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Extraction of $|V_{cb}|$ from two-body hadronic B decays

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We propose a method of extracting the Cabibbo–Kobayashi–Maskawa matrix element $|V_{cb}|$ from two-body hadronic decay processes of $B \to DK$ with precisely determined form factors of B meson semi-leptonic decays. The amplitude $\mathcal{M}(\bar{B}^0 \to D^+K^-)$ which does not include the effect of hadronic final state interactions can be theoretically evaluated by using factorization and form factors of semi-leptonic B decays. We can obtain all the amplitudes in an isospin relation $\mathcal{A}(B^- \to D^0K^-) = \mathcal{A}(\bar{B}^0 \to D^+K^-) + \mathcal{A}(\bar{B}^0 \to D^0\bar{K}^0)$, including the effect of hadronic final state interactions as well as $|V_{cb}|$, using the experimental data of branching fractions of these three processes with a truncation of the states which contribute to the hadronic final state interactions. The extracted value of $|V_{cb}|$ is $(37\pm6)\times10^{-3}$. The decay processes of $B \to DK^*$ and $B \to D^*K$ can also be used in the same way, and the extracted values of $|V_{cb}|$ are $(41\pm7)\times10^{-3}$ and $(42\pm9)\times10^{-3}$, respectively. This method becomes possible by virtue of recent precise determinations of the form factors of semi-leptonic B decays. The uncertainties of $|V_{cb}|$ by this method are expected to be reduced by the results of future B-factory experiments and lattice calculations.

Subject Index B51, B55, B56

1. Introduction

The precise determination of Cabibbo–Kobayashi–Maskawa (CKM) matrix elements [1,2] is one of the approaches to test the Standard Model, and to search for the physics beyond the Standard Model. The Standard Model predicts the unitarity relation of the CKM matrix,

$$\sum_{i=u,c,t} V_{ib}^* V_{id} = 0, \tag{1}$$

which gives a triangle in a complex plane. The existence of physics beyond the Standard Model may violate this relation. The sides and angles of this triangle will be precisely measured using various decay processes of B mesons in future B-factories [3,4]. In this work we focus on the determination of $|V_{cb}|$.

There are two main methods to extract the value of $|V_{cb}|$ from semi-leptonic B mesons. The method using inclusive decay data gives $|V_{cb}| = (42.00 \pm 0.65) \times 10^{-3}$ [5], and the method using exclusive decay data gives $|V_{cb}| = (38.71 \pm 0.75) \times 10^{-3}$ [6]. Though the difference between these values is within $3.3\,\sigma$, it is a problem in understanding the non-perturbative physics of QCD.[1] In fact, it has been pointed out that the proper parametrization of form factors is important [8,9]. In order to analyze exclusive decay processes $B \to D^{(*)}l\nu$ with a small amount of data, the Caprini–Lellouch–Neubert

[1] It has been pointed out that this problem cannot be solved by New Physics [7].

(CLN) parametrization of form factors [10] is precise enough. However, with much more data recently provided by the Belle Collaboration, not only q^2 distributions but also angular distributions [11], the Boyd–Grinstein–Lebed (BGL) parametrization of form factor [12] is better than the CLN parametrization, because the CLN parametrization may include about 10% errors from the absence of $\mathcal{O}(1/m_{c,b}^2)$ corrections [9]. Since the accuracy of recent lattice QCD results [13,14] is typically of the order of 1%, we need to use theoretical frameworks with correspondingly high precisions.[2]

In this work we intend to provide another method to extract $|V_{cb}|$, which may give new information to the above conflict in future. We propose that the hadronic decays of B mesons, especially two-body decays of $B \to DK$, $B \to DK^*$, and $B \to D^*K$, can be used to extract a precise value for $|V_{cb}|$ in future. The amplitude $\mathcal{M}(\bar{B}^0 \to D^+K^-)$ which does not include the effects of hadronic final state interactions can be theoretically evaluated by using the factorization, the form factors of semi-leptonic B decays, and the decay constant of the K^- meson. The form factors of semi-leptonic B decays are precisely determined by the latest Belle data [11,16] and the latest lattice QCD results [13,14] with the BGL parametrizations in Ref. [8]. The isospin symmetry provides the relation

$$\mathcal{A}(B^- \to D^0K^-) = \mathcal{A}(\bar{B}^0 \to D^+K^-) + \mathcal{A}(\bar{B}^0 \to D^0\bar{K}^0), \tag{2}$$

including the effects of hadronic final state interactions. We can extract these three amplitudes as well as the value of $|V_{cb}|$ by using the amplitude $\mathcal{M}(\bar{B}^0 \to D^+K^-)$ and the experimental values of three branching fractions, $\mathcal{B}(B^- \to D^0K^-)$, $\mathcal{B}(\bar{B}^0 \to D^+K^-)$, and $\mathcal{B}(\bar{B}^0 \to D^0\bar{K}^0)$ in Ref. [17]. In this procedure we need to truncate the states which contribute final state interactions: not including all the possible states, but including only two-body DK states. The processes of $B \to DK^*$ and $B \to D^*K$ can also be used in the same way. For $B \to D^*K$, we use the form factor obtained by the CLN parametrization with the latest data from the Belle Collaboration [11].

We emphasize that this method becomes possible only with recent precise determination of all the form factors of semi-leptonic B decay. More precise experimental data of the branching fractions of two-body hadronic B decays give a more precise value of $|V_{cb}|$. This method can be understood as an intermediate approach between inclusive and exclusive determination of $|V_{cb}|$, since it requires the use of several exclusive B-decay modes. It may be possible that this method will play an important role in the problem of $|V_{cb}|$ determinations with the results of future B-factory experiments and future precise lattice calculations, if the validity of the truncation of the states in final state interactions is established. In other words, once the value of $|V_{cb}|$ is precisely determined with semi-leptonic decays without any conflicts, this method will provide useful information to understand the final state interactions in two-body hadronic B decays.

In the next section we investigate the amplitudes of $B \to DK$ processes in detail, and propose a procedure to extract the value of $|V_{cb}|$. We also show that the same procedure applies to the processes of $B \to DK^*$ and $B \to D^*K$. In Sect. 3 we provide a numerical analysis of extracting the value of $|V_{cb}|$ from two-body hadronic B decays by our procedure. In Sect. 4 we provide a summary and discussion.

[2] The error with CNL parametrization comes from an excessive reduction of the number of parameters in form factors by using heavy quark symmetry. In fact, improvements are possible by including higher-order corrections (see, for example, Ref. [15]).

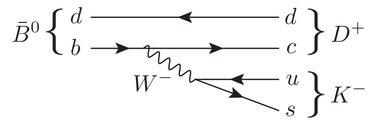

Fig. 1. Tree amplitude. For example, the M_1 and M_2 mesons are D^+ and K^-, respectively.

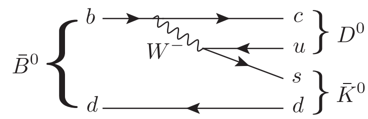

Fig. 2. Color-suppressed amplitude. For example, the M_1 and M_2 mesons are D^0 and $\bar{K}^0$, respectively.

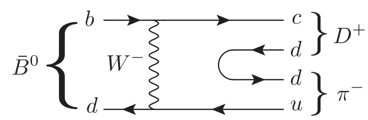

Fig. 3. Exchange amplitude where quark–antiquark pair creation occurs. For example, the M_1 and M_2 mesons are D^+ and π^-, respectively.

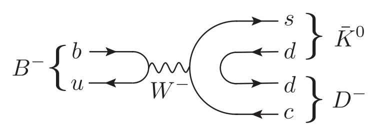

Fig. 4. W-annihilation amplitude. For example, the $\bar{B}$, M_1, and M_2 mesons are B^-, D^-, and $\bar{K}^0$, respectively.

2. Two-body hadronic decays of B mesons

Consider the hadronic two-body decay processes $\bar{B} \to M_1 M_2$, where M_1 and M_2 indicate D mesons and K or π mesons, respectively. The quark-level Feynman diagrams of these decays are classified into four topological types [18,19]. The amplitudes from the diagrams corresponding to each topological type are as follows:

(1) Tree amplitudes T: the diagrams have $b \to c$ weak current with the light degrees of freedom as spectator antiquarks of the $\bar{B}$ and M_1 mesons, and the W boson decays into the light quark–antiquark pairs which constitute the M_2 meson (see Fig. 1).
(2) Color-suppressed amplitudes C: the W boson decays into the light quark–antiquark pairs, the antiquark is included in the M_1 meson as the spectator of the c quark, and the quark constitutes the M_2 meson with the light degrees of freedom in the $\bar{B}$ meson (see Fig. 2).
(3) Exchange amplitudes E: the exchange of the W boson changes the flavor of the spectator of $\bar{B}$, and light quark–antiquark pair creation from gluons completes two mesons (see Fig. 3).
(4) W-annihilation amplitudes A: the $\bar{B}$ meson decays to a W boson and the W boson decays into a charm antiquark and a light quark, and they become constituents of M_1 and M_2 with a light quark and a light antiquark from gluons, respectively (see Fig. 4).

In this paper we do not consider the process which contains the contribution of A, since it does not include $|V_{cb}|$ and is, rather, relevant to $|V_{ub}|$.

In Table 1 we summarize all the hadronic two-body decays of $\bar{B}^0$ and B^- mesons which include the $b \to c$ transition, and the topologies of the corresponding amplitudes.

Table 1. Two-body hadronic decays and their amplitudes. Note that the decay mode $\bar{B}^0 \to D^+K^-$ is the only mode which is described by the diagram of tree topology only. The contributions of penguin diagrams are also listed.

Decay mode		Topologies			Penguin	Fraction (Γ_i/Γ) [17]
$\bar{B}^0 \to$	$D^+\pi^-$	T		E		$(2.52 \pm 0.13) \times 10^{-3}$
	$D^0\pi^0$		C	E		$(2.63 \pm 0.14) \times 10^{-4}$
	$D_s^+K^-$		C	E		$(2.7 \pm 0.5) \times 10^{-5}$
	D^+K^-	T				$(1.86 \pm 0.20) \times 10^{-4}$
	$D^0\bar{K}^0$		C			$(5.2 \pm 0.7) \times 10^{-5}$
	D^+D^-	T		E	yes	$(2.11 \pm 0.18) \times 10^{-4}$
	$D^+D_s^-$	T			yes	$(7.2 \pm 0.8) \times 10^{-3}$
$B^- \to$	$D^0\pi^-$	T	C			$(4.80 \pm 0.15) \times 10^{-3}$
	D^0K^-	T	C			$(3.74 \pm 0.16) \times 10^{-4}$
	D^0D^-	T	C		yes	$(3.8 \pm 0.4) \times 10^{-4}$
	$D^0D_s^-$	T	C		yes	$(9.0 \pm 0.9) \times 10^{-3}$

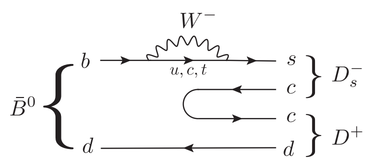

Fig. 5. Penguin diagram. The process of $\bar{B}^0 \to D_s^- D^+$ is shown as an example.

In general, several diagrams with different topologies contribute to the amplitudes for each decay process. We see that the amplitude of $\bar{B}^0 \to D^+K^-$ consists of a single diagram of topology T. The penguin diagrams (see Fig. 5) contribute only to the amplitude of $B \to DD$. For example, the amplitude of $\bar{B}^0 \to D^+D_s^-$ consists of T, with a pollution by a penguin diagram. Until the size of the contribution of the penguin diagram is clarified, we cannot use $\bar{B}^0 \to D^+D_s^-$ to extract $|V_{cb}|$ precisely.

We focus on the two-body decay $\bar{B}^0 \to D^+K^-$, which is described only by the diagram of topology T. The effective weak Hamiltonian [20] for the decay is

$$H_{\text{eff}} = \eta_{\text{EW}} \frac{G_{\text{F}}}{\sqrt{2}} V_{cb} V_{us}^* \left[C_1(\mu) O_1(\mu) + C_2(\mu) O_2(\mu)\right] + \text{h.c.}, \tag{3}$$

where η_{EW} is the electroweak correction, which represents the effects of short-distance QED correction. The factors $C_{1,2}$ are the Wilson coefficients, and $O_{1,2}$ are the current–current operators:

$$O_1 = \bar{c}^\alpha \gamma_\mu (1 - \gamma_5) b_\beta \bar{s}^\beta \gamma^\mu (1 - \gamma_5) u_\alpha, \tag{4}$$

$$O_2 = \bar{c}^\alpha \gamma_\mu (1 - \gamma_5) b_\alpha \bar{s}^\beta \gamma^\mu (1 - \gamma_5) u_\beta, \tag{5}$$

where α and β are color indices. The amplitude is given by the matrix element

$$\mathcal{A}(\bar{B}^0 \to D^+K^-) = \eta_{\text{EW}} \frac{G_{\text{F}}}{\sqrt{2}} V_{cb} V_{us}^* a_1(\mu) \left\langle D^+(p') K^-(p_K) \right| [\bar{c}\gamma_\mu(1-\gamma_5)b][\bar{s}\gamma^\mu(1-\gamma_5)u] \left| \bar{B}^0(p) \right\rangle, \tag{6}$$

where p, p', and p_K are four-momenta of $\bar{B}^0$, D^+, and K^-, respectively. The momentum p_K satisfies $q^2 = (p - p')^2 = p_K^2$. The factor $a_1(\mu) = C_2(\mu) + C_1(\mu)/3$ represents the effects of short-distance QCD correction, including short-distance non-factorizable QCD effects. The amplitude

$\mathcal{A}(\bar{B}^0 \to D^+K^-)$ also includes the effects of non-factorizable hadronic final state interactions (or rescattering effects), which are non-perturbative QCD effects. Now we introduce the amplitude $\mathcal{M}(\bar{B}^0 \to D^+K^-)$, which does not include the effects of hadronic final state interactions. The amplitude is given by factorizing the matrix element in Eq. (6), because only the diagram of topology T contributes. The final state is written by two independent asymptotic states of D^+ and K^- mesons, because we have temporarily neglected the effects of final state interactions, or long-distance non-factorizable QCD effects. The amplitude is written as

$$\mathcal{M}(\bar{B}^0 \to D^+K^-) = \eta_{\rm EW}\frac{G_{\rm F}}{\sqrt{2}}V_{cb}V_{us}^*a_1(\mu)\,\langle D^+(p')|\,\bar{c}\gamma_\mu(1-\gamma_5)b\,|\bar{B}^0(p)\rangle \times \langle K^-(p_K)|\,\bar{s}\gamma^\mu(1-\gamma_5)u\,|0\rangle\,. \tag{7}$$

The $B \to D$ part of the matrix element is given by

$$\langle D(p')|\,\bar{c}\gamma_\mu(1-\gamma_5)b\,|\bar{B}(p)\rangle = f_+(q^2)(p+p')_\mu + f_-(q^2)(p-p')_\mu, \tag{8}$$

where $f_\pm(q^2)$ are form factors of semi-leptonic decay of $\bar{B}^0$. Another matrix element is described as

$$\langle 0|\,\bar{u}\gamma_\mu(1-\gamma_5)s\,|K(p_K)\rangle = ip_{K\mu}f_{K^\pm}, \tag{9}$$

where $f_{K^\pm} = 155.6 \pm 0.4\,\mathrm{MeV}$ [17] is the decay constant of $K^\pm$ mesons. The absolute value of the amplitude is written as

$$|\mathcal{M}(\bar{B}^0 \to D^+K^-)| = \eta_{\rm EW}\frac{G_{\rm F}}{\sqrt{2}}|V_{cb}V_{us}^*|a_1(\mu)(m_{\bar{B}^0}^2 - m_{D^+}^2)f_{K^\pm}f_0(m_{K^\pm}^2), \tag{10}$$

where

$$f_0(q^2) = f_+(q^2) + \frac{q^2}{m_B^2 - m_D^2}f_-(q^2) \tag{11}$$

and the function of $f_0(q^2)$ is precisely determined in Ref. [8] for all possible q^2 regions. Notice that this amplitude depends only on one form factor, $f_0(q^2)$. If we could neglect the effect of hadronic final state interactions, the value of $|V_{cb}|$ could be straightforwardly extracted from the data of the decay rate in Ref. [17], since the decay rate is simply described as

$$\Gamma(\bar{B}^0 \to D^+K^-)|_{\text{no FSI}} = \frac{p^*}{8\pi m_B^2}|\mathcal{M}(\bar{B}^0 \to D^+K^-)|^2, \tag{12}$$

where

$$p^* = \frac{1}{2m_{\bar{B}^0}}\sqrt{\{m_{\bar{B}^0}^2 - (m_{D^+}+m_{K^-})^2\}\{m_{\bar{B}^0}^2 - (m_{D^+}-m_{K^-})^2\}}. \tag{13}$$

We obtain the value of $|V_{cb}| = (32.0 \pm 1.9) \times 10^{-3}$, which is inconsistent with the values determined by the inclusive and exclusive methods with semi-leptonic decays. This result indicates the failure of "naive factorization," and shows that the effect of hadronic final state interactions cannot be ignored and it is important to extract $|V_{cb}|$ from hadronic two-body B decays.[3]

[3] We have also neglected the effect of non-factorizable spectator quark scattering, which violates the factorization [21]. It has been shown in Ref. [22] that the effect is small in the heavy quark mass limit in the

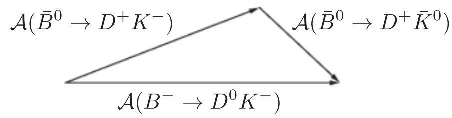

Fig. 6. Isospin relation of three amplitudes in a complex plane. We choose the direction of the amplitude $\mathcal{A}(B^- \to D^0K^-)$ as that of the real axis. Non-zero strong phases of δ_0 and δ_1 mean non-zero area of this triangle.

In order to consider the effect of hadronic final state interactions, we introduce a relation between decay amplitudes which follows from isospin symmetry. The amplitudes of $B^- \to D^0K^-$, $\bar{B}^0 \to D^+K^-$, and $\bar{B}^0 \to D^0\bar{K}^0$ are related by isospin symmetry as

$$\mathcal{A}(B^- \to D^0K^-) = \mathcal{A}(\bar{B}^0 \to D^+K^-) + \mathcal{A}(\bar{B}^0 \to D^0\bar{K}^0). \tag{14}$$

We expect that this relation should be satisfied within 1% accuracy, because the isospin breaking effect should be proportional to $(m_d - m_u)/\Lambda_{\rm QCD} \sim 0.02$ or $\alpha/\pi \sim 0.002$. We can represent this relation as a triangle on a complex plane (see Fig. 6). The isospin decompositions of these amplitudes are given by

$$\mathcal{A}(B^- \to D^0K^-) = A_1 = |A_1|e^{i\delta_1}, \tag{15}$$

$$\mathcal{A}(\bar{B}^0 \to D^+K^-) = \frac{1}{2}(A_1 + A_0) = \frac{1}{2}(|A_1|e^{i\delta_1} + |A_0|e^{i\delta_0}) \equiv \frac{1}{2}(|A_1| + |A_0|e^{i\delta_s})e^{i\delta_1}, \tag{16}$$

$$\mathcal{A}(\bar{B}^0 \to D^0\bar{K}^0) = \frac{1}{2}(A_1 - A_0) = \frac{1}{2}(|A_1|e^{i\delta_1} - |A_0|e^{i\delta_0}) = \frac{1}{2}(|A_1| - |A_0|e^{i\delta_s})e^{i\delta_1}, \tag{17}$$

where δ_0 and δ_1 are phases by the effect of hadronic final state interactions of the isospin 0 and 1 channels, respectively, and $\delta_s = \delta_0 - \delta_1$ is the physical strong phase. If there is no physical effect of hadronic final state interactions, $\delta_s = 0$ and the triangle of Fig. 6 collapses. In general, neglecting final state interactions results in not only vanishing phases of δ_0 and δ_1, but also changing the magnitudes of $|A_0|$ and $|A_1|$. If we truncate the states which contribute to the final state interactions by considering only two-body DK states, the relation

$$|\mathcal{M}(\bar{B}^0 \to D^+K^-)| = |\mathcal{A}(\bar{B}^0 \to D^+K^-)|_{\delta_{1,0}=0} = \frac{1}{2}(|A_1| + |A_0|) \tag{18}$$

is satisfied, because for each isospin channel there is only one final state. If we further include the states like $DK\pi\pi$, for example, the effect of final state interactions cannot be represented only by simple phases, and the magnitudes of $|A_0|$ and $|A_1|$ are also affected [23]. This truncation of the states, or neglecting inelastic final state interactions, is the main theoretical assumption in our method, except for isospin symmetry.

There is no justification of this assumption, since it has been known that the inelastic final state interactions are important in B decays in general [24,25]. To be precise, we need to describe $\alpha_0|A_0|$ and $\alpha_1|A_1|$ instead of naive $|A_0|$ and $|A_1|$ in Eq. (18), where α_0 and α_1 parametrize the changes of magnitudes of the amplitudes by neglecting the effects of inelastic final state interactions. A rough estimate $\alpha_0 \sim \alpha_1 \sim 0.8$ can be obtained by using the results of a global fit of the amplitudes and strong phases in Ref. [18], which means about 20% errors in our final results. This is a large error,

case that the spectator quark goes to a heavy meson, as in $\bar{B}^0 \to D^+K^-$ decay. We neglect the effect in this work, keeping in mind that we will need to include the small effect with precise experimental data in future.

comparable to the error from the present measurements of branching fractions. We certainly need to discover some methods to calculate α_0 and α_1 from first principles, but we leave this task for future work because of the large experimental errors in the measurements of branching fractions at this moment in time. Considering the other way around, if the value of $|V_{cb}|$ will be precisely extracted by other methods, our method will give a good place to investigate the final state interactions in two-body hadronic B decays.

Once the formula of Eq. (18) has been accepted, we can extract $|V_{cb}V_{us}^*|$ from the values of $|A_0|$ and $|A_1|$ which, as well as $\cos\delta_s$, can be extracted from the measurements of three decay rates.

Now we are going to extract $|V_{cb}|$ from the experimental values of decay fractions of three corresponding decay modes. From Eqs. (15), (16), and (17), the ratios of the decay fractions can be described as

$$\mathcal{R}_1 \equiv \frac{\mathcal{B}(\bar{B}^0 \to D^+K^-)}{\mathcal{B}(B^- \to D^0K^-)} = \frac{K_1}{4}\left(1 + 2\left|\frac{A_0}{A_1}\right|\cos\delta_s + \left|\frac{A_0}{A_1}\right|^2\right), \tag{19}$$

$$\mathcal{R}_2 \equiv \frac{\mathcal{B}(\bar{B}^0 \to D^0\bar{K}^0)}{\mathcal{B}(B^- \to D^0K^-)} = \frac{K_2}{4}\left(1 - 2\left|\frac{A_0}{A_1}\right|\cos\delta_s + \left|\frac{A_0}{A_1}\right|^2\right), \tag{20}$$

where the coefficients K_1 and K_2 are kinematical factors of

$$K_1 = \frac{\tau_{\bar{B}^0}m_{B^-}}{\tau_{B^-}m_{\bar{B}^0}} \cdot \frac{\sqrt{[1-(m_{D^+}/m_{\bar{B}^0} + m_{K^-}/m_{\bar{B}^0})^2][1-(m_{D^+}/m_{\bar{B}^0} - m_{K^-}/m_{\bar{B}^0})^2]}}{\sqrt{[1-(m_{D^0}/m_{B^-} + m_{K^-}/m_{B^-})^2][1-(m_{D^0}/m_{B^-} - m_{K^-}/m_{B^-})^2]}}, \tag{21}$$

$$K_2 = \frac{\tau_{\bar{B}^0}m_{B^-}}{\tau_{B^-}m_{\bar{B}^0}} \cdot \frac{\sqrt{[1-(m_{D^+}/m_{\bar{B}^0} + m_{\bar{K}^0}/m_{\bar{B}^0})^2][1-(m_{D^+}/m_{\bar{B}^0} - m_{\bar{K}^0}/m_{\bar{B}^0})^2]}}{\sqrt{[1-(m_{D^0}/m_{B^-} + m_{K^-}/m_{B^-})^2][1-(m_{D^0}/m_{B^-} - m_{K^-}/m_{B^-})^2]}}. \tag{22}$$

Equations (18), (19), and (20) are used to describe $|A_0|$ and $|A_1|$ in terms of $|\mathcal{M}(\bar{B}^0 \to D^+K^-)|$ as

$$|A_0| = \frac{2}{1+H^{-1}}|\mathcal{M}(\bar{B}^0 \to D^+K^-)|, \tag{23}$$

$$|A_1| = \frac{2}{1+H}|\mathcal{M}(\bar{B}^0 \to D^+K^-)|, \tag{24}$$

where

$$H = \left|\frac{A_0}{A_1}\right| = \sqrt{2(\mathcal{R}_1' + \mathcal{R}_2') - 1} \tag{25}$$

and $\mathcal{R}_i' \equiv \mathcal{R}_i/K_i$ with $i = 1, 2$. From Eqs. (19) and (20), $\cos\delta_s$ is described only by directly observable quantities as

$$\cos\delta_s = \frac{\mathcal{R}_1' - \mathcal{R}_2'}{H}. \tag{26}$$

From Eq. (24), the absolute value of the amplitude $|\mathcal{A}(B^- \to D^0K^-)| = |A_1|$ is given by

$$|\mathcal{A}(B^- \to D^0K^-)| = |V_{cb}V_{us}^*||\mathcal{M}'|\left|\frac{2}{1+H}\right|, \tag{27}$$

where

$$|\mathcal{M}'| = \frac{|\mathcal{M}(\bar{B}^0 \to D^+K^-)|}{|V_{cb}V_{us}^*|} = \eta_{\mathrm{EW}}\frac{G_{\mathrm{F}}}{\sqrt{2}}a_1(\mu)(m_{\bar{B}^0}^2 - m_{D^+}^2)f_{K^\pm}f_0(m_{K^\pm}^2) \tag{28}$$

is a known quantity. Finally, we get $|V_{cb}V_{us}^*|^2$ from the above equation and the value of the decay rate $\Gamma(B^- \to D^0K^-)$ as

$$|V_{cb}V_{us}^*|^2 = \frac{4\pi m_{B^-}\Gamma(B^- \to D^0K^-)}{\sqrt{[1-(r_1+r_2)^2][1-(r_1-r_2)^2]}}\frac{|1+H|^2}{|\mathcal{M}'|^2}, \tag{29}$$

where $r_1 = m_{D^0}/m_{B^-}$ and $r_2 = m_{K^-}/m_{B^-}$. This equation is used to extract $|V_{cb}V_{us}^*|^2$ from experimental data.

For $B \to DK^*$ and $B \to D^*K$, we can extract $|V_{cb}V_{us}^*|^2$ in the same way. The only major differences are the concrete forms of the amplitudes $\mathcal{M}(\bar{B}^0 \to D^+K^{*-})$ and $\mathcal{M}(\bar{B}^0 \to D^{*+}K^-)$.

For $\bar{B}^0 \to D^+K^{*-}$,

$$\begin{aligned}\mathcal{M}(\bar{B}^0 \to D^+K^{*-}) = \eta_{\mathrm{EW}}\frac{G_{\mathrm{F}}}{\sqrt{2}}V_{cb}V_{us}^*a_1(\mu)\,\langle D^+(p')|\,\bar{c}\gamma_\mu(1-\gamma_5)b\,|\bar{B}^0(p)\rangle \\ \times\,\langle K^{*-}(p_K)|\,\bar{s}\gamma^\mu(1-\gamma_5)u\,|0\rangle\end{aligned} \tag{30}$$

with the factorization procedure. The first matrix element in the amplitude is given in Eq. (8). The second matrix element in the amplitude is simply described as

$$\langle 0|\,\bar{u}\gamma_\mu(1-\gamma_5)s\,|K^*(p_K)\rangle = m_{K^*}f_{K^{*\pm}}\epsilon_\mu(p_{K^*}), \tag{31}$$

where $f_{K^{*\pm}}$ and $\epsilon_\mu(p_{K^*})$ are the decay constant and the polarization vector of $K^{*\pm}$ mesons, respectively. The polarization vector $\epsilon_\mu(p_{K^*})$ satisfies $\epsilon(p_{K^*})\cdot p_{K^*} = 0$. Then, we have

$$|\mathcal{M}(\bar{B}^0 \to D^+K^{*-})| = \eta_{\mathrm{EW}}\frac{G_{\mathrm{F}}}{\sqrt{2}}|V_{cb}V_{us}^*|a_1(\mu)2m_{\bar{B}^0}p^*f_{K^{*\pm}}f_+(m_{K^{*\pm}}^2). \tag{32}$$

Notice that this amplitude depends only on the form factor $f_+(q^2)$ instead of $f_0(q^2)$ in the case of $B \to DK$.

For $\bar{B}^0 \to D^{*+}K^-$, the amplitude is given by

$$\begin{aligned}\mathcal{M}(\bar{B}^0 \to D^{*+}K^-) = \eta_{\mathrm{EW}}\frac{G_{\mathrm{F}}}{\sqrt{2}}V_{cb}V_{us}^*a_1(\mu)\,\langle D^{*+}(p')|\,\bar{c}\gamma_\mu(1-\gamma_5)b\,|\bar{B}^0(p)\rangle \\ \times\,\langle K^-(p_K)|\,\bar{s}\gamma^\mu(1-\gamma_5)u\,|0\rangle\end{aligned} \tag{33}$$

with the factorization procedure. The first matrix element in this amplitude is described as [26]

$$\begin{aligned}&\langle D^{*+}(p')|\,\bar{c}\gamma_\mu(1-\gamma_5)b\,|\bar{B}^0(p)\rangle \\ &= \frac{2i\epsilon^{\mu\nu\alpha\beta}}{m_B+m_{D^*}}\epsilon_\nu^*p'_\alpha p_\beta V(q^2) - (m_B+m_{D^*})\left(\epsilon^{*\mu} - \frac{\epsilon^*\cdot q}{q^2}q^\mu\right)A_1(q^2) \\ &+ \frac{\epsilon^*\cdot q}{m_B+m_{D^*}}\left[(p+p')^\mu - \frac{m_B^2-m_{D^*}^2}{q^2}q^\mu\right]A_2(q^2) \\ &- 2m_{D^*}\frac{\epsilon^*\cdot q}{q^2}q^\mu A_0(q^2),\end{aligned} \tag{34}$$

Table 2. Inputs for the determination from $B \to DK$.

Input	Value	Reference
τ_{B^0}	$(1.520 \pm 0.004) \times 10^{-12}$ s	[17]
$\tau_{B^\pm}$	$(1.638 \pm 0.004) \times 10^{-12}$ s	[17]
$\mathcal{B}(\bar{B}^0 \to D^+K^-)$	$(1.86 \pm 0.20) \times 10^{-4}$	[17]
$\mathcal{B}(\bar{B}^0 \to D^0\bar{K}^0)$	$(5.2 \pm 0.7) \times 10^{-5}$	[17]
$\mathcal{B}(B^- \to D^0K^-)$	$(3.74 \pm 0.16) \times 10^{-4}$	[17]
$\lvert V_{us}\rvert$	0.2248 ± 0.0006	[17]
$f_{K^\pm}$	155.6 ± 0.4 MeV	[17]
$f_0(m_{K^\pm}^2)$	0.671 ± 0.012	[8]

Table 3. Sources of uncertainty of $|V_{cb}|$ from $B \to DK$.

Error source	Uncertainty [%]
$\Gamma(B^- \to D^0K^-)$	4.2
H	29.8
$f_0(m_{K^\pm}^2)$	3.6
$\lvert V_{cb}\rvert$	30.4

where $\epsilon_\mu(p')$ is the polarization vector of the D^* meson satisfying $\epsilon(p') \cdot p' = 0$, and $V(q^2)$, $A_1(q^2)$, $A_2(q^2)$, and $A_0(q^2)$ are form factors. Even though there are many form factors, we have a simple expression:

$$|\mathcal{M}(\bar{B}^0 \to D^{*+}K^-)| = \eta_{\mathrm{EW}} \frac{G_{\mathrm{F}}}{\sqrt{2}} |V_{cb}V_{us}^*| a_1(\mu) 2m_{\bar{B}^0} p^* f_{K^\pm} A_0(m_{K^\pm}^2). \tag{35}$$

Notice that this amplitude depends only on the form factor $A_0(q^2)$.

3. Numerical analyses and results

In our analysis we use the experimental data, masses and branching fractions in Ref. [17], and the form factors $f_{0,+}(q^2)$ in Ref. [8]. We use the value of the electroweak correction $\eta_{\mathrm{EW}} = 1.0066$ in Ref. [27], and the short-distance QCD correction $a_1(\mu) = 1.038$ at leading order with $\Lambda^{(5)}_{\overline{\mathrm{MS}}} = 225$ MeV and $\mu = 4.0$ GeV [20]. The accuracy of $a_1(\mu)$ is of the order of 1%. We do not consider the effect of isospin symmetry breaking, expecting that the effect is very small (within 1%).

From $B \to DK$ using Eq. (29) and the experimental data in Table 2, we obtain $|V_{cb}| = (37 \pm 6) \times 10^{-3}$ and $\cos\delta_s = 0.60 \pm 0.14$. Notice that the value of $|V_{cb}|$ is consistent with that determined by both the inclusive and exclusive methods with semi-leptonic decays. The uncertainty of $|V_{cb}|$ is about 30%, which is dominated by the experimental errors of the ratios $\mathcal{B}(\bar{B}^0 \to D^+K^-)/\mathcal{B}(B^- \to D^0K^-)$ and $\mathcal{B}(\bar{B}^0 \to D^0\bar{K}^0)/\mathcal{B}(B^- \to D^0K^-)$. Table 3 shows the sources of uncertainty of $|V_{cb}|$.

We find that the precise measurements of the branching fractions $\mathcal{B}(\bar{B}^0 \to D^+K^-)$, $\mathcal{B}(\bar{B}^0 \to D^0\bar{K}^0)$, and $\mathcal{B}(\bar{B}^- \to D^0\bar{K}^-)$ play an important role in the precise determination of $|V_{cb}|$ in our method. We note that the value of $|V_{cb}|$ is determined by using the form factor, which does not employ the CLN parametrization but the BGL parametrization. To compare the strong phase shift $\cos\delta_s$ with that from Ref. [18], we convert $\cos\delta_s$ to their $\cos\delta_c$, where δ_c is defined as the phase difference between $\mathcal{A}(\bar{B}^0 \to D^+K^-)$ and $\mathcal{A}(\bar{B}^0 \to D^0\bar{K}^0)$. Our result $\cos\delta_c = 0.43 \pm 0.16$ is consistent with that in Ref. [18] within errors.

Table 4. Inputs for the determination from $B \to DK^*$.

Input	Value	Reference
$\mathcal{B}(\bar{B}^0 \to D^+K^{*-})$	$(4.5 \pm 0.7) \times 10^{-4}$	[17]
$\mathcal{B}(\bar{B}^0 \to D^0\bar{K}^{*0})$	$(4.5 \pm 0.6) \times 10^{-5}$	[17]
$\mathcal{B}(B^- \to D^0K^{*-})$	$(5.3 \pm 0.4) \times 10^{-4}$	[17]
$f_{K^{*\pm}}$	205.6 ± 6.0 MeV	see text
$f_+(m^2_{K^{*\pm}})$	0.696 ± 0.012	[8]

Table 5. Inputs for determination from $B \to D^*K$.

Input	Value	Reference
$\mathcal{B}(\bar{B}^0 \to D^{*+}K^-)$	$(2.12 \pm 0.15) \times 10^{-4}$	[17]
$\mathcal{B}(\bar{B}^0 \to D^{*0}\bar{K}^0)$	$(3.6 \pm 1.2) \times 10^{-5}$	[17]
$\mathcal{B}(B^- \to D^{*0}K^-)$	$(4.20 \pm 0.34) \times 10^{-4}$	[17]
$A_0(m^2_{K^\pm})$	0.622 ± 0.062	see text

From $B \to DK^*$ we can obtain the value of $|V_{cb}|$ and the strong phase $\cos\delta_s$ in the same way. Using Eq. (32) and the experimental data in Table 4, we obtain $|V_{cb}| = (41 \pm 7) \times 10^{-3}$ and $\cos\delta_s = 0.82 \pm 0.20$.

This value of $|V_{cb}|$ is also consistent with both the inclusive and exclusive results. Notice that $\cos\delta_s$ is larger (δ_s is smaller) than that in $B \to DK$. This suggests that the effect of hadronic final state interactions between a pseudo-scalar meson and vector mesons is less important than in the case of two pseudo-scalar mesons. The corresponding value of $\cos\delta_c = -0.07 \pm 0.28$ is also consistent with that in Ref. [18] within errors. We have used the form factor with the BGL parametrization in Ref. [8]. The decay constant of the charged vector meson $f_{K^{*\pm}}$ is determined by the branching ratio of $\tau \to K^{*-}\nu_\tau$ [28]. Since the branching fraction is described as

$$\mathcal{B}(\tau \to K^{*-}\nu_\tau) = \frac{G_F^2 m_\tau |V_{us}|^2}{8\pi}\tau_\tau m^2_{K^{*\pm}} f^2_{K^{*\pm}} \left(1 - \frac{m_\tau^2}{2m^2_{K^*}}\right)\left(1 + \frac{m_{K^{*\pm}}}{m_\tau^2}\right)^2, \tag{36}$$

by using the measured values of $\mathcal{B}(\tau \to K^{*-}\nu_\tau) = (1.20 \pm 0.07) \times 10^{-2}$, $m_\tau = 1776.86 \pm 0.12$ MeV, and $\tau_\tau = (290.3 \pm 0.5) \times 10^{-15}$ s [17] we obtain $f_{K^{*\pm}} = 205.6 \pm 6.0$ MeV.

From $B \to D^*K$, in the same way, we obtain $|V_{cb}| = (42 \pm 9) \times 10^{-3}$ and $\cos\delta_s = 0.80 \pm 0.19$ using Eq. (35) and the experimental data in Table 5. This value of $|V_{cb}|$ is again consistent with those obtained by inclusive and exclusive determinations within errors. The value of the strong phase supports the previous suggestion that the effect of hadronic final state interactions is less important in the case with a vector meson in final state. The corresponding value $\cos\delta_c = 0.63 \pm 0.24$ is also consistent with Ref. [18] within errors. The form factor $A_0(q^2)$ is not given by the BGL parametrization, because there are no experimental data of the differential decay rate of $B \to D^*\tau\nu_\tau$, and also no lattice QCD calculations for the form factor. We have to use the form factor A_0, which is given by the CLN parametrization instead of the BGL parametrization by fully utilizing heavy quark symmetry. The CLN parametrization based on the heavy quark effective theory gives

$$A_0(q^2) = \frac{R_0(w)}{R_{D^*}} h_{A_1}(w), \tag{37}$$

Table 6. Summary of our results.

Mode	$\cos\delta_s$	$\lvert V_{cb}\rvert \times 10^3$
$B \to DK$	0.60 ± 0.14	37 ± 6
$B \to DK^*$	0.82 ± 0.20	41 ± 7
$B \to D^*K$	0.80 ± 0.19	42 ± 9

where $R_{D^*} = 2\sqrt{m_B m_{D^*}}/(m_B + m_{D^*})$,

$$h_{A_1}(w) = h_{A_1}(1)[1 - 8\rho^2_{D^*}z + (53\rho^2_{D^*} - 15)z^2 - (231\rho^2_{D^*} - 91)z^3], \tag{38}$$

$$R_0(w) = R_0(1) - 0.11(w-1) + 0.01(w-1)^2, \tag{39}$$

and w and z are kinetic variables defined as

$$w = \frac{m_B^2 + m_{D^*}^2 - q^2}{2m_B m_{D^*}}, \tag{40}$$

$$z = \frac{\sqrt{1+w} - \sqrt{2}}{\sqrt{1+w} + \sqrt{2}}. \tag{41}$$

The value of $h_{A_1}(1)$ has been obtained by the unquenched lattice QCD calculation [29]. The value of $R_0(1)$ can be obtained by using the relation, based on heavy quark symmetry [30,31],

$$R_3(1) \equiv \frac{R_2(1)(1-r) + r[R_0(1)(1+r) - 2]}{(1-r)^2} = 0.97, \tag{42}$$

if we know the value of $R_2(1)$, where $r = m_{D^*}/m_B$. The values of $R_2(1)$ and $\rho^2_{D^*}$ have been determined by the Belle Collaboration [11] from semi-leptonic $\bar{B}^0 \to D^{*+}l^-\bar{\nu}_l$ decay as $R_2(1) = 0.91 \pm 0.08$ and $\rho^2_{D^*} = 1.17 \pm 0.15$. In this way we obtain the value $R_0(1) = 1.08$ with the uncertainty of 10% considering unknown $\mathcal{O}(1/m_c^2)$ corrections. Our results are summarized in Table 6.

4. Conclusions

We have proposed a method of extracting the value of $|V_{cb}|$ from hadronic two-body B meson decays. The recent precise determination of the form factor $f_0(q^2)$ of semi-leptonic B meson decays in Ref. [8] allows us to perform this method with $B \to DK$ decay processes. The main theoretical assumption in our method, except for isospin symmetry, is that the effect of *inelastic* final state interactions is small. The small effect of non-factorizable spectator quark scattering has also been neglected, and should be included in cases with more precise experimental data. Specifically, we have neglected the possible states except for DK two-body states in final state interactions. The quantitative investigation of this truncation is future work which belongs to the efforts to understand non-perturbative QCD physics in hadronic decays. The effect of isospin symmetry breaking is not included, since it is negligibly small with the present precision of experimental data. In future, when the errors of branching fractions will be smaller and close to 1% accuracy as well as relevant form factors, we need to include the effect of isospin symmetry breaking. We have used form factors of semi-leptonic B meson decays which are determined by using the BGL parametrization in Refs. [8,9] for the extraction of $|V_{cb}|$ from $B \to DK$ and $B \to DK^*$. In the extraction of $|V_{cb}|$ from $B \to D^*K$ we had to use the CLN parametrization and heavy quark symmetry to obtain the form factor $A_0(q^2)$, which may contain possibly large uncertainties from higher-order corrections in heavy quark expansions.

Our final results are summarized in Table 6. The extracted values of $|V_{cb}|$ have about 30% uncertainties, and they are consistent with the values from both inclusive and exclusive semi-leptonic decays within errors. These consistent results show that our method is reasonable, at least with the present precisions. The experimental errors of the hadronic branching fractions, in particular $\mathcal{B}(\bar{B}^0 \to D^0\bar{K}^0)$, $\mathcal{B}(\bar{B}^0 \to D^0\bar{K}^{*0})$, and $\mathcal{B}(\bar{B}^0 \to D^{*0}\bar{K}^0)$, dominate the uncertainty of $|V_{cb}|$. We can expect that the uncertainty will become smaller with the results of future experiments and lattice calculations. It may be possible that this method will be the third one competing with conventional and established methods from inclusive or exclusive semi-leptonic B decays, if the problem of *inelastic* final state interactions is appropriately treated.

We have also examined the effects of hadronic final state interactions in two-body hadronic decays. The extracted strong phase shifts are consistent with the previous works of Refs. [18,32,33]. The strong phase in $B \to DK$ is larger than that in $B \to DK^*$ and $B \to D^*K$, which involve the vector meson in their final states (see Table 6). It is known in general that the final state interaction is more important for $B \to PP$ decays than $B \to PV$ decays, where P and V indicate pseudo-scalar and vector mesons. Here, we must note that the definitions of our phases are not exactly the same as in Refs. [18,32,33], and they coincide in the limit of negligible contribution of *inelastic* final state interactions. This fact will give a way to investigate the magnitude of the effect of *inelastic* final state interactions in the future. If the magnitude of $|V_{cb}|$ is precisely extracted by other methods in the future, our method will give a good place to investigate the final state interactions in two-body hadronic B-decays.

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