



SU(3) symmetry and its breaking effects in semileptonic heavy baryon decays

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ABSTRACT

We employ the flavor SU(3) symmetry to analyze semileptonic decays of anti-triplet charmed baryons (Λ_c^+ , $\Xi_c^{+,0}$) and find that the experimental data on $\mathcal{B}(\Lambda_c \rightarrow \Lambda \ell^+ \nu_\ell)$ implies $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (4.10 \pm 0.46)\%$ and $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \mu^+ \nu_\mu) = (3.98 \pm 0.57)\%$. When this prediction is confronted with recent experimental results from Belle collaboration $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (1.31 \pm 0.04 \pm 0.07 \pm 0.38)\%$ and $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \mu^+ \nu_\mu) = (1.27 \pm 0.06 \pm 0.10 \pm 0.37)\%$, it is found that the SU(3) symmetry is severely broken. We then consider the generic SU(3) breaking effects both in decay amplitudes and the mixing effect between the anti-triplet and sextet charmed baryons. We find that the pertinent data can be accommodated in different scenarios but the breaking effects are inevitably large. In some interesting scenarios, we also explore the testable implications in these scenarios which can be stringently tested with more data become available. Similar analyses are carried out for the semileptonic decays of anti-triplet beauty baryons to octet baryons and anti-triplet charmed baryons. The validity of SU(3) for these decays can also be examined when data become available.

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1. Introduction

Weak decays of heavy baryons carrying a charm quark have been studied extensively on both experimental and theoretical aspects [1], as they supply a platform for the study of strong and weak interactions in the standard model (SM). On the experimental side, data on charmed baryons decays from BESIII [2,3], Belle [4] and ALICE [5] collaborations provided important information to extract the CKM matrix element. Belle collaboration has provided a measurement of the Ξ_c^0 branching fractions very recently [4]:

$$\mathcal{B}_{\text{Belle}}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (1.31 \pm 0.04 \pm 0.07 \pm 0.38)\%,$$

$$\mathcal{B}_{\text{Belle}}(\Xi_c^0 \rightarrow \Xi^- \mu^+ \nu_\mu) = (1.27 \pm 0.06 \pm 0.10 \pm 0.37)\%, \quad (1)$$

which is about a factor of 2 more precise than the ALICE result:

$$\mathcal{B}_{\text{ALICE}}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (2.5 \pm 0.8)\%. \quad (2)$$

This comes from the ALICE measurement of $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e)/\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \pi^+) = 1.38 \pm 0.14 \pm 0.22$ [5] and Belle data $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- \pi^+) = 1.8 \pm 0.7\%$ [6]. We anticipate the difference between the above results can be clarified with the improvement of the experimental accuracy and the promising prospects on charmed baryons in the future. The available data on the decays from the anti-triplet heavy baryons to the octet baryons have been collected in Table 1, while the branching fraction $\mathcal{B}(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e) = (1.54 \pm 0.35)\%$ listed is obtained by averaging the Belle and ALICE data.

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Table 1

Experimental data and SU(3) symmetry analysis of anti-triplet charmed baryon decays. The SU(3) predictions for $\mathcal{B}(\Xi_c^- \rightarrow \Xi_c^0 \ell^+ \nu_\ell)$ are obtained by fitting the first two experimental data.

Channel	Branching ratio (%)	
	Experimental data	SU(3) symmetry
$\Lambda_c^+ \rightarrow \Lambda^0 e^+ \nu_e$	3.6 ± 0.4 [33]	3.6 ± 0.4 (input)
$\Lambda_c^+ \rightarrow \Lambda^0 \mu^+ \nu_\mu$	3.5 ± 0.5 [33]	3.5 ± 0.5 (input)
$\Xi_c^+ \rightarrow \Xi^0 e^+ \nu_e$	2.3 ± 1.5 [33]	12.17 ± 1.35
$\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e$	1.54 ± 0.35 [4,5]	4.10 ± 0.46
$\Xi_c^0 \rightarrow \Xi^- \mu^+ \nu_\mu$	1.27 ± 0.44 [4]	3.98 ± 0.57

On the theoretical side, one can apply the SU(3) flavor symmetry to analyze the semileptonic decays and obtain some model-independent relations among different decays [7–19]. For semileptonic charmed baryon decays, we have

$$\Gamma(\Xi_c^0 \rightarrow \Xi^- \ell^+ \nu_\ell) = \Gamma(\Xi_c^+ \rightarrow \Xi^0 \ell^+ \nu_\ell) = \frac{3}{2} \Gamma(\Lambda_c^+ \rightarrow \Lambda^0 \ell^+ \nu_\ell). \quad (3)$$

Since the irreducible amplitude can be extracted by fitting data, the SU(3) analysis bridges experimental data and the dynamical approaches like Lattice QCD [20–23] and model-dependent calculations [24–32]. We adopt the experiment data on Λ_c^+ semileptonic decays and the SU(3) relations with the lifetimes $\tau_{\Lambda_c^+} = 2.024 \times 10^{-13}$ s, $\tau_{\Xi_c^0} = 1.53 \times 10^{-13}$ s, $\tau_{\Xi_c^+} = 4.56 \times 10^{-13}$ s [33]. Then we obtain the branching ratios of $\Xi_c^{0,+}$ shown in Table 1, from which one can find an obvious deviation between experiments and theory.

It should be noted that the flavor SU(3) symmetry is an approximate symmetry, since u, d, and s quarks have different masses which breaks SU(3) symmetry [34]. For a more accurate analysis, SU(3) breaking effects should be included, which is the main focus of this work. Compared to the strange quark mass m_s , the up and down quark masses $m_{u,d}$ are much smaller and thus can be neglected. Therefore the s quark mass is the major source for flavor SU(3) symmetry breaking. In this work, we carry out an analysis with the leading-order SU(3) breaking effects on semileptonic anti-triplet charmed baryons decays and explore the scenarios in which recent experimental measurements can be consistently accommodated.

The rest of this paper is organized as follows. In Sec. 2, we give the theoretical framework for SU(3) symmetry and study symmetry breaking in semileptonic decays of anti-triplet heavy baryons for the process of $c \rightarrow d/s$. In Sec. 3, we also obtain numerical results using the SU(3) symmetry term and analyze the SU(3) symmetry breaking effect for the process of $b \rightarrow c/u$. A brief conclusion will be presented in the last section.

2. SU(3) symmetry for semileptonic anti-triplet charmed baryon decays

In the flavor SU(3) symmetry limit, hadron multiplets can be classified according to the SU(3) irreducible representation. Baryons with a charm quark and two light quarks can have $3 \otimes 3 = \bar{3} \oplus 6$ representations. The anti-triplet $\bar{3}$ semileptonic baryon (Λ_c^+ , Ξ_c^+ , Ξ_c^0) decays are our focus here, whose quark level Feynman diagrams are shown in Fig. 1(a). In the SM the low-energy effective Hamiltonian for these decays is given as

$$\mathcal{H}_{c \rightarrow d/s} = \frac{G_F}{\sqrt{2}} [V_{cq}^* \bar{q} \gamma^\mu (1 - \gamma_5) c \bar{\nu}_\ell \gamma_\mu (1 - \gamma_5) \ell] + h.c., \quad (4)$$

where $q = d, s$ and G_F is the Fermi-constant. V_{cq} is CKM matrix element. With the help of helicity amplitude method [35], the decay transition amplitude can be written as

$$\mathcal{A}(B_c \rightarrow B_q \ell^+ \nu_\ell) = \frac{G_F}{\sqrt{2}} V_{cq}^* \langle B_q | \bar{q} \gamma^\mu (1 - \gamma_5) c | B_c \rangle \langle \ell^+ \nu_\ell | \bar{\nu}_\ell \gamma^\nu (1 - \gamma_5) \ell | 0 \rangle g_{\mu\nu}, \quad (5)$$

with the decomposition of $g_{\mu\nu}$,

$$g_{\mu\nu} = - \sum_{\lambda=0,\pm 1} \epsilon_\mu^*(\lambda) \epsilon_\nu(\lambda) + \epsilon_\mu^*(t) \epsilon_\nu(t),$$

$$\epsilon_\mu(t) = \frac{q^\mu}{\sqrt{q^2}}, \quad (6)$$

where the $\epsilon_\mu(\lambda)$ is transverse ($\lambda = \pm 1$) or longitudinal ($\lambda = 0$) polarization states and $\epsilon_\mu(t)$ is timelike polarization states.

The above amplitude can be decomposed into the Lorentz invariant hadronic and leptonic matrix elements:

$$\mathcal{A}(B_c \rightarrow B_q \ell^+ \nu) = \frac{G_F}{\sqrt{2}} V_{cq}^* \langle B_q | \bar{q} \gamma^\mu (1 - \gamma_5) c | B_c \rangle \langle \ell^+ \nu_\ell | \bar{\nu}_\ell \gamma^\nu (1 - \gamma_5) \ell | 0 \rangle g_{\mu\nu} = \frac{G_F}{\sqrt{2}} V_{cq}^* \left(- \sum_{\lambda_w=0,\pm 1} H_{\lambda,\lambda_w} L_{\lambda_w} + H_{\lambda,t} L_t \right), \quad (7)$$

$$H_{\lambda,\lambda_w} = \langle B_q | \bar{q} \gamma^\mu (1 - \gamma_5) c | B_c \rangle \epsilon_\mu^*(\lambda_w),$$

$$L_{\lambda_w} = \langle \ell^+ \nu_\ell | \bar{\nu}_\ell \gamma^\nu (1 - \gamma_5) \ell | 0 \rangle \epsilon_\nu(\lambda_w),$$

where $H_{\lambda,\lambda_w}(L_{\lambda_w})$ is hadronic (leptonic) helicity amplitude, $\lambda_{(W)}(0, \pm 1, t)$ corresponds to the helicity of the daughter baryon (W) and the $\epsilon_\mu(\lambda_w)$ is the polarization vector of W boson.

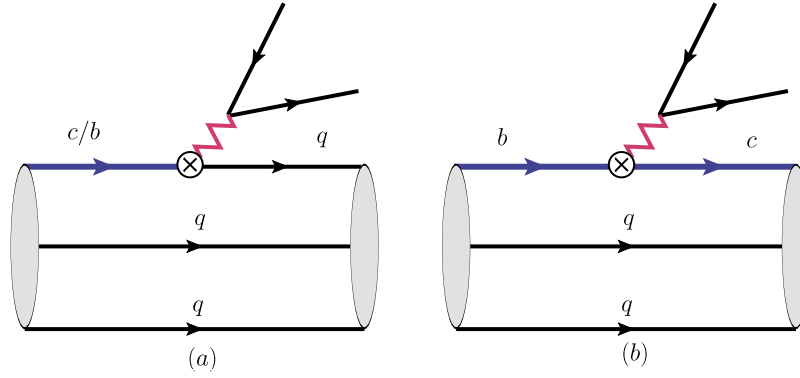


Fig. 1. The Feynman diagram of anti-triplet heavy baryons induced by $c \rightarrow d/s$, $b \rightarrow u$ (left), and $b \rightarrow c$ (right).

Table 2

Decay amplitudes charmed baryons Ξ_c and Λ_c decays into an octet baryons. These amplitudes can be obtained by expanding Eq. (10).

Channel	Amplitude
$\Lambda_c^+ \rightarrow \Lambda^0 \ell^+ \nu_\ell$	$-\sqrt{\frac{2}{3}} a_1^{\lambda, \lambda_w} V_{cs}^*$
$\Lambda_c^+ \rightarrow n \ell^+ \nu_\ell$	$a_1^{\lambda, \lambda_w} V_{cd}^*$
$\Xi_c^+ \rightarrow \Sigma^0 \ell^+ \nu_\ell$	$\frac{a_1^{\lambda, \lambda_w} V_{cd}^*}{\sqrt{2}}$
$\Xi_c^+ \rightarrow \Lambda^0 \ell^+ \nu_\ell$	$-\frac{a_1^{\lambda, \lambda_w} V_{cd}^*}{\sqrt{6}}$
$\Xi_c^+ \rightarrow \Xi^0 \ell^+ \nu_\ell$	$-a_1^{\lambda, \lambda_w} V_{cs}^*$
$\Xi_c^0 \rightarrow \Sigma^- \ell^+ \nu_\ell$	$a_1^{\lambda, \lambda_w} V_{cd}^*$
$\Xi_c^0 \rightarrow \Xi^- \ell^+ \nu_\ell$	$a_1^{\lambda, \lambda_w} V_{cs}^*$

In the SM, charmed baryons can decay into octet baryons. The SU(3) anti-triplet and octet matrix are denoted by

$$T_{c\bar{3}} = \begin{pmatrix} 0 & \Lambda_c^+ & \Xi_c^+ \\ -\Lambda_c^+ & 0 & \Xi_c^0 \\ -\Xi_c^+ & -\Xi_c^0 & 0 \end{pmatrix}, \quad T_8 = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda^0}{\sqrt{6}} \end{pmatrix}. \quad (8)$$

Tree operators of charm quark semileptonic decays into light quarks are categorized into $c \rightarrow d/s$. Therefore under the flavor SU(3) symmetry, the low-energy effective Hamiltonian can be decomposed in terms of H_3 shown as:

$$(H_3)^1 = 0, \quad (H_3)^2 = V_{cd}^*, \quad (H_3)^3 = V_{cs}^*. \quad (9)$$

The corresponding helicity amplitude can be written as:

$$H_{\lambda, \lambda_w} = a_1^{\lambda, \lambda_w} \times (T_{c\bar{3}})^{[ij]} (H_3)^k \epsilon_{ikm} (T_8)_j^m, \quad (10)$$

where the a_1^{λ, λ_w} represents SU(3) irreducible nonperturbative amplitude. The a_1^{λ, λ_w} can be expressed by the form factors

$$a_1^{\lambda, \lambda_w} = \bar{u}(\lambda) \left[f_1 \gamma^\mu + f_2 \frac{i\sigma^{\nu\mu}}{M_i} q^\nu + f_3 \frac{q^\mu}{M_i} \right] u(\lambda_i) \epsilon_\mu^*(\lambda_w) - \bar{u}(\lambda) \left[f'_1 \gamma^\mu + f'_2 \frac{i\sigma^{\nu\mu}}{M_i} q^\nu + f'_3 \frac{q^\mu}{M_i} \right] \gamma_5 u(\lambda_i) \epsilon_\mu^*(\lambda_w). \quad (11)$$

Here $u(\lambda_i)$ is the spinor of charmed baryons, $u(\lambda)$ is the spinor of the final state, and $f_i (i = 1, 2, 3)$, $f'_i (i = 1, 2, 3)$ are the form factors. In the heavy quark limit [36], the f_2 , f_3 , f'_2 , f'_3 are suppressed by $1/m_{B_c}$, and only one independent form factor exists if the large-recoil symmetry is adopted [37,38]. Actually, a previous calculation [25] also indicates that the form factor of vector parameter f_1 and axial-vector parameter f'_1 have dominant contributions to the heavy baryon decay processes. Thus we neglect f_2 , f'_2 , f_3 , and f'_3 in our later calculations. Expanding Eq. (10), one obtains the relations between the helicity amplitudes of different channels of anti-triplet charmed baryons, which are presented in Table 2. In the SU(3) symmetry limit, the branching fractions of $\Xi_c^+ \rightarrow \Xi^0 \ell^+ \nu_\ell$ and $\Xi_c^0 \rightarrow \Xi^- \ell^+ \nu_\ell$ can be predicted by using experimental data of $\Lambda_c^+ \rightarrow \Lambda^0 \ell^+ \nu_\ell$ which are given in Table 1. To shed further light on the decay dynamics, we take the pole model as an illustration to access the q^2 dependence of form factors [39]

$$f_i(q^2) = \frac{f_i}{1 - \frac{q^2}{m_p^2}}, \quad (12)$$

where $f_i = f_i(q^2 = 0)$ and $m_p = 2.061$ GeV, which is the average mass of D and D_s . The differential decay widths can be expressed by these form factors,

Table 3
Experimental and fit data of anti-triplet charmed baryons decays.

Channel	Branching ratio (%)	
	Experimental data	Fit data
$\Lambda_c^+ \rightarrow \Lambda^0 e^+ \nu_e$	3.60 ± 0.40	1.94 ± 0.18
$\Lambda_c^+ \rightarrow \Lambda^0 \mu^+ \nu_\mu$	3.5 ± 0.5	1.87 ± 0.176
$\Xi_c^+ \rightarrow \Xi^0 e^+ \nu_e$	2.3 ± 1.5	6.53 ± 0.60
$\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e$	1.54 ± 0.35	2.17 ± 0.20
$\Xi_c^0 \rightarrow \Xi^- \mu^+ \nu_\mu$	1.27 ± 0.44	2.09 ± 0.19
$\chi^2/d.o.f = 14.3$	$f_1 = 1.05 \pm 0.30$	$f'_1 = 0.11 \pm 0.95$

$$\frac{d\Gamma}{dq^2} = \frac{(m_\ell^2 - q^2)^2 \sqrt{\lambda} G_F^2 V_{SU(3)}^2}{284\pi^3 M^3 (q^2)^3} \left[(f_1(q^2))^2 \times (3s_+ m_\ell^2 (q^2 + s_-) + s_- (3q^2 + s_+) (m_\ell^2 + 2q^2)) \right. \\ \left. + (f'_1(q^2))^2 \times (3s_- m_\ell^2 (q^2 + s_+) + s_+ (3q^2 + s_-) (m_\ell^2 + 2q^2)) \right]. \quad (13)$$

Here $s_- = (M - M')^2 - q^2$, $s_+ = (M + M')^2 - q^2$, and $\sqrt{\lambda} = \sqrt{s_- s_+}$. M and M' are the mass of B_c and B_q , respectively. m_ℓ is the lepton mass. $V_{SU(3)}$ is the SU(3) factor coming from the coefficient of a_1^{λ, λ_w} in Table 2. For instance in the first process in Table 2, $V_{SU(3)} = -\sqrt{2/3} V_{cs}^*$.

Using the amplitudes, we can fit the parameters f_1 and f'_1 with experimental data. In the fit, we use the experimental values for the particle masses. The fitted results are shown in Table 3. Obviously, the χ^2 in fitting is too large to be considered as a good fit, which implies that the SU(3) symmetry is not a good symmetry for charmed baryon decays. In the previous fit, we have neglected the possible SU(3) breaking effects. Because the light u, d, and s quarks have different masses, the SU(3) symmetry is broken. Neglecting the masses of u and d quark the mass matrix M can be written as:

$$M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} \sim m_s \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = m_s \times \omega. \quad (14)$$

We can obtain the modified helicity amplitude as

$$H_{\lambda, \lambda_w} = a_1^{\lambda, \lambda_w} \times (T_{c\bar{3}})^{[ij]} (H_3)^k \epsilon_{ikm} (T_8)_j^m + a_2^{\lambda, \lambda_w} \times (T_{c\bar{3}})^{[in]} (H_3)^k \epsilon_{ikm} (T_8)_j^m \omega_n^j \\ + a_3^{\lambda, \lambda_w} \times (T_{c\bar{3}})^{[in]} (H_3)^k \epsilon_{kjm} (T_8)_i^m \omega_n^j + a_4^{\lambda, \lambda_w} \times (T_{c\bar{3}})^{[in]} (H_3)^k \epsilon_{jim} (T_8)_k^m \omega_n^j \\ + a_5^{\lambda, \lambda_w} \times (T_{c\bar{3}})^{[ij]} (H_3)^k \epsilon_{inm} (T_8)_j^m \omega_k^n. \quad (15)$$

The a_1^{λ, λ_w} is SU(3) symmetric irreducible nonperturbative amplitude and a_2^{λ, λ_w} , a_3^{λ, λ_w} , a_4^{λ, λ_w} , a_5^{λ, λ_w} are SU(3) symmetry breaking irreducible nonperturbative amplitudes, which are proportional to m_s . Furthermore, the SU(3) symmetry breaking terms in Eq. (15) include the contribution of anti-triplet and sextet charmed heavy baryons mixing terms which correspond to the contribution of a_2^{λ, λ_w} , a_3^{λ, λ_w} , and a_4^{λ, λ_w} .

2.1. Symmetry breaking in helicity amplitude

The SU(3) symmetry breaking irreducible nonperturbative amplitudes a_2^{λ, λ_w} , a_3^{λ, λ_w} , a_4^{λ, λ_w} , a_5^{λ, λ_w} in Eq. (15) can be decomposed in a similar way as that in Eq. (11). Again in our analysis, we only keep the vector and axial vector form factors.

Adding SU(3) symmetry breaking term and expanding the above formula in Eq. (15), one can obtain the amplitudes of different channels which are collected in the “amplitude I” column of Table 4. It can be seen that the parameters a_1^{λ, λ_w} and a_5^{λ, λ_w} always appear together in channels $\Lambda_c^+ \rightarrow \Lambda^0 \ell^+ \nu_\ell$, $\Xi_c^+ \rightarrow \Xi^0 \ell^+ \nu_\ell$ and $\Xi_c^0 \rightarrow \Xi^- \ell^+ \nu_\ell$. Therefore the SU(3) symmetry breaking irreducible nonperturbative amplitudes a_2^{λ, λ_w} , a_3^{λ, λ_w} , a_4^{λ, λ_w} , and a_5^{λ, λ_w} can be parametrized as

$$a_1^{\lambda, \lambda_w} + a_5^{\lambda, \lambda_w} = f_1(q^2) \times \bar{u}(\lambda) \gamma^\mu u(\lambda_i) \epsilon_\mu^*(\lambda_w) - f'_1(q^2) \times \bar{u}(\lambda) \gamma^\mu \gamma_5 u(\lambda_i) \epsilon_\mu^*(\lambda_w), \\ a_2^{\lambda, \lambda_w} - a_4^{\lambda, \lambda_w} = \delta f_1(q^2) \times \bar{u}(\lambda) \gamma^\mu u(\lambda_i) \epsilon_\mu^*(\lambda_w) - \delta f'_1(q^2) \times \bar{u}(\lambda) \gamma^\mu \gamma_5 u(\lambda_i) \epsilon_\mu^*(\lambda_w), \\ a_3^{\lambda, \lambda_w} = \Delta f_1(q^2) \times \bar{u}(\lambda) \gamma^\mu u(\lambda_i) \epsilon_\mu^*(\lambda_w) - \Delta f'_1(q^2) \times \bar{u}(\lambda) \gamma^\mu \gamma_5 u(\lambda_i) \epsilon_\mu^*(\lambda_w), \quad (16)$$

where the $a_2^{\lambda, \lambda_w} - a_4^{\lambda, \lambda_w}$ is the combination that appears in helicity amplitude $\Xi_c^+ \rightarrow \Xi^0 \ell^+ \nu_\ell$ and $\Xi_c^0 \rightarrow \Xi^- \ell^+ \nu_\ell$.

By using the replacement rule: $f_1^{(\prime)} \rightarrow f_1^{(\prime)} + \delta f_1^{(\prime)}$ of $\Xi_c^+ \rightarrow \Xi^0 \ell^+ \nu_\ell$ and $\Xi_c^0 \rightarrow \Xi^- \ell^+ \nu_\ell$, we can directly fit these parameters from the data. In doing the combination of $a_1^{\lambda, \lambda_w} + a_5^{\lambda, \lambda_w}$ together to fit data, the a_1^{λ, λ_w} in $\Lambda_c^+ \rightarrow \Lambda^0 \ell^+ \nu_\ell$, $\Xi_c^+ \rightarrow \Xi^0 \ell^+ \nu_\ell$ and $\Xi_c^0 \rightarrow \Xi^- \ell^+ \nu_\ell$ will need to be treated as $a_1^{\lambda, \lambda_w} - a_5^{\lambda, \lambda_w}$. One can take $a_1^{\lambda, \lambda_w} + a_5^{\lambda, \lambda_w}$, $a_2^{\lambda, \lambda_w} - a_4^{\lambda, \lambda_w}$, a_2^{λ, λ_w} , a_3^{λ, λ_w} , and a_5^{λ, λ_w} as independent parameters. The fitted results with two forms to access the q^2 dependence in form factors, pole model and constant, are given in Table 5 with a reasonable $\chi^2/d.o.f = 1.6$ and $\chi^2/d.o.f = 1.9$, respectively. It suggests that the SU(3) symmetry breaking effects generated by the light quark masses can improve the fit. Results for δf_1 and $\delta f'_1$ characterize the size of SU(3) symmetry breaking. From Table 5, one can find that SU(3) symmetry breaking effects to the differential decay width for the $\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e$ can reach as much as 50%, depending on the kinematics. Compared to the constant fit, the pole model fit results of form factors will be used in Table 5 since a relatively smaller χ^2 is obtained. The inadequacy of the experimental data at this stage prevents a direct analysis of different individual terms especially

Table 4

Decay amplitudes of charmed baryons anti-triplet decay into an octet baryon. The amplitudes in column I without c_1^{λ, λ_w} term come from Eq. (15). The effects of Ξ_c and Ξ'_c mixing can be obtained by adding terms proportional to c_1^{λ, λ_w} .

Channel	Amplitude I	Amplitude II
$\Lambda_c^+ \rightarrow \Lambda^0 l^+ \nu$	$-\sqrt{\frac{2}{3}}(a_1^{\lambda, \lambda_w} + a_5^{\lambda, \lambda_w})V_{cs}^*$	$-\sqrt{\frac{2}{3}}(a_1^{\lambda, \lambda_w} + a_5^{\lambda, \lambda_w})V_{cs}^*$
$\Lambda_c^+ \rightarrow n l^+ \nu$	$a_1 V_{cd}^*$	$a_1 V_{cd}^*$
$\Xi_c^+ \rightarrow \Sigma^0 l^+ \nu_\ell$	$\frac{(a_1^{\lambda, \lambda_w} + a_3^{\lambda, \lambda_w} - a_4^{\lambda, \lambda_w} - \frac{c_1^{\lambda, \lambda_w}}{\sqrt{2}}\theta)V_{cd}^*}{\sqrt{2}}$	$\frac{(a_1^{\lambda, \lambda_w} + a_3^{\lambda, \lambda_w} - a_4^{\lambda, \lambda_w})V_{cd}^*}{\sqrt{2}}$
$\Xi_c^+ \rightarrow \Lambda^0 l^+ \nu_\ell$	$-\frac{(a_1^{\lambda, \lambda_w} + 2a_2^{\lambda, \lambda_w} - a_3^{\lambda, \lambda_w} - a_4^{\lambda, \lambda_w} + \frac{3c_1^{\lambda, \lambda_w}}{\sqrt{2}}\theta)V_{cd}^*}{\sqrt{6}}$	$-\frac{(a_1^{\lambda, \lambda_w} + 2a_2^{\lambda, \lambda_w} - a_3^{\lambda, \lambda_w} - a_4^{\lambda, \lambda_w})V_{cd}^*}{\sqrt{6}}$
$\Xi_c^+ \rightarrow \Xi^0 l^+ \nu_\ell$	$-(a_1^{\lambda, \lambda_w} + a_2^{\lambda, \lambda_w} - a_4^{\lambda, \lambda_w} + a_5^{\lambda, \lambda_w} + \frac{c_1^{\lambda, \lambda_w}}{\sqrt{2}}\theta)V_{cs}^*$	$-(a_1^{\lambda, \lambda_w} + a_2^{\lambda, \lambda_w} - a_4^{\lambda, \lambda_w} + a_5^{\lambda, \lambda_w})V_{cs}^*$
$\Xi_c^0 \rightarrow \Sigma^- l^+ \nu_\ell$	$(a_1^{\lambda, \lambda_w} + a_3^{\lambda, \lambda_w} - a_4^{\lambda, \lambda_w} - \frac{c_1^{\lambda, \lambda_w}}{\sqrt{2}}\theta)V_{cd}^*$	$(a_1^{\lambda, \lambda_w} + a_3^{\lambda, \lambda_w} - a_4^{\lambda, \lambda_w})V_{cd}^*$
$\Xi_c^0 \rightarrow \Xi^- l^+ \nu_\ell$	$(a_1^{\lambda, \lambda_w} + a_2^{\lambda, \lambda_w} - a_4^{\lambda, \lambda_w} + a_5^{\lambda, \lambda_w} + \frac{c_1^{\lambda, \lambda_w}}{\sqrt{2}}\theta)V_{cs}^*$	$(a_1^{\lambda, \lambda_w} + a_2^{\lambda, \lambda_w} - a_4^{\lambda, \lambda_w} + a_5^{\lambda, \lambda_w})V_{cs}^*$

Table 5

Experimental data and fit results of anti-triplet charmed baryons decays with symmetry breaking term. The form factors f_1 and f_1' correspond to $a_1^{\lambda, \lambda_w} + a_5^{\lambda, \lambda_w}$. The form factors δf_1 and $\delta f_1'$ correspond to $a_2^{\lambda, \lambda_w} - a_4^{\lambda, \lambda_w}$.

Channel	Branching ratio (%)		
	Experimental data	Fit data (pole model)	Fit data (constant).
$\Lambda_c^+ \rightarrow \Lambda^0 e^+ \nu_e$	3.6 ± 0.4	3.61 ± 0.32	3.62 ± 0.32
$\Lambda_c^+ \rightarrow \Lambda^0 \mu^+ \nu_\mu$	3.5 ± 0.5	3.48 ± 0.30	3.45 ± 0.30
$\Xi_c^+ \rightarrow \Xi^0 e^+ \nu_e$	2.3 ± 1.5	3.89 ± 0.73	3.92 ± 0.73
$\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e$	1.54 ± 0.35	1.29 ± 0.24	1.31 ± 0.24
$\Xi_c^0 \rightarrow \Xi^- \mu^+ \nu_\mu$	1.27 ± 0.44	1.24 ± 0.23	1.24 ± 0.23
Fit parameter	$f_1 = 1.01 \pm 0.87, \delta f_1 = -0.51 \pm 0.92$		
(Pole model)	$f_1' = 0.60 \pm 0.49, \delta f_1' = -0.23 \pm 0.41$		
Fit parameter	$f_1 = 0.86 \pm 0.92, \delta f_1 = -0.25 \pm 0.88$		
(Constant)	$f_1' = 0.85 \pm 0.36, \delta f_1' = -0.43 \pm 0.50$		

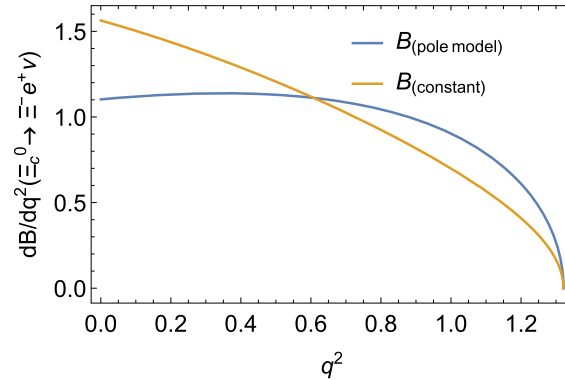


Fig. 2. The differential decay branching fraction dB/dq^2 for the $\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e$. The two curves are obtained by different treatments of form factors.

the a_2^{λ, λ_w} , a_3^{λ, λ_w} , and a_5^{λ, λ_w} . We hope that more experimental data can be accumulated to further examine the detailed sources of SU(3) symmetry breaking in the future.

In Fig. 2, we plot dB/dq^2 in $\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e$ with SU(3) symmetry form factors $f_i^{(\prime)}$ as constants or parametrized as in Eq. (12) to access the q^2 distribution. In both cases, the parameters in form factors are independently fitted. This can be tested by an experimental investigation in the future in addition to the branching ratio fitting in Table 5.

2.2. Symmetry breaking caused by the $\Xi_c^{0/+} - \Xi_c^{\prime 0/+}$ mixing

The inclusion of SU(3) breaking effects will lead to $\Xi_c^{0/+}$ and $\Xi_c^{\prime 0/+}$ to mix. Here $\Xi_c^{\prime 0/+}$ is a component field in the sextet T_{c6} ,

$$T_{c6} = \begin{pmatrix} \Sigma_c^{++} & \frac{\Sigma_c^+}{\sqrt{2}} & \frac{\Xi_c^{'+}}{\sqrt{2}} \\ \frac{\Sigma_c^+}{\sqrt{2}} & \Sigma_c^0 & \frac{\Xi_c^0}{\sqrt{2}} \\ \frac{\Xi_c^{'+}}{\sqrt{2}} & \frac{\Xi_c^0}{\sqrt{2}} & \Omega_c^0 \end{pmatrix}. \quad (17)$$

The mixing between anti-triplet charmed baryons to the sextet states is due to the following term expanding to the first order in m_s ,

$$H_{\lambda,\lambda_w}(T_{\bar{c}3}\omega \rightarrow T_{c6}) = d^{\lambda,\lambda_w} \times (T_{\bar{c}3})^{[ij]} \omega_j^k (T_{c6})_{\{ki\}}. \quad (18)$$

Expanding Eq. (18), one can find the mixing between $\Xi_c^{0(+)}$ and $\Xi_c^{0(+)}$, while other hadrons are not affected. The mixing angle θ can be introduced to define the mass eigenvalue state Ξ_c^0 and Ξ_c^+ ,

$$\Xi_c^{0/+mass} = \cos\theta \times \Xi_c^{0/+} + \sin\theta \times \Xi_c^{0/+'}, \quad (19)$$

where the angle θ is at the order $O(m_s)$. To the first order in m_s , $\cos\theta \sim 1$, $\sin\theta \sim \theta$.

To take into account the mixing effects for physical Ξ_c states, one needs to work out the sextet semileptonic decay amplitudes which are given to the first order in ω

$$\begin{aligned} H_{\lambda,\lambda_w} = & c_1^{\lambda,\lambda_w} \times (T_{c6})^{[ij]} (H_3)^k \epsilon_{ikm} (T_8)_j^m + c_2^{\lambda,\lambda_w} \times (T_{c6})^{[in]} (H_3)^k \epsilon_{ikm} (T_8)_j^m \omega_n^j \\ & + c_3^{\lambda,\lambda_w} \times (T_{c6})^{[in]} (H_3)^k \epsilon_{kjm} (T_8)_i^m \omega_n^j + c_4^{\lambda,\lambda_w} \times (T_{c6})^{[in]} (H_3)^k \epsilon_{jim} (T_8)_k^m \omega_n^j \\ & + c_5^{\lambda,\lambda_w} \times (T_{c6})^{[ij]} (H_3)^k \epsilon_{inm} (T_8)_j^m \omega_k^n. \end{aligned} \quad (20)$$

The helicity amplitude for anti-triplet charmed baryon with mass eigenvalue state Ξ_c^{mass} can be obtained by using Eq. (15), Eq. (19) and Eq. (20). At the leading order, the helicity amplitudes for the decay channel of mass eigenvalue states $\Xi_c^{0mass} \rightarrow \Xi^- \ell^+ \nu_\ell$ and $\Xi_c^{+mass} \rightarrow \Xi^0 \ell^+ \nu_\ell$ become

$$H_{\lambda,\lambda_w}^{mass} \propto V_{cs}^* (a_1^{\lambda,\lambda_w} + a_2^{\lambda,\lambda_w} - a_4^{\lambda,\lambda_w} + a_5^{\lambda,\lambda_w} + \frac{c_1^{\lambda,\lambda_w}}{\sqrt{2}} \theta), \quad (21)$$

where we have neglected the $O(m_s^2)$ and higher order corrections. The helicity amplitudes of other channels are listed in the ‘‘amplitude I’’ column of Table 4. In the table, the states in the first column are understood to be the mass eigenstates for the case with the mixing effect.

It is clear that the existing experimental data is insufficient to determine all these parameters. But one can see that by introducing the effective amplitude $a_4^{\lambda,\lambda_w} = a_4^{\lambda,\lambda_w} + c_1^{\lambda,\lambda_w} \theta / \sqrt{2}$ and $a_2^{\lambda,\lambda_w} = a_2^{\lambda,\lambda_w} + \sqrt{2} c_1^{\lambda,\lambda_w} \theta$, the effect of θ and c_1^{λ,λ_w} can be absorbed into a_2^{λ,λ_w} and a_4^{λ,λ_w} . The helicity amplitudes with a_2^{λ,λ_w} , a_4^{λ,λ_w} are listed in the ‘‘amplitude II’’ column of Table 4. Therefore, our fit results for the case without mixing effects are still valid, but the form factors δf_1 and $\delta f_1'$ correspond to the new effective amplitudes a_2^{λ,λ_w} , a_4^{λ,λ_w} .

Although several other form factors such as Δf_1 and $\Delta f_1'$ cannot be constrained due to the lack of experimental data, in some scenarios, we still estimate the branching ratios of some processes from the results in Table 4. We can estimate the branching fractions of $\Lambda_c^+ \rightarrow ne^+ \nu_e$ and $\Lambda_c^+ \rightarrow n\mu^+ \nu_\mu$:

$$\mathcal{B}(\Lambda_c^+ \rightarrow ne^+ \nu_e) = (0.520 \pm 0.046)\%, \quad \mathcal{B}(\Lambda_c^+ \rightarrow n\mu^+ \nu_\mu) = (0.506 \pm 0.045)\%, \quad (22)$$

by assuming a_5^{λ,λ_w} giving no contribution. If the process of $\Lambda_c^+ \rightarrow n\ell^+ \nu_\ell$ is measured by experiments, the contributions of a_5^{λ,λ_w} will be obtained. The branching fractions of $\Xi_c^+ \rightarrow \Sigma^0 \ell^+ \nu_\ell$, $\Xi_c^+ \rightarrow \Lambda^0 \ell^+ \nu_\ell$ and $\Xi_c^0 \rightarrow \Sigma^- \ell^+ \nu_\ell$ can also be estimated by assuming a_2^{λ,λ_w} , a_3^{λ,λ_w} , and a_5^{λ,λ_w} giving no contributions.

$$\begin{aligned} \mathcal{B}(\Xi_c^+ \rightarrow \Sigma^0 e^+ \nu_e) &= (0.496 \pm 0.046)\%, & \mathcal{B}(\Xi_c^+ \rightarrow \Sigma^0 \mu^+ \nu_\mu) &= (0.481 \pm 0.044)\%, \\ \mathcal{B}(\Xi_c^+ \rightarrow \Lambda^0 e^+ \nu_e) &= (0.067 \pm 0.013)\%, & \mathcal{B}(\Xi_c^+ \rightarrow \Lambda^0 \mu^+ \nu_\mu) &= (0.069 \pm 0.0213)\%, \\ \mathcal{B}(\Xi_c^0 \rightarrow \Sigma^- e^+ \nu_e) &= (0.333 \pm 0.031)\%, & \mathcal{B}(\Xi_c^0 \rightarrow \Sigma^- \mu^+ \nu_\mu) &= (0.323 \pm 0.029)\%. \end{aligned} \quad (23)$$

For the processes of $\Xi_c^+ \rightarrow \Sigma^0 \ell^+ \nu_\ell$, $\Xi_c^+ \rightarrow \Lambda^0 \ell^+ \nu_\ell$, $\Xi_c^0 \rightarrow \Sigma^- \ell^+ \nu_\ell$, once some of the processes are established in future experiments, we can fit the form factor $\Delta f_1^{(\prime)}$ which reflects the contribution of a_3^{λ,λ_w} . Then the branching fractions for the other processes depending on a_3^{λ,λ_w} can also be established.

3. SU(3) symmetry analysis in anti-triplet beauty baryons semileptonic decays

The anti-triplet beauty baryon semileptonic decays are governed by the Hamiltonian:

$$\mathcal{H}_{b \rightarrow u/c} = \frac{G_F}{\sqrt{2}} \left[V_{qb}^* \bar{q} \gamma^\mu (1 - \gamma_5) b \bar{\ell} \gamma_\mu (1 - \gamma_5) \nu_\ell \right] + h.c., \quad (24)$$

where $q = u, c$.

The $b \rightarrow c$ transition is an SU(3) singlet, while the $b \rightarrow u$ transition forms an SU(3) triplet H'_3 with $(H'_3)^1 = 1$ and $(H'_3)^{2,3} = 0$. The SU(3) matrix representation of anti-triplet beauty baryons are given as

$$T_{b\bar{3}} = \begin{pmatrix} 0 & \Lambda_b^0 & \Xi_b^0 \\ -\Lambda_b^0 & 0 & \Xi_b^- \\ -\Xi_b^0 & -\Xi_b^- & 0 \end{pmatrix}. \quad (25)$$

We write the helicity amplitude in SU(3) analysis in a similar fashion as what has been done for semileptonic charmed anti-triplet decays, as

Table 6
Amplitudes beauty baryons Ξ_b and Λ_b decays into octet and anti-triplet baryons.

Channel	Amplitude	Branching fraction (%)
$\Lambda_b^0 \rightarrow p \ell^- \bar{\nu}_\ell$	b_1^{λ, λ_w}	4.1 ± 1.0 (input) [33]
$\Xi_b^0 \rightarrow \Sigma^+ \ell^- \bar{\nu}_\ell$	$-b_1^{\lambda, \lambda_w}$	4.1 ± 1.0
$\Xi_b^- \rightarrow \Sigma^0 \ell^- \bar{\nu}_\ell$	$\frac{b_1^{\lambda, \lambda_w}}{\sqrt{2}}$	2.2 ± 0.5
$\Xi_b^- \rightarrow \Lambda^0 \ell^- \bar{\nu}_\ell$	$\frac{b_1^{\lambda, \lambda_w}}{\sqrt{6}}$	0.7 ± 0.2
$\Lambda_b^0 \rightarrow \Lambda_c^+ \ell^- \bar{\nu}_\ell$	$2e_1^{\lambda, \lambda_w}$	$6.2^{+1.4}_{-1.3}$ (input) [33]
$\Xi_b^0 \rightarrow \Xi_c^+ \ell^- \bar{\nu}_\ell$	$2e_1^{\lambda, \lambda_w}$	$6.2^{+1.4}_{-1.3}$
$\Xi_b^- \rightarrow \Xi_c^0 \ell^- \bar{\nu}_\ell$	$2e_1^{\lambda, \lambda_w}$	$6.6^{+1.5}_{-1.4}$

Table 7
SU(3) symmetry breaking amplitudes of beauty baryons Ξ_b and Λ_b decay into octet baryons and anti-triplet charmed baryons, respectively.

Channel	Amplitude
$\Lambda_b^0 \rightarrow p \ell^- \bar{\nu}_\ell$	b_1^{λ, λ_w}
$\Xi_b^0 \rightarrow \Sigma^+ \ell^- \bar{\nu}_\ell$	$-b_1^{\lambda, \lambda_w} + b_3^{\lambda, \lambda_w} - b_4^{\lambda, \lambda_w}$
$\Xi_b^- \rightarrow \Sigma^0 \ell^- \bar{\nu}_\ell$	$\frac{b_1^{\lambda, \lambda_w} - b_3^{\lambda, \lambda_w} - b_4^{\lambda, \lambda_w}}{\sqrt{2}}$
$\Xi_b^- \rightarrow \Lambda^0 \ell^- \bar{\nu}_\ell$	$\frac{b_1^{\lambda, \lambda_w} + 2b_2^{\lambda, \lambda_w} + b_3^{\lambda, \lambda_w} - b_4^{\lambda, \lambda_w}}{\sqrt{6}}$
$\Lambda_b^0 \rightarrow \Lambda_c^+ \ell^- \bar{\nu}_\ell$	$2e_1^{\lambda, \lambda_w}$
$\Xi_b^0 \rightarrow \Xi_c^+ \ell^- \bar{\nu}_\ell$	$2e_1^{\lambda, \lambda_w} + e_2^{\lambda, \lambda_w}$
$\Xi_b^- \rightarrow \Xi_c^0 \ell^- \bar{\nu}_\ell$	$2e_1^{\lambda, \lambda_w} + e_2^{\lambda, \lambda_w}$

$$H_{\lambda, \lambda_w} = b_1^{\lambda, \lambda_w} \times (T_{b\bar{3}})^{[ij]} (H'_3)^k \epsilon_{ikm} (T_8)_j^m + e_1^{\lambda, \lambda_w} \times (T_{b\bar{3}})^{[ij]} (T_{c\bar{3}})_{[ij]}, \quad (26)$$

where b_1^{λ, λ_w} and e_1^{λ, λ_w} are respectively similar to a_1^{λ, λ_w} in the previous section. The Feynman diagrams for the two term in H_{λ, λ_w} are shown in (a) and (b) of Fig. 1 respectively.

Expanding the H_{λ, λ_w} , one can obtain SU(3) amplitudes are listed in Table 6 and the SU(3) relations can be given as follows,

$$\begin{aligned} \Gamma(\Lambda_b^0 \rightarrow p \ell^- \bar{\nu}_\ell) &= \Gamma(\Xi_b^0 \rightarrow \Sigma^+ \ell^- \bar{\nu}_\ell) = 2\Gamma(\Xi_b^- \rightarrow \Sigma^0 \ell^- \bar{\nu}_\ell) = 6\Gamma(\Xi_b^- \rightarrow \Lambda^0 \ell^- \bar{\nu}_\ell), \\ \Gamma(\Lambda_b^0 \rightarrow \Lambda_c^+ \ell^- \bar{\nu}_\ell) &= \Gamma(\Xi_b^0 \rightarrow \Xi_c^+ \ell^- \bar{\nu}_\ell) = \Gamma(\Xi_b^- \rightarrow \Xi_c^0 \ell^- \bar{\nu}_\ell). \end{aligned} \quad (27)$$

Using the experimental data $\mathcal{B}(\Lambda_b^0 \rightarrow \Lambda_c^+ \ell^- \bar{\nu}_\ell) = (6.2^{+1.4}_{-1.3})\%$ and $\mathcal{B}(\Lambda_b^0 \rightarrow p \mu^- \bar{\nu}_\mu) = (4.1 \pm 1.0)\%$, we give the prediction in third column of Table 6.

For the processes we predicted, we expect them to be measured by Belle II and LHCb. The SU(3) symmetry of these processes will probably be tested. Due to the lack of experimental data at this stage, we can not explore the SU(3) symmetry breaking effects by fitting the form factors. We have also worked out how to include SU(3) symmetry breaking effects. The helicity amplitude including SU(3) symmetry breaking about b quark decays is given as:

$$\begin{aligned} H_{\lambda, \lambda_w} = & b_1^{\lambda, \lambda_w} \times (T_{b\bar{3}})^{[ij]} (H'_3)^k \epsilon_{ikm} (T_8)_j^m + b_2^{\lambda, \lambda_w} \times (T_{b\bar{3}})^{[in]} (H'_3)^k \epsilon_{ikm} (T_8)_j^m \omega_n^j \\ & + b_3^{\lambda, \lambda_w} \times (T_{c\bar{3}})^{[in]} (H'_3)^k \epsilon_{ikm} (T_8)_i^m \omega_n^j + b_4^{\lambda, \lambda_w} \times (T_{c\bar{3}})^{[in]} (H'_3)^k \epsilon_{ikm} (T_8)_j^m \omega_n^j + b_5^{\lambda, \lambda_w} \times (T_{c\bar{3}})^{[ij]} (H'_3)^k \epsilon_{ikm} (T_8)_j^m \omega_k^n \\ & + e_1^{\lambda, \lambda_w} \times (T_{b\bar{3}})^{[ij]} (T_{c\bar{3}})_{[ij]} + e_2^{\lambda, \lambda_w} \times (T_{b\bar{3}})^{[ij]} (T_{c\bar{3}})_{[kj]} \omega_i^k, \end{aligned} \quad (28)$$

where the b_1^{λ, λ_w} , e_1^{λ, λ_w} are SU(3) symmetry irreducible nonperturbative amplitude and b_2^{λ, λ_w} , b_3^{λ, λ_w} , b_4^{λ, λ_w} , b_5^{λ, λ_w} , e_2^{λ, λ_w} are SU(3) symmetry breaking irreducible nonperturbative amplitudes. Here we have written the b_i^{λ, λ_w} terms in a similar fashion as that for a_i^{λ, λ_w} terms. But the b_5^{λ, λ_w} term has no contribution, because $(H'_3)^k \omega_k^n$ is equal to zero. Expanding the formula above, we collected the SU(3) amplitudes in Table 7.

A number of relations for decay widths can be readily deduced from Table 7,

$$\begin{aligned} \Gamma(\Xi_b^- \rightarrow \Sigma^0 \ell^- \bar{\nu}_\ell) &= \frac{1}{2} \Gamma(\Xi_b^0 \rightarrow \Sigma^+ \ell^- \bar{\nu}_\ell), \\ \Gamma(\Xi_b^0 \rightarrow \Xi_c^+ \ell^- \bar{\nu}_\ell) &= \Gamma(\Xi_b^- \rightarrow \Xi_c^0 \ell^- \bar{\nu}_\ell). \end{aligned} \quad (29)$$

It can be seen from Eq. (29) that though the SU(3) symmetry breaking effects caused by the light quark mass are taken into account, there are still relations in these processes. These relations result from isospin symmetry which can only be broken if non-zero u and d quark masses with different values are included. We strongly suggest our experimental colleagues carry out measurements for these decays.

4. Conclusion

We have investigated the semileptonic decay of anti-triplet heavy baryons using SU(3) symmetry based on the latest experimental data. In the SU(3) symmetry limit, when fitting the available experimental data to the SU(3) symmetry analysis, we can only obtain a fit with a least $\chi^2/d.o.f = 14.3$ which means SU(3) symmetry is not a good symmetry for semileptonic charmed anti-baryon decays. We have then carried out detailed analyses with SU(3) symmetry breaking effect due to mass difference between s quark and u/d quark mass. In one scenario, we obtain a reasonable description of all relevant data with a $\chi^2/d.o.f = 1.6$. As an estimation, we give the branching ratios for $\Lambda_c^+ \rightarrow n\ell^+\nu_\ell$, $\Xi_c^+ \rightarrow \Sigma^0\ell^+\nu_\ell$, $\Xi_c^+ \rightarrow \Lambda^0\ell^+\nu_\ell$ and $\Xi_c^0 \rightarrow \Sigma^-\ell^+\nu_\ell$ in some scenarios.

We have also extended the analysis to the semileptonic decays of anti-triplet beauty baryons. However, the lack of experimental data prevents us from an in-depth study. Instead, we find a set of SU(3) relations in Eq. (27) and isospin relation in Eq. (29) between the decay widths of such processes. Our results will help to explore the physics behind these SU(3) symmetry breaking experimental data with more experimental data available in the future.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

- [1] M. Ablikim, et al., BESIII, Chin. Phys. C 44 (4) (2020) 040001, <https://doi.org/10.1088/1674-1137/44/4/040001>, arXiv:1912.05983 [hep-ex], for a recent review.
- [2] M. Ablikim, et al., BESIII, Phys. Rev. Lett. 115 (22) (2015) 221805, <https://doi.org/10.1103/PhysRevLett.115.221805>, arXiv:1510.02610 [hep-ex].
- [3] M. Ablikim, et al., BESIII, Phys. Lett. B 767 (2017) 42–47, <https://doi.org/10.1016/j.physletb.2017.01.047>, arXiv:1611.04382 [hep-ex].
- [4] Y.B. Li, et al., Belle, Phys. Rev. Lett. 127 (12) (2021) 121803, <https://doi.org/10.1103/PhysRevLett.127.121803>, arXiv:2103.06496 [hep-ex].
- [5] S. Acharya, et al., ALICE, arXiv:2105.05187 [nucl-ex].
- [6] Y.B. Li, et al., Belle, Phys. Rev. Lett. 122 (8) (2019) 082001, <https://doi.org/10.1103/PhysRevLett.122.082001>, arXiv:1811.09738 [hep-ex].
- [7] M. Gronau, O.F. Hernandez, D. London, J.L. Rosner, Phys. Rev. D 52 (1995) 6356–6373, <https://doi.org/10.1103/PhysRevD.52.6356>, arXiv:hep-ph/9504326 [hep-ph].
- [8] H.n. Li, C.D. Lu, F.S. Yu, Phys. Rev. D 86 (2012) 036012, <https://doi.org/10.1103/PhysRevD.86.036012>, arXiv:1203.3120 [hep-ph].
- [9] H.Y. Cheng, C.W. Chiang, A.L. Kuo, Phys. Rev. D 91 (1) (2015) 014011, <https://doi.org/10.1103/PhysRevD.91.014011>, arXiv:1409.5026 [hep-ph].
- [10] S.H. Zhou, Q.A. Zhang, W.R. Lyu, C.D. Lü, Eur. Phys. J. C 77 (2) (2017) 125, <https://doi.org/10.1140/epjc/s10052-017-4685-0>, arXiv:1608.02819 [hep-ph].
- [11] S. Müller, U. Nierste, S. Schacht, Phys. Rev. D 92 (1) (2015) 014004, <https://doi.org/10.1103/PhysRevD.92.014004>, arXiv:1503.06759 [hep-ph].
- [12] C.Q. Geng, Y.K. Hsiao, C.W. Liu, T.H. Tsai, J. High Energy Phys. 11 (2017) 147, [https://doi.org/10.1007/JHEP11\(2017\)147](https://doi.org/10.1007/JHEP11(2017)147), arXiv:1709.00808 [hep-ph].
- [13] C.Q. Geng, Y.K. Hsiao, C.W. Liu, T.H. Tsai, Phys. Rev. D 97 (7) (2018) 073006, <https://doi.org/10.1103/PhysRevD.97.073006>, arXiv:1801.03276 [hep-ph].
- [14] C.Q. Geng, C.W. Liu, T.H. Tsai, S.W. Yeh, Phys. Lett. B 792 (2019) 214–218, <https://doi.org/10.1016/j.physletb.2019.03.056>, arXiv:1901.05610 [hep-ph].
- [15] X.G. He, G.N. Li, Phys. Lett. B 750 (2015) 82–88, <https://doi.org/10.1016/j.physletb.2015.08.048>, arXiv:1501.00646 [hep-ph].
- [16] C.D. Lü, W. Wang, F.S. Yu, Phys. Rev. D 93 (5) (2016) 056008, <https://doi.org/10.1103/PhysRevD.93.056008>, arXiv:1601.04241 [hep-ph].
- [17] C.Q. Geng, C.W. Liu, T.H. Tsai, Phys. Lett. B 790 (2019) 225–228, <https://doi.org/10.1016/j.physletb.2019.01.025>, arXiv:1812.08508 [hep-ph].
- [18] M. Gronau, J.L. Rosner, Phys. Rev. D 89 (3) (2014) 037501, <https://doi.org/10.1103/PhysRevD.89.037501>, erratum: Phys. Rev. D 91 (11) (2015) 119902, arXiv:1312.5730 [hep-ph].
- [19] M.J. Savage, R.P. Springer, Phys. Rev. D 42 (1990) 1527–1543, <https://doi.org/10.1103/PhysRevD.42.1527>.
- [20] S. Meinel, Phys. Rev. Lett. 118 (8) (2017) 082001, <https://doi.org/10.1103/PhysRevLett.118.082001>, arXiv:1611.09696 [hep-lat].
- [21] S. Meinel, Phys. Rev. D 97 (3) (2018) 034511, <https://doi.org/10.1103/PhysRevD.97.034511>, arXiv:1712.05783 [hep-lat].
- [22] Q.A. Zhang, J. Hua, F. Huang, R. Li, Y. Li, C.D. Lu, P. Sun, W. Sun, W. Wang, Y.B. Yang, arXiv:2103.07064 [hep-lat].
- [23] W. Detmold, C. Lehner, S. Meinel, Phys. Rev. D 92 (3) (2015) 034503, <https://doi.org/10.1103/PhysRevD.92.034503>, arXiv:1503.01421 [hep-lat].
- [24] T.M. Aliev, S. Bilmis, M. Savci, Phys. Rev. D 104 (5) (2021) 054030, <https://doi.org/10.1103/PhysRevD.104.054030>, arXiv:2108.01378 [hep-ph].
- [25] Z.X. Zhao, Chin. Phys. C 42 (9) (2018) 093101, <https://doi.org/10.1088/1674-1137/42/9/093101>, arXiv:1803.02292 [hep-ph].
- [26] Y.L. Liu, M.Q. Huang, J. Phys. G 37 (2010) 115010, <https://doi.org/10.1088/0954-3889/37/11/115010>, arXiv:1102.4245 [hep-ph].
- [27] K. Azizi, Y. Sarac, H. Sundu, Eur. Phys. J. A 48 (2012) 2, <https://doi.org/10.1140/epja/i2012-12002-1>, arXiv:1107.5925 [hep-ph].
- [28] C.D. Lu, K. Ukai, M.Z. Yang, Phys. Rev. D 63 (2001) 074009, <https://doi.org/10.1103/PhysRevD.63.074009>, arXiv:hep-ph/0004213 [hep-ph].
- [29] H.Y. Cheng, B. Tseng, Phys. Rev. D 46 (1992) 1042, <https://doi.org/10.1103/PhysRevD.46.1042>, erratum: Phys. Rev. D 55 (1997) 1697.
- [30] R.N. Faustov, V.O. Galkin, Eur. Phys. J. C 76 (11) (2016) 628, <https://doi.org/10.1140/epjc/s10052-016-4492-z>, arXiv:1610.00957 [hep-ph].
- [31] C.F. Li, Y.L. Liu, K. Liu, C.Y. Cui, M.Q. Huang, J. Phys. G 44 (7) (2017) 075006, <https://doi.org/10.1088/1361-6471/aa68f1>, arXiv:1610.05418 [hep-ph].
- [32] P. Guo, H.W. Ke, X.Q. Li, C.D. Lu, Y.M. Wang, Phys. Rev. D 75 (2007) 054017, <https://doi.org/10.1103/PhysRevD.75.054017>, arXiv:hep-ph/0501058 [hep-ph].
- [33] P.A. Zyla, et al., Particle Data Group, PTEP 2020 (8) (2020) 083C01, <https://doi.org/10.1093/ptep/ptaa104>.
- [34] C.Q. Geng, Y.K. Hsiao, C.W. Liu, T.H. Tsai, Eur. Phys. J. C 78 (7) (2018) 593, <https://doi.org/10.1140/epjc/s10052-018-6075-7>, arXiv:1804.01666 [hep-ph].
- [35] F. Huang, Q.A. Zhang, arXiv:2108.06110 [hep-ph].
- [36] A.V. Manohar, M.B. Wise, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. 10 (2000) 1–191.
- [37] T. Mannel, Y.M. Wang, J. High Energy Phys. 12 (2011) 067, [https://doi.org/10.1007/JHEP12\(2011\)067](https://doi.org/10.1007/JHEP12(2011)067), arXiv:1111.1849 [hep-ph].
- [38] W. Wang, Phys. Lett. B 708 (2012) 119–126, <https://doi.org/10.1016/j.physletb.2012.01.036>, arXiv:1112.0237 [hep-ph].
- [39] Z.X. Zhao, Eur. Phys. J. C 78 (9) (2018) 756, <https://doi.org/10.1140/epjc/s10052-018-6213-2>, arXiv:1805.10878 [hep-ph].