligned} \quad (32)$$

Introducing (30), (31) in (29) we have

$$\begin{aligned} \varepsilon_{ijkl} \left(-\frac{10\alpha_1}{3} D\omega^k \omega^l \tau + 2\alpha_1 R^{km} \omega_m e^l - \frac{8}{3} v^2 \alpha_3 \omega^k \tau e^l - \alpha_1 R^{kl} dm \right. \\ \left. + 2\alpha_3 R^{km} \omega_m h^l - \frac{10}{3} \alpha_3 D\omega^k \omega^l \hat{\tau} - \alpha_3 R^{kl} dn \right) \\ = 0. \end{aligned} \quad (33)$$

Since $\tau^2 = 0$ we can write

$$\varepsilon_{ijkl} \left(-\frac{13\alpha_1}{3} D\omega^k \omega^l \tau - \frac{13}{3} \alpha_3 D\omega^k \omega^l \hat{\tau} \right) = 0, \quad (34)$$

which means that this equation is satisfied identically and therefore the space is a flat manifold as can be seen from the equations (26), (27).

Introducing eqs. (23) in (22) and (25) in (24), we obtain

$$\begin{aligned} \varepsilon_{ijkl} R^{ij} e^k e^l = \frac{6}{v^2} \left(k_1 \frac{\delta L_M}{\delta \tau} - \alpha k_2 \frac{\delta L_M}{\delta \hat{\tau}} \right), \\ \varepsilon_{ijkl} R^{ij} e^k \tau = \frac{3}{v^2} \left(k_1 \frac{\delta L_M}{\delta e^l} - \alpha \frac{\delta L_M}{\delta h^l} \right). \end{aligned} \quad (35)$$

Taking into account that

$$*(T_0)\delta\tau = \det(g)\delta_\delta^\sigma T^0_0 \delta\tau^\delta_\sigma dx^5, \tag{36}$$

$$\varepsilon_{ijkl0}R^{ij}e^ke^l\delta\tau = 2\det(g)(\delta_\delta^\sigma R - 2R^\sigma_\delta)\delta\tau^\delta_\sigma dx^5, \tag{37}$$

$$\varepsilon_{ijkl0}R^{ij}e^k\tau\delta e^l = -2\det(g)(\delta_\delta^\sigma R - 2R^\sigma_\delta)\delta e^\delta_\sigma dx^5, \tag{38}$$

and using $T^{(h)}_{00} = \rho^{(h)}$, $T^{(h)}_{ii} = 4p^{(h)}/c^2$, we find (with $R = 0$)

$$R_{00} = \frac{3}{2v^2}(k_1\rho^{(e)} - \alpha k_2\rho^{(h)}), \tag{39}$$

$$R_{00} = \frac{3}{c^2v^2}(k_1p^{(e)} - \alpha k_2p^{(h)}), \tag{40}$$

where (39) coincides with the results found in [6]

$$\nabla^2\phi = \frac{3}{2v^2}(k_1\rho^{(e)} - \alpha k_2\rho^{(h)}), \tag{41}$$

with $v = c/l$, $\beta_1 = \beta_2 = \kappa$, $k_1 = \kappa/8\alpha_3 = 8\pi G_5$, $k_2 = \kappa/24\alpha_3$, $\alpha = 3\alpha_1/\alpha_3$, $k_1 = 3k_2$. From (39), (40) we have,

$$2k_1p^{(e)} - 2\alpha k_2p^{(h)} = (k_1\rho^{(e)} - \alpha k_2\rho^{(h)})c^2, \tag{42}$$

$$2p^{(e)} - \frac{2\alpha}{3}p^{(h)} = \left(\rho^{(e)} - \frac{\alpha}{3}\rho^{(h)}\right)c^2.$$

Defining a density and an effective pressure as

$$p = \frac{p^{(e)}}{2} - \frac{\alpha}{6}p^{(h)}, \tag{43}$$

$$\rho = \frac{\rho^{(e)}}{2} - \frac{\alpha}{6}\rho^{(h)},$$

we find

$$p = \frac{\rho c^2}{2}, \tag{44}$$

and from (32), we have

$$\rho^{(h)} = -\frac{3}{\alpha}\rho^{(e)}. \tag{45}$$

From (39), (40) we can see

$$R_{00} = \frac{3}{4v^2}\left(k_1\rho^{(e)} - \alpha k_2\rho^{(h)} + 2k_1\frac{p^{(e)}}{c^2} - 2\alpha k_2\frac{p^{(h)}}{c^2}\right), \tag{46}$$

$$R_{00} = \nabla^2\phi = \frac{3k_1}{2v^2}\left(\rho + \frac{2p}{c^2}\right).$$

On the other hand, the interaction between the fluids is described by the following state equations

$$p^{(e)} = \omega^{(e)}\rho^{(e)}c^2, \tag{47}$$

$$p^{(h)} = \omega^{(h)}\rho^{(h)}c^2 = -\frac{3}{\alpha}\omega^{(h)}\rho^{(e)}c^2,$$

$$2k_1\omega^{(e)}\rho^{(e)} - 2\alpha k_2\omega^{(h)}\rho^{(h)} = k_1\rho^{(e)} - \alpha k_2\rho^{(h)},$$

$$2\left(k_1\omega^{(e)} + 3k_2\omega^{(h)}\right)\rho^{(e)} = (k_1 + 3k_2)\rho^{(e)}, \quad (48)$$

$$\omega^{(h)} = \frac{(k_1 + 3k_2)}{6k_2} - \frac{k_1}{3k_2}\omega^{(e)},$$

$$= 1 - \omega^{(e)}. \quad (49)$$

In the next section we will study a possible non-relativistic version of the results obtained in Ref. [1].

4. Newton–Chern–Simons cosmology

Following the formalism used in [8], we denote with (t, x^i) , the local coordinates where $i = 1, 2, 3, 4$ and $\tau = dx^0$, $h = \delta^{ij}\partial_i \otimes \partial_j$ are the temporal and spatial metric respectively. The matter is modeled as an ideal fluid with velocity u , which is a timelike unit vector. The vorticity $\Omega^{\alpha\beta}$ and the (rate of) strain $\Theta^{\alpha\beta}$ relative to a timelike unit vector field V , where $\tau(V) = 1$, i.e., $\tau_\alpha = g_{\alpha\beta}V^\beta$, are given by

$$\Omega^{\alpha\beta} = \frac{1}{2}(u_{;\lambda}^\alpha h^{\lambda\beta} - u_{;\lambda}^\beta h^{\lambda\alpha}),$$

$$\Theta^{\alpha\beta} = \frac{1}{2}(u_{;\lambda}^\alpha h^{\lambda\beta} + u_{;\lambda}^\beta h^{\lambda\alpha}). \quad (50)$$

The expansion rate and the (rate of) shear is the trace-free part of the strain are given by

$$\theta = h^{\alpha\beta}\Theta_{\alpha\beta}, \quad \sigma = \Theta - \frac{1}{4}\theta h \quad (51)$$

respectively.

It is possible to show that $\theta = u^\sigma_{;\sigma}$ and that the covariant derivative of the velocity can be decomposed as [8]

$$h_{\alpha\lambda}u_{;\beta}^\lambda = \Theta_{\alpha\beta} + \Omega_{\alpha\beta} + h_{\alpha\rho}V^\lambda u_{;\lambda}^\rho g_{\beta\sigma}V^\sigma, \quad (52)$$

and with the help of this last equation we can obtain the so called Raychaudhuri equation in the Newton–Chern–Simons gravity. Following Ref. [9] we start from the known identity (see also [10])

$$u_{;\beta;\gamma}^\alpha - u_{;\gamma;\beta}^\alpha = R_{\sigma\gamma\beta}^\alpha u^\sigma,$$

$$u_{;\alpha}^\beta u_{;\beta}^\alpha = \left(u_{;\beta}^\beta u_{;\alpha}^\alpha\right)_{;\alpha} - u_{;\alpha}^\beta u_{;\beta}^\alpha - R_{\alpha\beta} u^\alpha u^\beta, \quad (53)$$

where the two first terms on the right are given by [8]

$$\left(u_{;\beta}^\beta u_{;\alpha}^\alpha\right)_{;\alpha} = \text{div}(\nabla_\alpha u),$$

$$u_{;\alpha}^\beta u_{;\beta}^\alpha = h^{\rho\alpha}h^{\sigma\beta}(\Theta_{\rho\beta}\Theta_{\sigma\alpha} + \Omega_{\rho\beta}\Omega_{\sigma\alpha}), \quad (54)$$

and the last terms on the right is given by

$$R_{\alpha\beta}u^\alpha u^\beta = \frac{3k_1}{2\nu^2} \left(\rho + \frac{2p}{c^2} \right), \quad (55)$$

where we have used (46) together to the equations

$$\begin{aligned}
 R_{\alpha\beta} &= \frac{3k_1}{2v^2} \left(\rho + \frac{2p}{c^2} \right) \tau_\alpha \tau_\beta, \\
 \tau_\alpha \tau_\beta &= g_{\alpha\sigma} g_{\beta\rho} u^\sigma u^\rho.
 \end{aligned}
 \tag{56}$$

These results allow us to find the five dimensional Raychaudhuri equation for the Newton–Chern–Simons gravity

$$\operatorname{div}(\nabla_u u) = \nabla_u \theta + \frac{1}{4} \theta^2 + \sigma^{\alpha\beta} \sigma_{\alpha\beta} - \Omega^{\alpha\beta} \Omega_{\alpha\beta} + \frac{3k_1}{2v^2} \left(\rho + \frac{2p}{c^2} \right).
 \tag{57}$$

4.1. FLRW background

In this section we study the non-relativistic FLRW equations in the context of the Newton–Chern–Simons gravity.

The calculation of the Ricci tensor from its definition leads to the following result

$$\begin{aligned}
 R_{00} &= -\frac{1}{2} (h^{ij} \dot{h}_{ij})_{,0} - \frac{1}{4} h^{ij} \dot{h}_{jk} h^{kl} \dot{h}_{li} + 2h^{ij} \kappa_{0j,i} + \kappa^{ij} \kappa_{ij}, \\
 &= \frac{3k_1}{2v^2} \left(\rho + \frac{2p}{c^2} \right), \\
 R_{0i} &= h^{jk} \kappa_{ik,j} = 0, \\
 R_{ij} &= 0.
 \end{aligned}
 \tag{58}$$

The first equation is equivalent to the Raychaudhuri equation (57) for $u = V$, while the second equation is equivalent to $\Omega^{ij}_{,j} = 0$ for $u = V$, since for any u

$$\Omega_{\alpha\beta} = \frac{1}{2} (h_{ac} u^c_{,b} - h_{bc} u^c_{,a} - 2\kappa_{ab}).
 \tag{59}$$

For the other kinematical quantities we find

$$\Theta_{ab} = \frac{1}{2} (h_{ac} u^c_{,b} - h_{bc} u^c_{,a} + \dot{h}_{ab}),
 \tag{60}$$

$$\theta = u^a_{,a} + \frac{1}{2} h^{ab} \dot{h}_{ab},
 \tag{61}$$

$$\begin{aligned}
 \operatorname{div}(\nabla_u u) &= \dot{u}^a_{,a} + 2h^{ab} \kappa_{0a,b} + 2h^{bc} \kappa_{ac} u^a_{,b} \\
 &\quad + h^{bc} u^a_{,b} \dot{h}_{ac} + u^a u^b_{,ab} + u^a_{,b} u^b_{,a}.
 \end{aligned}
 \tag{62}$$

For an ideal fluid with pressure, the continuity equation and the Euler equation are respectively given by [11],

$$\dot{\rho} + \left[\left(\rho + \frac{p}{c^2} \right) u^i \right]_{,i} + \frac{1}{2} h^{ij} \dot{h}_{ij} \left(\rho + \frac{p}{c^2} \right) = 0,
 \tag{63}$$

and

$$\dot{u}^i + u^j u^i_{,j} + 2h^{ij} \kappa_{0j} + 2u^j \left(\frac{1}{2} h^{lk} \dot{h}_{kj} + h^{ik} \kappa_{jk} \right) + \left(\rho + \frac{p}{c^2} \right)^{-1} h^{ij} p_{,j} = 0.
 \tag{64}$$

When $\kappa_{ij} = 0$, we find that the equations (63), (64) and (58) take the form

$$\begin{aligned}
\dot{\rho} + \left[\left(\rho + \frac{p}{c^2} \right) u^i \right]_{;i} &= 0, \\
\dot{u}^i + u^j u^i_{;j} &= - \left(\rho + \frac{p}{c^2} \right)^{-1} p_{;i} + g^i, \\
-g^i_{;i} &= \frac{3k_1}{2v^2} \left(\rho + \frac{2p}{c^2} \right),
\end{aligned} \tag{65}$$

where $g^i = -2\kappa_{0i}$. So that we have arrived to the equations for Newton–Chern–Simons gravity coupled to an ideal fluid.

If now we assume that ρ and p are only functions of t (homogeneity), then the Euler equation implies

$$\nabla_u u = - \left(\rho + \frac{p}{c^2} \right)^{-1} \text{div}(ph) = 0, \tag{66}$$

and the continuity equation (63) shows that $u^i_{;i}$ depends only of the time t . These results lead to the following simplifications of the equation (57)

$$\dot{\theta} + \frac{1}{4}\theta^2 + \sigma^{ab}\sigma_{ab} - \Omega^{ab}\Omega_{ab} + \frac{3k_1}{2v^2} \left(\rho + \frac{2p}{c^2} \right) = 0. \tag{67}$$

Since we have used the fact that θ is a function that depends only on time, we have that (61) and (63) imply

$$\dot{\rho} + \theta \left(\rho + \frac{p}{c^2} \right) = 0. \tag{68}$$

Let us now consider a homogeneous and isotropic flat-FLRW background in the context of Newton–Chern–Simons gravity. This model is found using the following Ansatz

$$V = u, \quad h_{ij} = a^2(t)\delta_{ij}, \quad \Omega = 0, \tag{69}$$

which leads to

$$\theta = 4\frac{\dot{a}}{a}, \quad \sigma_{ab} = 0, \quad \dot{\theta} = 4 \left(\frac{\ddot{a}a - \dot{a}^2}{a^2} \right). \tag{70}$$

Here, a is the cosmic scale factor. Introducing these results in the equations (68) and (67) we obtain

$$\begin{aligned}
\dot{\theta} + \frac{1}{4}\theta^2 &= -\frac{3k_1}{2v^2} \left(\rho + \frac{2p}{c^2} \right), \\
\frac{\ddot{a}}{a} &= -\frac{3k_1}{8v^2} \left(\rho + \frac{2p}{c^2} \right).
\end{aligned} \tag{71}$$

In the following Section we will use the equations (71) to visualize the cosmologies that can be derived from the present five-dimensional scheme

4.2. Cosmological solutions

From now on we use units $k_1 = 8\pi G_5 = 1 = c$ and $v = 1/l$. The equations (71) are the conservation equation and the equation for the acceleration, respectively, where p is the pressure, ρ the energy density, $\theta = 4H$, $H = \dot{a}/a$ is the Hubble parameter and a is the cosmic scale factor. We immediately visualize the absence of the Friedmann constraint. This situation is analogous

to what happens in the projectable version of the Hořava–Lifshitz theory in (3+1)-dimensions [12]. From equations (71) it is possible to obtain the first integral

$$\frac{8}{3}v^2H^2 = \rho + \frac{C_0}{a^2}, \tag{72}$$

where C_0 is an integration constant. The term C_0/a^2 is not dark matter in the usual sense, but gravitationally behaves like a fluid whose pressure is $p = -(1/2)\rho$ which, as we shall see, corresponds to an evolutionary scheme with zero acceleration, which is a Milne universe. In General Relativity in (3+1)-dimensions, dark matter corresponds to $\rho(a) = \rho(a_0)(a_0/a)^3$. A term of this form is present in the Hořava–Lifshitz theory in (3+1)-dimension through $C(t)/a^3$. In Ref. [13], a realization of the Hořava–Lifshitz gravity as the dynamical Newton–Cartan geometry was discussed.

The scheme of equations in the projectable version of Hořava–Lifshitz theory in (3+1)-dimensions is given by the equations

$$\begin{aligned} \dot{\rho} + 3H(\rho + p) &= -Q, \\ 3\eta(2\dot{H} + 3H^2) &= p, \end{aligned} \tag{73}$$

where η is a dimensionless constant parameter associated with invariance under diffeomorphisms and Q represents the amount of energy non-conservation [14]. Here there is no Friedmann constraint. From these equations, it is straightforward to find the first integral

$$3\eta H^2 = \rho + \frac{C(t)}{a^3} \quad \text{with} \quad C(t) = C_0 + \int_{t_0}^t dt a^3 Q, \tag{74}$$

and $C(t)/a^3$ is not a real dark matter, but gravitationally it behaves like a fluid with $p = 0$.

Now, we return to the equations given in (71), which can be written in the form

$$\dot{\rho} + 4H(\rho + p) = 0, \tag{75}$$

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{3}{8v^2}(\rho + 2p). \tag{76}$$

Considering the barotropic relation $p = \omega\rho$, we can write (75) and (76) in the form

$$\dot{\rho} + 4H(1 + \omega)\rho = 0, \tag{77}$$

from which it is direct to obtain

$$\rho(a) = \rho(a_0) \left(\frac{a_0}{a}\right)^{4(1+\omega)}, \tag{78}$$

and

$$\dot{H} + H^2 = -\frac{3}{4v^2} \left(\omega + \frac{1}{2}\right)\rho, \tag{79}$$

from which we can see that $\ddot{a} \geq 0$ and that $\omega \leq -1/2$.

The equations (72) and (77), with $1 + z = a_0/a$ where z is the redshift parameter, implies that the Hubble parameter can be written as

$$H(z) = \sqrt{\frac{3\rho(0)}{8v^2}(1+z)^{4(1+\omega)} + \left(H^2(0) - \frac{3\rho(0)}{8v^2}\right)(1+z)^2}. \tag{80}$$

We consider now some particular cases:

(1) **Case when $\omega = 0$.** In this case $\rho(z) = \rho(0)(1+z)^4$ and

$$H(z) = \sqrt{\frac{3\rho(0)}{8v^2}(1+z)^2 + \left(H^2(0) - \frac{3\rho(0)}{8v^2}\right)(1+z)}, \quad (81)$$

where we can see that

$$H(z \rightarrow \infty) \rightarrow \infty \quad \text{and} \quad H(z \rightarrow -1) \rightarrow 0. \quad (82)$$

(2) **Case when $\omega = -1/2$.** In this case $\rho(z) = \rho(0)(1+z)^2$ and

$$H(z) = H(0)(1+z) \implies \ddot{a} = 0, \quad (83)$$

which correspond to a Milne universe.

(3) **Case when $\omega = -1$.** In this case $\rho(z) = \rho(0) = \text{const.}$, but according to (80)

$$H(z) = \sqrt{\frac{3\rho(0)}{8v^2} + \left(H^2(0) - \frac{3\rho(0)}{8v^2}\right)(1+z)^2}, \quad (84)$$

from which we see

$$H(z \rightarrow \infty) \rightarrow \infty \quad \text{and} \quad H(z \rightarrow -1) \rightarrow \sqrt{\frac{3\rho(0)}{8v^2}}, \quad (85)$$

and unlike to General Relativity in (3+1)-dimensions, we have $H(z) \neq \text{const.}$ for $\rho(z) = \text{const.}$

(4) **Case when $\omega < -1$.** In this case $\rho(z) = \rho(0)(1+z)^{-4(|\omega|-1)}$ and

$$H(z \rightarrow \infty) \rightarrow \sqrt{H^2(0) - \frac{3\rho(0)}{8v^2}}(1+z) \rightarrow \infty, \quad (86)$$

$$H(z \rightarrow -1) \rightarrow \sqrt{\frac{3\rho(0)}{8v^2}}(1+z)^{-2(|\omega|-1)} \rightarrow \infty. \quad (87)$$

We note that $H(z \rightarrow -1)$ diverges, when $\rho(z \rightarrow -1)$. However this not happens in a finite time how it happens in General Relativity in (3 + 1)-dimensions when we think, for instance, in a Little Big Rip future singularity [15].

Now, the first and second law of thermodynamics tell us, respectively,

$$TdS = d(\rho V) + pdV = (\rho + p)dV + Vd\rho, \quad (88)$$

$$T \frac{dS}{dt} = V[\dot{\rho} + 4H(\rho + p)]. \quad (89)$$

Since, according to (75), $\dot{\rho} + 4H(\rho + p) = 0$, we have an adiabatic evolution. This means that in Newton–Chern–Simons cosmology there is no Friedmann constraint and we have adiabatic evolution. On the other hand in Hořava–Lifshitz theory in (3+1)-dimensions there is no Friedmann constraint and, unlike of Newton–Chern–Simons cosmology, the evolution is non-adiabatic since $\dot{\rho} + 4H(\rho + p) = -Q \neq 0$ and therefore $dS/dt \neq 0$.

In summary we can say that we have presented cosmological schemes from the five-dimensional Einstein-Chern-Simons gravity theory. It could be interesting to use some process of compactification to project these results to (3 + 1)-dimensions and then compare them with the results obtained in the context of general relativity.

5. Final remarks

We have considered a five-dimensional action $S = \int L_{\text{ChS}}^{(5)} + \kappa L_M$ which is composed of a gravitational sector and a matter sector, where the gravitational sector is given by a Newton–Chern–Simons gravity action instead of the Einstein–Hilbert action and the matter sector is described by a perfect fluid. We have studied the implications of replacing the Einstein–Hilbert action by the Newton–Chern–Simons action on the cosmological evolution for a FLRW metric.

We have showed that the Newton–Chern–Simons cosmology is a sort of analogue of the projectable version of the Hořava–Lifshitz theory in (3+1)-dimension, although a term that contains Q is not present. We have found solutions and their asymptotic limits which show interesting properties. In addition, a phantom solution with a future singularity reminiscent of a Little Big Rip future singularity has been obtained. Finally, a brief revision of the adiabaticity in the cosmic evolution was made.

As we said at the end of the previous section, an interesting thing would be to do a compactification of five to four-dimensions in order to obtain generalized non-relativistic cosmologies to be compared with the respective schemes studied in the context of general relativity (work in progress).

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