Physics Letters B 820 (2021) 136534

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

The longitudinal structure function in the presence of QED effects

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ARTICLE INFO

Article history:
Received 20 September 2020
Received in revised form 7 June 2021
Accepted 18 July 2021
Available online 22 July 2021
Editor: G.F. Giudice

ABSTRACT

The highest precision for theoretical predictions at the high LHC energy requires the calculation of parton distribution function (PDFs) and the structure functions which include heavy quarks that include perturbative QCD and QED corrections. In this paper, the first, we review calculation of PDFs with QED corrections that we obtain them from the QCD⊗QED DGLAP evolution equations in Mellin space in NLO QCD and NLO QED approximations. We give a detailed explanation of the approach that we follow to determine the longitudinal structure functions up to $\mathcal{O}(\alpha_s^2)$ in QCD, $\mathcal{O}(\alpha\alpha_s)$ and $\mathcal{O}(\alpha^2)$ in QED approximations. We show how our method gives an exact representation for the total longitudinal structure functions, $F_L(x, Q^2)$, including the heavy quark structure functions in terms of the PDFs based on the Mellin transform techniques, valid to all orders in QED and QCD. In this work, we investigate the effects of the $\mathcal{O}(\alpha\alpha_s)$ and $\mathcal{O}(\alpha^2)$ corrections to PDFs on the longitudinal structure functions. We compare our results with APFEL and experimental data in the different values of Q^2. The results are shown that the QED corrections have a small but non-negligible impact on the structure functions.

Contents

1. Introduction

Determination of the parton distribution functions (PDFs) inside proton is needed for accurate comparisons of LHC cross sections with theoretical predictions. The topic of deep inelastic lepton-proton scattering (DIS) structure functions is crucial for understanding the internal structure of the proton. The longitudinal structure function F_L in proton is one of the important observable measured in deep inelastic scattering. It is proportional to the cross section for the interaction of the longitudinally polarized virtual photon with the proton. The issued values of proton structure function (F_2) at low value of x at HERA needed assumptions to be made about F_L or were limited to the kinematic region where the contribution from F_L was only repressed to be neglected. Measurements of the reduced cross section at the given values of x and Q^2 and different value of y allow F_2 and F_L to be extracted at the same time, as a result of that, eliminating the assumptions about F_L when extracting F_2. In addition, determining F_L is related to the gluon distribution function in proton. Then it is an important quantity. In the QCD modified parton model, the F_L structure function is non-zero, presented with contributions from quarks as well as gluons. At small values of x, the behavior of F_L is controlled more than anything else by gluon densities. Hence, the F_L structure function can be quickly calculated from the distribution of the gluon inside the proton when the gluon PDF is known [1–4].

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https://doi.org/10.1016/j.physletb.2021.136534

The Dokshitzer-GribovLipatov-Altarelli-Parisi (DGLAP) evolution equation [5–8] is the most fundamental one to investigate the evolution of the parton distribution function with x and Q^2. When a quark or gluon distribution function at initial value is given, can compute it for any value of Q^2 using this equation. The nucleon structure function relates the momentum distributions of the quarks and gluons which exist there. The study of the gluon distributions inside a hadron at small x is important since they involve dominant effect in this region. In fact the perturbative QCD predicts a very strong increasing of the gluon distribution in the small x region.

In recent years, many articles have discussed the effects of QED corrections on parton distribution functions and structure functions, and this topic has received much attention [9–15]. For this purpose, the DGLAP evolution equations require to be modified in the presence of QED effects. At first, the equation matrix elements should be changed with QED effects where relevant. The second is the introduction of an additional parton distribution quantify the photon content of the proton. Therefore, the generalization of DGLAP evolution equations is needed to account corrections correspond on $\mathcal{O}(\alpha)$, $\mathcal{O}(\alpha\alpha_s)$ and $\mathcal{O}(\alpha^2)$ orders. Florian et al. [16] have provided expressions for the Altarelli-Parisi splitting functions to high order ($O(\alpha\alpha_S)$) in QED. Also, Florian et al. [17] have computed the two-loop QED corrections to the Altarell-Parisi splitting functions, using a deconstructive algorithmic Abelianization of the NLO QCD corrections. The PDFs to include the QED corrections as well as parton PDFs have been calculated by the different groups such as MRST2004QED [18,19], NNPDF2.3QED [20], CT14QED [21], NNPDF3.1luxQED [22], NNPDF3.0QED [23] and MMHT2015QED [24]. The first publicly available set contains the photon PDFs with a parametrization based on radiation off of up and down quarks was MRST2004QED. The NNPDF2.3QED set uses the another parametrization, which was constrained by W, Z and Drell-Yan data at the LHC. This set updated in the new set as NNPDF3.0QED. The CT14QED set also use the radiative ansatz. In this way, for inelastic component of the photon PDF, the inelastic photon momentum fraction consider as a free parameter. This parameter is determined by comparison with DIS data that measured by the ZEUS Collaboration [25]. Recently, a new set that determines the photon PDF is NNPDF3.1luxQED in which the PDF is obtained from the lepton-photon structure function. They have determined the photon PDF and combine it with the quark and the other PDFs. This way reduce the uncertainties in the determination of the photon PDF. Also, the MMHT group has modified the partons, taking into account the effects of QED in their evolution. They calculated the photon PDF, $\gamma(x, Q^2)$, based on a similar methodology for the input to that of LUXQED group. They take into account their lower starting scale for the evolution than the LUXQED. Theoretical calculations are done, in particular the inclusion of the next to next leading order QCD and next to leading order QED corrections to the PDF evolution and computation of the DIS structure functions as implemented in APFEL program [26].

The paper is organized as it follows. The calculation of PDFs obtained from the DGLAP evolution equations with QED corrections in Mellin space is reviewed in Section 2. A brief description of the longitudinal structure functions of the proton is presented in Section 3. Also this section gives a detailed explanation of the method to calculate the Mellin moments as an analytical function of N. Section 4 describes our calculation and lists our results for the structure functions $F_L(x, Q^2)$ up to $\mathcal{O}(\alpha_s^2)$ in QCD, up to $\mathcal{O}(\alpha\alpha_s)$ and $\mathcal{O}(\alpha^2)$ in QED, in momentum space as a function of x. Finally, Section 5 gives our conclusion.

2. Review of the PDFs with QED effects

The level of accuracy obtained by the experimental measurements at the LHC requires the inclusion of QED and electroweak effects to the processes. Recent results showed that the contribution of photon distribution function has a significant important. Therefore, we have to consider the inclusion of QED in the calculations. Note that at this case the photon PDF does enter the DGLAP evolution equations. In this paper, we neglect the impact of lepton PDFs.

In this paper we focus the discussion on inclusion of QED corrections to the longitudinal structure functions in proton in terms of PDFs. To investigate the theoretical aspects of QED corrections implementation, it is required to upgrade the parton evolution equations. Here, we include QED corrections up to $\mathcal{O}(\alpha\alpha_s)$, $\mathcal{O}(\alpha^2)$ and up to $\mathcal{O}(\alpha_s^2)$ QCD computations. We resort to analytical solution of the combined QCD⊗QED DGLAP evolution equations in Mellin space where these calculations can be found with details in Ref. [11].

The fully form of QCD⊗QED DGLAP evolution equations are given by [12]

$$\begin{aligned}
\frac{\partial q_i(x,Q^2)}{\partial \ln Q^2} &= \sum_{j=1}^{n_f} P_{q_i q_j}(x)\otimes q_j(x,Q^2) \\
&\quad + \sum_{j=1}^{n_f} P_{q_i \bar{q}_j}(x)\otimes \bar{q}_j(x,Q^2) + P_{q_i g}(x)\otimes g(x,Q^2) \\
&\quad + P_{q_i\gamma}(x)\otimes\gamma(x,Q^2)\,, \\
\frac{\partial \bar{q}_i(x,Q^2)}{\partial \ln Q^2} &= \sum_{j=1}^{n_f} P_{\bar{q}_i q_j}(x)\otimes q_j(x,Q^2) \\
&\quad + \sum_{j=1}^{n_f} P_{\bar{q}_i \bar{q}_j}(x)\otimes \bar{q}_j(x,Q^2) + P_{\bar{q}_i g}(x)\otimes g(x,Q^2) \\
&\quad + P_{\bar{q}_i\gamma}(x)\otimes\gamma(x,Q^2)\,, \\
\frac{\partial g(x,Q^2)}{\partial \ln Q^2} &= \sum_{j=1}^{n_f} P_{g q_j}(x)\otimes q_j(x,Q^2) \\
&\quad + \sum_{j=1}^{n_f} P_{g \bar{q}_j}(x)\otimes \bar{q}_j(x,Q^2) + P_{gg}(x)\otimes g(x,Q^2)\,, \\
\frac{\partial \gamma(x,Q^2)}{\partial \ln Q^2} &= \sum_{j=1}^{n_f} P_{\gamma q_j}(x)\otimes q_j(x,Q^2) + \sum_{j=1}^{n_f} P_{\gamma \bar{q}_j}(x)\otimes \bar{q}_j(x,Q^2) \\
&\quad + P_{\gamma\gamma}(x)\otimes\gamma(x,Q^2)\,, \qquad (1)
\end{aligned}$$

where n_f is the number of active flavors and $P_{i,j}$ is the mixed QED and QCD splitting functions. It is defined as

$$P_{ij} = P_{ij}^{QCD} + P_{ij}^{QED} \quad i,j \equiv q,\bar{q},g,\gamma\,, \qquad (2)$$

where P_{ij}^{QCD} is related to the pure QCD splitting kernels that obtained, using the usual perturbative expansion,

$$P_{ij}^{QCD} = \alpha_s P_{ij}^{(1,0)} + \alpha_s^2 P_{ij}^{(2,0)} \quad i,j \equiv q,\bar{q},g,\gamma\,, \qquad (3)$$

and the terms correspond to the QED is written as it follows,

$$P_{ij}^{QED} = \alpha P_{ij}^{(0,1)} + \alpha\alpha_s P_{ij}^{(1,1)} + \alpha^2 P_{ij}^{(0,2)} \quad i,j \equiv q,\bar{q},g,\gamma\,, \qquad (4)$$

where the splitting kernels correspond on $\mathcal{O}(\alpha\alpha_s)$ and $\mathcal{O}(\alpha^2)$ orders can be found in Refs. [16,17]. As it obvious the first number in superscript denotes to the order of QCD correction and the second one represents the order of QED correction.

Hence, we determine the singlet, non-singlet, gluon and photon parton distribution functions inside proton in x space and for $Q^2 > Q_0^2$. The effects of photon-initiated contributions can be significant, its corrections range is up to 20% in the final PDFs. Consequently the photon distribution function at high scales of energy can not be ignored. Therefore the momentum sum rule constraint for the PDFs, modified to include the photon distribution function, can be written as

$$\int_0^1 dxx(\sum_i q_i(x,Q^2)+\bar{q}_i(x,Q^2)+g(x,Q^2)+\gamma(x,Q^2)) = 1\,. \quad (5)$$

An important test of the numerical evolve DGLAP equations with QED corrections is to check that this equation holds at all scales. As in the case of QCD, an important practical issue that needs to be addressed when solving the QED DGLAP evolution equations is the choice of the PDF basis. Instead the QCD⊗QED DGLAP evolution equations in Eq. (1), we use the new basis of distribution functions and correspond them, we write coupled and uncoupled evolution equations that introduce in Ref. [11]. We evolve a given set of PDFs from the initial condition at the scale Q_0 up to some other scale Q. We require the PDFs at the initial scale as input distribution function. Hence, we choose them at scale of $Q_0^2 = 2\,\text{GeV}^2$ from APFEL. The choice of Q_0^2 has no effect whatsoever on the results because the DGLAP equation evolves the input parametrization from the initial scale to the energy of the arbitrary point.

We perform the evolution equations in a fixed flavor number (FFN) scheme for all of PDFs. In this paper, we consider only five active flavors. We also assumed the symmetry between quarks-anti quarks distributions. Then the corresponding valence distributions vanish and we have $s=\bar{s}$, $c=\bar{c}$ and $b=\bar{b}$. In the following, we plot in Fig. 1 the parton distribution functions in x space where the PDFs have been evolved from $Q_0^2 = 2\,\text{GeV}^2$ up to $Q^2 = 10^4\,\text{GeV}^2$. It can be seen that the results for different PDFs indicate adequate behaviors, as expected. The normalization of the quark singlet is more tightly constrained from the DIS inclusive structure function data hence the sum rule, given by Eq. (5), affects on the gluon distribution function. Nonetheless we find that the back-reaction of QED corrections make influence on the quark and gluon PDFs which is however small but not negligible. To separate the QED corrections on PDFs from QCD ones and to compare them we also add in Fig. 1 the PDFs computing results with QED corrections. We also depict the valance quarks, sea quarks, gluon and photon distribution functions at different energy scales $Q^2 = 120\,, 1000\,, 8000\,\text{GeV}^2$ in three individual Figs. 2, 3 and 4, respectively. It is clear that increasing Q^2 lead to a decrease in value of the valance quark distributions. The other PDFs possess reliable features. Furthermore, the contribution of photon distribution function are increased with increasing the value of Q^2. It means that the photon distribution function is significant at high scales of Q^2. We also plot in Fig. 5 the ratio between the two calculations for all of PDFs to facilitate the visualization of their differences.

The photon PDF carries a non-zero amount of the total proton momentum, therefore it should be contributes to the momentum sum rule. We study the momentum fraction carried by the photon PDF,

$$<x>_\gamma (Q) \equiv \int_0^1 dxx(\gamma(x,Q^2))\,. \quad (6)$$

We computed the photon momentum fraction of Eq. (6) at $Q^2 = m_Z^2$ using different collaboration groups for PDF sets. We listed

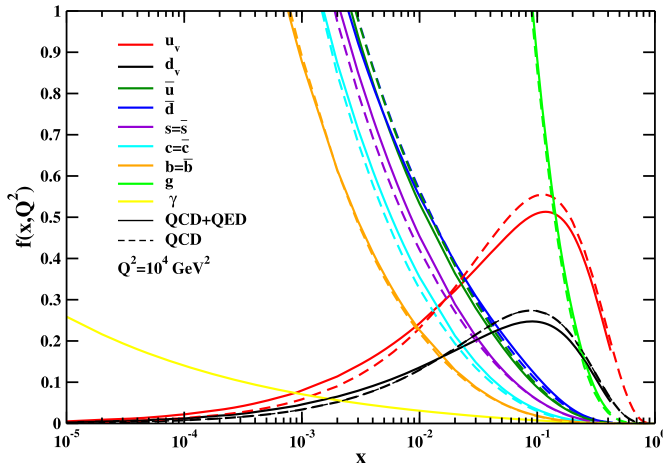

Fig. 1. The parton distribution functions at $Q^2 = 10^4\,\text{GeV}^2$ with and without QED corrections.

Table 1
The momentum fraction $\langle x\rangle_\gamma$ carried by photons in the proton at LHC scale $Q^2 = M_Z^2$, using PDFs sets from different collaboration groups.

	$\langle x\rangle_\gamma\ (Q^2 = M_Z^2)$
NNPDF2.3QED	0.256
NNPDF3.1luxQED	0.420
CT14QED	0.328
CT14QEDinc	0.478
APFEL	0.207
Our Model	0.259

in Table 1 the numerical values for $<x>_\gamma$, considering different groups. We found that the photon momentum fractions are unlike for these groups.

On the other hand in recent years different collaboration groups that study PDFs with QED corrections have reported relatively large value and in good agreement what we got for photon momentum fraction at $Q^2 = m_Z^2$ [22]. In practice, study on the resulting photon distributions shows various groups are in agreement with each other, differing only on the order of their small uncertainties.

In the next section, we present how one can calculate the DIS structure functions in terms of parton distribution functions.

3. DIS structure functions and QED corrections

In previous section, we showed that it is possible to solve analytically the DGLAP evolution equations with QED corrections up to $\mathcal{O}(\alpha_s^2)$ in QCD, $\mathcal{O}(\alpha\alpha_s)$ and $\mathcal{O}(\alpha^2)$ in QED approximation for the parton distribution functions directly using a method based on the Mellin transform. Now, we are going to calculate the proton and longitudinal structure functions, F_2 and F_L, respectively, in terms of the PDFs in this approximation. We present an analytical method for calculating the proton structure function in the $\mathcal{O}(\alpha_s^2)$ in QCD, $\mathcal{O}(\alpha\alpha_s)$ and $\mathcal{O}(\alpha^2)$ in QED approximations. We take into account the heavy quark contribution to the proton structure function. The deep inelastic electron-proton scattering (DIS) cross section, written in reduced form as

$$\sigma_r = F_2(x,Q^2) - \frac{y^2}{Y_+}F_L(x,Q^2)\,, \quad (7)$$

where $Y_+ \equiv 1+(1-y)^2$, and also here, $Q^2 = -q^2$ is the negative four-momentum squared transferred between the electron and the proton, and $x = Q^2/2qP$ denotes the Bjorken variable, where

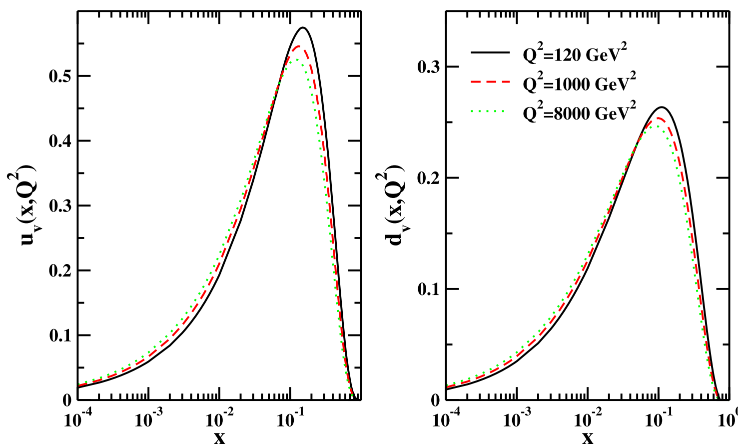

Fig. 2. Analytical results for valence quark distributions at three energy scales $Q^2 = 120$, 1000 and 8000 GeV^2, using DGLAP evolution equations.

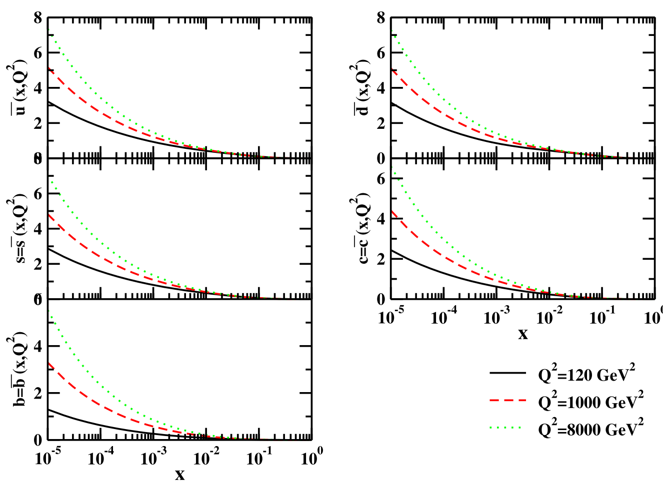

Fig. 3. Analytical results for sea quark distributions at three energy scales $Q^2 =$ 120 , 1000 and 8000 GeV^2, using DGLAP evolution equations.

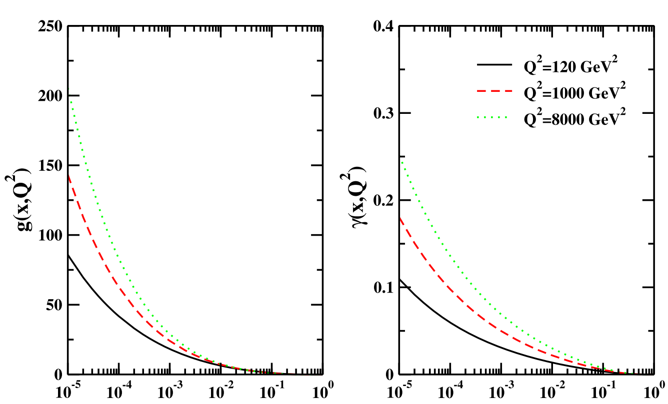

Fig. 4. Analytical results for gluon and photon distributions at three energy scales $Q^2 = 120$, 1000 and 8000 GeV^2, using DGLAP evolution equations.

P is the four-momentum of the proton. The two variables are related through the inelasticity of the scattering process, $y = Q^2/sx$, where s is the center-of-mass energy squared determined from the electron and proton beam energies.

In the following, we are going to analytically calculate the DIS structure functions in this approximation and compare them with the results of the other groups and experimental data. In order to diagonalize the convolution between the parton distribution functions and Wilson coefficients, we consider Mellin moments of the structure functions, as follows

$$F_i(N, Q^2) \equiv \int_0^1 dx x^{N-1} F_i(x, Q^2) = \sum_j C_i^j(N, \frac{m^2}{Q^2}) f_i(N, Q^2) , \tag{8}$$

where the definitions for coefficient moments of structure functions and of the parton distribution functions are as they follow:

$$C_i(N, \frac{m^2}{Q^2}) = \int_0^1 dx x^{N-1} C_i(x, \frac{m^2}{Q^2}) , \tag{9}$$

$$f_i(N, Q^2) = \int_0^1 dx x^{N-1} f_i(x, Q^2) . \tag{10}$$

Here, we have the total proton structure functions as $F_2^{p,total}(x, Q^2) = F_2^{p,light}(x, Q^2) + F_2^{heavy}(x, Q^2)$ in which it is assumed $F_2^{heavy}(x, Q^2) = F_2^c(x, Q^2) + F_2^b(x, Q^2)$, where $F_2^c(x, Q^2)$ and $F_2^b(x, Q^2)$ are the charm and bottom structure functions, respectively.

The light proton structure function, $F_2^{p,light}(x, Q^2)$, in Mellin space, up to the $\mathcal{O}(\alpha_s^2)$ in QCD, $\mathcal{O}(\alpha\alpha_s)$ and $\mathcal{O}(\alpha^2)$ in QED approximations, given by

$$F_2^{p,light}(n, Q^2) = F_2^{NS}(n, Q^2) + F_2^S(n, Q^2) + F_2^G(n, Q^2), \tag{11}$$

where the singlet F_2^S and gluon F_2^G contributions can be written, as follows:

$$F_2^S(n, Q^2) = (\frac{4}{9} 2\bar{u}(n, Q^2) + \frac{1}{9} 2\bar{d}(n, Q^2) + \frac{1}{9} 2\bar{s}(n, Q^2))(1 + \frac{\alpha_s}{4\pi} C_q^{(1)}(n))$$

$$F_2^G(n, Q^2) = (\frac{4}{9} + \frac{1}{9} + \frac{1}{9}) g(n, Q^2)(\frac{\alpha_s}{4\pi} C_g^{(1)}(n)) , \tag{12}$$

and the non-singlet F_2^{NS} contribution for three active flavors is given by

$$F_2^{NS}(n, Q^2) = (\frac{4}{9} u_v(n, Q^2) + \frac{1}{9} d_v(n, Q^2))(1 + \frac{\alpha_s}{4\pi} C_q^{(1)}(n)) \tag{13}$$

where the $C_q^{(1)}(n)$ and $C_g^{(1)}(n)$ are the next-to-leading order Wilson coefficient functions, derived in Mellin space. The next-to-leading order Wilson coefficient functions in Bjorken x space can be found

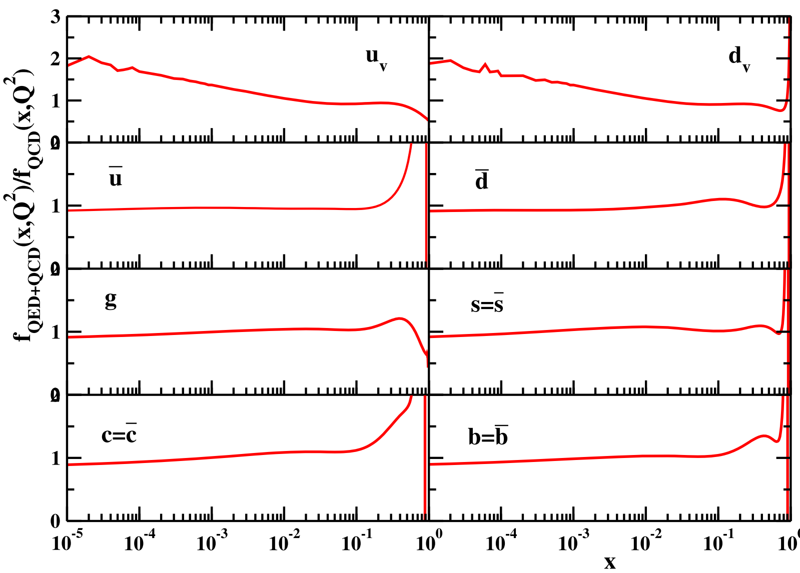

Fig. 5. The ratio of parton distribution functions at $Q^2 = 10^4$ GeV^2 with and without QED corrections.

in Ref. [27]. We have found the final desired solution of the proton structure function in x space, $F_2^{p,light}(x, Q^2)$, using the inverse Mellin transform [28], applying the appropriate change of variables. The next-to-leading order contribution of heavy quarks, $F_2^{c,b}(x, Q^2)$, to the proton structure function can be calculated in the fixed flavor number scheme (FFNS) approach [29–36].

The total longitudinal structure functions $F_L(N, Q^2)$ in proton consists of the light and heavy flavor contributions in Mellin space is given by [37,38]

$$\begin{aligned} F_L(N, Q^2) &= F_L^{light}(N, Q^2) + F_L^{heavy}(N, Q^2) \\ &= \left[C_L^{NS,light}(N, \alpha_s) + H_L^{NS}(N, \alpha_s, \frac{m^2}{Q^2})\right] q_{NS}(N, \mu^2) \\ &+ \left[C_L^{S,light}(N, \alpha_s) + H_L^{S}(N, \alpha_s, \frac{m^2}{Q^2})\right] q_S(N, \mu^2) \\ &+ \left[C_L^{g,light}(N, \alpha_s) + H_L^{NS}(N, \alpha_s, \frac{m^2}{Q^2})\right] g(N, \mu^2)\,. \end{aligned} \tag{14}$$

We choose $Q^2 = \mu^2$ as uniform factorization scale. The coefficient structure functions corresponding on the heavy quark contributions up to $\mathcal{O}(\alpha_s^2)$ in x space are given by [39,40]

$$\begin{aligned} H_{L,q}^{NS}(x, \alpha_s, \frac{m^2}{Q^2}) &= \alpha_s^2 \left[-\beta_{0,Q}\, C_{L,q}^{(1)} ln\left(\frac{Q^2}{m^2} + \hat{C}_{L,q}^{NS,(2)}\right)\right], \\ H_{L,q}^{PS}(x, \alpha_s, \frac{m^2}{Q^2}) &= \alpha_s^2 \hat{C}_{L,q}^{PS,(2)}, \\ H_{L,g}^{S}(x, \alpha_s, \frac{m^2}{Q^2}) &= \alpha_s \hat{C}_{L,g}^{(1)} + \alpha_s^2 \left[\frac{1}{2}\hat{P}_{qg}^{(0)} C_{L,q}^{(1)} ln\left(\frac{Q^2}{m^2}\right) + \hat{C}_{L,g}^{(2)}\right], \end{aligned} \tag{15}$$

where $H_{L,q}^S = H_{L,q}^{NS} + H_{L,q}^{PS}$.

Here, we showed how the electromagnetic proton structure functions $F_L(x, Q^2)$ could be determined in terms of the PDFs measured in electron-proton scattering experiments, and gave an obvious formula for the longitudinal structure function including all of terms up to next to leading order approximation. We have used these derivations and presented our results in the next section.

4. Results and discussions

In the Section 3, we present analytical method for the DIS structure functions up to next to leading order QCD and up to next to leading order QED approximation. We obtain the DIS structure functions with QED corrections in x space for the different values of Q^2. These functions are obtained in terms of PDFs as a set of functions for quark, antiquark and gluon PDFs that presented in Section 2.

The reduced cross section as measured via the analytically method is presented in Fig. 6. We present a comparison of your results on the reduced cross section obtained with the HERA results [41]. In Fig. 7, we present our predictions for $F_2(x, Q^2)$ for different value of Q^2. We compare those with experimental data. While our predictions for the proton structure function is in agreement with the data from the HERA collider experiments H1 [42], these observables are too inclusive to provide unambiguous evidence for DGLAP evolution equations.

In recently years, there are new data for the longitudinal structure functions, at low values of x that they have showed the longitudinal structure function is correspond on the gluon parton distribution function. An analytical formalism for the longitudinal structure function have been presented in the previous section. The comparison between our results for the longitudinal structure function, the APFEL predictions and experimental data [43–45] at different value of Q^2 is shown in Fig. 8. It should be noted that, in the presence of QED effects, we found good agreement between our results and experimental data [45–47]. In Fig. 9, we show the comparison of both our results and APFEL at high values of Q^2 at NLO in QCD and NLO in QED. The level of agreement between our predictions and APFEL is extremely good, we observe differences of 0.05% at most in all cases. We also in lower plots Fig. 9 present ratio results APFEL to our results. In conclusion, we found a good level of agreement for all comparison performed in this section. This guarantees that our model implements correctly the QCD⊗QED evolution, therefore it can be used in PDF fits.

The NLO QED corrections are the first photon-initiated corrections to the DIS structure functions. These corrections provide a direct handle on the photon PDF from DIS data. It is to be noted that, at LO in QED the photon PDF does not contribute directly to structure functions and it is only indirectly constraint from data through its coupling to the singlet PDF in the DGLAP evolution. When considering NLO QED corrections to DIS structure functions,

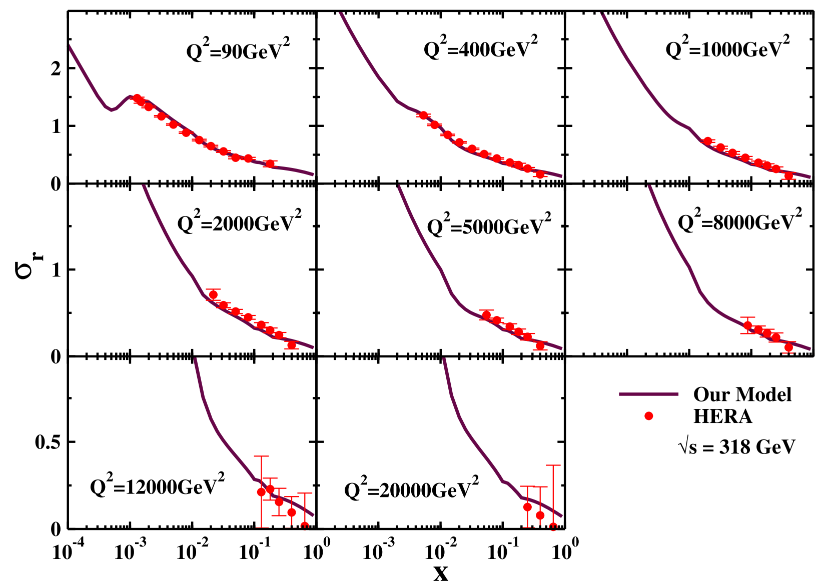

Fig. 6. Comparison of ep reduced cross sections, $\sigma_r(x, Q^2)$ with the available experimental data at the different values of Q^2 [41].

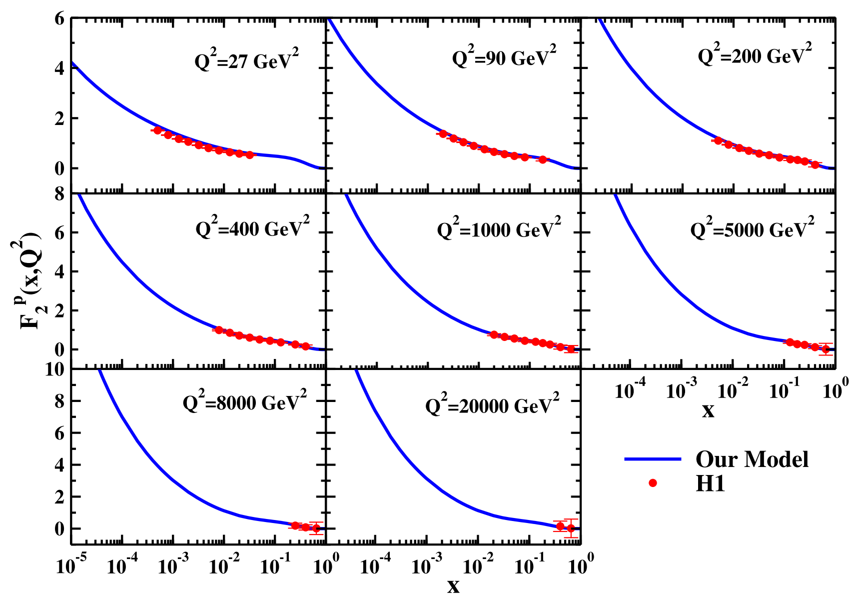

Fig. 7. The proton structure function $F_2(x, Q^2)$ at the different values of Q^2 in comparison with experimental data [42].

one has to include into the hard cross sections all the $\mathcal{O}(\alpha)$ diagrams where one single photon is either in the initial state or emitted from an incoming quark (or possibly an incoming lepton). Such diagrams are purely of QED origin and QCD contributions are not present.

To investigation the DIS structure functions with corrections up to $\mathcal{O}(\alpha_s^2)$ in QCD, $\mathcal{O}(\alpha\alpha_s)$ and $\mathcal{O}(\alpha^2)$ in QED, it is necessary to employ all of the $\mathcal{O}(\alpha)$ diagrams into the hard cross sections where one photon is either in the initial state or emitted from an incoming quark. These diagrams, being of the QED origin, have connected coefficient functions that can easily be extracted from the QCD expressions by properly regulating the color factors. The main problem of the addition of these corrections emerges from their flavor structure. Indeed, in the case of quarks, the isospin symmetry is broken because of the fact that the coupling of the photon is proportional to the squared charge of the parton to which it couples to a quark or a lepton. In the following, the neutral-current (NC) case, i.e. lepton and proton exchange a neutral boson, and the charged current (CC) case, where lepton and proton exchange a charged W boson, are addressed separately [48].

We now concentrate on the $\mathcal{O}(\alpha)$ contribution to the generic NC structure function F. Such correction can easily be derived from the structure of the $\mathcal{O}(\alpha_s)$ correction. The algorithm is very simple, for the coefficient functions one has:

$$C_{2,L}^{(\alpha)} = \frac{C_{2,L}^{(\alpha_s)}}{C_F},$$
$$C_g^{(\alpha)} = \frac{C_g^{(\alpha_s)}}{T_R}, \tag{16}$$

as there is no pure-singlet contribution at this order where $C_F = T_R = 1$ and $C_A = 0$ are the usual QCD color factors [49]. In order to make the corresponding structure functions, considering that the coupling between a photon and a quark of flavor q is proportional to e_q^2, the electroweak couplings should be modified as follows:

$$\widetilde{B}_q = B_q e_q^2, \tag{17}$$

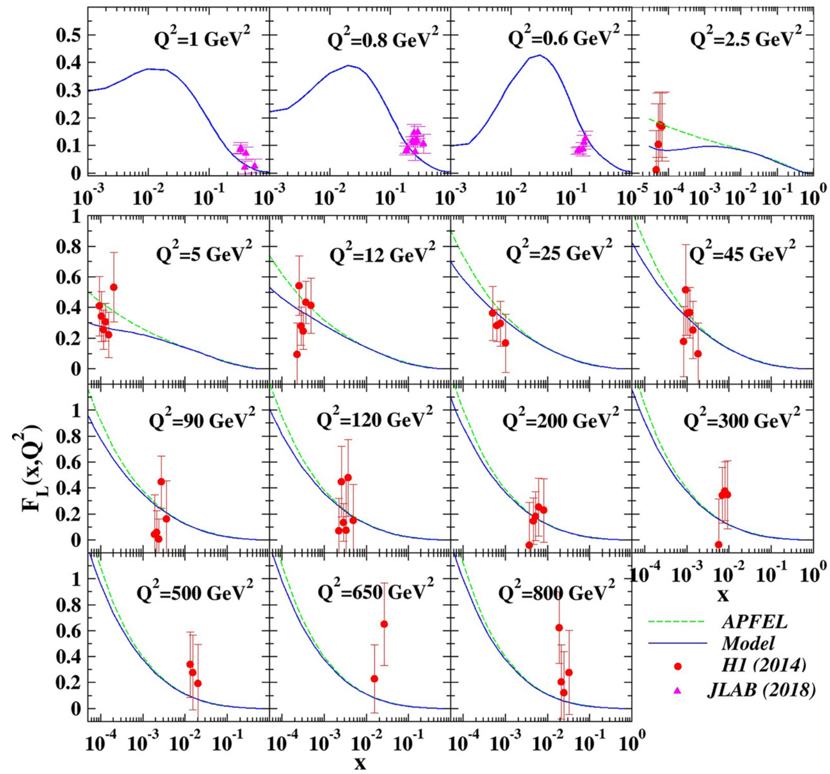

Fig. 8. The longitudinal structure function $F_L(x, Q^2)$ at the different values of Q^2 in comparison with the APFEL analysis and recent experimental data [43–45].

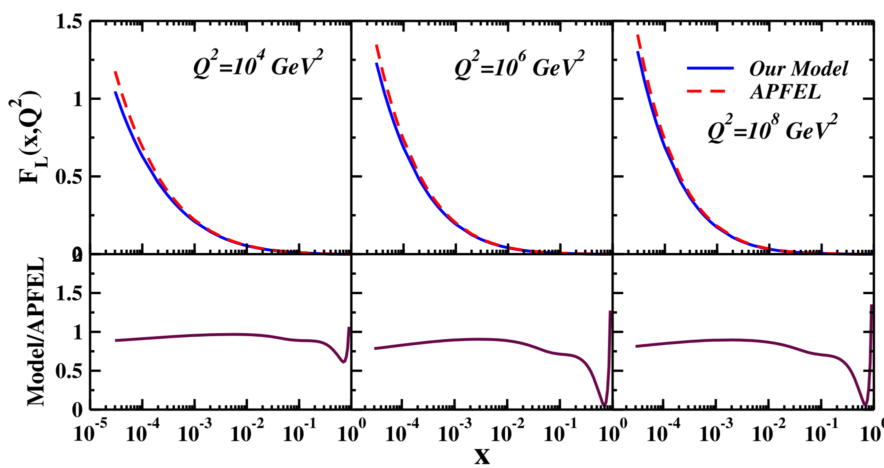

Fig. 9. Comparison of longitudinal structure function $F_L(x, Q^2)$ at $Q^2 = 10^4$ GeV^2, $Q^2 = 10^6$ GeV^2 and $Q^2 = 10^8$ GeV^2 with the results of APFEL analysis.

where B_q is the NC couplings and can be found in Ref. [50]. Following this instruction, taking into account the presented algorithm, one can write the $\mathcal{O}(\alpha)$ contribution to the light quark structure function as it follows:

$$F_{2,L}^{NC,(\alpha),light} = x\sum_q \widetilde{B}\left[C_{2,L;q}^{(\alpha)} \otimes (q+\bar{q}) + C_{2,L;\gamma}^{(\alpha)} \otimes \gamma\right]. \tag{18}$$

The heavy-quark components instead are:

$$F_{2,L}^{NC,(\alpha),c} = x\theta(Q^2 - m_c^2)B_c e_c^2\left[C_{2,L;c}^{(\alpha)} \otimes (c+\bar{c}) + C_{2,L;\gamma}^{(\alpha)} \otimes \gamma\right],$$
$$F_{2,L}^{NC,(\alpha),b} = x\theta(Q^2 - m_b^2)B_b e_b^2\left[C_{2,L;b}^{(\alpha)} \otimes (b+\bar{b}) + C_{2,L;\gamma}^{(\alpha)} \otimes \gamma\right]. \tag{19}$$

This feature is relevant to the composition of the FONLL general-mass structure functions.

For the charged current (CC) case the method to calculate the expressions of the $\mathcal{O}(\alpha)$ coefficient functions is accurately the same as in the NC case. On the other hand, this case is more complicated because the flavor structure of CC structure functions is more complex. As a first step, we write the $\mathcal{O}(\alpha_s)$ contribution to $F = F_2, F_L$ in a convenient way

$$F_{2,L}^{CC,(\alpha_s)} = x\sum_{\substack{U=u,c,t\\D=d,s,b}} |V_{UD}|^2\left[C_{2,L;q}^{(\alpha_s)} \otimes (D+\bar{U}) + 2C_{2,L;g}^{(\alpha_s)} \otimes g\right], \tag{20}$$

where V_{UD} are the elements of the CKM matrix.

We consider the impact of a factor e_q^2 every time that a quark of flavor q couples to a photon, the $\mathcal{O}(\alpha)$ corrections to the CC

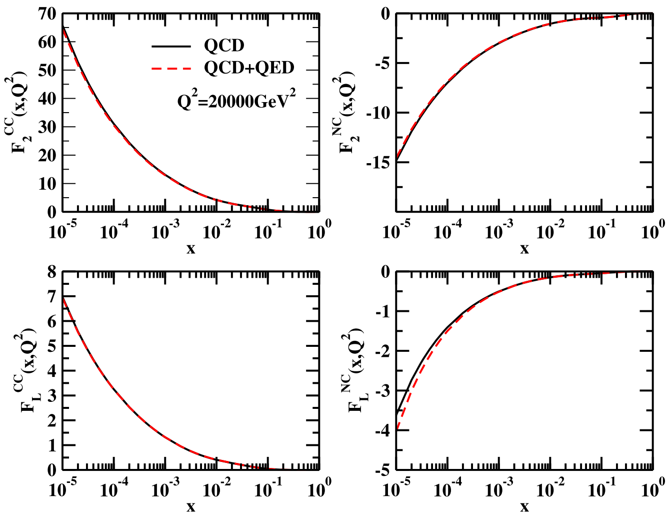

Fig. 10. Comparison of the DIS structure function in the neutral-current and charged-current with and without QED corrections at $Q^2 = 20000\ \text{GeV}^2$.

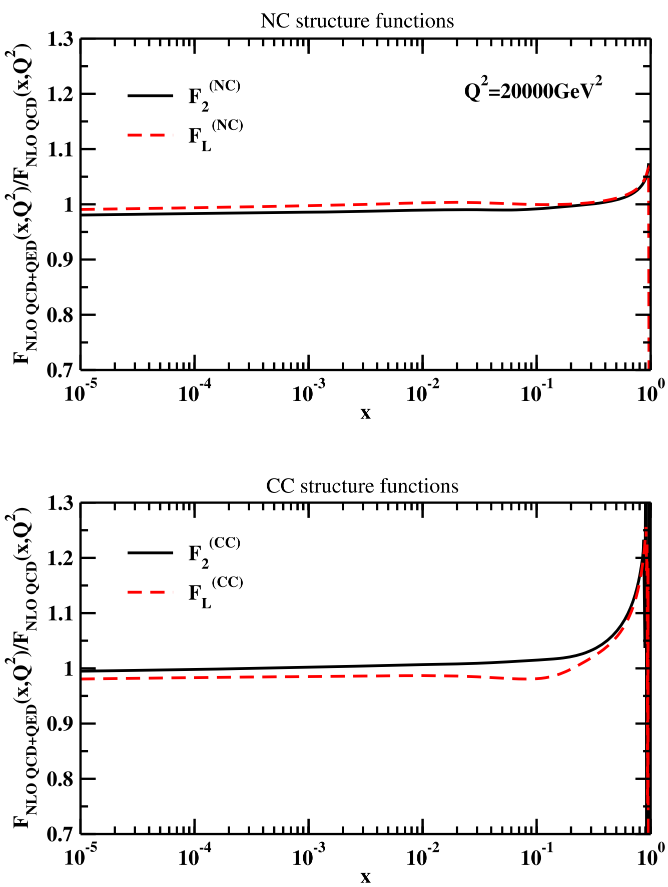

Fig. 11. The effects of the NLO QED corrections on the neutral-current (upper) and charged-current (lower plot) of the DIS structure function, $F_2(x, Q^2)$ and $F_L(x, Q^2)$, normalized to the pure QCD results.

structure functions F_2 and F_L for the production of a neutrino or an anti-neutrino are given by

$$F^{CC,(\alpha)}_{2,L} = x \sum_{\substack{U=u,c,t \\ D=d,s,b}} |V_{UD}|^2 \Big[C^{(\alpha)}_{2,L;q} \otimes (e_D^2 D + e_U^2 \bar{U}) + 2C^{(\alpha)}_{2,L;\gamma} \otimes \gamma \Big]. \tag{21}$$

As a result of the impact of the $\mathcal{O}(\alpha)$ correction on the DIS structure functions, utilizing this contribution, we have plotted its effect on of the pure QCD computation and QCD⊗QED corrections in Fig. 10. The plots are contained, evolving our results from $Q_0^2 = 2\ \text{GeV}^2$ to $Q^2 = 20000\ \text{GeV}^2$, including the full QED corrections discussed in the previous section. They also involve the resulting evolved PDFs to compute the NC and the CC DIS structure functions, including the $\mathcal{O}(\alpha)$ corrections to the coefficient functions discussed above. The predictions are displayed such as to be normalized to the pure QCD computation in which the QED corrections are absent both in the evolution and in the calculation of the structure functions. We show the effects of the NLO QED corrections on the neutral-current (upper plot) and charged-current (lower plot) of the DIS structure function, $F_2(x, Q^2)$ and $F_L(x, Q^2)$, normalized to the pure QCD results in Fig. 11. It is obvious that the impact of the QED corrections is very small specially in the low x region. In the large x region, the existence of a photon-initiated contribution has a more significant effect because of the suppression of the QCD distributions (quarks and gluon) relevant to the photon PDF. In conclusion, we emphasis that the general features perceived in this figure do not depend on the input PDF set.

5. Conclusion

In this work, we reviewed calculation of the PDFs obtained from the DGLAP evolution equations with corrections up to $\mathcal{O}(\alpha_s^2)$ in QCD, $\mathcal{O}(\alpha\alpha_s)$ and $\mathcal{O}(\alpha^2)$ in QED based on the Mellin transform techniques. Then, we have calculated the total longitudinal structure function in terms of these PDFs in this approximation. We have shown that the QED corrections up to these orders to the DGLAP evolution equations are important especially at high Q^2 where the photons can produce more partons. We have compared our results for the total longitudinal structure function, including the contribution heavy quarks, with those of the APFEL and experimental data. There is a nice agreement between them. Impact of QED corrections on structure functions lead the results on high accuracy. Our results indicated that the $\mathcal{O}(\alpha\alpha_s)$ and $\mathcal{O}(\alpha^2)$ corrections have a small effect on the total longitudinal structure functions but we can not ignore their impact on the structure functions.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

This project is supported financially by Iran National Science Foundation (INSF), grand No. 96009171.

Acknowledgements

The authors would like to thank the Iran National Science Foundation (INSF), grand No. 96009171, for its support of this study. The authors are also indebted the Yazd University for the provided facilities to do this project.

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