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Next-to-leading-logarithm k_T resummation for $B_c \to J/\psi$ decays

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ABSTRACT

We derive the k_T resummation for a transverse-momentum-dependent charmonium wave function, which involves the bottom quark mass m_b, the charm quark mass m_c, and the charm quark transverse momentum k_T, up to the next-to-leading-logarithm (NLL) accuracy under the hierarchy $m_b \gg m_c \gg k_T$. The resultant Sudakov factor is employed in the perturbative QCD (PQCD) approach to the $B_c \to J/\psi$ transition form factor $A_0^{B_c \to J/\Psi}(0)$ and the $B_c^+ \to J/\psi\pi^+$ decay. We compare the NLL resummation effect on these processes with the leading-logarithm one in the literature, and find a $(5-10)\%$ enhancement to the form factor $A_0^{B_c \to J/\Psi}(0)$ and a $(10-20)\%$ enhancement to the decay rate $BR(B_c^+ \to J/\Psi\pi^+)$. The improved k_T resummation formalism is applicable to the PQCD analysis of heavy meson decays to other charmonia.

1. Introduction

B_c meson decays contain rich heavy quark dynamics in both perturbative and nonperturbative regimes, that is worth of thorough exploration with high precision. It is thus crucial to develop an appropriate theoretical framework for analyzing B_c meson decays, for which data have been accumulated rapidly. A framework available in the literature is the perturbative QCD (PQCD) approach, which basically follows the conventional one for B meson decays: the dependence on the finite charm quark mass is included in hard decay kernels but neglected in the k_T resummation formula for meson wave functions [1–10]. Hence, a theoretical challenge from these decays is to derive the k_T resummation associated with energetic charm quarks with a finite mass. Such a rigorous k_T resummation formalism for a typical transition $B_c \to J/\psi$ was first attempted in [11]. The derivation relies on the power counting for the involved multiple scales, the bottom (charm) quark mass m_b (m_c) and the parton transverse momentum k_T. We have adopted the power counting rule proposed in [12], which obeys the hierarchy $m_b \gg m_c \gg k_T$. An intermediate impact of this hierarchy is that the large infrared logarithms $\ln(m_c/k_T)$, in addition to the ordinary ones $\ln(m_b/k_T)$, appear in the PQCD evaluation of B_c meson decays, and need to be resummed.

To proceed with the k_T resummation, we considered the $B_c \to J/\psi$ transition process, constructed the transverse-momentum-dependent (TMD) B_c and J/ψ meson wave functions in the k_T factorization theorem [13,14], and then performed the one-loop evaluation according to the wave-function definition as a nonlocal hadronic matrix element. The large logarithms attributed to the overlap of the collinear and soft radiative corrections were found to differ from those in B meson decays into light mesons [15], because of the additional charm quark scale. However, only the leading double logarithms from the correction to the quark-Wilson-line vertex in meson wave functions were captured in [11], namely, the k_T resummation for the $B_c \to J/\psi$ decays was achieved at the leading-logarithm (LL) accuracy so far. How the charm quark mass dependence in the LL k_T resummation affects the $B_c \to J/\psi$ transition form factor and the $B_c^+ \to J/\psi\pi^+$ branching ratio was then investigated [11].

In this letter we will complete the k_T resummation for the B_c and J/ψ meson wave functions up to the next-to-leading-logarithm (NLL) accuracy. Since the analysis involves the convolution with the corresponding hard decay kernel at the NLL accuracy, it is more convenient to perform the resummation in the impact parameter b space, which is conjugate to the transverse momentum k_T. We start with the one-loop calculation for the J/ψ meson wave function, from which all important logarithms are identified. It is found that these logarithms are grouped into two sets, $\ln(m_b b)$ and $\ln(m_c b)$, with their coefficients being identical but opposite in sign. It hints that the

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resummation technique can be applied to these two sets of logarithms separately: the k_T resummation is constructed for the first set, that for the second set can be inferred trivially via the replacement of the argument m_b by m_c, and the final result is given by the difference between them. Moreover, the resummation technique applied to the first set of logarithms is the same as the one applied to a light meson case [16] without the intermediate scale m_c. We emphasize that the matching to the one-loop J/ψ meson wave function is crucial for achieving the NLL accuracy. The NLL k_T resummation for the B_c meson wave function is then done in a similar way. At last, the NLL resummation effect is employed in the PQCD evaluation of the $B_c \to J/\psi$ transition form factor and the $B_c^+ \to J/\psi\pi^+$ branching ratio, and compared with the LL effect observed in [11].

2. B_c and J/ψ meson wave functions

In this section we construct the definitions of the TMD wave functions for the B_c and J/ψ mesons. Consider the $B_c(P_1) \to J/\psi(P_2)$ transition at the maximal recoil, where

$$P_1 = \frac{m_{B_c}}{\sqrt{2}}(1, 1, \mathbf{0}_T)\,, \qquad P_2 = \frac{m_{B_c}}{\sqrt{2}}(1, r^2_{J/\psi}, \mathbf{0}_T), \tag{1}$$

label the B_c and J/ψ meson momenta in the light-cone coordinates, respectively, with $r_{J/\psi} = m_{J/\psi}/m_{B_c}$, m_{B_c} ($m_{J/\psi}$) being the B_c (J/ψ) meson mass. Denote the anti-charm quark momenta as $k_1 = x_1 P_1$ in the B_c meson, and $k_2 = x_2 P_2$ in the J/ψ meson with the momentum fractions x_1 and x_2. Allowing the charm quarks to be off-shell only slightly, i.e., $k_1^2 - m_c^2 \sim k_2^2 - m_c^2 \sim O(m_c\Lambda_{\rm QCD})$, $\Lambda_{\rm QCD}$ being the QCD scale, we have postulated that the shapes of the B_c and J/ψ meson wave functions exhibit peaks around $x_1 \sim m_c/m_b$ and $x_2 \sim 1/2$, respectively [11]. A charm quark, carrying a longitudinal momentum initially, gains transverse momenta by radiating gluons [14], which generate the k_T dependence of a TMD wave function.

We point out that the power counting for a parton transverse momentum k_T is nontrivial, compared to the power counting for the fixed mass scales like m_b and m_c. First, the k_T factorization is suitable for a multi-scale process, such as the region of a small parton momentum fraction x in a process with a large scale Q. The small x introduces an additional intermediate scale $xQ^2 \sim Q\Lambda_{\rm QCD}$, respecting the hierarchy $Q^2 \gg xQ^2 \gg \Lambda^2_{\rm QCD}$. A parton k_T, being an integration variable in a k_T factorization formula, can take values of orders of the above scales. The criteria for applying the k_T factorization include: 1) the hard kernel of a considered process depends on the large scale Q^2 and the intermediate scale $Q\Lambda_{\rm QCD}$, but not on the small scale $\Lambda^2_{\rm QCD}$; 2) the factorization of the relevant TMD wave functions hold for a parton k_T at both the intermediate and small scales. Once these two criteria are satisfied, the k_T dependence in the hard kernel is not negligible, and a convolution between the hard kernel and the TMD wave functions can be derived. If the hard kernel involves only the large scale, the k_T dependence of the hard kernel can be neglected. It is then integrated out in the wave functions, and one is led to the collinear factorization. An integrated TMD wave function can be written as a convolution of its corresponding distribution amplitude with a perturbative kernel [17]. A typical example for establishing the k_T factorization at the one-loop level is referred to [18]: the matching between the QCD diagrams and the pion TMD wave function for the $\pi\gamma^* \to \gamma$ process gives the hard kernel in Eq. (40) of [18], that depends on Q^2 and $xQ^2 + k_T^2$, but not on k_T^2. Namely, the resultant hard kernel satisfies the first criterion, no matter which scale of k_T is. It can be shown that the eikonalization for factorizing the pion TMD wave function in Eq. (21) of [18] holds, as required by the second criterion, for both $k_T^2 \sim l_T^2 \sim O(Q\Lambda_{\rm QCD})$ and $O(\Lambda^2_{\rm QCD})$, which are lower than the dominant invariant Q^2 in that equation.

Since a TMD wave function contains the contributions characterized by both the intermediate and small scales, it is legitimate to further factorize the former out of the wave function, as the intermediate scale is regarded as being perturbative. Motivated by this observation, the joint resummation which organizes the mixed logarithms formed by the two invariants xQ^2 and k_T^2 has been performed for the pion wave function [19]. This resummation, like an ordinary k_T resummation, is justified perturbatively for the scale $k_T^2 \sim O(Q\Lambda_{\rm QCD})$. After this organization, the remaining pion wave function involves only the small scale $\Lambda^2_{\rm QCD}$. Similarly, it is also legitimate to further factorize the contribution characterized by an intermediate scale out of a hard kernel in the k_T factorization. This re-factorization yields the jet function defined in [20], through which the logarithms of xQ^2 are resummed to all orders.

Because more scales are involved in the $B_c \to J/\psi$ transition than in the $\pi\gamma^* \to \gamma$ process, there exist more leading infrared regions than in the latter. It has been argued [11] that a collinear region for the $B_c \to J/\psi$ transition is described by the power counting

$$l^\mu = (l^+, l^-, l_T) \sim \left(\frac{m_b}{m_c}\Lambda, \frac{m_c}{m_b}\Lambda, \Lambda\right), \tag{2}$$

where l is the momentum of a radiative gluon with the invariant mass squared of $O(\Lambda^2)$, and $\Lambda = m_c$ or k_T represents a lower scale. If the relation $m_b k_T \sim m_c^2$ is assumed [21], a collinear gluon momentum $l^\mu \sim (m_b k_T/m_c, m_c k_T/m_b, k_T)$ for $\Lambda = k_T$ will be equivalent to $l^\mu \sim (m_c, k_T^2/m_c, k_T)$. The soft radiation in the $B_c \to J/\psi$ transition is characterized by $l^\mu \sim (\Lambda, \Lambda, \Lambda)$ [11]. The dominant collinear (soft) enhancement is absorbed into the J/ψ (B_c) meson wave function, and the remaining contribution goes into a hard decay kernel. It is easy to see that the hard kernel for the $B_c \to J/\psi$ transition involves the scales down to m_c^2: the hard gluon invariant mass squared $(k_1 - k_2)^2$ contains $k_1^2 \sim k_2^2 \sim m_c^2$. It will be explained that the B_c and J/ψ meson wave functions can be factorized for the scales up to m_c^2. The above observations indicate that the k_T factorization is appropriate for the analysis of the $B_c \to J/\psi$ transition. As the scale m_c is regarded as perturbative [21], a hard piece with the scale m_c can be further factorized out of the wave-function definitions. Such a re-factorization has been applied to the light-cone distribution amplitudes of doubly-heavy mesons, which are then expressed as products of perturbatively calculable distribution parts and non-relativistic QCD (NRQCD) matrix elements [22,23].

The collinear gluons in the $B_c \to J/\psi$ transition can be collected by a gauge link, which is required by the gauge invariance of a TMD wave function. The one-loop diagram, in which a radiative gluon attaches to the valence b quark in the B_c meson and the valence c quark in the J/ψ meson, gives the collinear enhancement from Eq. (2). The b quark line is eikonalized in this region into $n^\nu/n\cdot l$, with ν labeling the gluon vertex, and a gauge link in the direction $n = (1, 1, \mathbf{0}_T)/\sqrt{2}$ along the B_c meson momentum is generated as shown in Fig. 1(a). The diagram, in which a radiative gluon attaches to the $\bar{c}$ quark in the B_c meson and the c quark in the J/ψ meson, also contains the collinear enhancement from Eq. (2). The eikonalization of the $\bar{c}$ quark line in the B_c meson results in a gauge link in the direction n shown in Fig. 1(b). The appearance of the gauge links in Figs. 1(c) and 1(d) can be explained similarly. The Fierz identity is

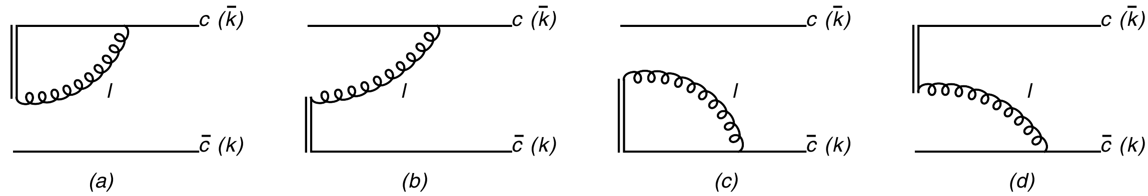

Fig. 1. One-loop vertex corrections to the J/ψ meson wave function.

then inserted to factorize the fermion flow [14] between the one-loop effective diagrams in Fig. 1 and the other parts of the transition process. Note that the above factorization procedure holds for k_T^2 up to the scale m_c^2, which is lower than the invariant mass squared of $O(m_b^2)$ ($O(m_b m_c)$) of the eikonalized b ($\bar{c}$) quark.

The J/ψ meson wave function $\Phi_{J/\psi}$ factorized out of the above transition process depends on two external vectors, the momentum P_2 and the direction n of the gauge link. Since the Feynman rule for the gauge link is scale-invariant in n, $\Phi_{J/\psi}$ must depend on n through the ratio of the Lorentz invariants, $\xi^2 = 4(P_2 \cdot n)^2/|n^2|$ [24,25]. We define the J/ψ meson wave function $\Phi_{J/\psi}(x, k_T, \xi^2, m_c)$ as

$$\Phi_{J/\psi}(x, k_T, \xi^2, m_c) = \int \frac{d^4 y}{(2\pi)^3} e^{-ik\cdot y} \langle 0|\bar{c}(y) W_y(n)^\dagger \not{n}_- W_0(n) c(0)|J/\psi(P_2)\rangle \delta(u\cdot y), \tag{3}$$

where $k = (xP_2^+, xP_2^-, \mathbf{k}_T)$ is the $\bar{c}$ quark momentum, and $y = (y^+, y^-, \mathbf{y}_T)$ denotes the coordinate of the $\bar{c}$ quark field. The projector $\not{n}_-$ with the null vector $n_- = (0, 1, \mathbf{0}_T)$ along the minus direction arises from the aforementioned insertion of the Fierz identity. The integration over the momentum orthogonal to the $\bar{c}$ quark momentum leads to the function $\delta(u\cdot y)$, with dimensionless vector $u = (-1/r_{J/\psi}^2, 1, \mathbf{0}_T)/\sqrt{2}$, which specifies the location of $\bar{c}$ on the y^+-y^- plane. Besides, $\Phi_{J/\psi}$ also depends on the factorization scale μ_f, which is not shown explicitly. The factor $W_y(n)$ represents the gauge link operator

$$W_y(n) = P\exp\left[-ig\int_0^\infty d\lambda n\cdot A(y+\lambda n)\right]. \tag{4}$$

A vertical link to connect the two links $W_y(n)$ and $W_0(n)$ at infinity is implicit. The removal of the gauge-link self-energy corrections [26] from the definition in Eq. (3) is understood, which is, however, irrelevant to the k_T resummation to be performed in the next section.

The soft region characterized by the power counting $l^\mu \sim (\Lambda, \Lambda, \Lambda)$ with Λ denoting m_c or k_T, also contributes dominantly to the one-loop diagrams discussed above. This contribution is factorized into the B_c meson wave function by eikonalizing the charm quark line on the J/ψ meson side, and the same gauge link in the direction n is chosen. Under the considered hierarchy $m_b \gg m_c$, the B_c meson wave function can be defined in a way similar to the B meson wave function,

$$\Phi_{B_c}(x, k_T, \xi^2, m_c) = \int \frac{d^4 y}{(2\pi)^3} e^{-ik\cdot y} \langle 0|\bar{c}(y) W_y(n)^\dagger \gamma_5 \not{n}_+ W_0(n) b(0)|B_c(P_1)\rangle \delta(u'\cdot y), \tag{5}$$

where $k = (xP_1^+, xP_1^-, \mathbf{k}_T)$ is the $\bar{c}$ quark momentum, and the projector $\gamma_5\not{n}_+$ arises from the insertion of the Fierz identity. The dimensionless vector $u' = (-1, 1, \mathbf{0}_T)/\sqrt{2}$, i.e., a vector in the z direction, is introduced to specify the location of $\bar{c}$ on the y^+-y^- plane: $\bar{c}$ is located on the time axis in this case. If one adopts an alternative power counting for the involved heavy quark masses, $m_b \sim m_c$, it will be more appropriate to define the B_c meson wave function in the effective theory of NRQCD rather than in QCD directly, since the separation between the bottom and charm quarks in coordinate space is much larger than $1/m_b$.

To identify the important logarithms in the J/ψ meson wave function, we calculate the one-loop effective diagrams displayed in Fig. 1 with an on-shell charm quark. Though the factorization of the J/ψ wave functions holds for k_T up to the scale m_c, we consider the hierarchy $m_b \gg m_c \gg k_T$, which generates the largest logarithms. Assume that the $\bar{c}$ quark carries the momentum $k = xP_2$, and the c quark carries $\bar{k} \equiv P_2 - k = (1-x)P_2$. Fig. 1(a), which does not induce a transverse momentum of the charm quark, gives the loop integral

$$\Phi_a^{(1)} = -\frac{i}{4} g^2 \mu_f^{2\epsilon} \int \frac{d^{4-2\epsilon} l}{(2\pi)^{4-2\epsilon}} \mathrm{tr}\left[\not{n}_+ \frac{\bar{\not{k}} + \not{l} + m_c}{(\bar{k}+l)^2 - m_c^2}\gamma_\nu \not{n}_-\right] \frac{1}{l^2 - m_g^2} \frac{n^\nu}{n\cdot l}, \tag{6}$$

with the color factor $C_F = 4/3$, the on-shell condition $\bar{k}^2 \approx m_c^2$, the factorization scale μ_f, the gluon momentum l, and the gluon mass m_g as an infrared regulator. The projectors $\not{n}_+$ and $\not{n}_-$ select the leading twist contribution. A straightforward computation yields

$$\Phi_a^{(1)} = \frac{\alpha_s}{4\pi} C_F \left[\frac{1}{\epsilon} + \ln\frac{4\pi\mu_f^2}{m_c^2 e^{\gamma_E}} - 2\ln\frac{(1-x)^2\xi^2}{m_c^2}\ln\frac{(1-x)^2\xi^2}{m_g^2} + \ln\frac{(1-x)^2\xi^2}{m_c^2} + 2 - \frac{\pi^2}{3}\right], \tag{7}$$

where $1/\epsilon$ represents an ultraviolet divergence and γ_E is the Euler constant. It is found that the collinear divergence regularized by the charm quark mass m_c and the soft divergence regularized by the gluon mass m_g overlap to produce the product of the corresponding logarithms in the above expression.

For Fig. 1(b), the transverse loop momentum l_T, flowing through the hard decay kernel, is not negligible in the k_T factorization as explained before. To facilitate the loop calculation, we apply the Fourier transformation to turn the convolution between the hard kernel and the J/ψ meson wave function into a product, and write the integral for the latter in the impact parameter b space as

$$\Phi_b^{(1)}=\frac{i}{4}g^2C_F\int\frac{d^4l}{(2\pi)^4}\exp(i\mathbf{l}_T\cdot\mathbf{b})\mathrm{tr}\left[\not n_+\frac{\bar{\not k}+\not l+m_c}{(\bar k+l)^2-m_c^2}\gamma_\nu \not n_-\right]\frac{1}{l^2-m_g^2}\frac{n^\nu}{n\cdot l}\,. \tag{8}$$

After performing the integration, we get

$$\Phi_b^{(1)}=\frac{\alpha_s}{4\pi}C_F\left[\frac{1}{2}\ln^2\frac{(1-x)^2\xi^2}{m_c^2}+2\ln\frac{(1-x)^2\xi^2}{m_c^2}\ln\frac{2(1-x)\xi}{bm_g^2e^{\gamma_E}}\right]. \tag{9}$$

Compared to Eq. (7), the above expression is free of an ultraviolet divergence due to the Fourier factor $\exp(i\mathbf{l}_T\cdot\mathbf{b})$. Note that the integration over the transverse momentum l_T in the presence of $\exp(i\mathbf{l}_T\cdot\mathbf{b})$ generates a Bessel function K_0, which can be approximated by a logarithmic function as its argument approaches to zero. Hence, Eq. (9) is valid only up to the logarithmic term, strictly speaking. This approximation works well enough for the matching between the NLL resummation and the one-loop result.

The sum of Eqs. (7) and (9) gives

$$\begin{aligned}\Phi_{a+b}^{(1)}&=\frac{\alpha_s}{4\pi}C_F\left[\frac{1}{\epsilon}+\ln\frac{4\pi\mu_f^2}{m_c^2e^{\gamma_E}}+\frac{1}{2}\ln^2\frac{(1-x)^2\xi^2}{m_c^2}-2\ln\frac{(1-x)^2\xi^2}{m_c^2}\ln\frac{(1-x)\xi be^{\gamma_E}}{2}+\ln\frac{(1-x)^2\xi^2}{m_c^2}+2-\frac{\pi^2}{3}\right]\\&=\frac{\alpha_s}{4\pi}C_F\left[\frac{1}{\epsilon}+\ln\frac{4\pi\mu_f^2}{m_c^2e^{\gamma_E}}-\frac{1}{2}\ln^2\frac{(1-x)^2\xi^2b^2e^{2\gamma_E-1}}{4}+\frac{1}{2}\ln^2\frac{m_c^2b^2e^{2\gamma_E-1}}{4}+2-\frac{\pi^2}{3}\right].\end{aligned} \tag{10}$$

It is seen in the first line that the infrared regulator m_g has been canceled as expected, and the soft scale has been replaced by $1/b$. It implies that the color transparency argument holds, and the soft divergences disappear in the summation over diagrams. The ultraviolet logarithm can be removed by choosing the factorization scale $\mu_f=m_c$, which defines the initial scale for the evolution of the J/ψ meson wave function in μ_f. In the second line we have reorganized the sum into the desired form: the logarithms are grouped into two sets, one containing $\ln^2(\xi b)$ and another containing $\ln^2(m_cb)$, as postulated in [11]. The difference between them arises only from the arguments $(1-x)\xi$ and m_c, and from the sign. This must be the case, because the collinear and soft divergences are regularized by the charm mass m_c and the impact parameter $1/b$, respectively, in the present calculation. Hence, the overlap of the corresponding logarithms should not generate $\ln^2(1/b)$, such that the above two sets of double logarithms have equal coefficients but with opposite signs. The sum of the contributions from Figs. 1(c) and 1(d) can be obtained simply by substituting the momentum fraction x for $(1-x)$ in Eq. (10),

$$\Phi_{c+d}^{(1)}=\frac{\alpha_s}{4\pi}C_F\left[\frac{1}{\epsilon}+\ln\frac{4\pi\mu_f^2}{m_c^2e^{\gamma_E}}-\frac{1}{2}\ln^2\frac{x^2\xi^2b^2e^{2\gamma_E-1}}{4}+\frac{1}{2}\ln^2\frac{m_c^2b^2e^{2\gamma_E-1}}{4}+2-\frac{\pi^2}{3}\right]. \tag{11}$$

A remark is in order. It has been elaborated recently [27] that the coefficient of a double logarithm associated with an on-shell parton is half of the coefficient in the off-shell case. Taking Fig. 1(a) as an example, we have evaluated its contribution for an energetic charm quark off-shell by $-k_T^2$, and obtained [11]

$$\Phi_a^{(1)}=\frac{\alpha_s}{4\pi}C_F\left[\frac{1}{\epsilon}+\ln\frac{4\pi\mu_f^2}{m_c^2e^{\gamma_E}}-\ln^2\frac{(1-x)^2\xi^2}{k_T^2}+\ln^2\frac{m_c^2}{k_T^2}+\ln\frac{(1-x)^2\xi^2}{m_c^2}+2-\frac{2}{3}\pi^2\right]. \tag{12}$$

Comparing Eq. (12) with Eq. (7), we indeed find that the coefficients of the double logarithms have been reduced to half in the on-shell case. We explain that the fixed-order calculation with an off-shell quark is required for the proof of the k_T factorization [14], in which the common parton virtuality $-k_T^2$ is adopted to regularize the infrared divergences in both QCD and effective diagrams. The k_T factorization holds, if the infrared logarithms $\ln k_T^2$ could be shown to cancel between these two sets of diagrams [14]. As deriving the k_T resummation formula, we consider an on-shell initial parton, which becomes virtual by transverse momenta through radiations. The resummation technique aims at collecting these radiations to all orders.

3. NLL k_T resummation

We first proceed with the NLL k_T resummation for the J/ψ meson wave function based on the complete one-loop results in the impact parameter space presented in the previous section, assuming $\xi\gg m_c\gg 1/b\gg\Lambda_{\mathrm{QCD}}$. The strategy is to focus only on the first set of logarithms $\ln(\xi b)$, whose treatment is similar to that of a light meson case, and then infer the resummation formula for the second set via the replacement of ξ by m_c. The choice of n is arbitrary in principle, which does not affect the collection of the collinear enhancement. This is the key observation for performing the resummation. We then study the variation of the J/ψ meson wave function with the gauge link direction n, which is equivalent to the variation with the dominant component P_2^+ of the J/ψ meson momentum via the scale ξ,

$$P_2^+\frac{d}{dP_2^+}\Phi_{J/\psi}=\xi\frac{d}{d\xi}\Phi_{J/\psi}=-\frac{n^2}{P_2\cdot n}P_2^\alpha\frac{d}{dn^\alpha}\Phi_{J/\psi}. \tag{13}$$

The technique of varying gauge links has been applied to the resummation of various types of logarithms, such as the rapidity logarithms in the B meson wave function [28], and the joint logarithms in the pion wave function [19]. The differentiation of each eikonal vertex and of its associated eikonal propagator on the gauge link with respect to n_α,

$$-\frac{n^2}{P_2\cdot n}P_2^\alpha\frac{d}{dn^\alpha}\frac{n^\mu}{n\cdot l}=\frac{n^2}{p\cdot n}\left(\frac{P_2\cdot l}{n\cdot l}n^\mu-P_2^\mu\right)\frac{1}{n\cdot l}\equiv\frac{\hat n^\mu}{n\cdot l}\,, \tag{14}$$

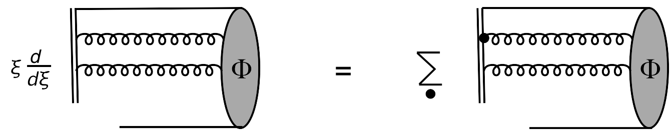

Fig. 2. Graphic representation of the derivative $\xi d\Phi_{J/\psi}/d\xi$.

Fig. 3. (a) soft function and (b) hard function at $O(\alpha_s)$.

leads to the derivative $\xi d\Phi_{J/\psi}/d\xi$ depicted in Fig. 2. The summation in Fig. 2 includes different attachments of the new vertex $\hat{n}^\mu$ defined by the last expression in Eq. (14), and represented by the symbol "•".

As stated before, terms of $O(m_c^2)$ can be dropped for the resummation of the first set of logarithms. If the loop momentum l is parallel to P_2, the factor $P_2 \cdot l$, being of $O(m_c^2)$, is negligible. When the second term P_2^μ in $\hat{n}^\mu$ is contracted with a vertex in $\Phi_{J/\psi}$, where all momenta are mainly parallel to P_2, the contribution from this collinear region is also of $O(m_c^2)$, and negligible. That is, the leading regions of l are soft and hard. According to [24], as the loop momentum flowing through the new vertex is soft, only the diagram with the new vertex being located at the outer most end of the gauge link dominates, and gives large single logarithms. In this soft region the subdiagram containing the new vertex can be factorized using the eikonal approximation, and the remainder is assigned to $\Phi_{J/\psi}$. This subdiagram is absorbed into a soft function K, whose $O(\alpha_s)$ contribution is displayed in Fig. 3(a). As the loop momentum flowing through the new vertex is hard, only the diagram with the new vertex being located at the inner most end of the gauge link dominates. In this region the subdiagram containing the new vertex is factorized into a hard function G, whose $O(\alpha_s)$ contribution is displayed in Fig. 3(b), and the remainder is identified to be $\Phi_{J/\psi}$.

We arrive at the differential equation in the impact parameter space

$$P_2^+ \frac{d}{dP_2^+}\Phi_{J/\psi} = 2\left[K(b\mu, \alpha_s(\mu)) + G(P_2^+/\mu, \alpha_s(\mu))\right]\Phi_{J/\psi}, \tag{15}$$

where the arguments of K and G specify their characteristic scales. Fig. 3(a) contributes

$$K = -ig^2 C_F \mu^\epsilon \int \frac{d^{4-\epsilon}l}{(2\pi)^{4-\epsilon}} \frac{\hat{n}^\mu}{n\cdot l} \frac{g_{\mu\nu}}{l^2} \frac{P_2^\nu}{P_2 \cdot l}[1 - \exp(i\mathbf{l}_T \cdot \mathbf{b})] - \delta K, \tag{16}$$

with δK being an additive counterterm. The Fourier factor $\exp(i\mathbf{l}_T \cdot \mathbf{b})$ appears in the second diagram of Fig. 3(a), because the loop momentum flows through $\Phi_{J/\Psi}$, such that the scale $1/b$ serves as an infrared cutoff of the loop integral in Eq. (16). The $O(\alpha_s)$ contribution to G from Fig. 3(b), where the soft subtraction is to avoid double counting of the soft contribution, is written as

$$G = -ig^2 C_F \mu^\epsilon \int \frac{d^{4-\epsilon}l}{(2\pi)^{4-\epsilon}} \frac{\hat{n}^\mu}{n\cdot l} \frac{g_{\mu\nu}}{l^2} \left[\frac{\not{P}_2 + \not{l}}{(P_2+l)^2}\gamma^\nu - \frac{P_2^\nu}{P_2\cdot l}\right] - \delta G\,, \tag{17}$$

where the charm quark mass m_c has been dropped as explained before, and δG is an additive counterterm. Choosing the subtraction scheme

$$\delta K = -\frac{\alpha_s}{2\pi} C_F \left[\frac{2}{\epsilon} + \ln(\pi e^{\gamma_E})\right] = -\delta G, \tag{18}$$

we get the soft and hard functions

$$K = -\frac{\alpha_s}{2\pi} C_F \ln(b^2\mu^2),$$
$$G = -\frac{\alpha_s}{2\pi} C_F \ln \frac{\xi^2 e^{2\gamma_E - 1}}{4\mu^2}. \tag{19}$$

Since K and G contain only single soft and ultraviolet logarithms, respectively, they can be treated by RG methods:

$$\mu \frac{d}{d\mu} K = -\lambda_K = -\mu \frac{d}{d\mu} G, \tag{20}$$

in which the anomalous dimension of K, $\lambda_K = \mu d\delta K/d\mu$, is given, up to two loops, by [29]

$$\lambda_K(\alpha_s) = \frac{\alpha_s}{\pi} C_F + \left(\frac{\alpha_s}{\pi}\right)^2 C_F \left[C_A\left(\frac{67}{36} - \frac{\pi^2}{12}\right) - \frac{5}{18} n_f\right], \tag{21}$$

with the number of quark flavors n_f and the color factor $C_A = 3$. As solving Eq. (20), we allow the scale μ to evolve to the infrared cutoff $1/b$ in K and to P_2^+ in G, and obtain the RG solution

Table 1
Dependence on the shape parameter β_{B_c} of the quantities $A_0^{B_c\to J/\psi}(0)$ and $\mathrm{BR}(B_c^+ \to J/\psi\pi^+)$ in the PQCD approach at the LL and NLL accuracy.

Quantities	$A_0^{B_c\to J/\psi}(0)$		$\mathrm{BR}(B_c^+ \to J/\psi\pi^+)$	
Shape parameter	LL	NLL	LL	NLL
$\beta_{B_c} = 0.8$ GeV	$0.488 - i0.095$	$0.511 - i0.147$	2.80×10^{-3}	3.10×10^{-3}
$\beta_{B_c} = 0.9$ GeV	$0.434 - i0.070$	$0.460 - i0.114$	2.10×10^{-3}	2.39×10^{-3}
$\beta_{B_c} = 1.0$ GeV	$0.384 - i0.053$	$0.414 - i0.090$	1.60×10^{-3}	1.87×10^{-3}
$\beta_{B_c} = 1.1$ GeV	$0.341 - i0.039$	$0.373 - i0.071$	1.23×10^{-3}	1.46×10^{-3}
$\beta_{B_c} = 1.2$ GeV	$0.306 - i0.029$	$0.339 - i0.057$	0.94×10^{-3}	1.16×10^{-3}

$$K(b\mu,\alpha_s(\mu)) + G(P_2^+/\mu,\alpha_s(\mu)) = K(1,\alpha_s(1/b)) + G(1,\alpha_s(P_2^+)) - \int_{1/b}^{P_2^+}\frac{d\bar\mu}{\bar\mu}\lambda_K(\alpha_s(\bar\mu)),$$
$$= -\frac{\alpha_s(P_2^+)}{2\pi}C_F\ln\frac{e^{2\gamma_E-1}}{2} - \int_{1/b}^{P_2^+}\frac{d\bar\mu}{\bar\mu}\lambda_K(\alpha_s(\bar\mu)). \tag{22}$$

The relation $\xi^2 = 2P_2^{+2}n^-/n^+ = 2P_2^{+2}$ for $n^+ = n^-$ specified in Eq. (3) has been inserted to get the initial condition $G(1,\alpha_s(P_2^+))$.

Substituting Eq. (22) into Eq. (15), we derive

$$\Phi_{J/\psi} = \exp\left[-\int_{1/b}^{(1-x)P_2^+}\frac{d\bar p}{\bar p}\left(\int_{1/b}^{\bar p}\frac{d\bar\mu}{\bar\mu}\lambda_K(\alpha_s(\bar\mu)) + \frac{\alpha_s(\bar p)}{2\pi}C_F\ln\frac{e^{2\gamma_E-1}}{2}\right)\right]$$
$$\times\exp\left[-\int_{1/b}^{xP_2^+}\frac{d\bar p}{\bar p}\left(\int_{1/b}^{\bar p}\frac{d\bar\mu}{\bar\mu}\lambda_K(\alpha_s(\bar\mu)) + \frac{\alpha_s(\bar p)}{2\pi}C_F\ln\frac{e^{2\gamma_E-1}}{2}\right)\right]\Phi_{J/\Psi}(x,b). \tag{23}$$

We have set the lower bound of the variable $\bar p$ to $1/b$, and the upper bounds to $(1-x)P_2^+$ and xP_2^+ for the integrals associated with Figs. 1(a) and 1(b), and Figs. 1(c) and 1(d), respectively, so that the initial condition $\Phi_{J/\Psi}(x,b)$ depends on x and b. As pointed out before, the k_T resummation formula for the second set of important logarithms can be inferred from Eq. (23) by substituting m_c for $(1-x)\xi$ and $x\xi$, namely, $m_c/\sqrt{2}$ for the upper bounds of $\bar p$, and flipping the signs of the integrands. Combining the two resummation formulas, we get the final result

$$\Phi_{J/\psi}(x,b,\xi,m_c) = \exp\left[-\int_{m_c/\sqrt{2}}^{(1-x)P_2^+}\frac{d\bar p}{\bar p}\left(\int_{1/b}^{\bar p}\frac{d\bar\mu}{\bar\mu}\lambda_K(\alpha_s(\bar\mu)) + \frac{\alpha_s(\bar p)}{2\pi}C_F\ln\frac{e^{2\gamma_E-1}}{2}\right)\right]$$
$$\times\exp\left[-\int_{m_c/\sqrt{2}}^{xP_2^+}\frac{d\bar p}{\bar p}\left(\int_{1/b}^{\bar p}\frac{d\bar\mu}{\bar\mu}\lambda_K(\alpha_s(\bar\mu)) + \frac{\alpha_s(\bar p)}{2\pi}C_F\ln\frac{e^{2\gamma_E-1}}{2}\right)\right]\Phi_{J/\psi}(x,b), \tag{24}$$

where the initial condition $\Phi_{J/\psi}(x,b)$ depends on the intermediate scale m_c only via the factorization scale, i.e., the argument of the strong coupling α_s implicitly. Expanding Eq. (24) to $O(\alpha_s)$ for a constant α_s, we reproduce all the logarithms in Eqs. (10) and (11). The remaining constant pieces will go into the $O(\alpha_s)$ hard decay kernel, when the one-loop J/ψ meson wave function and the one-loop decay amplitude are matched.

The above expression represents the complete NLL k_T resummation for the J/ψ meson wave function, which involves the three scale m_b, m_c and k_T. Compared to [11], we have included the so-called B term, i.e.., the second terms in the exponents in Eq. (24), and determined the order-unity coefficient associated with the lower bound of the variable $\bar p$ to be $1/\sqrt{2}$, both of which correspond to NLL effects. The inclusion of these NLL pieces requires a complete one-loop calculation of the J/ψ meson wave function in the impact parameter b space. The k_T resummation formula for the spectator charm quark in the B_c meson then reads

$$\Phi_{B_c}(x,b,\xi,m_c) = \exp\left[-\int_{m_c/\sqrt{2}}^{xP_1^-}\frac{d\bar p}{\bar p}\left(\int_{1/b}^{\bar p}\frac{d\bar\mu}{\bar\mu}\lambda_K(\alpha_s(\bar\mu)) + \frac{\alpha_s(\bar p)}{2\pi}C_F\ln\frac{e^{2\gamma_E-1}}{2}\right)\right]\Phi_{B_c}(x,b,m_c), \tag{25}$$

according to the second line of Eq. (24), for which the relevant large longitudinal component of the spectator momentum is xP_1^-. Because the upper and lower bounds of the integration variable $\bar p$ are both of $O(m_c)$, the resummation effect from Eq. (25) is less significant.

At last, we calculate the $B_c \to J/\psi$ transition form factor $A_0^{B_c\to J/\psi}(0)$ and the $B_c^+ \to J/\psi\pi^+$ branching ratio $\mathrm{BR}(B_c^+ \to J/\psi\pi^+)$ in the PQCD approach, taking into account the NLL k_T resummation effect from Eqs. (24) and (25). The explicit expressions for the above quantities, together with the input parameters and the models of the meson wave functions, can be found in [11]. The initial scale of the renormalization-group evolution for the meson wave functions, governed by the quark anomalous dimension [11], is modified from m_c to $m_c/\sqrt{2}$ for consistency. We adopt the one-loop running formula for the strong coupling α_s. It has been checked that the two-loop running causes only 1-2% reduction of the results from the one-loop running. The dependence of the quantities $A_0^{B_c\to J/\psi}(0)$ and $\mathrm{BR}(B_c^+ \to J/\psi\pi^+)$ on the shape parameter β_{B_c} of the B_c meson wave function in the range $[0.8, 1.2]$ GeV is presented in Table 1, and compared with that derived with the LL resummation effect [11]. The potential imaginary part of $A_0^{B_c\to J/\psi}(0)$, which is supposed to be a real object [30], increases a bit under the NLL resummation, but remains power suppressed. It is found that $A_0^{B_c\to J/\psi}(0)$ is enhanced by the NLL resummation effect by 5-10%, as β_{B_c} varies from 0.8 GeV to 1.2 GeV, and thus $\mathrm{BR}(B_c^+ \to J/\psi\pi^+)$ increases by about 10-20% accordingly. It implies that the NLL resummation effect is not negligible, and crucial for the determination of the B_c meson wave function, when relevant data are available in the future. The values in Table 1 are consistent with those from other approaches in the literature, which have been summarized in [11].

4. Conclusion

In this letter we have improved the k_T resummation for the $B_c \to J/\psi$ decays, which involve an additional intermediate charm scale compared with the $B \to \pi$ decays, to the NLL accuracy. We constructed the evolution equation for the TMD meson wave function by varying its associated gauge link, performed the k_T resummation by solving the evolution equation, and fixed the NLL pieces through the matching to the one-loop calculation. Our work represents the first NLL k_T resummation with the multiple scales m_b, m_c and k_T for B_c meson decays. It has been observed that the NLL resummation effect enhances the $B_c \to J/\psi$ transition form factor and the $B_c^+ \to J/\psi\pi^+$ branching ratio more than the LL resummation effect does. With more precise data from future experiments and the more accurate resummation formula obtained here, it is possible to determine the shape parameter of the B_c meson wave function, which can then be adopted to make reliable predictions for other decay modes. The above improved PQCD approach is also applicable to B and B_c meson decays to charmonia. Based on this work, we are ready to extend the k_T resummation with multiple scales to energetic charmed mesons, for which the current formalism is still preliminary.

Declaration of competing interest

We declare that we have no any conflicts of interest in this work.

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