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Stringy excited baryons in holographic quantum chromodynamics

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We analyze excited baryon states using a holographic dual of quantum chromodynamics that is defined on the basis of an intersecting D4/D8-brane system. Studies of baryons in this model have been made by regarding them as a topological soliton of a gauge theory on a five-dimensional curved spacetime. However, this allows one to obtain only a certain class of baryons. We attempt to present a framework such that a whole set of excited baryons can be treated in a systematic way. This is achieved by employing the original idea of Witten, which states that a baryon is described by a system composed of N_c open strings emanating from a baryon vertex. We argue that this system can be formulated by an Atiyah–Drinfeld–Hitchin–Manin-type matrix model of Hashimoto–Iizuka–Yi together with an infinite tower of the open string massive modes. Using this setup, we work out the spectra of excited baryons and compare them with the experimental data. In particular, we derive a formula for the nucleon Regge trajectory assuming that the excited nucleons lying on the trajectory are characterized by the excitation of a single open string attached on the baryon vertex.

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1. Introduction

Ever since the anti-de Sitter / conformal field theory (AdS/CFT) correspondence was proposed by Maldacena (for a review, see Ref. [1]), it has been recognized that it may provide us with a powerful tool for analyzing nonperturbative dynamics of non-Abelian gauge theories. One of the most intensive applications of the AdS/CFT correspondence is to the hadron physics of quantum chromodynamics (QCD). A key ingredient of hadron physics is how to understand spontaneous breaking of chiral symmetry. A holographic dual of QCD (in the top-down approach) with manifest chiral symmetry was presented in Refs. [2,3] on the basis of an intersecting D4/D8-brane configuration. It was argued there that chiral symmetry breaking is realized as a smooth interpolation of D8–anti-D8-brane ($\overline{\text{D8}}$) pairs in a curved background corresponding to D4-branes in type IIA supergravity. The associated Nambu–Goldstone mode (pion) is shown to arise from the five-dimensional gauge field on the interpolated D8-branes. This model is formulated in the large N_c and large 't Hooft coupling λ regime with $N_c \gg N_f$, where N_c and N_f are the numbers of colors and flavors, respectively, for the purpose of suppressing intricate stringy and quantum gravity effects. In spite of this approximation, the predictions of this model match well with various experimental data in low-energy hadron physics.

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In particular, it has been shown that the meson effective theory is given by a five-dimensional $U(N_f)$ gauge theory, and a tower of vector and axial-vector mesons, including ρ and a_1 mesons, appear as the Kaluza–Klein (KK) modes of the five-dimensional gauge field. Other mesons, including higher-spin mesons, are interpreted as excited open string modes attached on the D8-branes [4]. As they are described by an open string, nearly linear Regge trajectories with mild nonlinear corrections are obtained quite naturally, and it has been argued that the predicted meson spectrum agrees at least qualitatively with what is observed in nature.

The holographic model is also used to study the baryon sector. This is performed by noting that a baryon can be realized as a topological soliton in the five-dimensional gauge theory with a baryon number identified with a topological number. The original idea, due to Skyrme [5], is adding a so-called Skyrme term to the chiral Lagrangian of the massless pion. In the holographic model, the soliton solution is given by an instanton solution with the instanton number regarded as the baryon number [2]. Analysis of the moduli space quantum mechanics analogous to the work of Ref. [6] in the Skyrme model was performed in Refs. [7,8] to obtain the baryon spectrum and the static properties, respectively,[1] and again many of the results turned out to be consistent with the experimental data. However, one of the limitations in Ref. [7] is that it describes only a subclass of baryons with $I = J$ for $N_f = 2$. Here, J and I denote the spin and the isospin of a baryon, respectively. The reason for this limitation is clear: the moduli space approximation only takes into account the light degrees of freedom that correspond to the massless sector in the open string spectrum. We are led naturally to expect that incorporation of massive open string states enables us to obtain a larger class of baryons with $I \neq J$,[2] as was done in Ref. [4] for the meson sector.

The purpose of this paper is to examine holographic baryons following this line. To this end, we utilize the idea of Witten [15] that a holographic description of baryons is made by introducing a D-brane configuration, called a baryon vertex. In the present holographic model, we add a D4-brane that wraps around an $\mathbb{S}^4$ with N_c units of Ramond–Ramond (RR) flux over it. It was found in Refs. [15,16] that the RR flux forces N_c open strings to extend between the D4-brane and the D8-branes. The whole system is regarded as a holographic baryon. As a consistency check, the instanton solution is identical to the baryon vertex D4-brane in the context of the effective theory. The baryon states can be computed by working out a bound state of a many-body quantum mechanics that is defined from open strings attached on the baryon vertex. There are two types of open strings that should be taken into account. One of them is the 4-4 strings with both end points attached on the baryon vertex D4-brane, and the other is the 4-8 strings that extend between the D4-brane and one of the D8-branes. As shown in Refs. [17,18], the massless degrees of freedom that arise from these strings correspond to the instanton moduli space in the Atiyah–Drinfeld–Hitchin–Manin (ADHM) construction [19], and it is expected to be equivalent to the moduli space quantum mechanics in the soliton approach. This approach was proposed in Ref. [14], in which a matrix quantum mechanics describing multiple baryon systems was derived. Our main idea is to incorporate the massive open string states into this quantum mechanics to describe heavier baryons. Solving the bound state problem in quantum mechanics is highly involved in general. In this present case, however, we argue that taking the large-N_c limit makes the problem tractable. This is because the string coupling is of $\mathcal{O}(1/N_c)$ so that interactions among open strings are mostly negligible in the large-N_c limit.

[1] See also Refs. [9–12].

[2] For another approach to holographic baryons with $I \neq J$, see Ref. [13], which is based on the study of a matrix model formulated in Ref. [14].

The fundamental degrees of freedom in the quantum mechanics are given by massless and an infinite tower of massive modes of open strings attached on the baryon vertex D4-brane. The mass spectrum can be worked out by quantizing the open strings in the curved background of Eq. (2.1), but this is technically difficult to achieve. As suggested in Ref. [4], this problem gets simplified drastically by taking the limit $\lambda \gg 1$, where the spacetime curvature becomes negligible. Nontrivial curvature effects in the mass spectrum are incorporated perturbatively in $1/\lambda$ expansions. Using these results, the many-body quantum mechanics is formulated in a manner that is simple and powerful enough to study a wide range of holographic baryons quantitatively. As an application, we derive the mass formula of the nucleon and its excited states. We also discuss its implication to the nucleon Regge trajectory.

The organization of this paper is as follows. In Sect. 2, after giving a brief review of the holographic model of QCD with the emphasis on a baryon vertex, we compute the mass spectrum of the open strings attached on the baryon vertex and D8-branes. With this result, Sect. 3 formulates a many-body quantum mechanics that enables one to compute the mass spectrum of baryons that are missing in Ref. [7]. In Sect. 4, we compare the predictions of this model to experiments. We conclude in Sect. 5 with a summary and some comments about future directions. Some technical formulas that are used in the paper are summarized in Appendix A.

2. Holographic model of QCD and baryons

2.1. *Brief review of the model*

The holographic model of QCD we work with is constructed from an intersecting D4/D8-brane system [2,3]. The N_c D4-branes wrap around a circle on which a SUSY-breaking boundary condition is imposed, and yield gluons of gauge group $SU(N_c)$ on the worldsheet at low energy compared with the circle radius $1/M_{\rm KK}$. N_f D8- and $\overline{\rm D8}$-branes are placed at the anti-podal points of the SUSY-breaking circle. Quantization of D4–D8 and D4–$\overline{\rm D8}$ strings gives left- and right-handed quarks in the fundamental representation of $SU(N_c)$, respectively. This system has a manifest chiral $U(N_f)_L \times U(N_f)_R$ symmetry.

The holographic dual of this model is formulated by replacing the D4-branes with a solution of type IIA supergravity with a nontrivial dilaton ϕ [20]:

$$
\begin{aligned}
ds^2 &= \frac{4}{27}\lambda l_s^2\, d\tilde{s}^2, \\
d\tilde{s}^2 &= K(r)^{1/2}\eta_{\mu\nu}dx^\mu dx^\nu + K(r)^{-5/6}dr^2 + K(r)^{-1/2}r^2 d\theta^2 + \frac{9}{4}K(r)^{1/6}d\Omega_4^2,
\end{aligned} \tag{2.1}
$$

$$
e^\phi = \frac{\lambda^{3/2}}{3\sqrt{3}\pi N_c}K(r)^{1/4}. \tag{2.2}
$$

Here, $\mu, \nu = 0, 1, 2, 3$ denote the indices of the four-dimensional Minkowski spacetime where QCD is defined, $d\Omega_4^2$ is the metric of a unit $\mathbb{S}^4$, and $K = 1 + r^2$.[3] θ is the coordinate of the SUSY-breaking circle. In addition, there exist N_c units RR four-form flux over the $\mathbb{S}^4$:

$$
\frac{1}{2\pi}\int_{S^4} F_4 = N_c. \tag{2.3}
$$

[3] The radial coordinate r is related to $U/U_{\rm KK}$ used in Refs. [2,3] by $(U/U_{\rm KK})^3 = 1 + r^2$.

It is useful to define

$$z = r \sin\theta, \qquad y = r\cos\theta.$$

The metric in Eq. (2.1) is defined in the decoupling limit, where the dependence on l_s, the string length, factorizes as a prefactor. As a consequence, the string theory on this background is independent of l_s. This allows one to set

$$\alpha' \equiv l_s^2 = \frac{27}{4\lambda} \tag{2.4}$$

in units of $M_{\mathrm{KK}} = 1$ so that $ds^2 = d\tilde{s}^2$. (See Ref. [4] for more details on this point.) It follows that the stringy excitation modes have mass of $\mathcal{O}(\lambda^{1/2})$ and may be neglected at low energies for $\lambda \gg 1$.

Assuming $N_c \gg N_f$,[4] the D8-branes can be regarded as probes with no backreaction to the metric in Eq. (2.1) taken into account. It has been shown [2] that the D8- and $\overline{\mathrm{D8}}$-brane pairs interpolate with each other smoothly at $z = y = 0$, and the resultant D8-brane worldvolume is specified by the embedding equation $y = 0$. In this setup, the mesons are identified with the open strings attached on the D8-branes that can move along the z direction.

In order to incorporate baryon degrees of freedom into the model, we introduce a baryon vertex [15], which is given by a single D4-brane wrapping around $\mathbb{S}^4$ at $z = y = 0$. We refer to this D4-brane as a D4$_{\mathrm{BV}}$ in order to distinguish it from N_c color D4-branes. The RR flux in Eq. (2.3) forces N_c open strings to extend between the D8-branes and the baryon vertex. This configuration is identified with a single baryon. It is argued in Ref. [2] that this brane system is realized as an instanton solution on the D8-brane worldvolume theory. By analyzing the moduli space quantum mechanics corresponding to this instanton solution, Refs. [7,8] showed that aspects of the baryon dynamics are reproduced from this model both qualitatively and quantitatively. One of the limitations in this analysis, however, is that describing a baryon vertex as a classical solution of the $U(N_f)$ gauge theory on the D8-branes is valid only for low-lying baryons. This is because the $U(N_f)$ gauge theory is an effective theory of the D8-branes with only the massless degrees of freedom taken into account. In addition, the moduli space approximation only keeps light degrees of freedom in the fluctuations around the soliton solution. In fact, these are the main reasons why the analysis in Ref. [7] leads to only baryons with the spin J and isospin I equal to each other for the $N_f = 2$ case. For the purpose of obtaining more general baryons, we thus have to consider stringy effects in the baryon vertex.

2.2. *Quantization of open strings in a flat spacetime limit*

It is highly difficult to make a full quantization of a string that propagates in the curved background of Eq. (2.1) in the presence of the RR flux in Eq. (2.3). In order to circumvent this problem, we follow Ref. [4]. We first take the large-λ limit, where the curved background can be approximated with a ten-dimensional flat spacetime. Then, the baryon configuration reduces to a system with N_f D8-branes and a D4$_{\mathrm{BV}}$-brane with N_c open strings stretched between them in the flat background. For a technical reason, it is useful to formally T-dualize the system in the y direction. The D8/D4$_{\mathrm{BV}}$-brane system gets mapped to the D9/D5$_{\mathrm{BV}}$-brane configuration shown in Table 1. The 123z- and 6789-directions are labeled by indices M and i, respectively. The 6789-directions span $\mathbb{R}^4$, which results from the $\mathbb{S}^4$ that is decompactified for $\lambda \gg 1$. Quantization of a 9-5 and 5-5 string is performed

[4] For this, we mean that we consider N_f to be of $\mathcal{O}(1)$ and only take into account the leading terms in the $1/N_c$ expansion.

Table 1. D9/D5$_{\mathrm{BV}}$-brane system. $\tilde{y}$ is the T-dualized coordinate of y.

	0	1	2	3	z	$\tilde{y}$	6	7	8	9
$N_f \times$D9	○	○	○	○	○	○	○	○	○	○
D5$_{\mathrm{BV}}$	○					○	○	○	○	○

most easily by using a light-cone quantization, where the light-cone coordinate is taken to be $x^0 \pm \tilde{y}$. The manifest spacetime symmetry of the brane system is $SO(4)_{123z} \times SO(4)_{6789}$.

We first study the light-cone quantization of a 9-5 string. The equations of motion (EOM) of the worldsheet boson in the 6789-directions is solved in terms of Fourier expansions with an integer modding, while that in the 123z-directions in terms of those with a half-integer modding, because of the boundary conditions imposed on them. For the worldsheet fermions in the Neveu–Schwarz (Ramond) sector, the solutions of the EOM in the 6789-directions are written in terms of Fourier expansions with a half-integer (integer) modding, while those in the 123z-directions are written in terms of those with an integer (half-integer) modding. It follows that the Neveu–Schwarz (NS) ground state is degenerate due to the fermion zero modes, belonging to a spinor representation of $SO(4)_{123z}$. The Ramond (R) ground state is degenerate too, and belongs to a spinor representation of $SO(4)_{6789}$. We label an irreducible representation of $SO(4)_{123z} \simeq (SU(2)_L \times SU(2)_R)/\mathbb{Z}_2$ by (s_L, s_R), where s_L and s_R are the spin of $SU(2)_L$ and $SU(2)_R$, respectively. The (integer spin) representation of $SO(4)_{6789}$ is labeled by Young tableaux as $\mathbf{1}$, $\Box_4$, $\begin{smallmatrix}\Box\\\Box\end{smallmatrix}_6$, $\Box\!\Box_9$, etc., where the subscripts denote the dimensions. Then, the low-lying 9-5 string states in the NS sector with the Gliozzi–Scherk–Olive (GSO) projection imposed are summarized in Table 2. Although it is not manifest in the light-cone quantization, the six-dimensional Lorentz symmetry on the D5$_{\mathrm{BV}}$-brane worldvolume allows one to summarize the massive excitations into the irreducible representations of the little group $SO(5)_{\tilde{y}6789}$, which contains $SO(4)_{6789}$ as a subgroup. Table 3 gives a list of the low-lying 9-5 string states in the NS sector in terms of $SO(5)_{\tilde{y}6789}$.

We next study the mass spectrum of a 5-5 string using the light-cone quantization. The worldsheet bosons can be Fourier expanded with an integer modding for both 123z- and 6789-directions. The worldsheet fermions in the NS (R) sector can be Fourier expanded with a half-integer (integer) modding for the 123z- and 6789-directions. The physical ground state in the NS sector is massless and given by $\psi^M_{-1/2}|0\rangle_{\mathrm{NS}}$ and $\psi^i_{-1/2}|0\rangle_{\mathrm{NS}}$. Here, $|0\rangle_{\mathrm{NS}}$ is tachyonic, being GSO-projected out. The first excited 5-5 string states in the NS sector that survive the GSO projection are given by acting on $|0\rangle_{\mathrm{NS}}$ with a set of the creation operators with total excitation number equal to $3/2$. These have the mass squared $(3/2 - 1/2)/l_s^2 = 1/l_s^2$ and are listed in Table 4.

As in the 9-5 string states, any massive state of the 5-5 string is summarized into an irreducible representation of $SO(4)_{123z} \times SO(5)_{\tilde{y}6789}$. It is found that the first excited states with $N_{55} = 1$ in Table 4 are rearranged as

$$\begin{gathered}[(1,0)\oplus(0,1)]\,\Box_5, \quad (1/2,1/2)\begin{smallmatrix}\Box\\\Box\end{smallmatrix}_{10}, \quad (1/2,1/2)\,\Box_5, \quad (0,0)\begin{smallmatrix}\Box\\\Box\\\Box\end{smallmatrix}_{10}, \\ (0,0)\,\Box\!\Box_{15}, \quad (1/2,1/2)\,\mathbf{1}, \quad [(0,0)\oplus(1,1)]\;\mathbf{1}, \end{gathered} \tag{2.5}$$

Table 2. Low-lying 9-5 string states in the NS sector. α_{-r} denotes the Fourier mode of a worldsheet scalar and ψ_{-r} that of a worldsheet fermion. $M = 1, 2, 3, z$ and $i = 6, 7, 8, 9$ are the vector indices for $SO(4)_{123z}$ and $SO(4)_{6789}$, respectively. a and $\dot{a}$ are the undotted and dotted spinor indices of $SO(4)_{123z} \simeq (SU(2)_L \times SU(2)_R)/\mathbb{Z}_2$, corresponding to the doublet representation of $SU(2)_L$ and $SU(2)_R$, respectively. N_{95} is the total excitation number of a 9-5 string state with the mass squared equal to N_{95}/l_s^2.

		$SO(4)_{123z}$	$SO(4)_{6789}$
$N_{95} = 0$	$\lvert a\rangle_{\mathrm{NS}}$	$(1/2, 0)$	$\mathbf{1}$
$N_{95} = 1/2$	$\alpha^M_{-1/2}\lvert a\rangle_{\mathrm{NS}}$	$(0, 1/2) \oplus (1, 1/2)$	$\mathbf{1}$
	$\psi^i_{-1/2}\lvert \dot{a}\rangle_{\mathrm{NS}}$	$(0, 1/2)$	$\square_4$
$N_{95} = 1$	$\alpha^i_{-1}\lvert a\rangle_{\mathrm{NS}}$	$(1/2, 0)$	$\square_4$
	$\psi^M_{-1}\lvert \dot{a}\rangle_{\mathrm{NS}}$	$(1/2, 0) \oplus (1/2, 1)$	$\mathbf{1}$
	$\alpha^M_{-1/2}\alpha^N_{-1/2}\lvert a\rangle_{\mathrm{NS}}$	$(1/2, 0) \oplus (1/2, 1) \oplus (3/2, 1)$	$\mathbf{1}$
	$\alpha^M_{-1/2}\psi^i_{-1/2}\lvert \dot{a}\rangle_{\mathrm{NS}}$	$(1/2, 0) \oplus (1/2, 1)$	$\square_4$
	$\psi^i_{-1/2}\psi^j_{-1/2}\lvert a\rangle_{\mathrm{NS}}$	$(1/2, 0)$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}_6$

Table 3. Low-lying 9-5 string states in the NS sector (states in Table 2) classified by $SO(4)_{123z} \times SO(5)_{\tilde{y}6789}$.

	$SO(4)_{123z} \times SO(5)_{\tilde{y}6789}$
$N_{95} = 0$	$(1/2, 0)\,\mathbf{1}$
$N_{95} = 1/2$	$(0, 1/2)\,\square_5 \oplus (1, 1/2)\,\mathbf{1}$
$N_{95} = 1$	$(1/2, 0)\begin{smallmatrix}\square\\\square\end{smallmatrix}_{10} \oplus (1/2, 0)\,\square_5 \oplus (1/2, 1)\,\square_5$
	$\oplus\,(1/2, 0)\,\mathbf{1} \oplus (1/2, 1)\,\mathbf{1} \oplus (3/2, 1)\,\mathbf{1}$

where the Young tableaux are those of $SO(5)_{\tilde{y}6789}$. In fact, these states are obtained as the decomposition of $\square\square_{44} \oplus \begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}_{84}$ of $SO(9)$, which is the same as the first excited 9-9 string states considered in Ref. [4].

2.3. *Symmetries in the presence of a baryon vertex*

Reference [4] discusses that the D4/D8-brane system has discrete symmetries that are identified with those in massless QCD. The parity P and charge conjugation C are given by

$$P = I_{123z}, \qquad C = I_{z89}\Omega\,(-1)^{F_L}, \tag{2.6}$$

respectively, where $I_{i_1 i_2 \cdots}$ is spacetime involution along the $i_1, i_2, \ldots$ directions, Ω is a worldsheet parity, and F_L is a spacetime fermion number in the left-moving sector of a string worldsheet. A

Table 4. Low-lying 5-5 string states in the NS sector. N_{55} is the total excitation number of a 5-5 string state with mass squared equal to N_{55}/l_s^2.

		$SO(4)_{123z}$	$SO(4)_{6789}$
$N_{55}=0$	$\psi^M_{-1/2}\lvert 0\rangle_{\text{NS}}$	$(1/2,1/2)$	$\mathbf{1}$
	$\psi^i_{-1/2}\lvert 0\rangle_{\text{NS}}$	$(0,0)$	$\square_4$
$N_{55}=1$	$\psi^M_{-1/2}\psi^N_{-1/2}\psi^L_{-1/2}\lvert 0\rangle_{\text{NS}}$	$(1/2,1/2)$	$\mathbf{1}$
	$\psi^M_{-1/2}\psi^N_{-1/2}\psi^i_{-1/2}\lvert 0\rangle_{\text{NS}}$	$(1,0)\oplus(0,1)$	$\square_4$
	$\psi^M_{-1/2}\psi^i_{-1/2}\psi^j_{-1/2}\lvert 0\rangle_{\text{NS}}$	$(1/2,1/2)$	$\begin{smallmatrix}\square\\\square\end{smallmatrix}_6$
	$\psi^i_{-1/2}\psi^j_{-1/2}\psi^k_{-1/2}\lvert 0\rangle_{\text{NS}}$	$(0,0)$	$\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix}_4$
	$\psi^M_{-3/2}\lvert 0\rangle_{\text{NS}}$	$(1/2,1/2)$	$\mathbf{1}$
	$\psi^i_{-3/2}\lvert 0\rangle_{\text{NS}}$	$(0,0)$	$\square_4$
	$\alpha^M_{-1}\psi^N_{-1/2}\lvert 0\rangle_{\text{NS}}$	$(0,0)\oplus(1,0)\oplus(0,1)\oplus(1,1)$	$\mathbf{1}$
	$\alpha^M_{-1}\psi^i_{-1/2}\lvert 0\rangle_{\text{NS}}$	$(1/2,1/2)$	$\square_4$
	$\alpha^i_{-1}\psi^M_{-1/2}\lvert 0\rangle_{\text{NS}}$	$(1/2,1/2)$	$\square_4$
	$\alpha^i_{-1}\psi^j_{-1/2}\lvert 0\rangle_{\text{NS}}$	$(0,0)$	$\square_4\oplus\begin{smallmatrix}\square\\\square\end{smallmatrix}_6\oplus\square\square_9$

D4$_{\text{BV}}$-brane placed at $x^1=x^2=x^3=y=z=0$[5] is invariant under P, while it is mapped to a $\overline{\text{D4}}_{\text{BV}}$-brane under C. To see the latter, note that when the $\mathbb{Z}_2$ action generated by C is gauged, a background has an O6-plane at $z=x^8=x^9=0$, and it is known that the D4$_{\text{BV}}$-brane has to be paired with a $\overline{\text{D4}}_{\text{BV}}$-brane in the presence of the O6-plane [21]. This is consistent with the fact that the baryon is invariant under the parity, up to sign of the wavefunction, while it is mapped to an anti-baryon under the charge conjugation.

In order to see how P acts on the NS ground state of the 9-5 string considered in Sect. 2.2, it is useful to write the parity operator in a bosonized form. We note that the worldsheet fermions of a 9-5 string can be expressed using free worldsheet complex scalars H^1 and H^2 as

$$\psi^1 \pm i\psi^2 = e^{\pm iH^1}, \qquad \psi^3 \pm i\psi^z = e^{\pm iH^2}.$$

Parity acts on the worldsheet fermions as

$$\psi^M \to -\psi^M,$$

which in turn induces the transformation of H^1, H^2 as

$$(H^1, H^2) \to (H^1 + (2n_1+1)\pi, H^2 + (2n_2+1)\pi), \tag{2.7}$$

[5] For a D4$_{\text{BV}}$-brane wrapped on $\mathbb{S}^4$, it can be shown that $y=z=0$ is energetically favored and realized in the classical minimal energy configuration. It may be located anywhere in $\mathbb{R}^3 \ni x^{1,2,3}$, because of the translational invariance. Here we just put it at $x^1=x^2=x^3=0$ to have a P-invariant configuration.

with a choice of $n_1, n_2 \in \mathbb{Z}$. The vertex operator corresponding to the NS ground state of a 9-5 string is given by

$$e^{i(s_1 H^1 + s_2 H^2)}, \tag{2.8}$$

up to a ghost sector that is invariant under P, with $s_1 = s_2 = \pm 1/2$ for $|a\rangle_{\mathrm{NS}}$ and $s_1 = -s_2 = \pm 1/2$ for $|\dot{a}\rangle_{\mathrm{NS}}$. Therefore, the parity transformation in Eq. (2.7) acts as the chirality operator on the spinor representation of $SO(4)_{123z}$ up to a sign ambiguity. We choose n_1 and n_2 in Eq. (2.7) such that $|a\rangle_{\mathrm{NS}}$ and $|\dot{a}\rangle_{\mathrm{NS}}$ are parity even and odd, respectively. With this convention, the parity of the proton and the neutron turn out to be even. This is consistent with the conventional choice of the parity in QCD, in which the parity of quarks are chosen to be even. For a $\overline{\mathrm{D5}}_{\mathrm{BV}}$-brane, which represents an anti-baryon, since the GSO projection is opposite, the parity of the the NS ground state is odd. This is again consistent with the fact that the anti-quarks have odd parity.

Then, the parity of the excited states can be computed by using the transformation laws of the creation operators that act on the ground state. Namely, ψ^M_{-r} and α^M_{-r} with $M = 1, 2, 3, z$ are parity-odd and ψ^i_{-r} and α^i_{-r} with $i = 6, 7, 8, 9$ are parity-even operators.

In addition to these symmetries, the D4/D8-system admits a discrete symmetry that has no counterpart in QCD. This is called τ-parity[6] and defined as

$$P_\tau = I_{y9}\,(-1)^{F_L}. \tag{2.9}$$

As discussed in Ref. [4], both the quarks that originate from 4-8 and 4-$\bar{8}$ strings in the open string picture and the gluons that originate from the 4-4 strings are even under τ-parity. This implies that all the states that can be interpreted as the genuine color singlet states of QCD have to be τ-parity even as well. There are τ-parity-odd states in the spectrum of the bound states in our model. However, such states are artifacts of the model which do not have counterparts in QCD, and we will not consider them in the following.

Assuming that the $\mathrm{D4}_{\mathrm{BV}}$-brane is placed at $y = 0$, one can show that the $\mathrm{D4}_{\mathrm{BV}}$-brane is invariant under the τ-parity P_τ. To see this, we note that I_{y9} maps the $\mathrm{D4}_{\mathrm{BV}}$ to a $\overline{\mathrm{D4}}_{\mathrm{BV}}$ and $(-1)^{F_L}$ maps it back to a $\mathrm{D4}_{\mathrm{BV}}$.

For the purpose of reading off the τ-parity of an open string state, it is useful to work in the T-dualized description used in Sect. 2.2. When the y-direction is T-dualized, P_τ is mapped to

$$\widetilde{P}_\tau = I_{9\tilde{y}}, \tag{2.10}$$

where $\tilde{y}$ is the T-dualized coordinate of y. This is simply a 180° rotation in the 9-$\tilde{y}$ plane and it is easy to find the action of $\widetilde{P}_\tau$ from the representation of $SO(5)_{\tilde{6}789}$ listed in Table 3 and Eq. (2.5).

In addition to the τ-parity discussed above, we can also use the $SO(5)$ isometry of $\mathbb{S}^4$ in the background to single out the open string states that could be used to construct a baryon in QCD. It is easy to see that both quarks and gluons are invariant under this $SO(5)$, and hence the baryons in QCD have to be an $SO(5)$ singlet. In the flat spacetime limit, the requirement of the $SO(5)$ invariance amounts to demanding that the states be $SO(4)_{6789}$-singlet and carry no momentum along the 6789-directions. In the T-dualized picture, we should also impose the condition that the momentum along $\tilde{y}$ is zero, since the original y direction is not compactified and there is no winding mode along y.

[6] τ-parity was originally introduced in Ref. [22] in the context of glueball spectrum and then generalized to the system with quarks in Ref. [4].

Table 5. 9-5 string states that could contribute to genuine QCD baryons. All the states belong to the fundamental representation of the flavor $U(N_f)$ symmetry and have the unit charge with respect to the $U(1)$ gauge symmetry on the $\mathrm{D4_{BV}}$-brane. The massive 9-5 string states are labeled by $j = 1, 2, \ldots$, and will be used in Sect. 4.1.

	$SO(4)_{123z}$	$SU(2)_J$	Parity	label j
$N_{95} = 0$	$(1/2, 0)$	$1/2$	$+$	
$N_{95} = 1/2$	$(1, 1/2)$	$3/2 \oplus 1/2$	$-$	1
$N_{95} = 1$	$(1/2, 0)$	$1/2$	$+$	2
	$(1/2, 1)$	$3/2 \oplus 1/2$	$+$	3
	$(3/2, 1)$	$5/2 \oplus 3/2 \oplus 1/2$	$+$	4

Therefore, among the open string states obtained in Sect. 2.2, we only consider the states that are invariant under $SO(4)_{6789}$ and the τ-parity $\widetilde{P}_\tau$, and carry no momentum along the $\tilde{y}6789$ directions.

2.4. Summary of the results

We first derive the 9-5 string states that meet the conditions discussed in the last subsection. The requirement of $SO(4)_{6789}$ invariance implies that the R sector must be removed because all the states in the R sector are $SO(4)_{6789}$-nonsinglet. It follows from the τ-parity condition that among the $SO(4)_{6789}$-singlet NS states, only those with an even number of the spacetime index $\tilde{y}$ are allowed. The NS ground state satisfies these conditions. For the first excited states (those with $N_{95} = 1/2$) listed in Table 3, only the state with $(s_L, s_R) = (1, 1/2)$ is allowed. From the second excited states with $N_{95} = 1$, we pick up

$$(1/2, 0)\mathbf{1} \oplus (1/2, 1)\mathbf{1} \oplus (3/2, 1)\mathbf{1}.$$

Finally, we set the momenta along the $\tilde{y}6789$ direction to zero, which is equivalent to omitting the dependence of the corresponding wavefunctions on $\tilde{y}$ and $x^{6,7,8,9}$. These results are summarized in Table 5, where we also list the representation (spin) of $SU(2)_J$, which is related to the $SO(3)_{123}$ subgroup of $SO(4)_{123z}$ by $SU(2)_J/\mathbb{Z}_2 \simeq SO(3)_{123}$. Note that $SO(4)_{123z}$ symmetry appears only in the flat spacetime limit and it is broken to $SO(3)_{123}$ due to the z-dependence of the background. The masses of these states in the flat spacetime limit are proportional to the excitation number N_{95} as

$$m^2 = \frac{N_{95}}{\alpha'} = \frac{4\lambda}{27} N_{95} \quad (N_{95} = 0, 1/2, 1, \ldots), \tag{2.11}$$

where we have used the relation in Eq. (2.4).

The quantum field corresponding to the 9-5 massless state is denoted by ω^I_α, which reduces to a function of time t only as discussed above. Here, $\alpha = 1, 2$ is the spin index for $SU(2)_J$ and $I = 1, 2, \ldots, N_f$ is the index for the flavor $U(N_f)$ symmetry.

Next, we discuss the 5-5 string states. As in the 9-5 string case, all the R states are non-singlet under $SO(4)_{6789}$ and thus ruled out. The NS massless states that satisfy all the conditions are given by $\psi^M_{-1/2}\,|0\rangle_{\mathrm{NS}}$ $(M = 1, 2, 3, z)$ only. The corresponding fields are denoted as X^M. Again, these fields reduce to functions of t. Among the first excited states with $N_{55} = 1$ listed in Eq. (2.5), the following states satisfy all the conditions:

$$2\,(0, 0) \oplus (1/2, 1/2) \oplus (1, 1). \tag{2.12}$$

Table 6. 5-5 string states that could contribute to genuine QCD baryons. All the states are singlet under the flavor $U(N_f)$ symmetry and neutral under the $U(1)$ gauge symmetry on the $D4_{BV}$-brane. The massive 5-5 strings are labeled by $k = 1, 2, \ldots$, and will be used in Sect. 4.1.

	$SO(4)_{123z}$	$SU(2)_J$	Parity	Label k
$N_{55} = 0$	$(1/2, 1/2)$	$1 \oplus 0$	$-$	
$N_{55} = 1$	$2\,(0, 0)$	$0 \oplus 0$	$+$	$1, 2$
	$(1/2, 1/2)$	$1 \oplus 0$	$-$	3
	$(1, 1)$	$2 \oplus 1 \oplus 0$	$+$	4

Note here that there are two $(0, 0)$ states, and one of them comes from $(0, 0)_{\square\square\,15}$ in Eq. (2.5) with two $\tilde{y}$ indices. The masses are given by

$$m^2 = \frac{N_{55}}{\alpha'} = \frac{4\lambda}{27} N_{55} \quad (N_{55} = 0, 1, 2, \ldots). \tag{2.13}$$

The results for the 5-5 strings are summarized in Table 6.

3. One-baryon quantum mechanics

In the previous section, we obtained the spectrum of the open strings attached on the baryon vertex $D4_{BV}$-brane.[7] Here, we write down the quantum mechanical ((0 + 1)-dimensional) action for these open string degrees of freedom. This action is a generalization of the quantum mechanical action obtained in a solitonic approach of the baryons in holographic QCD [7], which is related to that of the collective coordinates in the Skyrme model [6], and the nuclear matrix model formulated in [14], which is obtained by considering the ground states in the open string spectrum. The baryon states are obtained by quantizing this system. In this section we give the general procedure to obtain the baryon spectrum including the contributions from the excited open string states. The explicit construction of some of the low-lying baryon states will be given in Sect. 4.

3.1. The action

The action for the open string states attached on the baryon vertex $D4_{BV}$-brane is written as

$$S = \int dt\, (L_0 + L_m), \tag{3.1}$$

where L_0 is the Lagrangian for the ground states while L_m is the part that involves the excited states. L_0 is derived in Ref. [14] as

$$L_0 = \frac{M_0}{2} \left[\dot{X}^2 + |D_0 w|^2 - V_{\rm ADHM}(w) - V_0(X, w) \right] + N_c A_0, \tag{3.2}$$

where $w = (w^I_\alpha)$ is a complex $N_f \times 2$ matrix variable with a spin ($SU(2)_J$) index $\alpha = 1, 2$ and a flavor ($SU(N_f)_{\rm flavor}$) index $I = 1, \ldots, N_f$, $X = (X^M)$ ($M = 1, 2, 3, z$) is a real four-component variable, and A_0 the $U(1)$ gauge field on the $D4_{BV}$-brane; w and X correspond to the ground state for

[7] In this and the following sections, we consider the original $D8/D4_{BV}$ system, rather than the T-dualized version ($D9/D5_{BV}$ system) considered in the previous section. Therefore, the 9-5 and 5-5 strings in the previous section correspond to 8-4 and 4-4 strings, respectively.

8-4 strings and 4-4 strings, respectively. The value of X represents the position of the $\mathrm{D4_{BV}}$-brane in the four-dimensional space parametrized by (x^1, x^2, x^3, z). The dot denotes the time derivative as $\dot{X} \equiv \frac{d}{dt}X$ and

$$D_0 w \equiv \dot{w} - iA_0 w \equiv \frac{dw}{dt} - iA_0 w \tag{3.3}$$

is the covariant derivative. The potential terms are given by

$$V_{\mathrm{ADHM}}(w) = c\left(\mathrm{tr}(\vec{\tau}\, w^\dagger w)\right)^2 = c\left(2|w^\dagger w|^2 - (|w|^2)^2\right), \tag{3.4}$$

$$V_0(X, w) = m_z^2 (X^z)^2 + \gamma |w|^2 + \frac{v}{|w|^2}. \tag{3.5}$$

Here, $\vec{\tau} = (\tau^1, \tau^2, \tau^3)$ is the Pauli matrix and we have used the notation $|a|^2 \equiv \mathrm{tr}(a^\dagger a) = \sum_{\alpha,I} (a^\dagger)_I^\alpha a_\alpha^I$ for a complex matrix $a = (a_\alpha^I)$. M_0, c, m_z, γ, and v are constants; M_0, c, and m_z are related to the number of colors N_c and the 't Hooft coupling λ as[8]

$$M_0 = \frac{\lambda N_c}{27\pi}, \qquad c = \frac{\lambda^2}{3^6 \pi^2}, \qquad m_z^2 = \frac{2}{3}. \tag{3.6}$$

The potential V_{ADHM} in Eq. (3.4) is obtained by integrating out the auxiliary fields in Ref. [14]. The condition $V_{\mathrm{ADHM}}(w) = 0$ is equivalent to the ADHM constraints for the ADHM construction of the self-dual instanton solution. The first term of V_0 in Eq. (3.5) represents the fact that the $\mathrm{D4_{BV}}$-brane is attracted to the origin in the z-direction due to the curved background. The second and third terms in Eq. (3.5) are added rather phenomenologically. γ is chosen to be $\gamma = 1/6$ in Ref. [14] so that the second term in Eq. (3.5) recovers the corresponding term in the soliton approach [7]. The third term in Eq. (3.5) was not present in Ref. [14], but one could add it to have more flexibility. We treat γ and v as unspecified parameters for the moment.[9]

L_m is the Lagrangian with the excited states obtained in Sect. 2. It can be written as

$$L_m = \frac{M_0}{2}\left[\sum_j \left(|D_0 \Psi_j|^2 - m_j^2 |\Psi_j|^2\right) + \sum_k \left(\dot{\Phi}_k^2 - m_k^2 \Phi_k^2\right) + L_{\mathrm{int}}\right], \tag{3.7}$$

where Ψ_j and Φ_k denote the fields corresponding to the excited states created by 8-4 strings and 4-4 strings, respectively. We call these "massive fields" in the following. The indices j and k label all the excited states, and m_j^2 and m_k^2 are the mass squared of these states given in Eqs. (2.11) and (2.13), which are of order $1/\alpha' \sim \mathcal{O}(\lambda)$. The Ψ_j are complex fields that couple with the $U(1)$ gauge field A_0 with the unit charge, while the Φ_k are real fields, which are neutral under the $U(1)$ gauge symmetry. L_{int} gives the interaction terms for the massive fields that may also contain massless fields. We put the overall factor $M_0/2$ by convention so that all the fields have the dimension of length. Since the

[8] There is a mass parameter M_{KK} that gives the mass scale of the model. We mainly work in the $M_{\mathrm{KK}} = 1$ unit. The M_{KK} dependence can be easily recovered by dimensional analysis.

[9] One motivation to add these terms is to accommodate possible additional energy contributions from the gauge fields on the D8-branes. The second and third terms in Eq. (3.5) mimic the ρ-dependent energy contributions from the gauge fields in Ref. [7]. Note that we should not trust this potential near $w = 0$ when $v \neq 0$, since the third term in Eq. (3.5) diverges at $w = 0$. As we will see in Sects. 3.3 and 3.4, the wavefunctions of the baryon states that we are mostly interested in peak away from $w = 0$ and we expect that it does not affect the main features of the analysis.

evaluation of the interaction terms including the massive states is beyond the scope of this paper, we assume that the contribution from $L_{\rm int}$ is small as far as the qualitative features of the baryon spectrum are concerned. In Sect. 3.7 we argue that though most of the possible terms in $L_{\rm int}$ are suppressed in the large-N_c limit, there are some terms that could survive even in the large-N_c limit.

3.2. *Gauss law constraint and Hamiltonian*

To quantize our system, we follow the approach developed recently in Ref. [13]. We take the $A_0 = 0$ gauge and impose the EOM for A_0 (Gauss law constraint) as a physical state condition on the Hilbert space. The Gauss law constraint can be written as

$$q_w + \sum_j q_j = N_c, \tag{3.8}$$

where

$$q_w \equiv \frac{M_0}{2}{\rm tr}(i(\dot{w}^\dagger w - w^\dagger \dot{w})), \qquad q_j \equiv \frac{M_0}{2} i(\dot{\Psi}_j^\dagger \Psi_j - \Psi_j^\dagger \dot{\Psi}_j). \tag{3.9}$$

These q_w and q_j correspond to the charge associated with the phase rotation symmetries $w \to e^{i\alpha_w} w$ and $\Psi_j \to e^{i\alpha_j}\Psi_j$, respectively, which are approximate symmetries that exist when the interaction term $L_{\rm int}$ is neglected. The Gauss law constraint in Eq. (3.8) represents the fact that N_c open strings have to be attached on the D4$_{\rm BV}$-brane, and q_j is interpreted as the number of excited open strings associated with Ψ_j.[10]

It is interesting to note that the Gauss law constraint in Eq. (3.8) implies that the spin of the baryon state is half-integer or integer for odd or even N_c, respectively.[11] Indeed, the wavefunction for the baryon state satisfying the Gauss law constraint in Eq. (3.8) is of the form[12]

$$\psi(X, w, w^\dagger, \Psi_j, \Psi_j^\dagger, \Phi_k) = \underbrace{w_{\alpha_1}^{I_1} \cdots w_{\alpha_{q_w}}^{I_{q_w}} \Psi_{j_1} \cdots \Psi_{j_{N_c - q_w}}}_{N_c} \widetilde{\psi}(X, w^\dagger w, \Psi_j^\dagger \Psi_{j'}, \Psi_j^\dagger w, w^\dagger \Psi_j, \Phi_k). \tag{3.10}$$

Here, $\widetilde{\psi}$ is a $U(1)$-invariant wavefunction that is written only through $U(1)$ invariants. Because 8-4 strings (w and Ψ_j) and 4-4 strings (X and Φ_k) carry half-integer and integer spin, respectively, $\widetilde{\psi}$ can only have an integer spin and the spin of the state in Eq. (3.10) is $N_c/2 \mod \mathbb{Z}$.

Omitting $L_{\rm int}$, the Hamiltonian in the $A_0 = 0$ gauge is given by

$$H = H_0 + H_m, \tag{3.11}$$

with

$$H_0 = \frac{1}{2M_0}(P_X^2 + |P_w|^2) + \frac{M_0}{2}(V_{\rm ADHM}(w) + V_0(X, w)), \tag{3.12}$$

$$H_m = \sum_j \left(\frac{1}{2M_0}|P_{\Psi_j}|^2 + \frac{1}{2}M_0 m_j^2 |\Psi_j|^2\right) + \sum_k \left(\frac{1}{2M_0}P_{\Phi_k}^2 + \frac{1}{2}M_0 m_k^2 \Phi_k^2\right), \tag{3.13}$$

[10] Both q_w and q_j can be negative. The sign reflects the orientation of the fundamental string attached on the D4$_{\rm BV}$-brane.

[11] See Refs. [13,23] for related discussions.

[12] Here, we discuss the cases with $0 \le q_w \le N_c$ for simplicity. Other cases can also be discussed in a similar way.

where P_X, P_w, P_{Ψ_j}, and P_{Φ_k} are the momenta conjugate to X, w, Ψ_j, and Φ_k, respectively. H_m in Eq. (3.13) is simply a collection of harmonic oscillators associated with the excited open string states obtained in Sect. 2. The quantum mechanics for H_0 in Eq. (3.12) has been studied in Refs. [13,14], though the part with w is treated in a different way in the following.

3.3. H_0 for $N_f = 2$

We are particularly interested in the cases with $N_f = 2$, in which w is a 2×2 complex matrix and can be parametrized as

$$w = Y_0 1_2 + i\vec{Y} \cdot \vec{\tau}, \quad (Y = (Y_0, \vec{Y}) \in \mathbb{C}^4), \tag{3.14}$$

where 1_2 is the 2×2 unit matrix. Y transforms as the (complex) four-dimensional vector representation of $SO(4) \simeq (SU(2)_I \times SU(2)_J)/\mathbb{Z}_2$, where $SU(2)_J$ and $SU(2)_I = SU(N_f)_{\text{flavor}}$ with $N_f = 2$ corresponds to the spin and isospin groups, respectively. The kinetic term for w in Eq. (3.12) is written as

$$\frac{1}{2M_0}|P_w|^2 = -\frac{1}{4M_0}\Delta_Y, \tag{3.15}$$

where $\Delta_Y = 4\frac{\partial^2}{\partial \overline{Y}_A \partial Y_A}$ is the Laplacian in $\mathbb{C}^4$.

Using the relations

$$|w|^2 = 2|Y|^2, \qquad |w^\dagger w|^2 = 4(|Y|^2)^2 - 2|Y^2|^2, \tag{3.16}$$

where $Y^2 \equiv Y_0^2 + \vec{Y} \cdot \vec{Y}$ and $|Y|^2 \equiv |Y_0|^2 + \vec{Y}^\dagger \cdot \vec{Y}$, the ADHM potential can be written as

$$V_{\text{ADHM}}(Y) = 4c\left((|Y|^2)^2 - |Y^2|^2\right). \tag{3.17}$$

The minimum of this potential is parametrized by

$$Y = e^{i\theta} y \quad (\theta \in \mathbb{R},\ y \in \mathbb{R}^4). \tag{3.18}$$

Note that y together with X correspond to the collective coordinates of the one-instanton configuration considered in Ref. [7]. More explicitly,

$$\rho \equiv \sqrt{y^2}, \qquad a \equiv y/\rho \tag{3.19}$$

corresponds to the size and the $SU(2)$ orientation of the instanton solution, respectively.[13] One way to include the components that are orthogonal to the directions along Eq. (3.18) is to parametrize Y as[14]

$$Y = e^{i\theta}(y + i\widetilde{y}) \quad (\theta \in \mathbb{R},\ y, \widetilde{y} \in \mathbb{R}^4) \tag{3.20}$$

with

$$\widetilde{y} = \beta_a i \Sigma^a a \quad ((\beta_a) = (\beta_1, \beta_2, \beta_3) \in \mathbb{R}^3), \tag{3.21}$$

[13] Using the relation $a^2 = 1$, one can show that $\boldsymbol{a} \equiv a_0 1_2 + i\vec{a} \cdot \vec{\tau}$ is an element of $SU(2)$. This $\boldsymbol{a}$ is also related to the collective coordinate of the Skyrmion for $N_f = 2$ [6].

[14] The notation y and $\widetilde{y}$ in this section should not be confused with that in Sect. 2.2.

where Σ^a ($a = 1, 2, 3$) are the generators of $SU(2)_I$ acting on y, which are chosen to be pure imaginary anti-symmetric matrices. See Appendix A for the explicit forms. One can easily show that

$$y \cdot \widetilde{y} = 0, \qquad \widetilde{y}^2 = \beta^2, \tag{3.22}$$

where $\beta^2 = \beta_a\beta_a$ and the ADHM potential becomes

$$V_{\mathrm{ADHM}}(Y) = 16c\,\rho^2\beta^2. \tag{3.23}$$

Note that the parametrization in Eq. (3.20) has a redundancy induced by the $\mathbb{Z}_2$ transformation

$$\theta \to \theta + \pi, \qquad y \to -y. \tag{3.24}$$

When the wavefunction is written in terms of θ, y, and β_a instead of Y, we should impose the invariance of the wavefunction under this $\mathbb{Z}_2$ transformation.

In this paper we consider the cases that β takes small values so that V_{ADHM} does not generate an additional mass term for ρ. One important observation is that the kinetic term of the Hamiltonian in Eq. (3.12) contains a term as

$$-\frac{1}{2M_0\rho^2}\frac{\partial^2}{\partial\theta^2} \tag{3.25}$$

for $\beta^2 \ll \rho^2$ [see Eq. (3.29)]. Since q_w is the generator of the phase rotation of Y, we have the relation

$$q_w = -i\frac{\partial}{\partial\theta} \tag{3.26}$$

in the quantum mechanics. When we consider the cases with $\sum_j q_j \sim \mathcal{O}(1)$, q_w has to be of $\mathcal{O}(N_c)$ because of the Gauss law constraint in Eq. (3.8). In such cases, the term in Eq. (3.25) gives a potential of the form

$$-\frac{1}{2M_0\rho^2}\frac{\partial^2}{\partial\theta^2} \sim \frac{N_c}{\lambda\rho^2} \tag{3.27}$$

up to a numerical factor in the large-N_c limit, which has the effect of pushing ρ to have a larger value. Let ρ_0 be the value of ρ that minimizes the effective potential given by adding this term to V_0 in Eq. (3.5). Assuming that the third term in Eq. (3.5) is either negligible or of the same order as Eq. (3.27), i.e. $v \sim \mathcal{O}(\lambda^{-2})$, we find $\rho_0^2 \sim \mathcal{O}(\lambda^{-1})$, which is consistent with the results in Refs. [7,9,10]. We will shortly obtain an explicit expression for ρ_0 in the large-N_c limit [see Eq. (3.32)], and show that it has the effect of generating a large mass term for β_a in the next subsection.

3.4. Large-N_c limit

Now, let us figure out which terms in H_0 are important in the large-N_c limit. First, we decompose ρ as $\rho = \rho_0 + \delta\rho$, and regard $M_0^{1/2}\delta\rho$, $M_0^{1/2}\beta_a$, and a to be order 1 variables,[15] which means that

$$\delta\rho \sim \beta_a \sim \mathcal{O}(\lambda^{-1/2}N_c^{-1/2}), \qquad \frac{\partial}{\partial\rho} \sim \frac{\partial}{\partial\beta_a} \sim \mathcal{O}(\lambda^{1/2}N_c^{1/2}). \tag{3.28}$$

Then, the leading ($\mathcal{O}(\lambda N_c^2)$) and subleading ($\mathcal{O}(\lambda N_c)$) terms in the Laplacian Δ_Y turn out to be

$$\Delta_Y \simeq \left(\frac{1}{\rho^2} + \frac{3\beta^2}{\rho_0^4}\right)\frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial\rho^2} + \left(\frac{\partial}{\partial\beta_a}\right)^2 + \mathcal{O}(\lambda N_c^{1/2}). \tag{3.29}$$

Keeping these terms, the Hamiltonian for ρ and β_a becomes

$$\begin{aligned} H_0|_{\rho,\beta} &\simeq \frac{1}{4M_0}\left[\frac{q_w^2}{\rho^2} + \frac{3q_w^2}{\rho_0^4}\beta^2 - \frac{\partial^2}{\partial\rho^2} - \left(\frac{\partial}{\partial\beta_a}\right)^2\right] \\ &\quad + \frac{M_0}{2}\left[16c\,\rho^2\beta^2 + 2\gamma(\rho^2+\beta^2) + \frac{v}{2(\rho^2+\beta^2)}\right] \\ &\simeq 2M_0\gamma\rho_0^2 - \frac{1}{4M_0}\left[\frac{\partial^2}{\partial\rho^2} + \left(\frac{\partial}{\partial\beta_a}\right)^2\right] + M_0\left(\omega_{\delta\rho}^2\delta\rho^2 + \omega_\beta^2\beta^2\right), \end{aligned} \tag{3.30}$$

where

$$\omega_{\delta\rho}^2 = 4\gamma, \qquad \omega_\beta^2 = 8c\rho_0^2 + \gamma - \frac{v}{4\rho_0^4} + \frac{3q_w^2}{4M_0^2\rho_0^4}. \tag{3.31}$$

Here we have imposed the condition that ρ_0 minimizes the potential for ρ, which reads

$$\rho_0^2 = \frac{1}{2}\sqrt{\frac{1}{\gamma}\left(\frac{q_w^2}{M_0^2} + v\right)}. \tag{3.32}$$

The Hamiltonian in Eq. (3.30) is a sum of the harmonic oscillators for ρ and β_a.

A few comments are in order. First, $\omega_{\delta\rho}^2$ coincides with m_z^2 in Eq. (3.6) for $\gamma = 1/6$ used in Ref. [14], which is consistent with Ref. [7]. Second, the value of ρ_0 in Eq. (3.32) agrees with that in Ref. [14] when $q_w = N_c$ and $v = 0$. However, as pointed out in Ref. [14], it is larger than the value in Refs. [7,9,10] by a factor of 5/4. One can adjust the value of v as $v = -\frac{N_c^2}{5M_0^2}$ to match with the value in Refs. [7,9,10]. Third, on the right-hand side of ω_β^2 in Eq. (3.31), the first term $8c\rho_0^2$ is of order λ, while the other terms are of order 1. Recall that the masses of the excited open string states are $m^2 \propto 1/\alpha' \sim \mathcal{O}(\lambda)$. This means that although β_a arises as the ground states (the open string states with $N_{95} = 0$), it acquires a large mass comparable to the massive excited states due to the ADHM potential in Eq. (3.4) together with the Gauss law constraint in Eq. (3.8).

[15] This is equivalent to writing down the Lagrangian in terms of the canonically normalized fields $\widetilde{\delta\rho} \equiv M_0^{1/2}\delta\rho$ and $\widetilde{\beta}_a \equiv M_0^{1/2}\beta_a$ and taking the large-N_c limit with these fields kept finite. On the other hand, a satisfies $a^2 = 1$ by definition, and hence we regard it as an order 1 variable. We also assume here that quantum numbers for the baryon state such as spin and isospin are all order 1, except for q_w which is assumed to be of order N_c as discussed around Eq. (3.26).

3.5. Mass formula

As argued in Sect. 3.3, the Hamiltonian is reduced to a collection of harmonic oscillators in the large-N_c limit, which can be easily solved. Then, the masses of the baryons are obtained as

$$M = M_0^* + m_z n_z + \omega_{\delta\rho} n_\rho + \omega_\beta \sum_{a=1}^{3} n_\beta^a + \sum_j m_j (n_j^\Psi + n_j^{\overline{\Psi}}) + \sum_k m_k n_k^\Phi, \tag{3.33}$$

where n_z, n_ρ, n_β^a, n_j^Ψ, $n_j^{\overline{\Psi}}$, and n_k^Φ are non-negative integers corresponding to the excitation levels of the harmonic oscillators associated with X^z, $\delta\rho$, β_a, Ψ_j, $\overline{\Psi}_j$, and Φ_k, respectively; m_z, $\omega_{\delta\rho}$, and ω_β are given in Eqs. (3.6) and (3.31); m_j and m_k are the masses for the corresponding open string states given in Eqs. (2.11) and (2.13), respectively; and M_0^* is a (q_w-dependent) constant whose classical value is

$$M^*_{0\,\text{classical}} = (1 + 2\gamma\rho_0^2) M_0, \tag{3.34}$$

where the first term M_0 comes from the tension of the D4$_{\text{BV}}$-brane placed at $y = z = 0$ and the second term $2\gamma\rho_0^2 M_0$ is the first term in Eq. (3.30). It also contains the contributions from the zero-point energies of all the fields in the system, including those neglected in Sect. 2. Since there are infinitely many fields involved, it is not easy to evaluate it explicitly.[16] For this reason, we leave M_0^* as an unknown parameter and focus on the mass differences.

Note that the mass in Eq. (3.33) implicitly depends on the value of q_w through the parameters M_0^* and ω_β. Because the Gauss law constraint in Eq. (3.8) implies that q_w is related to n_j^Ψ and $n_j^{\overline{\Psi}}$ by

$$q_w + \sum_j (n_j^\Psi - n_j^{\overline{\Psi}}) = N_c, \tag{3.35}$$

these parameters are state dependent.

As a consistency check, one can show that the formula in Eq. (3.33) agrees with the leading-order terms in the baryon mass formula obtained in Ref. [7] when $q_w = N_c$ and $n_\beta = n_j^\Psi = n_j^{\overline{\Psi}} = n_k^\Phi = 0$. In fact, the baryon mass formula in Ref. [7] can be written as

$$M = M_0 + \sqrt{\frac{(\ell+1)^2}{6} + \frac{2}{15} N_c^2} + \frac{2(n_\rho + n_z) + 2}{\sqrt{6}} \tag{3.36}$$

$$\simeq M_0 + 2M_0\gamma\rho_0^2 + \frac{(\ell+1)^2}{4M_0\rho_0^2} + \omega_{\delta\rho} n_\rho + m_z n_z + \frac{1}{2}(\omega_{\delta\rho} + m_z) + \mathcal{O}(N_c^{-3}), \tag{3.37}$$

where $\ell \in \mathbb{Z}_{\geq 0}$ is related to the spin J and isospin I as $I = J = \ell/2$. The ℓ dependence appears because the Laplacian in the y-space,

$$\Delta_y \equiv \left(\frac{\partial}{\partial y_A}\right)^2 = \frac{1}{\rho^3}\partial_\rho(\rho^2 \partial_\rho) + \frac{1}{\rho^2}\Delta_{S^3}, \tag{3.38}$$

contains the Laplacian on S^3 parametrized by a, denoted by Δ_{S^3}, whose eigenvalue is $-\ell(\ell+2)$. In Eq. (3.29) we have neglected this contribution, though it also appears in Δ_Y if we keep the $\mathcal{O}(N_c^0)$ term.

[16] As pointed out in Ref. [7], a similar problem also appears in the soliton approach.

In Ref. [7] ℓ was chosen to be odd (or even) for odd (or even) N_c by hand, so that the spin of the baryon obtained in the soliton approach is consistent with that in the quark model, as it is also the case for the Skyrme model with $N_f = 2$. In our case, this condition is replaced with $\ell \equiv q_w$ (mod 2), which automatically follows from the fact that the eigenfunction of Δ_{S^3} is given by

$$T^{(\ell)}(a) \equiv C^{A_1\cdots A_\ell} a_{A_1}\cdots a_{A_\ell}, \tag{3.39}$$

where $C^{A_1\cdots A_\ell}$ is a traceless symmetric tensor of rank ℓ, and θ appears in the wavefunction as an overall factor $e^{iq_w\theta}$. As explained around Eq. (3.24), the wavefunction has to be invariant under the $\mathbb{Z}_2$ transformation in Eq. (3.24), which implies $\ell \equiv q_w$ (mod 2).

3.6. Wavefunctions of the baryon states

As discussed above, the Hamiltonian of the one-baryon quantum mechanics is a collection of infinitely many harmonic oscillators in the large-N_c limit. The eigenfunction can be written as a product of a function of X, a, $\delta\rho$, and β_a and a function of Ψ_j, $\Psi_j^\dagger$, and Φ_k as

$$\psi(X, a, \delta\rho, \beta_a, \Psi_j, \Psi_j^\dagger, \Phi_k) = \psi_0(X, a, \delta\rho, \beta_a)\, \psi_m(\Psi_j, \Psi_j^\dagger, \Phi_k). \tag{3.40}$$

We call ψ_0 and ψ_m wavefunctions for the massless and massive sectors, respectively.[17]

The massless sector wavefunction ψ_0 can be written as

$$\psi_0 = e^{i\vec{p}\cdot\vec{X}}\, T^{(\ell)}(a)\psi_{n_z}(X^z)\psi_{n_\rho}(\delta\rho)\psi_{n_\beta}(\beta_a), \tag{3.41}$$

where $e^{i\vec{p}\cdot\vec{X}}$ is the wavefunction for the plane wave with momentum $\vec{p}$, $T^{(\ell)}(a)$ is defined in Eq. (3.39), and ψ_{n_z}, ψ_{n_ρ}, and ψ_{n_β} are the eigenfunctions of the harmonic oscillators for X^z, $\delta\rho$, and β_a with the excitation numbers n_z, n_ρ, and n_β^a, respectively. We set $\vec{p} = 0$ in the following for simplicity. We also use the bra–ket notation as

$$|\psi_0\rangle = |\ell, n_z, n_\rho, n_\beta^a, q_w\rangle\,. \tag{3.42}$$

Here, q_w is included in the notation to remember that the massless sector wavefunction also depends on q_w.

If ψ_{n_β} is trivial, ψ_0 agrees with the large-N_c limit of the wavefunction obtained in Ref. [7]. As shown in Ref. [7], $T^{(\ell)}(a)$ has a degeneracy of $(\ell+1)^2$ that corresponds to the states in the representation of $I = J = \ell/2$. The mass formula in Eq. (3.33) appears to be independent of ℓ, because the ℓ dependence is a subleading effect in the large-N_c limit. Upon taking finite-N_c effects into account, we expect that the energy is an increasing function of ℓ, as in Ref. [7].[18]

Note that since X^z is parity odd, ψ_{n_z} has parity $(-1)^{n_z}$. As mentioned in Sect. 3.4, $\omega_{\delta\rho}$ coincides with m_z^2 for $\gamma = 1/6$ and hence the states with $(n_\rho, n_z) = (1, 0)$ and $(n_\rho, n_z) = (0, 1)$ are degenerate. This implies a degeneracy between parity-even and -odd states for those with $(n_\rho, n_z) \neq (0, 0)$. This could be a hint toward an understanding of the parity-doubling phenomenon in the excited baryons.[19]

[17] Although X^z, $\delta\rho$, and β_a have mass terms in the Hamiltonian in Eqs. (3.12) and (3.30), we consider them to be in the massless sector because these modes originate from the the massless open string states in the flat spacetime limit.

[18] If we set $N_c = 3$ in the mass formula in Eq. (3.36) given in Ref. [7], the expansion as in Eq. (3.37) is not justified for $\ell > 1$. This suggests that the ℓ dependence is actually important to compare with the realistic QCD. (See Ref. [7] for further discussion.)

[19] See, e.g., Ref. [25] for a review.

ψ_{n_β} is a wavefunction for a three-dimensional harmonic oscillator with respect to β_a ($a = 1, 2, 3$). The energy contribution in the mass formula in Eq. (3.33) for this part is $\omega_\beta n_\beta$, with

$$n_\beta \equiv \sum_{a=1}^{3} n_\beta^a. \tag{3.43}$$

The degeneracy is

$$\frac{1}{2}(n_\beta + 1)(n_\beta + 2), \tag{3.44}$$

and the eigenspace for a given n_β can be decomposed into a direct sum over the states with isospin $I = 0, 2, \ldots, n_\beta$ or $I = 1, 3, \ldots, n_\beta$ for even or odd n_β, respectively. For example, for the state with $\ell = 1$ and $n_\beta = 1$, the massless wavefunction ψ_0 has spin $1/2$ and isospin $1/2 \otimes 1 = 3/2 \oplus 1/2$.

The wavefunction for the massive sector is given by the eigenfunctions of the harmonic oscillators associated with Ψ_j, $\Psi_j^\dagger$, and Φ_k, which is written in the bra–ket notation as

$$|\psi_m\rangle = |n_j^\Psi, n_j^{\overline{\Psi}}, n_k^\Phi\rangle\,. \tag{3.45}$$

In order to classify these states, we introduce the notation

$$\mathcal{N} = \mathcal{N}_{84} + \mathcal{N}_{44}, \tag{3.46}$$

which we call the level of a baryon, with

$$\mathcal{N}_{84} = \sum_j (n_j^\Psi + n_j^{\overline{\Psi}}) N_{84}^{(j)}, \qquad \mathcal{N}_{44} = \sum_k n_k^\Phi N_{44}^{(k)}, \tag{3.47}$$

where $N_{84}^{(j)}$ and $N_{44}^{(k)}$ are the excitation numbers for Ψ_j and Φ_k given in Tables 5 and 6, respectively.[20] It will become increasingly complicated to extract the spin and isospin for the states with larger $\mathcal{N}$. We will give some explicit examples of the baryon states in Sect. 4.

3.7. *Comments on L_{int}*

Here, we make some comments on L_{int} in Eq. (3.7). First, we classify L_{int} depending on the order of the massive fields multiplied and assume that each term contains at least two massive fields so that the trivial configuration $\Psi_j = \Phi_k = 0$ is a solution of the EOM for the massive fields. Note that the overall factor M_0 in the Lagrangian in Eq. (3.7) is proportional to N_c, which reflects the fact that the leading terms of the open string action are given by the string worldsheet of disk topology. As always, we neglect the loop corrections of string theory which are suppressed by $1/N_c$. Then, L_{int} is order 1 in the $1/N_c$ expansion with fixed λ. If one writes down the Lagrangian using canonically normalized massive fields

$$\widetilde{\Psi}_j \equiv \sqrt{M_0}\Psi_j, \qquad \widetilde{\Phi}_k \equiv \sqrt{M_0}\Phi_k, \tag{3.48}$$

one finds that all the terms with more than two massive fields are suppressed in the large-N_c limit. Therefore, the terms in L_{int} that survive in the large-N_c limit are quadratic with respect to the massive

[20] N_{84} and N_{44} correspond to N_{95} and N_{55} in Sect. 2, respectively.

Table 7. Nucleon and lightest baryons with $I = 1/2$ and $J^P = (n + 1/2)^{(-)^n}$ $(n = 0, 1, \ldots, 5)$. Data taken from the baryon summary table in Ref. [24].

Baryons	N	N(1520)	N(1680)	N(2190)	N(2220)	N(2600)
J^P	$1/2^+$	$3/2^-$	$5/2^+$	$7/2^-$	$9/2^+$	$11/2^-$
Mass [MeV]	939	1510∼1520	1680∼1690	2140∼2220	2250∼2320	2550∼2750

fields. For the same reason, it should not contain $\dot{w}$, $\dot{X}$, or X. Then, the possible terms consistent with the $U(1)$ gauge symmetry are schematically written as

$$w^n(w^\dagger)^n\Psi_j^\dagger\Psi_{j'}, \quad w^n(w^\dagger)^n\Phi_k^\dagger\Phi_{k'}, \quad w^n(w^\dagger)^{n+2}\Psi_j\Psi_{j'}, \quad w^n(w^\dagger)^{n+1}\Psi_j\Phi_k, \tag{3.49}$$

with properly contracted indices and their complex conjugates. As we have seen in Sects. 3.3 and 3.4, w is treated as an order 1 variable and these terms may appear even in the large-N_c limit.

One might think that these terms are perhaps suppressed for large λ. Unfortunately, however, the answer is no. Consider, for example, a term proportional to $|w|^{2n}|\Psi_j|^2 \propto |Y|^{2n}|\Psi_j|^2$ for $N_f = 2$. As observed in Sect. 3.4, the leading term in Y is $Y \sim \rho_0 a \sim \mathcal{O}(\lambda^{-1/2})$. Recall that all the fields have the dimension of length in our convention. To have the correct dimensions, there should be an appropriate number of α' or $M_{\rm KK}$ in the coefficient of Eq. (3.49) to saturate the correct dimension of $L_{\rm int}$. A possible term is of the form

$$L_{\rm int} \sim \alpha'^{-n-1}|Y|^{2n}|\Psi_j|^2 \sim \lambda|\Psi_j|^2, \tag{3.50}$$

which shifts the mass for Ψ_j in the same order as the original mass term. This is the same mechanism as the mass generation of β_a discussed in Sect. 3.4. $L_{\rm int}$ may also induce mixing terms as well, and the diagonalization of the mass matrix may become very complicated. Because we do not know the explicit form of $L_{\rm int}$ we are not able to evaluate it explicitly, and leave the detailed analysis including $L_{\rm int}$ for future research.

4. Comparison with experiments

4.1. Regge trajectory

Here, we focus on the baryons listed in Table 7, which are the lightest baryons with $I = 1/2$ and $J^P = (n + 1/2)^{(-)^n}$ $(n = 0, 1, \ldots, 5)$ found in the experiments.

These baryons have been considered to be described by an excited (rotating) open string with a quark and diquark pair attached on the two end points [26–29].[21] An analogous object in our model is a D4$_{\rm BV}$-brane with $(N_c - 1)$ 8-4 strings in the ground state and only one 8-4 string being excited as J increases. The aim of this subsection is to discuss whether our model gives us plausible predictions assuming that this is the correct interpretation. More explicitly, the lightest one in Table 7, which is the nucleon (proton or neutron), is identified with $q_w = N_c$, $\ell = 1$, and $n_\rho = n_z = n_j^\Psi = n_j^{\overline{\Psi}} = n_k^\Phi = 0$.[22] The excited nucleons with spin $J \geq 3/2$ in Table 7 are interpreted as the highest spin state among those with $q_w = N_c - 1$, $\ell = 0$, $n_\rho = n_z = n_{j'}^{\overline{\Psi}} = n_k^\Phi = 0$, and $n_{j'}^\Psi = \delta_{j'j}$ for some j. These states are

[21] For earlier and closely related works, see Refs. [30–32]. See also Refs. [33–36] for related works based on quark–diquark models.

[22] Here, we consider N_c to be a large odd number. Recall that the condition $\ell \equiv q_w \pmod 2$ has to be satisfied (see Sect. 3.5).

most likely to be the lightest state among the highest spin states with isospin 1/2 for each level. Let us discuss if the quantum numbers and the masses of these states are consistent with the experimental data with this interpretation.

The states we consider are labeled uniquely by the level $\mathcal{N}$ introduced in Eq. (3.46). Let $M_{\mathcal{N}}$ denote the baryon mass for a given $\mathcal{N}$. The nucleon corresponds to the case $\mathcal{N} = 0$, which has $J^P = 1/2^+$ and mass given by

$$M_{\mathcal{N}=0} = M_0^*(q_w = N_c, \ell = 1). \tag{4.1}$$

Here, M_0^* is considered to be a function of q_w and ℓ, as argued in Sect. 3.5. As it is technically hard to compute the quantum $M_0^*(q_w, \ell)$, we regard it as an unknown parameter.

For $\mathcal{N} \geq 1/2$, because $\ell = 0$, the massless sector has vanishing spin and isospin. Then, the total spin of the excited baryons with $\mathcal{N} \geq 1/2$ is fixed by the massive sector. Let the excitation number of the excited 8-4 string be $N_{84}^{(j)}$, which is to be identified with the level $\mathcal{N}$ for the excited nucleons as seen before. For each $\mathcal{N} = 1/2, 1, 3/2, 2, \ldots$, the highest spin states are contained in the states of the form

$$\left(\alpha_{-1/2}^{M_1} \cdots \alpha_{-1/2}^{M_{2\mathcal{N}}} - (\text{trace parts})\right) |a, I\rangle_{\rm NS}\,, \tag{4.2}$$

which belongs to the spin $(\mathcal{N}, \mathcal{N}) \otimes (1/2, 0)$ representation of $SU(2)_L \times SU(2)_R$. Here, we have included the flavor index I to show that it is an isospin 1/2 state for $N_f = 2$. Decomposing this under the vector-like subgroup $SU(2)_J \subset SU(2)_L \times SU(2)_R$, one finds that the highest spin is given by $J = 2\mathcal{N} + 1/2$. The parities of these excited nucleons are given by $P = (-)^{2\mathcal{N}}$, because the state in Eq. (4.2) has parity $(-)^{2\mathcal{N}}$ and the massless sector is parity even for $n_z = 0$. Therefore, the spin, isospin, and parity for the excited nucleon states constructed above are consistent with those in Table 7.

The baryon mass formula in Eq. (3.33) implies that the masses for these excited nucleons with $J \geq 3/2$ states are

$$M_{\mathcal{N}} = M_0' + \sqrt{\frac{\mathcal{N}}{\alpha'}} = M_0' + \frac{1}{\sqrt{2\alpha'}}\sqrt{J - \frac{1}{2}}, \tag{4.3}$$

where $M_0' \equiv M_0^*(q_w = N_c - 1, \ell = 0)$. This formula can be recast as a formula for spin J as a function of mass M:

$$J = 2\alpha'(M - M_0')^2 + \frac{1}{2}. \tag{4.4}$$

It has been observed that, when the spin J is plotted as a function of the mass squared M^2, the excited nucleon states listed in Table 7 lie on a linear trajectory that satisfies

$$J = \alpha_0 + \alpha' M^2, \tag{4.5}$$

with $\alpha_0|_{\rm exp} \simeq -0.3$ and $\alpha'|_{\rm exp} \simeq 0.9\,{\rm GeV}^{-2}$. Our formula in Eq. (4.4) is a nonlinear function with respect to M^2, and one would think it disagrees with the observation. However, choosing

$$\alpha' \simeq 0.6\,{\rm GeV}^{-2}, \qquad M_0' \simeq 0.5\,{\rm GeV}, \tag{4.6}$$

we get the plot shown in Fig. 1, which shows that it can fit the data reasonably well. Due to the nonlinear term in Eq. (4.4), the trajectory in Figure 1 is curved toward the left and the value of mass

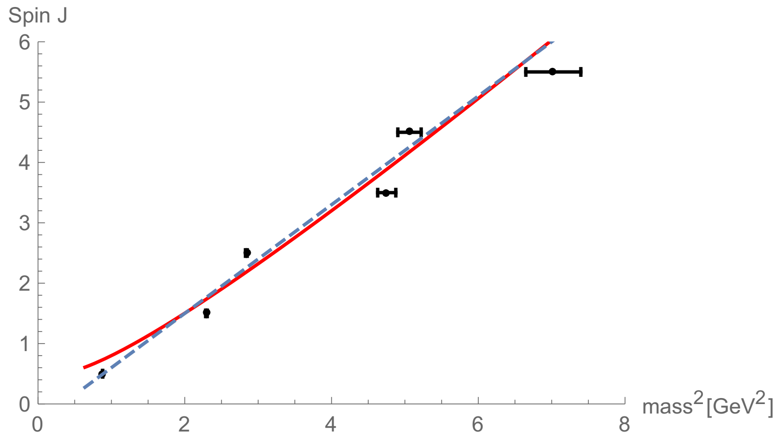

Fig. 1. A plot of Eq. (4.4) compared with the experimental data. The dots with error bars represent the data listed in Table 7. The solid line is the plot of Eq. (4.4) with $\alpha' \simeq 0.6\,\mathrm{GeV}^{-2}$ and $M_0' \simeq 0.5\,\mathrm{GeV}$, while the dashed line is the linear trajectory of Eq. (4.5) with $\alpha_0 \simeq -0.3$ and $\alpha' \simeq 0.9\,\mathrm{GeV}^{-2}$.

squared for $J = 1/2$ becomes significantly smaller compared to that of the nucleons (proton or neutron). This is, however, not a problem of the formula in Eq. (4.4) as it is derived for the states with $J \geq 3/2$. Our expression for the nucleon mass is given in Eq. (4.1). Though we are not able to predict its value, this observation suggests that the difference between Eq. (4.1) and M_0',

$$M_{\mathcal{N}=0} - M_0' = M_0^*(q_w = N_c, \ell = 1) - M_0^*(q_w = N_c - 1, \ell = 0), \tag{4.7}$$

is positive, as expected.[23]

We emphasize that the values in Eq. (4.6) should not be considered to be an accurate estimate, because we have neglected all the $1/N_c$ and $1/\lambda$ corrections, as well as the possible contributions from the interaction term in Eq. (3.50) for the massive fields. Nevertheless, let us make a few comments here on the value of α'. In Refs. [2,3], the parameters M_{KK} and λ were chosen to be

$$M_{\mathrm{KK}} \simeq 949\,\mathrm{MeV}, \qquad \lambda \simeq 16.6 \tag{4.8}$$

to fit the experimental values of the ρ-meson mass and the pion decay constant. If we use these values and the relation in Eq. (2.4), we obtain $\alpha' \simeq 0.452\,\mathrm{GeV}^{-2}$, which is a bit small compared with the value in Eq. (4.6). On the other hand, the value of α' evaluated from the Regge slope of the ρ-meson trajectory is $\alpha'|_{\mathrm{exp}} \simeq 0.88\,\mathrm{GeV}^{-2}$. In Ref. [8], the ρ-meson Regge behavior is analyzed theoretically using the same holographic model of QCD as in the present paper. It was argued there that the ρ-meson trajectory has some nonlinear corrections similar to that in Eq. (4.4), and the value of α' that fits well with the experimental data turned out to be around $1.1\,\mathrm{GeV}^{-2}$. The value of α' in Eq. (4.6) is close to neither of these values, though it is not too far from them. It is important to resolve this discrepancy by making a more accurate estimate of α'.

Note that the slope α' of the linear Regge trajectory in Eq. (4.5) for the excited nucleons is very close to that of the ρ-mesons. This is one of the motivations for conjecturing that both of them

[23] To get a rough estimate, one could try to evaluate it by assuming that the ℓ dependence is small and the mass difference $\Delta M_0^* \equiv M_{\mathcal{N}=0} - M_0'$ is entirely determined by Eq. (3.34). Then, one gets $\Delta M_0^* = 2M_0\gamma(\rho_0^2|_{q_w=N_c} - \rho_0^2|_{q_w=N_c-1})$. For $\gamma = 1/6$ and $\nu = 0$, using Eq. (3.32) we get $\Delta M_0^* = M_{\mathrm{KK}}/\sqrt{6}$, where we have recovered the M_{KK} dependence by dimensional analysis. Using the value of M_{KK} in Eq. (4.8), this is estimated as 387 MeV.

are described by open strings with some particles attached on the end points, as investigated in Refs. [26–29]. Our description is similar to these models in that only one of the N_c strings attached on the baryon vertex gets excited while the rest remain in the ground state. This system may be approximated with a single open string by regarding the effect of the baryon vertex as a massive end point. However, a clear distinction from the models in Refs. [26–29] is that the mass of the end point in the present model is of $\mathcal{O}(N_c)$ and considered to be much heavier than the energy scale determined by the string tension. In fact, it is not difficult to verify that a rotating open string with a massive end point of mass M_0 has a classical energy E that reduces in the heavy end point limit to

$$J = 2\alpha'(E - M_0)^2, \tag{4.9}$$

which agrees with Eq. (4.4) up to an additive constant $1/2$ and the contributions from the zero-point energy in M_0'.[24] We note that the difference between the mass formula in Eq. (4.4) and Eq. (4.9) is due to quantum $1/N_c$ corrections.

4.2. More about excited baryon states

In this subsection we show some examples of low-lying excited baryons that are obtained in a manner explained in the previous sections. For simplicity, we set $(n_\rho, n_z) = (0, 0)$ and $n^a_\beta = 0$. The states in the massless and massive sectors are denoted by $|\ell, q_w\rangle$ and $|n^\Psi_j, n^{\overline{\Psi}}_j, n^\Phi_k\rangle$, respectively, where only nonvanishing quantum numbers are indicated explicitly for notational simplicity.

We start from the sector $\mathcal{N} = 1/2$. This sector is constructed only from the excitation of a single 8-4 string with $N^{(1)}_{84} = 1/2$, because any 4-4 excited state has $N^{(k)}_{44} \geq 1$.[25] The corresponding field Ψ_1 belongs to $(1, 1/2)_-$ under $SU(2)_L \times SU(2)_R$ with the subscript denoting parity, and yields a harmonic oscillator with angular frequency given by $m_1 = \sqrt{1/(2\alpha')}$. The condition $\mathcal{N}_{84} = 1/2$ is satisfied when $(n^\Psi_1, n^{\overline{\Psi}}_1) = (1, 0)$ or $(0, 1)$. The excited states $|n^\Psi_1 = 1\rangle$ and $|n^{\overline{\Psi}}_1 = 1\rangle$ have the same energy eigenvalue of H_m, with $SU(2)_L \times SU(2)_R$ spin given by $(1, 1/2)_-$. We consider only the former, because this leads to $q_w = N_c - 1$ so that Eq. (3.34) shows that the corresponding massless sector has less energy compared to that with $q_w = N_c + 1$, which corresponds to $|n^{\overline{\Psi}}_1 = 1\rangle$. As q_w is even, the massless sector is allowed to have $\ell = 0, 2, 4, \ldots$. We first consider the case $\ell = 0$, which yields the lightest state in the massless sector $|\ell = 0, q_w\rangle$, which belongs to the trivial $SU(2)_L \times SU(2)_I$ representation and has even parity because $n_z = 0$. Hence, the tensor product state of $|\ell = 0, q_w\rangle$ with $|n^\Psi_1 = 1\rangle$ has $SU(2)_L \times SU(2)_R \times SU(2)_I$ spin given by

$$(1, 1/2)^{1/2}_- \otimes (0, 0)^0_+ = (1, 1/2)^{1/2}_-, \tag{4.10}$$

where the superscripts represent the isospin. The tensor product state of $|\ell = 2, q_w\rangle$ with $|n^\Psi_1 = 1\rangle$ decomposes under $SU(2)_L \times SU(2)_R \times SU(2)_I$ as

$$(1, 1/2)^{1/2}_- \otimes (1, 0)^1_+ = [(2, 1/2) \oplus (1, 1/2) \oplus (0, 1/2)]^{3/2}_- \oplus [(2, 1/2) \oplus (1, 1/2) \oplus (0, 1/2)]^{1/2}_- .$$

It is straightforward to decompose all these states in terms of $SU(2)_J \subset SU(2)_L \times SU(2)_R$; the results are summarized in Table 8. Note that $(3/2)^{1/2}_-$ appearing in the first row is identified with N(1520) in the previous section.

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[24] For a systematic treatment of the classical motion of rotating strings with massive end points, see Ref. [29].

[25] $N^{(j)}_{84}$ and $N^{(k)}_{44}$ are the excitation numbers for Ψ_j and Φ_k, respectively. Ψ_j with $j = 1, 2, 3, 4$ and Φ_k with $k = 1, 2, 3, 4$ are listed in Tables 5 and 6, respectively.

Table 8. Excited baryon states for $\mathcal{N} = 1/2$.

Product states	$SU(2)_L \times SU(2)_R \times SU(2)_I$	$SU(2)_J \times SU(2)_I$
$\lvert\ell = 0, q_w = N_c - 1\rangle \otimes \lvert n_1^\Psi = 1\rangle$	$(1, 1/2)_-^{1/2}$	$(3/2)_-^{1/2} \oplus (1/2)_-^{1/2}$
$\lvert\ell = 2, q_w = N_c - 1\rangle \otimes \lvert n_1^\Psi = 1\rangle$	$[(2, 1/2) \oplus (1, 1/2) \oplus (0, 1/2)]_-^{3/2}$ $\oplus [(2, 1/2) \oplus (1, 1/2) \oplus (0, 1/2)]_-^{1/2}$	$[(5/2) \oplus 2(3/2) \oplus 2(1/2)]_-^{3/2}$ $\oplus[(5/2) \oplus 2(3/2) \oplus 2(1/2)]_-^{1/2}$

We next turn to discussing the $\mathcal{N} = 1$ states. This is possible only when $(\mathcal{N}_{84}, \mathcal{N}_{44}) = (1, 0)$ or $(0, 1)$. The first condition is further divided into two cases: (i) $(n_1^\Psi, n_1^{\overline{\Psi}}) = (2, 0), (1, 1), (0, 2)$ and (ii) $(n_j^\Psi, n_j^{\overline{\Psi}}) = (1, 0), (0, 1)$ for $j = 2, 3, 4$. Note that the 8-4 massive states with $j = 2, 3, 4$ are given by the three states with $N_{84} = 1$ listed in Table 5. Again, we focus on the lightest states in each case, implying that we pick up only $(n_1^\Psi, n_1^{\overline{\Psi}}) = (2, 0)$ and $(n_j^\Psi, n_j^{\overline{\Psi}}) = (1, 0)$ with $j = 2, 3, 4$. The second condition $(\mathcal{N}_{84}, \mathcal{N}_{44}) = (0, 1)$ is solved by $n_k^\Phi = 1$ with $k = 1, 2, 3, 4$, with the rest of the excitation numbers set to zero. Note that the 4-4 string states labeled by $k = 1, 2, 3, 4$ are given by the excited states with $N_{44} = 1$ shown in Table 6. As no excitation is made by any 8-4 string mode, this case gives $q_w = N_c$.

Let us now work out the baryon states for the above three cases. For the first case, the state in the massive sector is given by $\lvert n_1^\Psi = 2\rangle$, which transforms under $SU(2)_L \times SU(2)_R \times SU(2)_I$ as

$$\left[(1, 1/2)_-^{1/2} \otimes (1, 1/2)_-^{1/2}\right]_{\text{symmetrized}} = \left[(0, 1)^1 \oplus (0, 0)^0 \oplus (2, 1)^1 \oplus (2, 0)^0 \oplus (1, 1)^0 \oplus (1, 0)^1\right]_+ . \quad (4.11)$$

The massless sector for this case is characterized by $q_w = N_c - 2 = \text{odd}$. We are thus allowed to set $\ell = 1$ as the lightest state, whose $SU(2)_L \times SU(2)_R \times SU(2)_I$ spin is given by $(1/2, 0)_+^{1/2}$. By taking the tensor product of this state $\lvert\ell = 1, q_w = N_c - 2\rangle$ with $\lvert n_1^\Psi = 2\rangle$, we find the following baryon states:

$$[(1/2, 0) \oplus (1/2, 1) \oplus (3/2, 0) \oplus (3/2, 1) \oplus (5/2, 1)]_+^{3/2} \oplus [2(1/2, 0) \oplus 2(1/2, 1) \oplus 2(3/2, 0) \oplus 2(3/2, 1) \oplus (5/2, 0) \oplus (5/2, 1)]_+^{1/2}. \quad (4.12)$$

For the second case, we take the massive sector state to be $\lvert n_j^\Psi = 1\rangle$ with $j = 2, 3, 4$. This corresponds to $q_w = N_c - 1 = \text{even}$. We can take $\ell = 0, 2, 4 \ldots$. The state $\lvert\ell = 0, q_w\rangle$ has a trivial spin so that the tensor product of this state with $\lvert n_j^\Psi = 1\rangle$ has the same spin as $\lvert n_j^\Psi = 1\rangle$. The massless sector with $\ell = 2$ has $SU(2)_L \times SU(2)_R \times SU(2)_I$ spin given by $(1, 0)^1$. The tensor product of this state with $\lvert n_j^\Psi = 1\rangle$ is easy to evaluate for each $j = 2, 3, 4$.

Finally, the massive sector for the third case is characterized by the four states $\lvert n_k^\Phi = 1\rangle$ with $k = 1, 2, 3, 4$. As noted before, this corresponds to $q_w = N_c = \text{odd}$ so that odd ℓ is allowed. We pick up $\ell = 1$, which is expected to give the lightest state among those with odd ℓ, and take its tensor product with $\lvert n_k^\Phi = 1\rangle$. Note that any 4-4 string state has a vanishing isospin. The same computation is easy to perform for the next-lightest state with $\ell = 3$.

All the results are summarized in Table 9. Decomposing these states in terms of $SU(2)_J \subset SU(2)_L \times SU(2)_R$ is straightforward.

Now we discuss possible identifications of the states listed in Tables 8 and 9 with the baryons found in experiments. Because we have not been able to derive the ℓ dependence in the baryon mass

Table 9. Excited baryon states for $\mathcal{N} = 1$.

Product states	$SU(2)_L \times SU(2)_R \times SU(2)_I$
$\lvert \ell = 1, q_w = N_c - 2\rangle \otimes \lvert n_1^\Psi = 2\rangle$	$[(1/2,0) \oplus (1/2,1) \oplus (3/2,0) \oplus (3/2,1) \oplus (5/2,1)]_+^{3/2}$ $\oplus[2(1/2,0) \oplus 2(1/2,1) \oplus 2(3/2,0) \oplus 2(3/2,1) \oplus (5/2,0) \oplus (5/2,1)]_+^{1/2}$
$\lvert \ell = 0, q_w = N_c - 1\rangle \otimes \lvert n_2^\Psi = 1\rangle$	$(1/2,0)_+^{1/2}$
$\lvert \ell = 0, q_w = N_c - 1\rangle \otimes \lvert n_3^\Psi = 1\rangle$	$(1/2,1)_+^{1/2}$
$\lvert \ell = 0, q_w = N_c - 1\rangle \otimes \lvert n_4^\Psi = 1\rangle$	$(3/2,1)_+^{1/2}$
$\lvert \ell = 2, q_w = N_c - 1\rangle \otimes \lvert n_2^\Psi = 1\rangle$	$[(3/2,0) \oplus (1/2,0)]_+^{3/2} \oplus [(3/2,0) \oplus (1/2,0)]_+^{1/2}$
$\lvert \ell = 2, q_w = N_c - 1\rangle \otimes \lvert n_3^\Psi = 1\rangle$	$[(3/2,1) \oplus (1/2,1)]_+^{3/2} \oplus [(3/2,1) \oplus (1/2,1)]_+^{1/2}$
$\lvert \ell = 2, q_w = N_c - 1\rangle \otimes \lvert n_4^\Psi = 1\rangle$	$[(5/2,1) \oplus (3/2,1) \oplus (1/2,1)]_+^{3/2} \oplus [(5/2,1) \oplus (3/2,1) \oplus (1/2,1)]_+^{1/2}$
$\lvert \ell = 1, q_w = N_c\rangle \otimes \lvert n_1^\Phi = 1\rangle$	$(1/2,0)_+^{1/2}$
$\lvert \ell = 1, q_w = N_c\rangle \otimes \lvert n_2^\Phi = 1\rangle$	$(1/2,0)_+^{1/2}$
$\lvert \ell = 1, q_w = N_c\rangle \otimes \lvert n_3^\Phi = 1\rangle$	$[(1,1/2) \oplus (0,1/2)]_-^{1/2}$
$\lvert \ell = 1, q_w = N_c\rangle \otimes \lvert n_4^\Phi = 1\rangle$	$[(3/2,1) \oplus (1/2,1)]_+^{1/2}$
$\lvert \ell = 3, q_w = N_c\rangle \otimes \lvert n_1^\Phi = 1\rangle$	$(3/2,0)_+^{3/2}$
$\lvert \ell = 3, q_w = N_c\rangle \otimes \lvert n_2^\Phi = 1\rangle$	$(3/2,0)_+^{3/2}$
$\lvert \ell = 3, q_w = N_c\rangle \otimes \lvert n_3^\Phi = 1\rangle$	$[(2,1/2) \oplus (1,1/2)]_-^{3/2}$
$\lvert \ell = 3, q_w = N_c\rangle \otimes \lvert n_4^\Phi = 1\rangle$	$[(5/2,1) \oplus (3/2,1) \oplus (1/2,1)]_+^{3/2}$

formula of Eq. (3.33), we have to rely on some qualitative arguments. Our guiding principles are as follows. First, we expect that the states with the same ℓ, q_w, and $\mathcal{N}$ are nearly degenerate. Second, for a given (ℓ, q_w), the states with $\mathcal{N} = 1$ are heavier than those with $\mathcal{N} = 1/2$. Third, for a given $\mathcal{N}$, the mass is an increasing function of both ℓ and q_w except for the state with $n_1^\Psi = 2$ listed in the first row of Table 9, which is expected to be heavier than the others according to the baryon mass formula in Eq. (3.33).[26]

The predictions for the low-lying excited baryons with $I = 1/2$ are summarized in Table 10, whose data are taken from Tables 8 and 9. We will not attempt to relate the states with $J = 1/2$ in this table to those in the baryon summary table in Ref. [24] here, because these might be regarded as excited states with nonvanishing n_ρ, n_z, and n_β without excitations in the massive sector.[27] Note that the states with $(3/2)_-^{1/2}$ and $(5/2)_+^{1/2}$ in the first and third rows of Table 10 are identified with N(1520) and N(1680), respectively, in Sect. 4.1. The $(5/2)_-^{1/2}$ state at $\mathcal{N} = 1/2$ is expected to be the lightest state with this quantum number and hence it may be identified with N(1675), which is the lightest baryon with the same quantum number listed in the baryon summary table. Then, the $(3/2)_-^{1/2}$ states

[26] Here, we have assumed that $M_0^*|_{q_w=N_c} - M_0^*|_{q_w=N_c-2}$ is smaller than $(\sqrt{2}-1)/\alpha'$, which can be justified for large λ.

[27] Some such states were already discussed in Ref. [7].

Table 10. Low-lying excited baryons with $I = 1/2$. Here we have omitted the states with $n_1^\Psi = 2$ in Table 9. The blue-colored states are identified with excited baryons lying on the nucleon Regge trajectory in Sect. 4.1.

Level	States	$SU(2)_J \times SU(2)_I$
$\mathcal{N} = 1/2$	$\lvert \ell = 0, q_w = N_c - 1\rangle \otimes \lvert n_1^\Psi = 1\rangle$	$[(3/2) \oplus (1/2)]_-^{1/2}$
	$\lvert \ell = 2, q_w = N_c - 1\rangle \otimes \lvert n_1^\Psi = 1\rangle$	$[(5/2) \oplus 2(3/2) \oplus 2(1/2)]_-^{1/2}$
$\mathcal{N} = 1$	$\lvert \ell = 0, q_w = N_c - 1\rangle \otimes \lvert n_{2,3,4}^\Psi = 1\rangle$	$[(5/2) \oplus 2(3/2) \oplus 3(1/2)]_+^{1/2}$
	$\lvert \ell = 1, q_w = N_c\rangle \otimes \lvert n_{1,2,3,4}^\Phi = 1\rangle$	$[(5/2) \oplus 2(3/2) \oplus 4(1/2)]_+^{1/2} \oplus [(3/2) \oplus 2(1/2)]_-^{1/2}$
	$\lvert \ell = 2, q_w = N_c - 1\rangle \otimes \lvert n_{2,3,4}^\Psi = 1\rangle$	$[(7/2) \oplus 3(5/2) \oplus 6(3/2) \oplus 5(1/2)]_+^{1/2}$

at the second row are expected to have mass nearly equal to N(1675). A natural candidate for one of them is N(1700).[28]

As for the $\mathcal{N} = 1$ states, we find that the $(3/2)_+^{1/2}$ states in the third row of Table 10 are expected to have mass nearly equal to N(1680). A natural candidate for one of them is N(1720).[29] Since the fourth row has larger values of ℓ and q_w compared with the third row, the $(5/2)_+^{1/2}$ state in the fourth row is expected to be heavier than N(1680) and N(1720). A natural candidate for it is N(1860), though this state has not been established in experiments. If this is the case, the $(3/2)_\pm^{1/2}$ states in the fourth row are expected to be nearly degenerate with N(1860). These states could be identified with N(1900) and N(1875). The baryon states in the fifth row contain a state with $(7/2)_+^{1/2}$. The only baryon with this quantum number listed in the baryon summary table is N(1990), though this is not considered to be established. Then, the $(5/2)_+^{1/2}$ and $(3/2)_+^{1/2}$ states in the fifth row of Table 10 could be identified with N(2000) and N(2040), respectively, which are again poorly established in experiments.

Unfortunately, the identification we have made is not a clear one-to-one correspondence. There is more than one candidate state in the model for many of the baryons listed in the baryon summary table. In particular, the degeneracy of the states in Table 10 does not match the experimental data perfectly. Furthermore, as mentioned in the footnotes, some of the baryons may be identified with the states that are not listed in Table 10. This lack of a one-to-one correspondence could in part be because all the excited baryons we consider are unstable resonances (for finite N_c), and many of them, in particular the heavier ones, are probably not easy to identify in experiments. Furthermore, some of the states in Tables 10 and 11 could be artifacts of the model. Although, as discussed in Sect. 2, we have imposed invariance with respect to the $SO(5)$ symmetry and τ-parity to get rid of artifacts, we are not able to show that this is sufficient to exclude all of them. It is expected that incorporation of full $1/\lambda$ corrections into the baryon mass formula makes the artifacts of the model infinitely heavy in the $M_{\text{KK}} \to \infty$ ($\lambda \to 0$) limit with Λ_{QCD} kept fixed. However, the extrapolation to the small-λ regime is a notoriously difficult problem in the holographic description, because we have to deal with all the stringy corrections in a highly curved spacetime. A similar observation was

[28] There are other possibilities for this identification. For example, $\lvert \ell = 0, n_\rho = 1, q_w = N_c - 1\rangle \otimes \lvert n_1^\Psi = 1\rangle$ and $\lvert \ell = 3, n_z = 1, n_\beta = 1, q_w = N_c\rangle$ also have $(3/2)_-^{1/2}$ components that could be identified with N(1700).

[29] As in the case of N(1700), $\lvert \ell = 0, n_z = 1, q_w = N_c - 1\rangle \otimes \lvert n_1^\Psi = 1\rangle$ and $\lvert \ell = 3, n_\rho = 1, n_\beta = 1, q_w = N_c\rangle$ also have $(3/2)_+^{1/2}$ components that could be identified with N(1720).

Table 11. Low-lying excited baryons with $I = 3/2$. We have omitted the states with $n_1^\Phi = 2$ and $\ell = 3$ in Table 9.

Level	States	$SU(2)_J \times SU(2)_I$
$\mathcal{N} = 1/2$	$\lvert\ell = 2, q_w = N_c - 1\rangle \otimes \lvert n_1^\Psi = 1\rangle$	$[(5/2) \oplus 2(3/2) \oplus 2(1/2)]_-^{3/2}$
$\mathcal{N} = 1$	$\lvert\ell = 2, q_w = N_c - 1\rangle \otimes \lvert n_{2,3,4}^\Psi = 1\rangle$	$[(7/2) \oplus 3(5/2) \oplus 6(3/2) \oplus 5(1/2)]_+^{3/2}$

also made in Ref. [4]. We leave as an open problem the study of a dictionary between the theoretical predictions and the experimental data in more detail.

We also examine the mass spectrum of Δ baryons with isospin $I = 3/2$. The theoretical predictions for this case are summarized in Table 11, whose data is taken from Tables 8 and 9. It is natural to identify the $(5/2)_-^{3/2}$ and $(7/2)_+^{3/2}$ states in the first and second rows of Table 11 with the lightest Δ baryons having the same quantum numbers listed in the baryon summary table, which are $\Delta(1930)$ and $\Delta(1950)$, respectively. This suggests that the $(5/2)_+^{3/2}$ states in the second row of Table 9 are nearly degenerate with $\Delta(1950)$. A good candidate to be identified with one of these states is $\Delta(1905)$. However, this identification is problematic: although our formula in Eq. (3.33) suggests that the $\mathcal{N} = 1$ states are significantly heavier than the $\mathcal{N} = 1/2$ states, $\Delta(1930)$ and $\Delta(1950)$ are nearly degenerate and $\Delta(1905)$ is even lighter than $\Delta(1930)$.

5. Conclusions

We have discussed stringy excited baryons using the holographic dual of QCD on the basis of an intersecting D4/D8-brane system. A key step to this end is to work on the whole system of a baryon vertex without describing it by a topological soliton on an effective five-dimensional gauge theory. We formulated this system as a many-body quantum mechanics that is composed of the ADHM-type matrix model of Hashimoto–Iizuka–Yi [14] and an infinite number of open string massive modes. This is done by relying on an approximation that is valid in the large-N_c and -λ regime. The resultant quantum mechanics provides us with a powerful framework for making a systematic analysis of excited baryons, including those with $I \neq J$ that are difficult to obtain in the soliton picture.

By construction, it would be too ambitious for the theoretical predictions from the present model to match the experimental data to good accuracy. Interestingly, we have seen that the present model reproduces a qualitative feature of the nucleon Regge trajectory. It has been argued that the stringy excited baryons to be identified with the excited nucleons are interpreted as a rotating open string with a massive end point. Such a picture of baryon Regge trajectories has been studied extensively in the literature [26–29]. It is worth emphasizing that the massive end point in this model is due to a $\mathrm{D4_{BV}}$, having a mass of $\mathcal{O}(N_c)$. The Regge trajectory formula in Ref. (4.4) that we proposed in this paper is not given by a simple, linear relation between the spin and the mass squared because of the heavy end point.

We conclude this paper by making some comments about future directions. First, it is important to improve the theoretical accuracy of the model by incorporating the interacting terms in L_{int} that have been neglected for technical difficulties. It would be almost impossible to fix the mass terms of the mass fields Ψ_j and Φ_k precisely, because infinitely many higher-order terms could contribute to a single mass term, as discussed in Sect. 3.7. Instead, what may be performed immediately is to take into account the effects of the mixing terms like $\Psi_j \Psi_j$ in the baryon mass formula. With these mixing terms, q_j is not a conserved charge any more so that an exact diagonalization of H_m in a manner consistent with the Gaussian constraint appears highly involved. It would be interesting to compute the perturbative effect of the mixing terms into the mass formula.

One of the unsatisfactory points is that the values of the parameters γ and ν in the potential of Eq. (3.5) are not determined from first principles. Though it is possible to adjust them to fit the results in the soliton picture as in Ref. [7], a derivation within our framework is desired to make sure that all the parameters can be fixed, in principle, without any ambiguities. Compared with the soliton picture, the origin of the potential in Eq. (3.5) is expected to be due to the energy contribution from the $U(N_f)$ gauge field on the flavor D8-branes in the presence of a baryon vertex. It would be interesting to examine this in more detail.

Finally, it would be of great interest to apply the results in this paper to a more complicated system made out of multiple baryon and anti-baryon vertices. A typical example is given by a stringy realization of tetraquarks. It would be nice to try to formulate a holographic model for tetraquarks following this paper and compare the theoretical predictions with experiments.

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Appendix A. $SO(4) \simeq SU(2)_I \times SU(2)_J$

The generators of the Lie algebra of $SO(4) \simeq (SU(2)_I \times SU(2)_J)/\mathbb{Z}_2$ can be chosen as

$$
\begin{aligned}
i\Sigma^1 &= i\sigma_2 \otimes \sigma_1 = \begin{pmatrix} & \sigma_1 \\ -\sigma_1 & \end{pmatrix}, \\
i\Sigma^2 &= -i\sigma_2 \otimes \sigma_3 = \begin{pmatrix} & -\sigma_3 \\ \sigma_3 & \end{pmatrix}, \\
i\Sigma^3 &= i\mathbf{1}_2 \otimes \sigma_2 = \begin{pmatrix} i\sigma_2 & \\ & i\sigma_2 \end{pmatrix}, \\
i\widetilde{\Sigma}^1 &= -i\sigma_1 \otimes \sigma_2 = \begin{pmatrix} & -i\sigma_2 \\ -i\sigma_2 & \end{pmatrix}, \\
i\widetilde{\Sigma}^2 &= -i\sigma_2 \otimes \mathbf{1}_2 = \begin{pmatrix} & -\mathbf{1}_2 \\ \mathbf{1}_2 & \end{pmatrix}, \\
i\widetilde{\Sigma}^3 &= i\sigma_3 \otimes \sigma_2 = \begin{pmatrix} i\sigma_2 & \\ & -i\sigma_2 \end{pmatrix}.
\end{aligned}
$$

$\{\Sigma^a\}$ and $\{\widetilde{\Sigma}^a\}$ satisfy the same algebra as the Pauli matrices,

$$
\Sigma^a\Sigma^b = \delta^{ab} + i\epsilon^{abc}\Sigma^c, \qquad \widetilde{\Sigma}^a\widetilde{\Sigma}^b = \delta^{ab} + i\epsilon^{abc}\widetilde{\Sigma}^c,
$$

and they commute with each other,

$$\Sigma^a \widetilde{\Sigma}^b = \widetilde{\Sigma}^b \Sigma^a.$$

$\{i\Sigma^a\}_{a=1,2,3}$ and $\{i\widetilde{\Sigma}^a\}_{a=1,2,3}$ are the generators of $SU(2)_I$ and $SU(2)_J$, respectively.

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