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Letter

Evidence for weak-coupling holography from the gauge/gravity correspondence for D*p*-branes

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Gauge/gravity correspondence is regarded as a powerful tool for the study of strongly coupled quantum systems, but its proof is not available. An unresolved issue that should be closely related to the proof is what kind of correspondence exists, if any, when gauge theory is weakly coupled. We report progress about this limit for the case associated with Dp-branes ($0 \leq p \leq 4$), namely, the duality between the $(p+1)$D maximally supersymmetric Yang–Mills theory and superstring theory on the near-horizon limit of the Dp-brane solution. It has been suggested by supergravity analysis that the two-point functions of certain operators in gauge theory obey a power law with the power different from the free-field value for $p \neq 3$. In this work, we show for the first time that the free-field result can be reproduced by superstring theory on the strongly curved background. The operator that we consider is of the form $\mathrm{Tr}(Z^J)$, where Z is a complex combination of two scalar fields. We assume that the corresponding string has the worldsheet spatial direction discretized into J bits, and use the fact that these bits become non-interacting when 't Hooft coupling is zero.

Subject Index B16, B21, B25, B29, B34

Introduction Gauge/gravity correspondence [1] is a proposal for concrete realizations of the holographic principle [2]. There have been exciting developments in which quantum gravity on various spacetimes is proposed to be equivalent to gauge theories defined at the spatial boundary. Gauge/gravity correspondence is now being applied to theories beyond the original proposals based on string/M-theory, in such areas as nuclear physics and condensed matter physics (see, e.g., Refs. [3–6] for reviews). It is widely regarded as a powerful tool that allows one to study strongly coupled quantum theory by gravity that is simple.

Gauge/gravity correspondence has not been proven yet, and we do not know how (or why) it works. An unresolved issue that should be closely related to the proof is what kind of correspondence exists, if any, when gauge theory is weakly coupled. There have been very few studies[1] on this limit, compared to the active studies on the other strong-coupling limit. The purpose of this letter is to point out an intriguing fact about this limit, which may suggest a possibility for a new approach towards establishing holography in the weak-coupling limit.

[1] For studies on the weak-coupling limit of the same SYM theories as the one studied here, see Refs. [7–16]. There have been studies of somewhat different theories based on higher-spin symmetry: For AdS_3/CFT_2, see, e.g., Refs. [17,18] and references therein; for AdS_4/CFT_3 involving 3D vector models, see, e.g., the reviews [19,20].

We consider gauge/gravity correspondence associated with two descriptions of Dp-branes, namely the conjectured equivalence between the $(p+1)$D maximally supersymmetric $SU(N)$ Yang–Mills theory and superstring theory on the near-horizon limit of the Dp-brane solution [21]. The former is the low-energy effective theory on the Dp-brane worldvolume based on the open-string lowest mode, and the latter gravity description is based on the closed-string degrees of freedom.

The $p=3$ case is the well studied AdS$_5$/CFT$_4$ correspondence between the (3+1)D $\mathcal{N}=4$ supersymmetric $SU(N)$ Yang–Mills theory and superstring theory on AdS$_5\times S^5$ [1]. We will consider the general case with $0\le p\le 4$. For $p\neq 3$, there is no conformal invariance, and exact results are hard to obtain, but there have been quantitative studies (especially for the $p=0$ case) [22–30,32,33,35]. Correlation functions at strong 't Hooft coupling have been obtained by tree-level supergravity. The results for $p=0$ have been confirmed by Monte Carlo simulations [28,29][2], providing strong evidence for the gauge/gravity correspondence without conformal symmetry.

We will consider gauge theory at zero temperature, and study the two-point function of a single-trace operator with large angular momentum J on the transverse S^{8-p}, which was introduced by Berenstein, Maldacena, and Nastase (BMN) [36] in the study of strings on the plane wave background. We show that the free-field result of gauge theory can be reproduced by string theory, by making an assumption that a string is composed of J "bits", which is natural in view of previous work [16,36–41]. In the following, we will first review gauge/gravity correspondence for general p at strong 't Hooft coupling, so that readers who are not familiar with this topic can follow the line of argument. The main result about weak 't Hooft coupling is presented near the end.

The background The near-horizon limit of the metric and the dilaton for the zero-temperature Dp-brane solution in the string frame are

$$\begin{aligned} ds^2 &= H^{-1/2}\left(-dt^2+dx_a^2\right)+H^{1/2}\left(dr^2+r^2d\Omega_{8-p}^2\right),\\ e^{\phi} &= g_sH^{\frac{3-p}{4}},\quad H=\frac{q}{r^{7-p}} \end{aligned}\tag{1}$$

where $a=1,\dots,p$ and $q=\tilde{c}_pg_sN\ell_s^{7-p}$ with $\tilde{c}_p=2^{6-p}\pi^{(5-p)/2}\Gamma(7-p)/2$. The integer N is the number of Dp-branes, and ℓ_s is the string length. The Yang–Mills coupling is given by $g_{\rm YM}^2=(2\pi)^{p-2}g_s\ell_s^{p-3}$.

For $p=3$, the near-horizon geometry is AdS$_5\times S^5$; for general p, it is related to AdS$_{p+2}\times S^{8-p}$ by Weyl rescaling,

$$ds^2=H^{1/2}r^2\left[\left(\frac{2}{5-p}\right)^2\left(\frac{dt^2+dx_a^2+dz^2}{z^2}\right)+d\Omega_{8-p}^2\right],\tag{2}$$

where the radial variable z in the Poincaré coordinates for AdS$_{p+2}$ is defined by

$$z=\frac{2}{5-p}(g_sN)^{1/2}\ell_s^{(7-p)/2}r^{-(5-p)/2}.\tag{3}$$

The boundary is at $z\to 0$. The distance $|\Delta x|$ in gauge theory roughly corresponds to the region $z\lesssim|\Delta x|$ [42,43], as we can see, e.g., in the geodesic approximation.

[2] See also recent numerical studies of gauge theories at finite temperature [34,35].

For $p \neq 3$, the dilaton and the curvature depend on the radial position, and the background does not have AdS isometries; correspondingly, the gauge theory does not have conformal invariance. Nevertheless, the representation (2) is useful; since null geodesics are not affected by the Weyl factor, supergravity modes, which are massless in 10 dimensions, show similar behavior (obeying a power law) to the conformal case at tree level.

In this work, we always assume that the string coupling is small, $e^{\phi} \ll 1$, so that we can ignore string loop effects. For $p < 3$, this is satisfied in the near-boundary (UV) region, $z \ll (g_s N)^{-1/(3-p)} N^{2(5-p)/((7-p)(3-p))} \ell_s$, and, for $p > 3$, the center (IR) region, $(g_s N)^{-1/(3-p)} N^{2(5-p)/((7-p)(3-p))} \ell_s \ll z$. If curvature is small in string units, string higher excitations can be ignored. For $p < 3$, this is satisfied in the IR region, $(g_s N)^{-1/(3-p)} \ell_s \ll z$, and for $p > 3$, the UV region, $z \ll (g_s N)^{-1/(3-p)} \ell_s$. If we take $N \to \infty$ with the 't Hooft coupling fixed but large $g_s N \gg 1$, the above two conditions are satisfied in almost the whole near-horizon region [23]. In that case, the tree-level supergravity is expected to be valid. When we take 't Hooft coupling to be small, we should consider strongly curved background.

The operator We will focus on the two-point functions of a "BMN operator" [36],

$$\mathcal{O} = \mathrm{Tr}\left(Z^J\right), \tag{4}$$

where $Z = X^8 + iX^9$ is a complex combination of two of the $(9-p)$ scalar fields (that arise from dimensional reduction of the (9+1)D gauge field) in the $(p+1)$D maximally supersymmetric Yang–Mills theory. The integer J corresponds to a quantum number for the $SO(9-p)$ symmetry (angular momentum along a great circle in the X^8–X^9 plane).

For $p = 3$, this operator is one of the BPS operators, which belong to a short representation of the superconformal algebra. Its scaling dimension is given by the free-field value $\Delta = J$, and is protected against quantum corrections. The BPS operators correspond to supergravity modes.

For $p \neq 3$, the analogs of the BPS operators, which are related to the ones for $p = 3$ by "T-duality" (or dimensional reduction), couple to supergravity modes [44–46]. The full spectrum of the supergravity modes has been obtained for the D0-brane background [23,24], and the corresponding operators in the $(0+1)$D SYM have been identified [23,24] with the help of "generalized conformal symmetry" [47][3]. The operator (4) belongs to T_J^{++} defined in Ref. [44], and corresponds to the supergravity mode called s_J^3 in Ref. [23].

Supergravity analysis We assume that the general relation due to Gubser, Klebanov, Polyakov [48], and Witten [49] (GKPW) between the bulk partition function $Z[\phi_0]$ and the generating functional for correlation functions in gauge theory holds:

$$Z[\phi_0] = \langle e^{\int d^{p+1}x \phi_0(x)\mathcal{O}(x)} \rangle. \tag{5}$$

Here, ϕ_0 is the boundary condition for a bulk field ϕ, imposed at the $(p+1)$D boundary of the AdS$_{p+2}$-like space, and $\mathcal{O}$ is the operator that couples to ϕ_0. Calculations are performed in the Euclidean signature. In the limit of weak string coupling, the bulk partition function is given by the classical

[3] This is a symmetry realized when the coupling constant is allowed to vary. To the author's knowledge, this symmetry is not powerful enough to constrain the scaling exponent w.r.t. the coordinate separation, unlike true superconformal symmetry.

action, $Z[\phi_0] = e^{-S_{\mathrm{SG}}[\phi_0]}$, and can be calculated by tree-level supergravity using bulk-to-boundary propagators. By applying the GKPW prescription to the near-horizon Dp-brane background, the two-point function of the operator (4) has been found to be [23]

$$\langle \mathcal{O}(x')\mathcal{O}(x)\rangle = \frac{\delta}{\delta\phi_0(x')}\frac{\delta}{\delta\phi_0(x)}e^{-S_{\mathrm{SG}}[\phi_0]} \sim \frac{1}{|x-x'|^{\frac{4J}{5-p}+c_p}}, \tag{6}$$

where we have ignored an overall constant factor. The constant c_p in the exponent takes the value[4]

$$c_p = -\frac{(3-p)^2}{5-p}. \tag{7}$$

The correlator (6) obeys a power law[5], even though the coupling constant g_{YM} has a dimension for $p \neq 3$. The power is different from the free-field value for $p \neq 3$. Strong-coupling dynamics together with supersymmetric cancellations should be responsible for this behavior in gauge theory, but the mechanism is not understood analytically yet. For $p = 0$, this power law has been confirmed by Monte Carlo simulation [28,29].

Geodesic approximation Supergravity modes are massless in 10D spacetime, but when we regard them as Kaluza–Klein modes, angular momentum J on S^{8-p} corresponds to mass[6] $m = \frac{2}{5-p}J$ in AdS_{p+2}. In the large-J limit, the geodesic approximation can be used to obtain the correlator.

We will consider Euclidean AdS_{p+2}. Then, a geodesic of a massive particle connects x_i and x_f on the boundary (separated, e.g., in the Euclidean time direction). We will regulate the infinite volume of the AdS_{p+2}, and put the boundary at the radial position $z = 1/\Lambda$. The geodesic is a half sphere $x^2 + z^2 = \tilde{\ell}^2$ in the Poincaré coordinates. In terms of the proper time τ, it is given by

$$z = \frac{\tilde{\ell}}{\cosh\tau}, \quad x = \tilde{\ell}\tanh\tau, \tag{8}$$

where $\tilde{\ell}$ is a parameter corresponding to the distance of the two points along the boundary $|x_i - x_f| = 2\tilde{\ell}$. The geodesic reaches the boundary at an infinite proper time; thus we introduce a cutoff $-T \leq \tau \leq T$. The relation between the cutoffs Λ and T is given from Eq. (8) as $e^T \sim 2\tilde{\ell}\Lambda = |x_f - x_i|\Lambda$.

The two-point function is obtained by evaluating the geodesic length:

$$\langle \mathcal{O}(x_f)\mathcal{O}(x_i)\rangle = e^{-m\int_{-T}^{T}d\tau} = e^{-\frac{4}{5-p}JT} = \frac{1}{\left(\Lambda|x_f - x_i|\right)^{\frac{4J}{5-p}}}. \tag{9}$$

The exponent indeed agrees with the leading (J-dependent) exponent in Eq. (6) obtained by the GKPW prescription.

Worldsheet analysis The null geodesic in 10D spacetime, given by Eq. (8) together with the S^{8-p} part, is a classical solution of the closed-string worldsheet theory, in which the string is in a point-like

[4] This value of c_p has been deduced from the available supergravity results for $p = 0$ and $p = 3$.
[5] Correlation functions for operators corresponding to string higher modes are predicted to be exponential functions [25,27].
[6] For Kaluza–Klein reduction of a massless particle on Eq. (2), the Weyl factor plays no role, and we effectively obtain a massive particle action on the true AdS_{p+2} [25,26]. The factor $2/(5-p)$ is due to the ratio of the radii of AdS_{p+2} and S^{8-p} [25,26].

configuration. Fluctuations around the null geodesic can be studied by expanding the string action to the quadratic order around the classical solution[7]. After eliminating unphysical modes by gauge fixing and the use of constraints, we get eight bosonic and eight fermionic fields, which are massive on the worldsheet [25,26][8]. The origin of their mass is the curvature of the background spacetime.

The bosonic part of the quadratic action is [25,26][9]

$$S^{(2)} = \frac{1}{4\pi\ell_s^2}\int d\tau \int_0^{2\pi\tilde{\alpha}} d\sigma \Big\{ \dot{x}_a^2 + \tilde{r}^{p-3}(\tau) {x'_a}^2 + m_x^2 x_a^2 + \dot{y}_i^2 + \tilde{r}^{p-3}(\tau) y_i'^2 + m_y^2 y_i^2 \Big\}, \tag{10}$$

where $a = 1, \ldots, p+1$, and $i = p+2, \ldots, 8$. The radius of the σ direction is proportional to the angular momentum, $\tilde{\alpha} \propto J$ [25,26,29]. The terms involving the spatial derivative ∂_σ have a factor $\tilde{r}(\tau) \equiv 2\cosh\tau/\{(5-p)\tilde{\ell}\}$, which depends on the position on the geodesic, so the frequencies of string excited states are time dependent. The mass for the bosonic fields depends on whether the field describes fluctuations along AdS_{p+2} (m_x) or S^{8-p} (m_y) [25]:

$$\begin{aligned} m_x &= 1 \qquad (p+1 \text{ fields}), \\ m_y &= \frac{2}{5-p} \qquad (7-p \text{ fields}). \end{aligned} \tag{11}$$

The quadratic action for fermions has been obtained similarly, starting from the Green–Schwarz action [26]. The mass for the fermionic fields is[10]

$$m_f = \frac{7-p}{2(5-p)} \qquad (8 \text{ fields}). \tag{12}$$

The ground state of the closed string corresponds to the operator (4) [36]. The classical amplitude (9) is corrected by contributions from the fluctuations[11]. The frequencies of the lowest (σ-independent) modes are equal to the masses (11) and (12). Their contribution to the zero-point energy is

$$E_0 = \frac{1}{2}\big((p+1)m_x + (7-p)m_y - 8m_f\big) = -\frac{(3-p)^2}{2(5-p)}. \tag{13}$$

[7] In order to have Euclidean AdS and to keep the correct number of physical degrees of freedom, we take one of the S^{8-p} directions timelike. This prescription was introduced in Ref. [50], and is called double Wick rotation.

[8] This analysis is very similar to the one performed by BMN [36]. They study a string (particle) moving around the center of Lorentzian AdS, and identity the AdS energy (in the global coordinates) with the scaling dimension in the gauge theory. For $p = 3$, our analysis based on the proposal of Ref. [50] gives essentially the same results as BMN (see Refs. [50–52] for further discussion). But for $p \neq 3$ without conformal symmetry, BMN's procedure would not be applicable; thus we should use our approach based on a string (particle) that reaches the boundary.

[9] We have fixed the worldsheet metric as $\sqrt{h}h^{\tau\tau} = (\sqrt{h}h^{\sigma\sigma})^{-1} = \tilde{r}^{(3-p)/2}(\tau)$ [25]. With this choice, m_x, m_y are constant, but $x_a'^2$, $y_i'^2$ get τ-dependent coefficients. If we take the conformal gauge [25], m_x, m_y would be τ dependent. The final result does not depend on the gauge choice.

[10] The gauge-fixed action has worldsheet supersymmetry only for $p = 3$; thus the masses of bosons and fermions are different for $p \neq 3$.

[11] The classical action for a massless particle vanishes, but Eq. (9) is obtained by evaluating the Routh function (where only an angular direction is Legendre transformed from the Lagrangian to make J an independent variable) [25].

By including this correction[12] in the classical result (9), we recover the GKPW result [26],

$$\langle \mathcal{O}(x_f)\mathcal{O}(x_i)\rangle = e^{-2(m+E_0)T} = \frac{1}{(\Lambda|x_f - x_i|)^{\frac{4J}{5-p}+c_p}}, \tag{14}$$

with the value of c_p given in Eq. (7).

Free-field result from the bulk Up to now, we have reviewed the framework for gauge/gravity correspondence for the Dp-brane and the analyses are expected to be valid at strong 't Hooft coupling. Assuming the validity of this framework, let us now consider what happens at zero 't Hooft coupling.

We make one additional assumption: we assume that the operator (4) corresponds to a superstring whose worldsheet spatial direction is discretized into J bits[13]. This is natural if we recall the proposal by BMN [36] (see also subsequent work [16,37–41]) that string excitations on the ground state (4) are represented in gauge theory by inserting "impurities" (fields X_i with $i \neq 8, 9$, or derivatives ∂_a with $a = 0, 1, \ldots, p$) into the sequence of Z.

Weak 't Hooft coupling corresponds to strongly curved backgrounds. In this limit, the string tension is much smaller compared to the scale of the curvature of the background. If string tension is strictly zero (corresponding to zero 't Hooft coupling), no binding force exists between bits, and we can think of the string as a collection of J non-interacting bits (particles). In this case, the zero-point energy would be the sum of contributions from J bits, i.e., J times E_0, which we obtained in Eq. (13). By including this correction, the correlator now becomes

$$\langle \mathcal{O}(x_f)\mathcal{O}(x_i)\rangle = e^{-2(m+JE_0)T} = \frac{1}{(\Lambda|x_f - x_i|)^{(p-1)J}}. \tag{15}$$

This is the free-field result: the mass dimension of a scalar field in $(p+1)$ dimensions is $(p-1)/2$, and the operator $\mathcal{O}$ consists of J scalar fields.

Discussion We have shown that the free-field result of the $(p+1)$D maximally supersymmetric Yang–Mills theory can be reproduced from the bulk string theory for the two-point function of a particular operator $\mathrm{Tr}Z^J$. Our result is based on a natural assumption that the string corresponding to this operator is made of J bits. We regard this result as strong evidence that gauge/gravity correspondence works for general p and also at weak 't Hooft coupling.

This result suggests a very concrete picture of strings at strong curvature, i.e., at substringy distances: The mode that we usually consider to be a graviton is really a collection of bits. It is important to fully elucidate its consequences. (This is not particular to $p \neq 3$; it is equally important for $p = 3$.)

It remains to be seen whether our approach can be promoted to a calculational framework applicable for finite 't Hooft coupling. Having reproduced a zero 't Hooft coupling result from the bulk, it is hoped that perturbative expansions of gauge theory in terms of 't Hooft coupling can be reproduced as well[14]. Discretized string action is not unique; thus one should identify the interactions between bits appropriately for this purpose.

[12] Contributions from higher modes (with wave number $n \geq 1$ along σ) will cancel between fermions and bosons for $\ell_s \to 0$, since the difference in Eqs. (11) and (12) is unimportant in this limit.

[13] Strings with discretized worldsheets appear in attempts to reformulate large-N gauge theory by Thorn [53–55]. They also appear in a proposal by Nielsen and Ninomiya [56,57] for reformulating string theory by treating left and right movers as independent. The direct relationship of such work with ours is not clear at the moment.

[14] See Refs. [7,16,51,52] for previous work in that direction.

Let us make some comments to clarify the meaning of our proposal. First, in our proposal, we are assuming $N \to \infty$. In this limit in which we can ignore string loops, we should be able to treat the background as fixed and non-fluctuating, even though it is strongly curved[15]. (In that sense, we are not really in the quantum gravity regime.) Second, in the present work, we considered the large-J limit, which allowed us to perform a semi-classical study of string theory, with its gauge symmetry completely fixed. It is important to formulate the correspondence at weak 't Hooft coupling for finite J, without the semi-classical approximation and gauge fixing, whether or not it is calculationally feasible.

A crucial question is whether one can construct a consistent string theory with a discrete worldsheet. By discretization we may lose characteristic features of string theory[16] such as the following ones. First, open–closed string duality is due to the presence of an infinite tower of modes on the worldsheet. By discretization, the closed-string level number is cut off at $n \lesssim J$. This is not necessarily a problem, since the SYM theory consists of only the lowest modes of the open string. It is very important to examine whether $n \lesssim J$ is just the right number of degrees of freedom necessary for the duality in this context to hold. Second, the elimination of the UV divergence in string theory is due to modular invariance, which is a remnant of conformal symmetry. It is not at all clear what its counterpart is in a discretized theory. For our proposal at $N \to \infty$, the problem of string loop divergence is not relevant, but if we want to extend it to finite N, this problem would be very important.

To gain insight into the above issues, it would be helpful to extend our analysis to more general cases. One direction would be to a class of operators with spins along the AdS_{p+2} direction. These correspond to macroscopic strings rotating in AdS [59][17]. Another direction of extension would be to finite temperature. Finding the correct type of discretized string theory in this case may shed light on the Hagedorn behavior [60].

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[15] In general, there could be α' corrections to the background, but we expect the near-horizon Dp-brane background to be protected against this due to supersymmetry, as is the case for $\mathrm{AdS}_5 \times S^5$.

[16] See Ref. [58] for an illuminating account of string theory, with strong emphasis on general features and physical interpretation.

[17] To the author's knowledge, this has not been studied for $p \neq 3$ (even for strong 't Hooft coupling); it would be necessary to reformulate the correspondence in the Euclidean signature and look for strings that reach the boundary as is done in this work.

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