Use of Z polarization in $e^+e^- \to ZH$ to measure the triple-Higgs coupling

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Abstract

It is shown that a certain angular asymmetry of charged leptons produced in the decay of Z in the process $e^+e^- \to ZH$, related to a tensor polarization component of the Z, can be used to constrain the anomalous triple-Higgs coupling, independent of the other anomalous couplings like the ZZH coupling which dominate at tree level. This is because the angular asymmetry is odd under naïve time reversal, and hence dependent on loop-level contributions. At a future e^+e^- collider like the International Linear Collider (ILC), for example, a limit of about 3.4 might be placed on the ratio of the actual triple Higgs coupling to that predicted in the standard model for a centre-of-mass energy of 500 GeV and an integrated luminosity of 30 fb^{-1} with electron and positron beams having longitudinal polarization of -80% and $+30\%$, respectively.

1. Anomalous triple-Higgs coupling

Since the discovery of the Higgs boson, all-out efforts are being made to determine the properties of the Higgs boson, especially its couplings to fermions and gauge bosons, with increasing

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accuracy. The results seem to be in agreement with the predictions of the standard model (SM) to a good degree of accuracy. An important aspect to pin down the theory as the SM is to measure the scalar self-couplings with great accuracy and to check their agreement with the predictions of the SM.

The scalar self-couplings correspond to λ_3 and λ_4 in the following terms in the scalar Lagrangian

$$\mathcal{L}_{3H} = -\lambda_3 H^3, \tag{1}$$

$$\mathcal{L}_{4H} = -\lambda_4 H^4. \tag{2}$$

In the SM, these couplings are related to the physical Higgs mass m_H and the scalar vacuum expectation value v by

$$\lambda_3^{SM} = \lambda_4^{SM} v; \;\; \lambda_4^{SM} = \frac{m_H^2}{2v^2}. \tag{3}$$

Future experiments at the LHC as well as at the proposed lepton colliders will determine λ_3 and λ_4 with greater precision and would be able to check the SM relations of eq. (3). It is possible that the correct full theory is not the SM, but an extension of the SM. In that case, the above couplings could be the couplings in an effective theory, and they may not obey the relations (3). The deviations of these couplings from their SM values have been discussed in the context of the standard model effective field theory (SMEFT), where effective interactions induced by new physics are written in terms of higher-dimensional operators, suppressed by a high-energy scale, the effective theory presumed to be valid at energies much lower than this scale. Thus, for example, $\mathcal{L}_{3H}$ would get a contribution from a dimension-six operator, $-\lambda_6(H^\dagger H)^3$, see for example, [1].

A determination of the triple-Higgs coupling λ_3 can be carried out through a process where two (or more) Higgs bosons are produced. First of all, such a process needs a high centre-of-mass (c.m.) energy. Moreover, it has been found that in the SM [2], there is destructive interference between the one-loop diagrams contributing to the process $gg \to HH$, making the total cross section extremely small at a hadron collider. Thus, the accuracy of the determination of λ_3 is low.

A suggestion was made by McCullough [3] that the triple-Higgs coupling could be measured through its contribution in one-loop diagrams in single-Higgs production. The process considered in [3] was $e^+e^- \to ZH$, in which, it was assumed that only λ_3 deviates from its SM value,

$$\lambda_3 = \lambda_3^{SM}(1+\kappa). \tag{4}$$

κ can be nonzero in an SMEFT, for example, in the presence of dimension-six ϕ^6 operators. The conclusion was that it would be possible to put a limit on the fractional deviation κ which is of the order of 28% at e^+e^- c.m. energy of 240 GeV, with an integrated luminosity $\mathcal{L}$ of 10 ab^{-1}, expected to be available at TLEP (currently known as FCC-ee) [4]. It was shown [5] that the sensitivity of $e^+e^- \to ZH$ can be improved with polarized beams and, in particular, for the polarization combination ($P_{e^-} = -0.8$, $P_{e^+} = +0.3$) the accuracy to measure κ is about 57% for $\sqrt{s} = 250$ GeV and $\mathcal{L} = 2$ ab^{-1}, envisaged at the ILC, as compared to 70% for the case of unpolarized beams. This estimate is based on the assumption, made for the sake of concreteness, that there are no other contributions to an effective ZZH vertex.

Unfortunately, anomalous ZZH couplings, as for example, characterized by dimension-six operators in SMEFT, can contribute at tree level to the cross section for $e^+e^- \to ZH$, and hence

this contribution can overwhelm the triple-Higgs contribution. So this sensitivity is possible only provided the tree-level contributions are known, or eliminated somehow. The extraction of κ from the cross section would then be possible, for example, making use of measurement at more than one energy, using the different energy dependences of the two contributions [3], or using details of Z decay distributions.

We examine here the possibility that Z polarization in $e^+e^- \to ZH$ can be used to measure κ with either less sensitivity to the tree-level ZZH coupling, or, perhaps, independent of it. Using a very useful and simple relation first formulated in [6], the polarization parameters of the Z produced in various processes can be accessed through angular asymmetries of the decay products of the Z, especially a charged lepton pair [6–10]. This property was applied specifically to the process $e^+e^- \to ZH$ in [9]. The interesting feature of Z polarization and the consequent decay asymmetry that we would like to exploit here is that one particular asymmetry gets contribution from only the absorptive part of the amplitude. It is therefore sensitive to the loop-level contribution of the triple-Higgs coupling. This particular CP-even angular asymmetry is odd under time reversal T.[1] The CPT theorem then requires the existence of an absorptive part in the amplitude for this asymmetry to be nonzero. This asymmetry then automatically measures the interference between the tree amplitude and the loop amplitude. Since the tree-level ZZH coupling at the lowest order is the SM coupling, the asymmetry is a measure of the loop contribution when restricting to linear order in anomalous couplings.[2]

While it is true that this asymmetry gets contribution from whichever loop amplitude contributes to the process, we are mainly interested in the triple-Higgs coupling which is likely to be only mildly constrained. As compared to other couplings entering loop amplitudes, like the top Yukawa coupling or the WWH coupling, which would already be well known and close to their SM values, the triple-Higgs coupling would contribute the dominant uncertainty.

There have been various suggestions for the construction of e^+e^- colliders with c.m. energy ranging from a few hundred GeV to a few TeV. After the discovery of the Higgs boson with mass of about 125 GeV, the dominant suggestion is to construct a linear collider, named the International Linear Collider, which would first operate at a c.m. energy of 250 GeV, enabling precise measurement of Higgs properties, through an abundant production of a ZH final state. There have also been other proposals, as for example, the Compact Linear Collider (CLIC), the Future Circular Collider (FCC-ee) and Circular Electron Positron Collider (CEPC) where electron and positron beams would be collided, providing a clean environment to study couplings of SM particles, and possibly look for new physics, if any. In the context of e^+e^- colliders, particularly the ILC, the possibility of utilizing beam polarization and its advantages has received much attention. Suitable longitudinal beam polarization could help in improving the sensitivity for many different processes and suppressing unwanted background [12,13].

The use of polarization in the context of Higgs properties is also discussed in [14,15]. It is expected that at the ILC, polarizations of 80% and 30% would be possible respectively for electron and positron beams for c.m. energy of 250 GeV [13].

On the experimental side, polarization of weak gauge bosons has been measured at the LHC in W + jet production [16,17], Z + jet production [18,19], in W production in the decay of top quarks [20], and more recently in WZ production [21] and same-sign WW production [22]. The

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[1] We refer here to naïve time reversal, that is, a reversal of the directions of all spins and momenta, but not an interchange of initial and final states as is required by genuine time reversal. Henceforth T will refer to naïve time reversal.

[2] The fact that naïve T-odd quantities can be used for studying loop induced triple-Higgs coupling has been made use of in [11], though they do not relate it to Z polarization variables.

gauge-boson polarizations and helicity fractions are inferred from the angular distributions of the fermions to which the gauge bosons decay [23].

Various methods have been suggested for investigating the Higgs self-coupling. In a recent study [24], it is shown how the trilinear Higgs coupling can be constrained from di-Higgs production with a $b\bar{b}b\bar{b}$ final state by using deep learning at a future high luminosity LHC (HL-LHC) run. With this method, a constraint of $-0.8 < \lambda_3/\lambda_3^{\text{SM}} < 6.6$ at 66% CL may be set with 3000 fb^{-1} of HL-LHC data. Furthermore, for HL-LHC runs, using $b\bar{b}\gamma\gamma$ channel for HH, the trilinear coupling can be constrained to $1.00 < \lambda_3 < 6.22$ at the 95% CL [25]. Including various other decay channels, a precision range of $3.4 - 7.7\%$ is proposed to be obtained for a 30 ab^{-1} integrated luminosity [26]. (See also [27].)

Modification of the Higgs coupling or a dimension-six operator in the SMEFT produces linear growth in energy of processes involving vector bosons which are longitudinally polarized and Higgs in the final state [28]. A detailed analysis in the context of lepton and hadron colliders for processes such as $W_L W_L \rightarrow W_L W_L H$ and $W_L W_L \rightarrow HHH$, is contained in Ref. [29].

The current experimental bound from di-Higgs production obtained by ATLAS on the ratio of Higgs self coupling to its SM value is $-5.0 < \lambda_3/\lambda_3^{SM} < 12.0$ [30].

For e^+e^- colliders such as the FCC-ee [31], ILC [32], the CEPC [33], and the CLIC [34], there have been several proposals for using c.m. energies that cover a wide range starting from a few hundred GeV to a few TeV, as well as for methods to determine trilinear Higgs coupling [35,36].

2. Loop contribution of the triple-Higgs coupling in $e^+e^- \rightarrow ZH$

The process $e^+e^- \rightarrow ZH$ involves a ZZ^*H vertex, which can be written as

$$\Gamma_{\mu\nu}^{ZZH} = g_Z m_Z [(1 + \mathcal{F}_1) g_{\mu\nu} + \mathcal{F}_2 k_{1\nu} k_{2\mu}], \tag{5}$$

where k_1, k_2 are the momenta (assumed directed inwards) of the gauge bosons carrying the respective polarization indices μ, ν. This form assumes that the gauge bosons are either on-shell, satisfying $k_{i\mu}\epsilon^\mu(k_i) = 0$ ($i = 1, 2$), or couple to a conserved current, so that the terms with $k_{1\mu}$ or $k_{2\nu}$ can be dropped. Here m_Z is the Z mass, and $g_Z = g_W/\cos\theta_W$, θ_W being the weak mixing angle. The quantities $\mathcal{F}_{1,2}$ are functions of bilinear invariants constructed from the momenta.

Isolating the contribution of the triple-Higgs coupling λ_3, the form factors $\mathcal{F}_{1,2}$ for the process $Z^* \rightarrow ZH$ (we ignore the subprocess $W^{+*}W^{-*} \rightarrow H$, since the measurement of the gauge boson polarization requires decay into charged leptons) can be written at one-loop order in terms of the Passarino-Veltman (PV) functions [37] as follows.

$$\mathcal{F}_1(k_1^2, k_2^2) = \frac{\lambda_3^{\text{SM}}(1+\kappa)}{(4\pi)^2} \left(-3B_0 - 12(m_Z^2 C_0 - C_{00}) - \frac{9m_H^2}{2}(\kappa + 1)B_0' \right), \tag{6}$$

$$\mathcal{F}_2(k_1^2, k_2^2) = \frac{\lambda_3^{\text{SM}}(1+\kappa)}{(4\pi)^2} 12(C_1 + C_{11} + C_{12}). \tag{7}$$

For the process $Z^* \rightarrow ZH$, the arguments of the PV functions are

$$B_0 \equiv B_0(m_H^2, m_H^2, m_H^2), \quad C_0 \equiv C_0(m_H^2, s, m_Z^2, m_H^2, m_H^2, m_Z^2), \tag{8}$$

and analogously for the functions B_0' and the tensor coefficients C_1, C_{11} and C_{12}.

The above expressions are to be evaluated to first order in the parameter κ for consistency, as there would be higher-loop contributions at order κ^2 which are not being included. Use has been made of the package LoopTools [38] to evaluate the PV integrals.

3. *Z* polarization parameters and lepton angular asymmetries

An earlier work [9] considered Z polarization in $e^+e^- \to ZH$ in the presence of anomalous ZZH couplings, and discussed the role of angular asymmetries of leptons produced in Z decay. We use the formalism of that work in the presence of not only tree-level anomalous coupling, but include the loop-induced couplings involving the triple-Higgs coupling.

We now use the notation of [9], rather than the one used in eq. (5), for ease of comparison. The two notations are related to each other in the way described below. The vertex $Z_\mu(q) \to Z_\nu(k)H$ in the process $e^+e^- \to Z^*(q) \to Z(k)H$ is written with the Lorentz structure

$$\Gamma^{ZZH}_{\mu\nu} = \frac{g}{\cos\theta_W} m_Z \left[a_Z g_{\mu\nu} + \frac{b_Z}{m_Z^2} \left(q_\nu k_\mu - g_{\mu\nu} q \cdot k \right) + \frac{\tilde{b}_Z}{m_Z^2} \epsilon_{\mu\nu\alpha\beta} q^\alpha k^\beta \right] \tag{9}$$

where g is the $SU(2)_L$ coupling and θ_W is the weak mixing angle. The a_Z and b_Z terms are invariant under CP, while the $\tilde{b}_Z$ term is CP violating. In the SM, at tree level, the coupling $a_Z = 1$, whereas the other two couplings b_Z and $\tilde{b}_Z$ vanish. We will now take a_Z and b_Z each to be a sum of a tree-level contribution from SMEFT and a loop-level contribution from diagrams including an effective triple-Higgs coupling, and neglect the CP-violating coupling $\tilde{b}_Z$. With this in mind, we are using a modified notation, and now a_Z and b_Z will include also contribution from $\mathcal{F}_1$ and $\mathcal{F}_2$ discussed above:

$$a_Z = a_Z^0 + \mathcal{F}_1 - (q \cdot k)\mathcal{F}_2, \tag{10}$$
$$b_Z = b_Z^0 - m_Z^2 \mathcal{F}_2. \tag{11}$$

In a low-energy SMEFT with a cut-off scale Λ, a_Z and b_Z would receive tree-level contributions, represented above by the subscript 0, from the SM and from dimension-six terms like $\Phi^\dagger \Phi F_{\mu\nu} F^{\mu\nu}/\Lambda^2$, where Φ is the scalar doublet field and $F_{\mu\nu}$ is the field strength tensor.

The non-zero helicity amplitudes in the limit of massless initial states are [9]

$$M(-,+,+) = \frac{g^2 m_Z \sqrt{s}(c_V + c_A)}{2\sqrt{2}\cos^2\theta_W(s - m_Z^2)} \left[a_Z - \frac{\sqrt{s}}{m_Z^2}(E_Z b_Z + i\tilde{b}_Z|\vec{p}_Z|) \right] \tag{12}$$
$$\times (1 - \cos\theta)$$

$$M(-,+,-) = \frac{g^2 m_Z \sqrt{s}(c_V + c_A)}{2\sqrt{2}\cos^2\theta_W(s - m_Z^2)} \left[a_Z - \frac{\sqrt{s}}{m_Z^2}(E_Z b_Z - i\tilde{b}_Z|\vec{p}_Z|) \right] \tag{13}$$
$$\times (1 + \cos\theta)$$

$$M(-,+,0) = \frac{g^2 E_Z \sqrt{s}(c_V + c_A)}{2\cos^2\theta_W(s - m_Z^2)} \left[a_Z - \frac{\sqrt{s}}{E_Z} b_Z \right] \sin\theta \tag{14}$$

$$M(+,-,+) = \frac{-g^2 m_Z \sqrt{s}(c_V - c_A)}{2\sqrt{2}\cos^2\theta_W(s - m_Z^2)} \left[a_Z - \frac{\sqrt{s}}{m_Z^2}(E_Z b_Z + i\tilde{b}_Z|\vec{p}_Z|) \right] \tag{15}$$
$$\times (1 + \cos\theta)$$

$$M(+,-,-) = \frac{-g^2 m_Z \sqrt{s}(c_V - c_A)}{2\sqrt{2}\cos^2\theta_W(s - m_Z^2)} \left[a_Z - \frac{\sqrt{s}}{m_Z^2}(E_Z b_Z - i\tilde{b}_Z|\vec{p}_Z|) \right] \tag{16}$$
$$\times (1 - \cos\theta)$$

$$M(+,-,0) = \frac{g^2 E_Z \sqrt{s}(c_V - c_A)}{2\cos^2\theta_W(s - m_Z^2)} \left[a_Z - \frac{\sqrt{s}}{E_Z} b_Z \right] \sin\theta \tag{17}$$

Here the first two entries in M denote the signs of the helicities of the e^- and e^+, respectively, and the third entry is the Z helicity. θ is the polar angle of the Z relative to the e^- direction as the z axis. c_V and c_A are respectively the vector and axial-vector leptonic couplings of the Z, given by

$$c_V = \frac{1}{2}(-1 + 4\sin^2\theta_W), \quad c_A = -\frac{1}{2}. \tag{18}$$

The density matrix elements for $e^- e^+ \to ZH$ for the Z spin summed over the e^+ and e^- helicities are derived from the helicity amplitudes, setting $\tilde{b}_Z = 0$ are given by

$$\rho(\pm, \pm) = \frac{g^4 m_Z^2 s}{8\cos^4\theta_W (s - m_Z^2)^2} \left[(c_V + c_A)^2 (1 \mp \cos\theta)^2 \right.$$
$$\left. + (c_V - c_A)^2 (1 \pm \cos\theta)^2 \right] \left| a_Z - b_Z \frac{E_Z \sqrt{s}}{m_Z^2} \right|^2 \tag{19}$$

$$\rho(0, 0) = \frac{g^4 E_Z^2 s}{2\cos^4\theta_W (s - m_Z^2)^2} \sin^2\theta \, (c_V^2 + c_A^2) \left| a_Z - b_Z \frac{\sqrt{s}}{E_Z} \right|^2 \tag{20}$$

$$\rho(\pm, \mp) = \frac{g^4 m_Z^2 s}{4\cos^4\theta_W (s - m_Z^2)^2} \sin^2\theta \, (c_V^2 + c_A^2) \left| a_Z - b_Z \frac{E_Z \sqrt{s}}{m_Z^2} \right|^2 \tag{21}$$

$$\rho(\pm, 0) = \frac{g^4 m_Z E_Z s}{4\sqrt{2}\cos^4\theta_W (s - m_Z^2)^2} \sin\theta$$
$$\times \left[(c_V + c_A)^2 (1 \mp \cos\theta) - (c_V - c_A)^2 (1 \pm \cos\theta) \right]$$
$$\times \left[|a_Z|^2 - a_Z b_Z^* \frac{\sqrt{s}}{E_Z} - a_Z^* b_Z \frac{\sqrt{s} E_Z}{m_Z^2} + \frac{s}{m_Z^2} |b_Z|^2 \right]. \tag{22}$$

Here $+$, $-$ and 0 denote the Z helicities.

We do not display the somewhat longer expressions for the density matrix elements taking into account the polarizations P_L and $\bar{P}_L$ of the electron and positron beams, respectively. However, the expressions are more compact on integration over $\cos\theta$, and these are displayed here. We also include the appropriate phase space factor, so that the expressions are normalized to give the correct total cross section σ as the trace of the density matrix:

$$\sigma = \sigma(+, +) + \sigma(-, -) + \sigma(0, 0). \tag{23}$$

The integrated density matrix is given by

$$\sigma(\pm, \pm) = \frac{2(1 - P_L \bar{P}_L) g^4 m_Z^2 |\vec{k}_Z|}{192\pi \sqrt{s} \cos^4\theta_W (s - m_Z^2)^2} (c_V^2 + c_A^2 - 2 P_L^{\text{eff}} c_V c_A)$$
$$\times \left| a_Z - b_Z \frac{E_Z \sqrt{s}}{m_Z^2} \right|^2 \tag{24}$$

$$\sigma(0, 0) = \frac{2(1 - P_L \bar{P}_L) g^4 E_Z^2 |\vec{k}_Z|}{192\pi \sqrt{s} \cos^4\theta_W (s - m_Z^2)^2} (c_V^2 + c_A^2 - 2 P_L^{\text{eff}} c_V c_A)$$

$$\times \left| a_Z - b_Z \frac{\sqrt{s}}{E_Z} \right|^2 \tag{25}$$

$$\sigma(\pm, \mp) = \frac{(1 - P_L \bar{P}_L) g^4 m_Z^2 |\vec{k}_Z|}{192\pi \sqrt{s} \cos^4 \theta_W (s - m_Z^2)^2} (c_V^2 + c_A^2 - 2P_L^{\text{eff}} c_V c_A)$$

$$\times \left| a_Z - b_Z \frac{E_Z \sqrt{s}}{m_Z^2} \right|^2 \tag{26}$$

$$\sigma(\pm, 0) = \frac{(1 - P_L \bar{P}_L) g^4 m_Z E_Z |\vec{k}_Z|}{256\sqrt{2}\sqrt{s} \cos^4 \theta_W (s - m_Z^2)^2} (2c_V c_A - P_L^{\text{eff}} (c_V^2 + c_A^2))$$

$$\times \left[|a_Z|^2 - a_Z b_Z^* \frac{\sqrt{s}}{E_Z} - a_Z^* b_Z \frac{\sqrt{s} E_Z}{m_Z^2} + \frac{s}{m_Z^2} |b_Z|^2 \right]. \tag{27}$$

In the above equations, $P_L^{\text{eff}} = (P_L - \bar{P}_L)/(1 - P_L \bar{P}_L)$, and the indices $+, -$ and 0 denote the Z helicities.

It was shown that certain angular asymmetries of the charged lepton produced in the decay of the Z can be simply related to vector and tensor polarizations of the Z [6–9] and hence to the spin density matrix of the Z in the process. Each of the asymmetries considered was found to be dominated by one of the anomalous couplings $\delta a_Z \equiv \text{Re}\, a_Z - 1$, $\text{Re}\, b_Z$ and $\text{Im}\, b_Z$ to linear order in the anomalous couplings. Now that we include loop contributions of the triple-Higgs couplings, these asymmetries could be measured experimentally to put limits on linear combinations of the anomalous ZZH couplings and the anomalous triple-Higgs coupling κ.

Here we choose a particular angular asymmetry, A_{yz}, the only one found to be proportional to the imaginary part of the anomalous coupling b_Z. A_{yz} is defined as

$$A_{yz} \equiv \frac{\sigma(\cos\theta^* \sin\phi^* > 0) - \sigma(\cos\theta^* \sin\phi^* < 0)}{\sigma(\cos\theta^* \sin\phi^* > 0) + \sigma(\cos\theta^* \sin\phi^* < 0)} \tag{28}$$

and is related to the tensor polarization component

$$T_{yz} = \frac{-i\sqrt{3}\{[\sigma(0,+) - \sigma(+,0)] - [\sigma(-,0) - \sigma(0,-)]\}}{4\sigma} \tag{29}$$

by

$$A_{yz} = \frac{2}{\pi}\sqrt{\frac{2}{3}} T_{yz}. \tag{30}$$

Here, $\sigma(i, j)$ is the integral of $\rho(i, j)$ over the Z azimuthal angle, σ being the trace of $\sigma(i, j)$. The angles θ^* and ϕ^* are polar and azimuthal angles of the lepton in the rest frame of the Z. The Z rest frame is reached by a combination of boosts and rotations from the laboratory frame. In the laboratory frame, the e^- momentum defines the positive z axis, and the production plane of Z is defined as the xz plane. While boosting to the Z rest frame, the xz plane is kept unchanged. Then, the angles θ^* and ϕ^* are measured with respect to the would-be momentum of the Z.

Let us understand why the asymmetry A_{yz} will be proportional to $\text{Im}\, b_Z$. For that we need to know the transformation properties of $\cos\theta^* \sin\phi^*$ under CPT. Under naïve time reversal T, all momenta change sign. With the above definitions, it can be seen that under T, the Z momentum in the laboratory frame changes sign, as also the momentum of the decay charged lepton, so the value of θ^* unchanged. On the other hand, since the normal to the xz plane, which is along

$\vec{p}_{e^-} \times \vec{p}_Z$, does not change under T, the y component of the decay-lepton momentum changes sign because $\vec{p}_{\ell^-}$ itself changes sign. This makes the asymmetry A_{yz} a T-odd asymmetry. At the same time, the momenta of the $e^- e^+$ pair in the laboratory frame, as also those of the final charged lepton pair in the Z rest frame, are invariant under CP. The momentum of the Z does change sign under CP, and also the normal to the xz plane changes direction. Therefore, $\cos\theta^*$ and $\sin\phi^*$ both change sign. The combination $\cos\theta^* \sin\phi^*$ then remains invariant under CP. The asymmetry A_{yz} is thus odd under CPT. As remarked earlier, for a CPT-odd quantity to get a nonzero value, it should get contribution from an absorptive amplitude. In our case, the only absorptive part which can interfere at linear order with the SM contribution is from the imaginary part of b_Z. It can be checked that all other asymmetries are either even under CPT, or else odd under CP and so not possible at one-loop level.

Making use of expression for the density matrix elements derived in the presence of anomalous ZZH couplings a_Z and b_Z (since no CP violation is possible at one-loop level, we do not consider $\tilde{b}_Z$), we find that the asymmetry A_{yz} is given by

$$A_{yz} = \left(\frac{2 c_V c_A - P_L^{\text{eff}}(c_V^2 + c_A^2)}{4(c_V^2 + c_A^2 - 2 P_L^{\text{eff}} c_V c_A)} \right) \left(\frac{|\vec{k}_Z|^2 \sqrt{s}}{(E_Z^2 + m_Z^2) m_Z} \right) \left(\frac{\text{Im}\,(a_Z^* b_Z)}{|a_Z|^2} \right). \tag{31}$$

As can be seen, the asymmetry is proportional to Im b_Z. Hence it will not get contributions from the tree-level couplings, if they are assumed to be real. As discussed earlier, in SM extensions like the 2HDM, a_Z is real, though it could be different from unity at tree level, and $b_Z = 0$ at tree level. Im b_Z does get contribution at one-loop from the triple-Higgs coupling. In fact, it can be written down as

$$\text{Im}\, b_Z = -m_Z^2 \text{Im}\,\mathcal{F}_2. \tag{32}$$

A_{yz} can therefore be used to determine the triple-Higgs coupling independent of the tree-level anomalous ZZH couplings. a_Z also gets contribution from the triple-Higgs coupling at loop level, but working to linear order in the anomalous couplings, Im $(a_Z^* b_Z)/|a_Z|^2$ is simply Im b_Z.

We now evaluate the contribution of the triple-Higgs coupling to the asymmetry A_{yz}. The tree-level b_Z, if any, being real, will not contribute to the asymmetry. The contribution of a_Z, appearing in denominator of the asymmetry, will be restricted, for consistency to $a_Z = 1$, since the contribution of any anomalous coupling $\delta a_Z = a_Z - 1$ will appear only as δa_Z^2, and hence can be neglected.

We include the possibility of longitudinal electron polarization of 80% and positron polarization of 30% for the ILC, and only polarized electrons with 80% polarization for CLIC. There are studies which consider the possibility of incorporating polarized beams at CEPC as well as FCC-ee, which we hope will allow their advantage to be utilized. Since we find the most advantageous configuration as $(P_{e^-}, P_{e^+}) = (-0.8, +0.3)$, we show results of only this combination. Moreover, for simplicity and uniformity, we assume that the full luminosity to be available for the polarized beam combination, though many staging possibilities have been considered while planning future experiments.

That such a combination of beam polarization enhances the asymmetry can be seen from the following arguments. As we argued earlier, A_{yz} is odd under T. Now, all momenta transform in the same way under parity P as under T, i.e., they reverse their signs. In addition, under P, the helicities change sign. Thus, if P is a symmetry of the theory, A_{yz} being odd under P would vanish. Our theory is not symmetric under P, since the left-hand and right-handed couplings of Z to leptons are different. But because $c_V \approx -0.06$ is numerically small, there is an approximate

P invariance, and the asymmetry turns out to be proportional to c_V, and therefore small. In the presence of significant beam polarizations which are opposite in sign for the e^- and e^+, the P symmetry is no longer an approximate symmetry. There is quite a large P violation, and as a result, A_{yz} is greatly enhanced.

We can see how this works out in practice from explicit expressions. A_{yz} arises from a combination of the imaginary parts of the density matrix elements $\rho(\pm, 0)$. It can be seen from eqs. (12)-(17) that the contribution to these matrix elements from the e^- and e^+ helicity combinations $(-, +)$ and $(+, -)$ occur with opposite signs, in addition to the different couplings $(c_V + c_A)$ and $(c_V - c_A)$, respectively. Since numerically c_V is much smaller than c_A in magnitude, this results in a partial cancellation in the calculation of $\rho(+, 0)$ as well as that of $\rho(-, 0)$, giving a coupling dependence of $2c_V c_A$ in the numerator of the asymmetry as compared to $c_V^2 + c_A^2$ in the cross section appearing in the denominator of the asymmetry. In the presence of polarization, the factors $(c_V + c_A)^2$ and $(c_V - c_A)^2$ get different polarization dependent factors, preventing the partial cancellation, and thus enhancing the asymmetry.

For an asymmetry, the estimated error takes the form

$$\delta A = \frac{\sqrt{1 - A_{SM}^2}}{\sqrt{\sigma_{SM} \mathcal{L}}} \tag{33}$$

with σ_{SM} being the SM cross section for the process $e^+ e^- \to Z^* H \to \ell \bar{\ell} H$ ($\ell = e, \mu$) at a collider with integrated luminosity $\mathcal{L}$ and A_{SM} is the corresponding value of asymmetry in the SM.

The efficiency of measurement of the cross section for ZH production with Z decaying into lepton pairs is taken to be 0.4% for $\sqrt{s} = 240$ GeV and luminosity 10 ab^{-1} [4]. It is appropriately scaled for other luminosities. Efficiencies for cross section measurement quoted in earlier works are 0.9% for ILC at 250 GeV and luminosity 2 ab^{-1} [39,40], 3.8% at CLIC for $\sqrt{s} = 350$ GeV and luminosity 500 fb^{-1} [41], and 0.5% at CEPC for $\sqrt{s} = 240$ or 250 GeV and luminosity 5.6 ab^{-1} [42].

For our numerical calculations we make use of the following values of parameters: $m_Z = 91.1876$ GeV, $m_W = 80.379$ GeV, $m_H = 125.0$ GeV, $\sin^2 \theta_W = 0.22$ and $G_F = 1.1663787 \times 10^{-5}$ GeV^{-2}.

4. Results

The results are as follows.

We first list in Table 1 values of Im b_Z which arise from the one-loop triple-Higgs contribution for the value of $\kappa = 1$ for various values for c.m. energy. We present in Table 2 the asymmetry A_{yz} and the limits that could be obtained using this asymmetry for several colliders, with different energies and integrated luminosities. In cases where polarized beams are likely to be available, we include the results with unpolarized beams, as well with e^- and e^+ polarizations of -0.8 and $+0.3$ respectively. In case of CLIC, we give the result with only electron beams polarized.

The limit on κ from A_{yz} with the same ILC parameters as mentioned, *viz.*, $\sqrt{s} = 250$ GeV and $\mathcal{L} = 2$ ab^{-1}, and using the same sensitivity for Z measurement as obtained from the literature, comes out to be about 119. However, this limit has the advantage that it is strictly independent of the tree-level contribution to the ZZH coupling. To maintain this advantage and get a better limit, both energy and luminosity need to be pushed up. Thus, for $\sqrt{s} = 500$ GeV, it is 4.16 for $\mathcal{L} = 10$ ab^{-1} and 2.40 for $\mathcal{L} = 30$ ab^{-1}.

Table 1

Values of Im b_Z from one-loop triple-Higgs coupling contributions for $\kappa = 1$ and various values of $\sqrt{s}$.

$\sqrt{s}$ (GeV)	Im b_Z (for $\kappa = 1$)
240	-3.62×10^{-4}
250	-4.91×10^{-4}
350	-8.22×10^{-4}
365	-8.09×10^{-4}
380	-7.93×10^{-4}
500	-6.13×10^{-4}

Table 2

Values of the asymmetry A_{yz} for $\kappa = 1$ and 1 σ limits on κ from A_{yz} at various colliders with different energies and luminosities. Values for CLIC are shown for unpolarized beams, as well as for e^- beams polarized to -80%. In case of ILC, values are shown for unpolarized beams as well as with e^- and e^+ beam polarizations of -0.8 and $+0.3$, respectively.

| Collider | c.m. energy (GeV) | $10^4 \times A_{yz}$ | | Luminosity (ab^{-1}) | Limit | |
		unpolarized beams	polarized beams		unpolarized beams	polarized beams
CEPC	240	-0.159		10	506	
CEPC	240	-0.159		20	358	
CLIC	380	-2.88	-10.6	0.5	124	31.0
FCC	240	-0.159		10	506	
FCC	250	-0.314		5	362	
FCC	365	-2.64		1.5	78.2	
ILC	250	-0.314	-1.23	2	573	119
ILC	250	-0.314	-1.23	5	362	75.3
ILC	350	-2.39	-9.38	30	19.4	4.03
ILC	500	-4.00	-15.7	4	31.6	6.57
ILC	500	-4.00	-15.7	10	20.0	4.16
ILC	500	-4.00	-15.7	30	11.5	2.40

While our main interest is to use measurements which are independent of the tree-level anomalous couplings, for the sake of completeness, we also list in Table 3 the limits (in percent) that can be obtained on κ from the measurement of the cross section, under the assumption that there are no tree-level anomalous contributions, i.e., $\delta a_z = 0$ and $b_Z = 0$. Table 4 lists the SM cross sections at various c.m. energies, including the contribution of the one-loop triple-Higgs contribution.

5. Conclusions

We have investigated the possibility of using a tensor polarization variable T_{yz} of the Z in the process $e^+ e^- \to ZH$ and a decay-lepton asymmetry A_{yz} related to this variable to constrain the loop-level contribution to the anomalous triple-Higgs coupling κ, independent of tree-level anomalous ZZH coupling. A_{yz}, an asymmetry dependent on the lepton azimuthal angle in a

Table 3

Cross sections (in fb) for $\kappa = 1$ and $1\ \sigma$ limits (in per cent) on κ from the cross section at various colliders with different energies and luminosities, assuming no anomalous tree-level ZZH contribution. Values for CLIC are shown for unpolarized beams, as well as e^- beams polarized to -80%. In case of ILC, values are shown for unpolarized beams as well as with e^- and e^+ beams polarizations of -0.8 and $+0.3$, respectively.

| Collider | c.m. energy (GeV) | Cross section (fb) | | Luminosity (ab^{-1}) | Limit (%) | |
		unpolarized beams	polarized beams		unpolarized beams	polarized beams
CEPC	240	247		10	27.3	
CEPC	240	247		20	19.3	
CLIC	380	107	127	0.5	1334	1223
FCC	240	247		10	27.3	
FCC	250	246		5	44.3	
FCC	365	117		1.5	506	
ILC	250	246	368	2	70.0	57.1
ILC	250	246	368	5	44.3	36.1
ILC	350	129	194	30	81.3	66.4
ILC	500	56.8	85.2	4	311	254
ILC	500	56.8	85.2	10	197	161
ILC	500	56.8	85.2	30	114	92.8

Table 4

Values of the SM cross sections at various c.m. energies, including the contribution of the one-loop triple-Higgs contribution, for unpolarized and polarized beams. In the polarized case e^- and e^+ polarizations are respectively -0.8 and $+0.3$, except in the case of the for 380 GeV, where the e^+ polarization is zero.

| c.m. energy (GeV) | Cross section (fb) | |
	unpolarized	polarized
240	243	365
250	242	363
340	138	207
350	129	194
365	117	176
380	107	127
500	56.9	85.4

specific frame, is found to be odd under naïve time reversal operation, and is therefore proportional to the absorptive part of the amplitude. It therefore isolates the loop-level contributions. In case the triple-Higgs contribution is the dominant one, we calculate the asymmetry and the possible limit on κ for several collider energies and luminosities.

It is seen that a considerably high luminosity of at least 30 fb^{-1} is needed to constrain anomalous triple-Higgs coupling even at the level of 200% to 400%. The better limit of about 240% requires a c.m. energy of 500 GeV. The use of longitudinally polarized beams is also seen as absolutely essential to reach these sensitivities. Longitudinal beam e^- polarization of -80% and e^+ polarization of $+30\%$ improve the limits by a factor between 4 and 5.

While the limit which could be obtained by measuring the cross section for the same combination of experimental parameters is much better, about 93%, this is possible only if the tree-level ZZH contribution is known to be negligible.

Note that we have used the formalism of [6], which entails a particular choice of frame of reference, as also the charged-lepton decay channels for the Z. It is possible that by employing some different frame, as also including hadronic decay channels of the Z, the sensitivity may be improved. This investigation would require detailed numerical simulations, which we do not attempt in this work.

CRediT authorship contribution statement

Kumar Rao: Conceptualization, Formal analysis, Methodology, Software. **Saurabh D. Rindani:** Conceptualization, Formal analysis, Methodology, Software. **Priyanka Sarmah:** Conceptualization, Formal analysis, Methodology, Software. **Balbeer Singh:** Conceptualization, Formal analysis, Methodology, Software.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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