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Non-zero θ_{13} and δ_{CP} phase with A_4 flavor symmetry and deviations to tri-bi-maximal mixing via $Z_2 \times Z_2$ invariant perturbations in the neutrino sector

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Abstract

In this work, a flavour theory of a neutrino mass model based on A_4 symmetry is considered to explain the phenomenology of neutrino mixing. The spontaneous symmetry breaking of A_4 symmetry in this model leads to tribimaximal mixing in the neutrino sector at a leading order. We consider the effect of $Z_2 \times Z_2$ invariant perturbations in neutrino sector and find the allowed region of correction terms in the perturbation matrix that is consistent with 3σ ranges of the experimental values of the mixing angles. We study the entanglement of this formalism on the other phenomenological observables, such as δ_{CP} phase, the neutrino oscillation probability $P(\nu_\mu \to \nu_e)$, the effective Majorana mass $|m_{ee}|$ and $|m_{\nu e}^{eff}|$. A $Z_2 \times Z_2$ invariant perturbations in this model is introduced in the neutrino sector which leads to testable predictions of θ_{13} and CP violation. By changing the magnitudes of perturbations in neutrino sector, one can generate viable values of δ_{CP} and neutrino oscillation parameters. Next we investigate the feasibility of charged lepton flavour violation in type-I seesaw models with leptonic flavour symmetries at high energy that leads to tribimaximal neutrino mixing. We consider an effective theory with an $A_4 \times Z_2 \times Z_2$ symmetry, which after spontaneous symmetry breaking at high scale which is much higher than the electroweak scale leads to charged lepton flavour violation processes once the heavy Majorana neutrino mass degeneracy is lifted either by renormalisation group effects or by a soft breaking of the A_4 symmetry. In this context the implications for charged lepton flavour violation processes like $\mu \to e\gamma$, $\tau \to e\gamma$, $\tau \to \mu\gamma$ are discussed.

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1. Introduction

Ever since the discovery of neutrino oscillations, the aspects of lepton masses, mixings and flavour violation [1] have been an active topic of research and there have been a lot of updates on the results from a long ongoing series of global fits to neutrino oscillation data [2–5]. Neutrino flavor conversion was first detected in solar [6] and atmospheric neutrinos [7]. This discovery led to the Nobel prize in Physics in 2015 [8,9] and was confirmed by subsequent results from the KamLAND reactor experiment [10] as well as long baseline accelerator experiments.

The neutrinos change their flavour as they propagate in space and this phenomenon is known as neutrino oscillation which occurs since the flavour gauge eigenstates of neutrinos are mixture of mass eigenstates. The mixing is described by PMNS matrix which can be parameterised in terms of three neutrino mixing angles and CP violating phases. The experimental discovery of neutrino oscillations constitutes not only neutrino mass squared differences, but the probability of nearly degenerate neutrino spectrum is also contemplated. Further neutrino oscillation has triggered the experimental and theoretical endeavour to understand the aspects of lepton masses, mixings and flavour violation in SUSY GUTs theories. The massive neutrinos are produced in their gauge eigenstates (ν_α) which is related to their mass eigenstate (ν_i), where the gauge eigenstates take part in gauge interactions.

$$|\nu_\alpha\rangle = \sum U_{\alpha_i}|\nu_i\rangle \tag{1}$$

where, $\alpha = e, \mu, \tau$, ν_i is the neutrino of distinct mass m_i.

In the physics of the dynamics of neutrino mass generation in the leptonic sector, the flavour problem of particle physics, is one of the open challenges that the field of high energy physics faces today.

Since the flavour mixing happens due to the mixing between mass and flavour eigenstates, neutrinos have nondegenerate mass. To put into effect this idea into a renormalisable field theory what so ever symmetry used in generating neutrino mass degeneracy must be broken. In this work A_4 symmetry [11–15] which is the group of the even permutation of four objects or equivalently that of a tetrahedron used to maintain this degeneracy is broken spontaneously to produce the spectrum of different charged lepton masses.

Many inferences have been intended to guess the actual pattern of lepton mixings. Some of the phenomenological pattern of neutrino mixings incorporate for example, Tri-bimaximal (TBM) [13–15], Trimaximal (TM1/TM2) and bi-large mixing patterns.

Over the past two decades, a lot of theoretical and experimental works have been going on, which aimed at grasping the structure of lepton mixing matrix [16]. Solar and atmospheric angle as conferred by accelerator and reactor data indicated that the mixing in the lepton sector is very different from quark mixings, given the large values of θ_{12} and θ_{23}. These observations were soon encrypted in the tribimaximal (TBM) mixing ansatz presented by Harrison, Perkins, and Scott [17] and also [18] described by.

$$U_{PMNS} \simeq \begin{bmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix} = U_{TBM} \tag{2}$$

where, $Sin\,\theta_{13} = 0$. In this educated guess, mixing angles have $Sin\,\theta_{12} = \frac{1}{3}$, $\theta_{23} = \frac{\pi}{4}$ and $Sin^2\,\theta_{23} = \frac{1}{2}$ whose perspective is good bearing in mind the latest neutrino oscillation global fit. Since the TBM ansatz was first proposed so it became a touchstone convention for inspiring the pattern of lepton masses and mixings. Unfortunately, it envisages $Sin\,\theta_{13} = 0$ and hence zero leptonic CP violation phase in neutrino oscillation. Infact, data from reactors have stipulated that such sterling TBM ansatz can not be the correct description of nature, since the reactor mixing angle θ_{13} has been confirmed to be non-zero to a very high significant content [19,20]. Further, till now it is becoming increasingly apparent that there has been compelling evidence for CP violation in neutrino oscillations, allocating further hint that alteration or change of TBM mixing ansatz is vital.

Neutrino oscillation experiments are a probe to measure neutrino mixing and mass spectrum since the oscillation probability $P(\nu_\mu \to \nu_e)$ depends on mixing angles, Dirac CP Violation phase and the mass square differences m^2_{21}, m^2_{23}. Results from earlier experiments stipulate that θ_{13} is very small, almost zero and the lepton mixing matrix follows the TBM (tri-bimaximal mixing) ansatz. This ansatz tells $Sin\,\theta_{13} = 0$, $Sin^2\,\theta_{23} = \frac{1}{2}$, $Tan^2\,\theta_{12} = \frac{1}{2}$. One can conclude the neutrino mixing matrix as TBM type, with small deviations or corrections to it due to perturbation in the charged-lepton or neutrino sector. Current experimental observations of fairly large θ_{13} [2–5], deviated neutrino mixing a little away from TBM ansatz, but close to the predictions of non-zero θ_{13} and $\delta_{CP} = \pm\frac{\pi}{2}$. One can correlate the CP violation in neutrino oscillation with the octant of the atmospheric mixing angle θ_{23}. In this paper, we would like to address a model based on A_4 symmetry which gives non-zero θ_{13}, $\delta_{CP} = \pm\frac{\pi}{2}$ and $Sin^2\,\theta_{23} = 0.57$ via perturbations in the form of $Z_2 \times Z_2$ invariant symmetry in the neutrino sector at leading order. In order to take into account the deviations in mixing angles at a leading order consistent with the experimental results, we add a perturbation in neutrino sector in the form of $Z_2 \times Z_2$ invariant symmetry including second order corrections in the PMNS matrix.

The predictions of vanishing θ_{13} by TBM is owing to its invariance under $\mu - \tau$ exchange symmetry [1]. Small explicit breaking of $\mu - \tau$ symmetry can generate large Dirac CP violating phase in neutrino oscillations [21]. Also some studies in the context of corrections to TBM mixing in A_4 symmetry is presented in [22]. All CP violations (both Dirac and Majorana types) emerge from a common origin in neutrino seesaw. $\mu - \tau$ symmetry breaking shares the common origin with all CP violations, since in the limit of $\mu - \tau$ symmetry mixing angle θ_{13} becomes non-zero and thus CP conservation takes place [23]. Studies on common origin of soft $\mu - \tau$ symmetry and CP breaking in neutrino seesaw, the origin of matter, baryon asymmetry, hidden flavor symmetry are vividly illustrated in [23,24]. $\mu - \tau$ symmetry and its breaking together with CP violation, correlation between θ_{13}, θ_{23} and θ_{12}, in connection to hidden flavor symmetry (including $Z_2 \times Z_2$) are extensively studied in [23–26]. Octahedral symmetric group O_h is described as the flavor symmetry of neutrino-lepton sector. Here the residual symmetries are $Z_2^{\mu-\tau} \bigotimes Z_2^s$ and Z_l^4 and it prescribe the neutrinos and charged leptons, respectively [25]. Studies on the notion of constrained maximal CP violation (CMCPV) which predicts the features $\delta_{CP} = \frac{-\pi}{2}$ and $\theta_{23} = \frac{\pi}{4}$ and their origin in the context of flavor symmetry is presented in [26]. With the discovery of non-zero value of the reactor mixing angle θ_{13} by reactor experiments RENO [27] and Daya Bay [28] the texture of tri-bimaximal mixings can be generalised.

A minimally asymmetric Yukawa texture based on the Frobenius group T_{13} and SU(5) GUT are presented in [29–32], where the neutrino masses and mixing angles are predicted from TBM seesaw mixing, (up to a sign), the CP violating phase (1.32π) and Jarlskog invariant are determined in agreement with current global fits with definite prediction for neutrinoless double beta experiments, and baryon asymmetry is explained from flavoured leptogenesis.

The essence of knowing the exact symmetries behind the observed pattern of neutrino oscillations is one of the challenging tasks in particle physics.

In this work we propose a A_4 family symmetry [33] − the symmetry group of even permutations of 4 objects or equivalently that of a tetrahedron, which is used here to obtain neutrino mixing predictions within fundamental theories of neutrino mass. This A_4 family symmetry was first introduced as a possible family symmetry for the quark sector [34] and is now mostly used for the lepton sector [35–39]. During last two decades many neutrino oscillation experiments like KamLAND [40], LBL+ATM+REAC [41], SOL, LBL+ATM, REAC, LBL, (LBL+REAC) and ATM [42] are being performed and the oscillation parameters are being measured to a very good precision. On the light of discovery of non zero θ_{13}, the neutrino mass model dictating TBM mixing pattern needs necessary modifications.

The discrete family symmetry groups necessitates the need of special vacuum alignment condition to implement tribimaximal mixing pattern ansatz. Also one can generate deviations from TBM mixing pattern by adding symmetry breaking terms in the interactive Lagrangian of the specific discrete family symmetry group. This results in partial and complete symmetry breaking. Residual symmetries exist in neutrino and charged lepton sectors after such perturbations.

In this work we also carry out studies on lepton flavour violation decay $\mu \to e\gamma$, $\tau \to \mu\gamma$ and $\tau \to e\gamma$ in $G_{SM} \times A_4 \times U(1)_X$ incorporating $Z_2 \times Z_2$ invariant perturbation in both charged lepton sector and neutrino sector as discussed below, and hence one can guess the sensitivity to test the observation of sleptons and sparticles at future run of LHC. These charged lepton flavour violation rates depend on the form of Dirac neutrino yukawa couplings as fixed by most favourable predicted value of Dirac CPV phase of this work and on the details of soft SUSY breaking parameters and Tanβ. We have used the Higgs mass as measured at LHC, non zero reactor mixing angle θ_{13} for neutrinos, and latest present and future constraints on BR$(\mu \to e\gamma)$ [43].

Persuaded by the prerequisite for departing from the simplest first-order form for the TBM ansatz, Eq. (2), here we propose a generalised version of the TBM ansatz in which the new ansatz is realised in a model based on A_4 group as suggested in [15] by breaking A_4 symmetry spontaneously to Z_2 in the neutrino sector, which correctly accounts for the non-zero value of θ_{13} and introduces CP violation. We then incorporate a real $Z_2 \times Z_2$ perturbations in the neutrino sector leading to feasible values of θ_{13} and δ_{CP}. This results in predictions of neutrino oscillation parameters and leptonic CPV phase that will be tested at upcoming neutrino experiments. Appendix A summarises the A_4 algebra.

2. The A_4 model

We take a type I SeeSaw model based on A_4 symmetry [15]. Let us limit ourselves to only leptonic sector. The field consists of three left handed $SU(2)_L$ gauge doublets, three right handed charged gauge singlets, three right handed neutrino gauge singlets. In addition there exists also four Higgs doublets ϕ_i $(i = 1, 2, 3)$ and ϕ_0 and three scalar singlets. The above fields can be represented under various irreducible representations as it shown in Table 1.

The Yukawa Lagrangian of the leptonic fields of the model $G_{SM} \times A_4 \times U(1)_X$ [44] is

$$L = L_{Charged\ leptons\ Dirac} + L_{Neutrino\ Dirac} + L_{Neutrino\ Majorana} \tag{3}$$

Table 1
Allocations under various irreducible representations of $SU(2)_L$, $U(1)_Y$ and A_4.

Fields	$SU(2)_L$	$U(1)_Y$	A_4	Representation
Left Handed Doublets	$\frac{1}{2}$	$Y=-1$	$\underline{3}$	Y_{iL}
Right Handed Charged Lepton Singlets	0	$Y=-2$	$\underline{1}\oplus\underline{1'}\oplus 1''$	l_{iR}
Right Handed Neutrino Singlets	0	$Y=0$	$\underline{3}$	ν_{iR}
Higgs Doublet	$\frac{1}{2}$	$Y=1$	$\underline{3}$	ϕ_i
Higgs Doublet	$\frac{1}{2}$	$Y=1$	$\underline{1}$	ϕ_0
Real Gauge Singlet	0	$Y=0$	$\underline{3}$	F_i

where, G_{SM} is the standard model gauge symmetry, $G_{SM}=U(1)_Y\times SU(2)_L\times SU(3)_C$. Now,

$$\begin{aligned}
&L_{\textit{Charged leptons Dirac}}\\
&\quad=-[h_1\left(\bar{Y}_{1L}\phi_1\right)l_{1R}+h_1\left(\bar{Y}_{2L}\phi_2\right)l_{1R}+h_1\left(\bar{Y}_{3L}\phi_3\right)l_{1R}+h_2\left(\bar{Y}_{1L}\phi_1\right)l_{2R}\\
&\qquad+\omega^2\left\{h_2\left(\bar{Y}_{2L}\phi_2\right)l_{2R}\right\}+\omega\left\{h_2\left(\bar{Y}_{3L}\phi_3\right)l_{2R}\right\}+h_3\left(\bar{Y}_{1L}\phi_1\right)l_{3R}\\
&\qquad+\omega\left\{h_3\left(\bar{Y}_{2L}\phi_2\right)l_{3R}\right\}+\omega^2\left\{h_3\left(\bar{Y}_{3L}\phi_3\right)l_{3R}\right\}]+hc
\end{aligned}\tag{4}$$

where,

$$\omega=exp(\frac{2\pi i}{3})=-\frac{1}{2}+i\frac{\sqrt{3}}{2}$$

$$L_{\textit{Neutrino Dirac}}=-h_0\left(\bar{Y}_{1L}\nu_{1R}\right)\bar{\phi_0}-h_0\left(\bar{Y}_{2L}\nu_{2R}\right)\bar{\phi_0}-h_0\left(\bar{Y}_{3L}\nu_{2R}\right)\bar{\phi_0}+h.c\tag{5}$$

and

$$\begin{aligned}
&L_{\textit{Neutrino Majorana}}\\
&\quad=-\frac{1}{2}[\left\{M\nu_{1R}^TC^{-1}\nu_{1R}+M\nu_{2R}^TC^{-1}\nu_{2R}+M\nu_{3R}^TC^{-1}\nu_{3R}\right\}+h.c\\
&\qquad+h_FF_1\nu_{2R}^TC^{-1}\nu_{3R}+h_FF_1\nu_{3R}^TC^{-1}\nu_{2R}+h_FF_2\nu_{3R}^TC^{-1}\nu_{1R}\\
&\qquad+h_FF_2\nu_{1R}^TC^{-1}\nu_{3R}+h_FF_3\nu_{1R}^TC^{-1}\nu_{2R}+h_FF_3\nu_{2R}^TC^{-1}\nu_{1R}]
\end{aligned}\tag{6}$$

where C is the charge conjugation matrix. $L_{\textit{Charged leptons Dirac}}$ is the Dirac mass matrix in the charged leptonic fields, $L_{\textit{Neutrino Dirac}}$ is the Dirac mass matrix in the neutrino sector, $L_{\textit{Neutrino Dirac}}$ is the Dirac mass matrix in neutrino sector.

The model here is accompanied by an additional $U(1)_X$ symmetry which prevents the existence of Yukawa interactions of the form $\bar{Y_{iL}}\nu_{iR}\tilde{\phi_i}$ and $\bar{Y_{iL}}\nu_{iR}\tilde{\phi_0}$ as $Y_{iL}, l_{iR}, \tilde{\phi_0}$ have quantum numbers $X=1$ and all other fields have quantum numbers $X=0$. This phenomenology disfavours Nambu Goldstone boson to arise in this case as $U(1)_X$ symmetry does not break spontaneously but explicitly. Thus the Yukawa Lagrangian for the leptonic sector are of the form as described by Eq. (3), (4), (5), (6) under the symmetry $G_{SM}\times A_4\times U(1)_X$.

Some studies on Cosmological Domain Wall Problem are done in [45], where it is shown that if a discrete symmetry is embedded with a continuous gauge or global gauge group, (in this case only $U(1)_X$), then on account of the phenomenon *Lazarides-Shafi mechanism* the electroweak phase transition of the apparent discrete symmetry A_4 (which is a subgroup of the centre of the continuous lie group), results in only a network of domain walls bounded by strings to form and then quickly collapse.

Here a symmetry of the form $U(1)_X$ exists. Under this symmetry Y_{iL}, l_{iR} and ϕ_i have quantum numbers $X=1$ and all other fields have $X=0$. This symmetry does not permit the terms like, $\bar{Y}_L \nu_R \tilde{\phi_i}$ which are invariant under $G_{SM} \times A_4$ and contributes to Dirac mass matrix for neutrinos. Spontaneous symmetry breaking leads to the following Vacuum expectation values for scalars, $\upsilon_1, \upsilon_2, \upsilon_3$ for $\phi_i^{\prime s}$, u_i for $F_i^{\prime s}$, υ_0 for ϕ_0. Let, $\upsilon_1, \upsilon_2, \upsilon_3 = \upsilon$. Here $Y_{iL} = (\nu_i, l_i) \sim \underline{3}$, $l_{iR} \sim \underline{1}, \underline{1}', \underline{1}''$, $\phi_i = (\phi_i^0, \phi_i^-) \sim \underline{3}$, $(i = 1, 2, 3)$. Along with these vacuum expectation values, the superpotential for different mass terms are:

$$-\bar{l_L} M_l^0 l_R - \bar{\nu_L} M_D \nu_R + \frac{1}{2}\nu_R^T C^{-1} M_R \nu_R + h.c$$

where,

$$M_l^0 = \begin{bmatrix} h_1\upsilon_1 & h_2\upsilon_1 & h_3\upsilon_1 \\ h_1\upsilon_2 & h_2\omega^2\upsilon_2 & h_3\omega\upsilon_2 \\ h_1\upsilon_3 & h_2\omega\upsilon_3 & h_3\omega^2\upsilon_3 \end{bmatrix}, \quad M_R = \begin{bmatrix} M & h_s u_3 & h_s u_2 \\ h_s u_3 & M & h_s u_1 \\ h_s u_2 & h_s u_1 & M \end{bmatrix} \tag{7}$$

and $M_D = h_0 \upsilon_0 I$. A special vacuum alignment is needed in tribimaximal mixing, which is given by

$$\upsilon_1 = \upsilon_2 = \upsilon_3 = \upsilon, \quad u_1 = u_3 = 0, \quad \text{and} \quad h_F u_2 = M'. \tag{8}$$

The charged lepton mass matrix M_l^0 is now put in a diagonal form by the transformation,

$$M_l^{0d} = U_\omega M_l^0 I \tag{9}$$

where M_l^{0d} is the diagonal form of M_l^0. Here

$$U_\omega^{CW} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix} \quad \text{and} \quad \omega = exp(\frac{i2\pi}{3}) = -\frac{1}{2} + i\frac{\sqrt{3}}{2}. \tag{10}$$

Now,

$$\begin{aligned} M_l^{0d} &= \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix} \begin{bmatrix} h_1\upsilon & h_2\upsilon & h_3\upsilon \\ h_1\upsilon & h_2\omega^2\upsilon & h_3\omega\upsilon \\ h_1\upsilon & h_2\omega\upsilon & h_3\omega^2\upsilon \end{bmatrix} \\ &= \frac{1}{\sqrt{3}} \begin{bmatrix} 3h_1\upsilon & 0 & 0 \\ 0 & 3h_2\upsilon & 0 \\ 0 & 0 & 3h_3\upsilon \end{bmatrix} = \begin{bmatrix} \sqrt{3}h_1\upsilon & 0 & 0 \\ 0 & \sqrt{3}h_2\upsilon & 0 \\ 0 & 0 & \sqrt{3}h_3\upsilon \end{bmatrix} \end{aligned} \tag{11}$$

Or,

$$M_l^{0d} = \begin{bmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{bmatrix}. \tag{12}$$

M_R is diagonalised by the orthogonal transformation,

$$\begin{aligned} U_\nu M_R U_\nu^\dagger &= \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} M & 0 & M' \\ 0 & M & 0 \\ M' & 0 & M \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix} \\ &= \begin{bmatrix} M - M' & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M + M' \end{bmatrix}. \end{aligned} \tag{13}$$

The choice of the vacuum alignment for scalar fields is to break A_4 spontaneously along two incompatible directions. (111) with residual symmetry Z_3 and (100) with residual symmetry Z_2. The vacuum alignment breaks A_4 in charged lepton sector coupling only with ϕ_i to Z_3 group. Also the vacuum alignment breaks A_4 in neutrino sector coupling only with ϕ_0 and F, the residual symmetry is Z_2 group. The PMNS matrix, tribimaximal with phases is

$$\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega \end{bmatrix}\begin{bmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -i \end{bmatrix}. \tag{14}$$

3. Perturbations in neutrino sector

In this section, we consider the effect of perturbations to mass matrices due to higher order corrections in the form of $Z_2 \times Z_2$ invariant perturbations. In the model discussed till now, the PMNS matrix has the tribimaximal form with zero θ_{13} and zero δ_{CP} phase. To generate non-zero values for these, small perturbation in the form of $Z_2 \times Z_2$ symmetry is added to our model. We first instigate a symmetry breaking term in the charged lepton sector which is invariant under the symmetry $Z_2 \times Z_2$, which is a normal subgroup of A_4 with four elements. The three non trivial singlet representation of $Z_2 \times Z_2$ are $\widehat{\underline{1}}'''(1,1,-1,-1)$, $\widehat{\underline{1}}''(1,-1,1,-1)$, $\widehat{\underline{1}}'(1,-1,-1,1)$, whereas the one trivial singlet representation has the form $\widehat{\underline{1}}(1,1,1,1)$. The breaking of A_4 triplet into $Z_2 \times Z_2$ irreducible representations is given as

$$(\underline{3}) \text{ of } A_4 \longrightarrow (\widehat{\underline{1}}' \oplus \widehat{\underline{1}}'' \oplus \widehat{\underline{1}}''') \text{ of } Z_2 \times Z_2, \tag{15}$$

$$(\underline{1}'', \underline{1}', \underline{1}) \text{ of } A_4 \longrightarrow (\widehat{\underline{1}}) \text{ of } Z_2 \times Z_2. \tag{16}$$

If we want to break A_4 into $Z_2 \times Z_2$ irreducible representations, it could be breaking of A_4 triplet of right handed neutrino singlets into $\widehat{\underline{1}}(1,1,1,1)$ trivial representations of $Z_2 \times Z_2$.

The general $Z_2 \times Z_2$ invariant perturbations are of the form

$$h_1 \bar{Y}_L M_1 \phi l_{1R} + h_2 \bar{Y}_L M_2 \phi l_{2R} + h_3 \bar{Y}_L M_3 \phi l_{3R} \tag{17}$$

where, $\bar{Y}_L$, ϕ are the three dimensional reducible representations of $Z_2 \times Z_2$ and l_R's are the trivial singlets. Since we have considered here $Z_2 \times Z_2$ invariant perturbations, then the matrices M_1, M_2, M_3 must commute with the matrices in Eq. (A.11) in Appendix A. The U_{e3} element of the PMNS matrix in its TBM form is zero because the 11 and 13 elements of U_ω are same. The perturbation terms in Eq. (17) can disturb the balance between the 11 and 13 elements of U_ω and this phenomenology leads to non-zero θ_{13}. The value of θ_{13} relates to the elements of the mass matrices, M_1, M_2, M_3. We prefer the form of M_i's as $M_i = diag(\bar{z}, 0, \omega^{i-1}z)$ to generate simple form of perturbed charged lepton mass matrices M_l. z is a complex number and $|z| < 1$. After spontaneous symmetry breaking the resulting $M_l = M_l^0 + \delta M_l$ where, M_l^0 has the form in Eq. (7), and δM_l has the form

$$\delta M_l = \begin{bmatrix} h_1 \upsilon \bar{z} & h_2 \upsilon \bar{z} & h_3 \upsilon \bar{z} \\ 0 & 0 & 0 \\ h_1 \upsilon z & h_2 \upsilon z \omega & h_3 \upsilon z \omega^2 \end{bmatrix}. \tag{18}$$

δM_l arises from the higher order effects of the theory. We parameterise all the higher order perturbations in the terms of the complex number z [15]. There is no residual symmetry remaining in the charged lepton sector after the spontaneous symmetry breaking. Constraining U_ω to be unitary, we limit z as

$$z = -1 \pm \sqrt{1 - S^2} + iS. \tag{19}$$

For $z < 1$ one gets perturbation to be of the order S. Using the parametrisation $S = Sin\,\alpha$, U_ω to

$$U_\omega = \frac{1}{\sqrt{3}} \begin{bmatrix} e^{i\alpha} & 1 & e^{-i\alpha} \\ e^{i\alpha} & \omega & \omega^2 e^{-i\alpha} \\ e^{i\alpha} & \omega^2 & \omega e^{-i\alpha} \end{bmatrix}, \tag{20}$$

we introduce a $Z_2 \times Z_2$ invariant perturbations in the neutrino sector, and study its influence on θ_{13} and δ_{CP}. The perturbing matrix is diagonal since it should satisfy $Z_2 \times Z_2$ symmetry. We chose the perturbation [46] to be as follows:

$$M \nu_R^T C^{-1} \begin{bmatrix} \frac{1}{\rho} e^- i\varphi & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\rho} e^- i\varphi \end{bmatrix} \nu_R$$

where, $\frac{1}{\rho} e^- i\varphi$ characterises the soft breaking of A_4. M is A_4 invariant soft term in the Lagrangian. The perturbing term is A_4 breaking but $Z_2 \times Z_2$ invariant soft term in the Lagrangian. The perturbed matrix is now

$$\begin{bmatrix} M + \frac{1}{\rho} e^- i\varphi M & 0 & M' \\ 0 & M & 0 \\ M' & 0 & M - \frac{1}{\rho} e^- i\varphi M \end{bmatrix}. \tag{21}$$

We can diagonalise it by rotation angle x, where,

$$Tan\,2x = \frac{M'}{\frac{1}{\rho} e^- i\varphi M}. \tag{22}$$

Thus, we see that the ratio, $\frac{M'}{M}$ is a physical observable in the rotation angle, $Tan\,2x$ which helps us to diagonalise the perturbing matrix Eq. (21). The introduction of the perturbing terms like $\frac{1}{\rho} e^- i\varphi$ and the ratio, $\frac{M'}{M}$ has helped us to derive non zero θ_{13} and other neutrino oscillation parameters in terms of x, α, ρ, Eq. (24)-(37). The input range of Majorana phases used in our calculation is from 0 to 2π. The heavy right handed Majorana neutrino used here for the computation of various LFV decay rates like $\mu \to e\gamma$, $\tau \to \mu\gamma$, $\tau \to e\gamma$ is 10^{15} GeV.

The most interesting feature of our work is that, from Eq. (24)-(37) we can extract meaningful extract of current pattern of neutrino flavour mixing in the sense that the favoured value of δ_{CP} phase from our results attached with non-zero θ_{13} can induce signatures of various decay rates of charged lepton flavour violation processes like $\mu \to e\gamma$, $\tau \to \mu\gamma$, $\tau \to e\gamma$ after spontaneous symmetry breaking of our model $G_{SM} \times A_4 \times U(1)_X$ incorporating $Z_2 \times Z_2$ invariant perturbations into account. The sleptons and gauginos so constrained are shown in Figs. 11–14. The prospect to test these sparticles at future run of LHC will favour or rule out our model.

After introducing $Z_2 \times Z_2$ perturbations in both charged leptonic sector and neutrino sector [46–48], and setting U_ω as defined by Eq. (20), PMNS matrix after perturbation becomes

$$U_{PMNS} = \frac{1}{\sqrt{3}} \begin{bmatrix} e^{i\alpha} & 1 & e^{-i\alpha} \\ e^{i\alpha} & \omega & \omega^2 e^{-i\alpha} \\ e^{i\alpha} & \omega^2 & \omega e^{-i\alpha} \end{bmatrix} \begin{bmatrix} Cos\,x & 0 & -Sin\,x \\ 0 & 1 & 0 \\ Sin\,x & 0 & Cos\,x \end{bmatrix}. \tag{23}$$

From above it is seen that, after computation matching with the actual PMNS matrix one gets,

$$\begin{aligned} \mathrm{Sin}\,\theta_{13} e^{-i\delta_{CP}} &= \frac{1}{\sqrt{3}}\left(e^{-i\alpha}\mathrm{Cosx} - e^{i\alpha}\,\mathrm{Sinx}\right) \\ &= \frac{1}{\sqrt{3}}\left(\mathrm{Cos}\,\alpha\,\mathrm{Cosx} - \mathrm{Cos}\,\alpha \mathrm{Sinx} - i\,\mathrm{Sin}\,\alpha \mathrm{Cosx} - i \mathrm{Sin}\,\alpha\,\mathrm{Sinx}\right). \end{aligned} \tag{24}$$

Therefore,

$$\mathrm{Sin}\,\theta_{13} Cos\delta_{CP} = \frac{1}{\sqrt{3}}\left(\mathrm{Cos}\,\alpha\,\mathrm{Cosx} - \mathrm{Cos}\,\alpha\,\mathrm{Sinx}\right), \tag{25}$$

and

$$\mathrm{Sin}\,\theta_{13} Sin\,\delta_{CP} = \frac{1}{\sqrt{3}}\left(\mathrm{Sin}\,\alpha \mathrm{Cosx} + \mathrm{Sin}\,\alpha\,\mathrm{Sinx}\right). \tag{26}$$

Squaring and adding Eq. (25) and Eq. (26) we get

$$\begin{aligned} & Sin^2\theta_{13}(Cos^2\delta_{CP} + Sin^2\delta_{CP}) \\ &= \frac{1}{(\sqrt{3})^2}\left\{(Cos\,\alpha\,Cos\,x - Cos\,\alpha\,Sin\,x)^2 + (Sin\,\alpha\,Cos\,x + Sin\,\alpha\,Cos\,x)^2\right\} \\ &= \frac{1}{3}\Big\{Cos^2\alpha\,Cos^2x + Cos^2\alpha\,Sin^2x - 2Cos^2\alpha Cos\,x\,Sin\,x + Sin^2\alpha\,Cos^2x \\ &\quad + Sin^2\alpha\,Sin^2x + 2Sin^2\alpha Cos\,x\,Sin\,x\Big\} \\ &= \frac{1}{3}\{1 - Cos\,2\alpha\,Sin\,2x\} = \frac{1}{3}\left\{1 - Sin\,2x(1 - 2Sin^2\alpha)\right\} = \frac{1}{3}\left\{1 - Sin\,2x(1 - 2S^2)\right\} \\ &= \frac{1}{3}\left\{1 - \frac{1}{\sqrt{1 + Cot^2 2x}}(1 - 2S^2)\right\} = \frac{1}{3}\left\{1 - \frac{1}{\sqrt{1 + \kappa^2}}(1 - 2S^2)\right\} \\ &= \frac{1}{3}\left\{1 - (1 + \frac{-1}{2}\kappa^2)(1 - 2S^2)\right\} = \frac{\kappa^2}{6} + \frac{2}{3}S^2 - \frac{\kappa^2 S^2}{3}, \end{aligned} \tag{27}$$

or

$$Sin^2\theta_{13} = \frac{1}{3}\left(1 - Cos\,2\alpha\,Sin\,2x\right) = \frac{\kappa^2}{6} + \frac{2}{3}S^2 - \frac{\kappa^2 S^2}{3}, \tag{28}$$

where perturbations in neutrino sector is defined by

$$\kappa = \frac{1}{\rho} e^{-} i\varphi \frac{M}{M'} = Cot2x, \tag{29}$$

and $S = Sin\,\alpha$. Therefore, in terms of soft breaking parameters, one gets

$$Sin^2\theta_{13} = \frac{1}{6\rho^2} e^{-} 2i\varphi \frac{M^2}{M'^2} + \frac{2}{3} Sin^2\alpha - \frac{1}{3\rho^2} e^{-} 2i\varphi \frac{M^{2\,2}}{M'^2}\,Sin^2\alpha. \tag{30}$$

Similarly one has,

$$Sin^2\theta_{12}\,Cos^2\theta_{13} = \frac{1}{3}, \tag{31}$$

and also

$$Cos^2\,\theta_{13} = \frac{1}{3}(2 + Sin\,2x\,Cos\,2\alpha). \tag{32}$$

Thus,

$$Sin^2\,\theta_{12} = \frac{1}{2 + Sin\,2x\,Cos\,2\alpha}. \tag{33}$$

Inflating the above expression for $Sin^2\,\theta_{12}$ upto the order κ^2 and S^2, one gets

$$\begin{aligned} Sin^2\,\theta_{12} &= \frac{1}{3} + \frac{2}{9}S^2 + \frac{\kappa^2}{18} - \frac{\kappa^2 S^2}{27} \\ &= \frac{1}{3} + \frac{2}{9}Sin^2\alpha + \frac{1}{18\rho^2}e^-2i\varphi\frac{M^2}{M'^2} - \frac{1}{27\rho^2}e^-2i\varphi\frac{M^2}{M'^2}Sin^2\alpha. \end{aligned} \tag{34}$$

Similarly we get in terms of soft breaking parameters, after computation matching with the actual PMNS matrix,

$$\begin{aligned} Sin^2\,\theta_{23} &= \frac{\sqrt{3}\,Sin\,2x\,Sin\,2\alpha + 2 + Sin\,2x\,Cos2\alpha}{4 + Sin\,2x\,Cos\,2\alpha} \\ &= 0.5 + \frac{Sin\,\alpha}{\sqrt{3}} - \frac{1}{3\sqrt{3}\rho^2}e^-2i\varphi\frac{M^2}{M'^2}Sin\,\alpha. \end{aligned} \tag{35}$$

Finally, we have

$$\begin{aligned} &Cos\,\delta_{CP} \\ &= \sqrt{1 - \frac{Cos^2\,2x\,(2 + Cos\,2\alpha\,Sin\,2x)^2}{\left(1 - Cos^2\,2\alpha\,Sin^2\,2x\right)\left[4 + 4Cos2\alpha\,Sin\,2x + (-1 + 2Cos4\alpha)Sin^2 2x\right]}} \end{aligned} \tag{36}$$

or

$$\begin{aligned} Cos\,\delta_{CP} &= \sqrt{1 - \frac{(-\kappa)^2}{4\,Sin^2\,\alpha + \kappa^2 - 16\frac{\kappa^2 Sin^2\,\alpha}{3}}} \\ &= \sqrt{1 - \frac{\frac{1}{\rho^2}e^-2i\varphi\frac{M^2}{M'^2}^2}{4Sin^2\,\alpha + \frac{1}{\rho^2}e^-2i\varphi\frac{M^2}{M'^2}^2 - 16\frac{\frac{1}{\rho^2}e^-2i\varphi\frac{M^2}{M'^2}^2 Sin^2\,\alpha}{3}}}, \end{aligned} \tag{37}$$

keeping the leading powers in numerators and denominator. For no perturbation in neutrino sector, the value of δ_{CP} becomes zero as κ tends to zero. Owing to perturbation inhibition only in neutrino sector, one sets $S = 0$ or $\delta_{CP} = \pm\frac{\pi}{2}$. In conformity with T2k data from the ν_e appearance data, the value of δ_{CP} is favoured to be in the lower half plane. In the wake of perturbations only in the neutrino sector, one gets

$$Sin\,\delta_{CP} = -Sin\frac{\pi}{2}. \tag{38}$$

This implies $S \longrightarrow 0$ or κ is positive and perturbations in charged leptonic sector is imperceptible.

As we work in the type I seesaw framework, the heavy Majorana neutrino mass scale M is much higher than the electroweak scale and the light neutrinos are simply given by the well-known effective mass matrix

$$m_\nu = -M_D M_R^{-1} m_D^T, \tag{39}$$

where M_D is the Dirac − type neutrino mass matrix in the weak basis. The charged lepton masses are real and diagonal.

$$M_D = \upsilon_0 U_\omega^\dagger Y_\nu, \tag{40}$$

where υ_0 denotes the vacuum expectation value of the usual SM Higgs doublet, $\langle\phi_0\rangle = \upsilon_0$. We thus compute the form of Y_ν, the Dirac neutrino yukawa couplings (DNY) from $M_D = h_0\upsilon_0 I$ and Eq. (40), after spontaneous breaking of A_4 symmetry incorporating $Z_2 \times Z_2$ invariant perturbations in the neutrino sector and thus generating non zero θ_{13} and δ_{CP}.

$$Y_\nu = h_0 |U_\omega^\dagger|^{-1}. \tag{41}$$

Here h_0 is the arbitrary coupling constant giving the lepton masses. Taking h_0 of the order of 0.08 we can construct a user defined neutrino Yukawa coupling Y_ν corresponding to favoured value of α from our results.

Charged lepton flavour violation (CLFV) and hence neutrino oscillations and mixings are real phenomenon. In terms of low energy observables, the lepton flavour violating entries in the SO(10) SUSY GUT framework can be expressed as

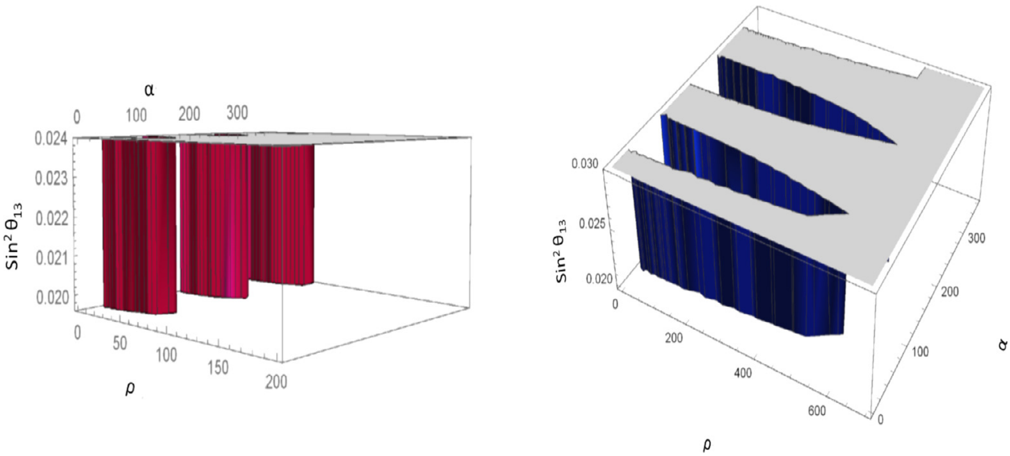
$$\left(m_{\tilde{L}}^2\right)_{i\neq j} = \frac{-3m_o^2 + A_o^2}{8\pi^2} \sum_k \left(Y_\nu^\star\right)_{ik} \left(Y_\nu\right)_{jk} log\left(\frac{M_X}{M_{R_k}}\right), \tag{42}$$

here M_X is the GUT scale, M_{R_k} is the scale of the k^{th} heavy RH majorana neutrino, m_0 and A_0 are universal soft mass and trilinear terms at the high scale. Y_ν are the Dirac neutrino Yukawa couplings. The flavor violating off-diagonal entries at the weak scale are found by using Y_ν. The branching ratio of a charged lepton flavour violating decay [1] $l_i \to l_j$ is

$$\mathrm{BR}\left(l_i \to l_j + \gamma\right) \approx \alpha^3 \frac{|\delta_{ij}^{LL}|^2}{G_F^2 M_{SUSY}^4} Tan^2\beta\, \mathrm{BR}\left(l_i \to l_j \nu_i \tilde{\nu_j}\right). \tag{43}$$

The most interesting feature of this work is that we predicted form of Dirac neutrino Yukawa coupling Y_ν corresponding to favoured value of $\alpha \sim 60^\circ$ which could realise signatures of rare CLFV decays like $\mu \to e\gamma$, $\tau \to \mu\gamma$, $\tau \to e\gamma$ after spontaneous symmetry breaking of our model $G_{SM} \times A_4 \times U(1)_X$ incorporating $Z_2 \times Z_2$ invariant perturbations into account. In this context we used the value of Higgs mass as measured at LHC, latest global data on the reactor mixing angle θ_{13} for neutrinos, and latest constraints on BR($\mu \to e\gamma$) as projected by MEG at PSI and MEG II PSI [43] planning to achieve sensitivity to BR($\mu \to e\gamma$) $\sim 10^{-14}$. Beyond Standard Model Physics that could yield CLFV embraces SUSY sparticles, Z' vector bosons with flavour non diagonal couplings which is an indication of lepton flavour violation.

Signatures of CLFV could be tested by the late 2020s at next run of High − Luminosity LHC, if SUSY sparticles are observed within few TeV range, as discussed in detail below. It is worth mentioning here that, during last run of LHC, no SUSY partner of SM has been observed, and this could point to a high scale SUSY theory. Split SUSY offers a dark matter candidate and unifies the fundamental forces at high energies, it doesn't address the stability of Higgs boson. To show that the model predicts the neutrino mixing angles compatible with the observed data, we obtain the allowed parameter space for the correction terms in the perturbation matrix in the form of $Z_2 \times Z_2$ invariant symmetry compatible with the 3σ range of the observed ν oscillation data by varying the parameters. Considering, definite values for $\frac{M'}{M} = 10^{-2}, 10^{-3}$ and setting forth



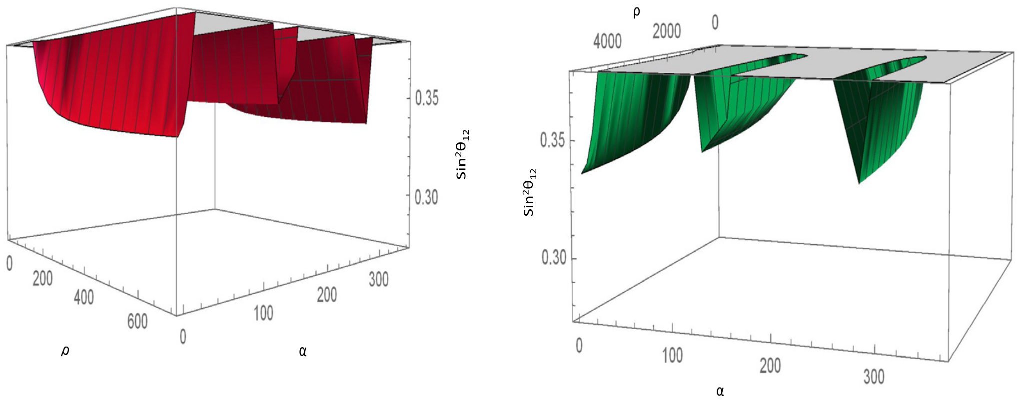
(a) (b)

Fig. 1. In Fig. 1(a) the points in $\rho-\alpha$ space which satisfy the 3σ constraints on $Sin^2\theta_{13}$ for $\frac{M'}{M}=10^{-2}$ are presented. Fig. 1(b) shows the points in $Sin^2\theta_{13}-\rho-\alpha$ space corresponding to the 3σ bounds on $Sin^2\theta_{13}$ for $\frac{M'}{M}=10^{-3}$.

the values for soft breaking phase as $\varphi=0,\pm\frac{\pi}{2},\pi$, we generate the soft breaking parameter space for ρ and α from the 3σ constraints on mixing angles, $Sin^2\theta_{12}$, $Sin^2\theta_{23}$, $Sin^2\theta_{13}$, and CP violating Phase, δ_{CP}. As can be seen from Eq. (30) that to generate non-zero values of θ_{13}, soft breaking phase φ should take the values as $0,\pm\frac{\pi}{2},\pi$ as φ is present in the form of $\frac{1}{6\rho^2}e^-2i\varphi$ in the expression of θ_{13}. $e^-2i\varphi$ will take real values only for $\varphi=0,\pm\frac{\pi}{2},\pi$, since, other values of φ, such as $\pm\frac{\pi}{4}$ will generate imaginary values of θ_{13} in the form of $i\,Sin2\varphi$ which is absurd. Similarly Eq. (34), Eq. (35) and Eq. (37) will take real favoured values of mixing angles θ_{12}, θ_{23} and CP violating phase δ_{CP} respectively for $\varphi=0,\pm\frac{\pi}{2},\pi$.

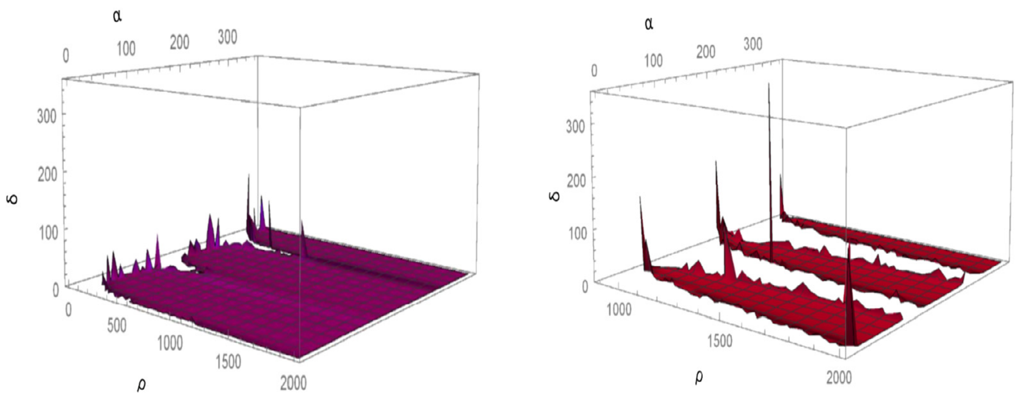
Owing to the Eq. (37) one finds that the value of δ_{CP} depends on the comparative supremacy between the parameters $Sin\,\alpha$ and κ or ρ. This obsession between the parameters $Sin\,\alpha$ and κ or ρ in procuring the mixing angles and CP violation phase, δ_{CP} in terms of Eqs. (28), (29), (30), (34), (35), (37) by virtue of perturbations in neutrino sector and charged leptonic sector has been plotted in Figs. 1–10.

In Fig. 1 the predicted dependence of $Sin^2\theta_{13}$ on the soft breaking parameter, ρ and α is shown. Owing to the constrained nature of the mixing angle, $Sin^2\theta_{13}$ varying within its 3σ range indicated by current neutrino oscillation global fit [49], one finds the correlation between mixing angle, $Sin^2\theta_{13}$, and ρ, α breaking parameter space corresponding to $\frac{M'}{M}=10^{-2}$ and $\frac{M'}{M}=10^{-3}$ in the left and right panel respectively. Owing to $Sin\,\alpha$ and $Z_2\times Z_2$ perturbations in the neutrino sector the allowed range of ρ space lies in the range $[0,50]$ and $[0,560]$ for $\frac{M'}{M}=10^{-2}$ and $\frac{M'}{M}=10^{-3}$ respectively. Fig. 2, shows the variation of $Sin^2\theta_{12}$ within its 3 σ range with respect to the soft breaking parameter, ρ and α. The curtailment of the mixing angle, $Sin^2\theta_{12}$ differing within its 3σ range allowed by the current neutrino oscillation global fit, one finds the whole experimentally allowed range of ρ, α breaking parameter space corresponding to $\frac{M'}{M}=10^{-2}$ and $\frac{M'}{M}=10^{-3}$ in the left and right panel respectively. On account of $Sin\,\alpha$ and $Z_2\times Z_2$ perturbations in the neutrino sector the predicted range of ρ space lies in the range $[0,600]$ and $[1500,4500]$ for $\frac{M'}{M}=10^{-2}$ and $\frac{M'}{M}=10^{-3}$ respectively. From Fig. 1, one finds that the value of $Sin^2\theta_{13}=0.0216$ is near $\rho=560$. For this value of ρ, the change in $Sin^2\theta_{12}$



(a) (b)

Fig. 2. In Fig. 2(a) the allowed range of $\rho - \alpha$ space which satisfy the 3σ constraints on $Sin^2\theta_{12}$ for $\frac{M'}{M} = 10^{-2}$ are shown. Fig. 2(a) shows the correlation between ρ and α space corresponding to the 3σ bounds on $Sin^2\theta_{12}$ for $\frac{M'}{M} = 10^{-3}$.



(a) (b)

Fig. 3. In Fig. 3(a), the values of δ_{CP} within its 3σ bounds phase for different regions in α space for $\frac{M'}{M} = 10^{-2}$ are shown. Fig. 3(b) shows the values of δ_{CP} within its 3σ bounds as indicated by current neutrino oscillation global fit [49] for different regions in α space for $\frac{M'}{M} = 10^{-3}$.

is comparatively small $(\sim 6)\%$. Thus, one finds that the value of ρ is comparatively large but the parameter $\rho e^{-}i\varphi = \kappa \frac{M'}{M}$ appraising the perturbation in the neutrino sector is utterly small because $\frac{M'}{M} << 1$ and also owing to the contributions from $\frac{1}{\rho}$ factor. In the limiting case, $Sin\alpha \rightarrow 0$, the value of $Sin^2\theta_{23}$ becomes $\frac{1}{2}$.

The value of δ_{CP} depends on the relative predominance between the parameters $Sin\alpha$ and κ or ρ. This dependance is plotted in Figs. 3, 5. From the right panel in Fig. 3 the results of our present analysis suggests δ_{CP} violation phase to be around 144°, corresponding to $\frac{M'}{M} = 10^{-3}$ with $\alpha \sim 60°$. The analysis of NOνA results shows a preference for $\delta_{CP} \sim 0.8\pi$ suggests our present analysis of δ_{CP} phase $\sim 144°$ exactly coincides with the preferred value. The separate analysis

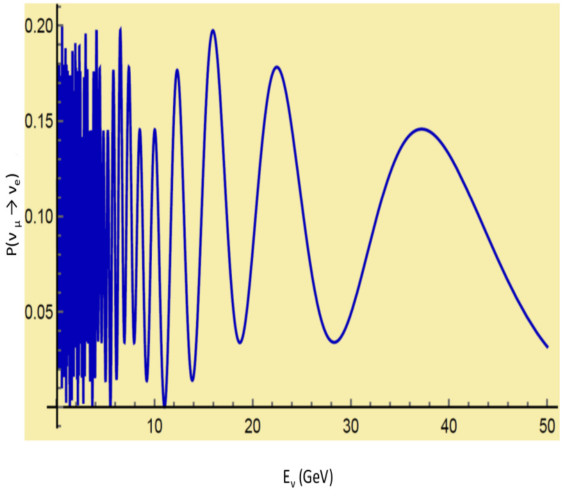


Fig. 4. In Fig. 4 the allowed range of electron neutrino appearance probability at T2K which covers a more restricted region is depicted. The blue region is our model prediction.

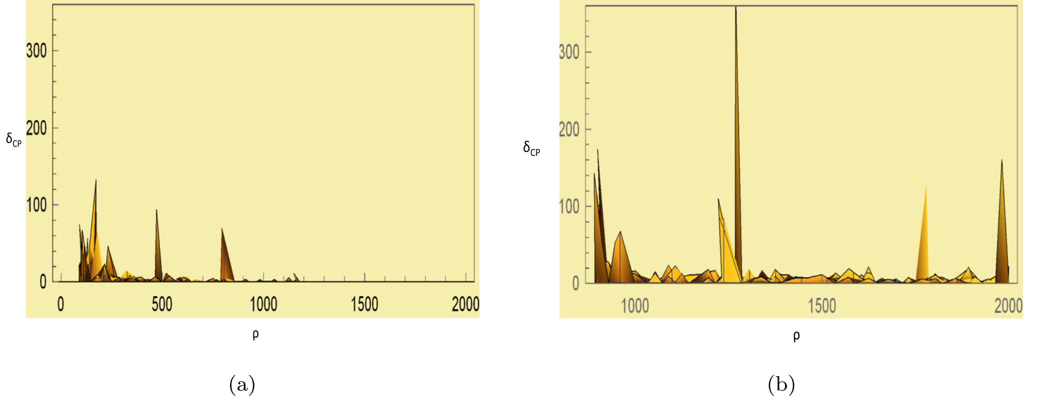
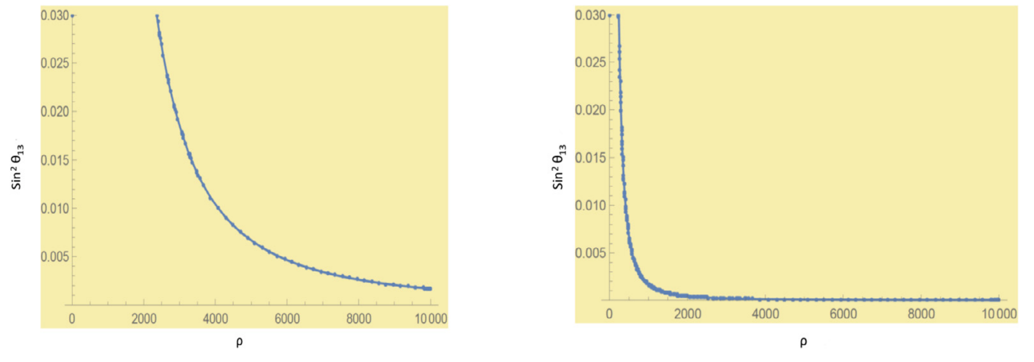


Fig. 5. Fig. 5(a) presents the values of δ_{CP} within its 3σ bounds phase for different regions in ρ space for $\frac{M'}{M} = 10^{-2}$. In Fig. 5(b) the values of δ_{CP} within its 3σ bounds as indicated by current neutrino oscillation global fit [49], for different regions in ρ space for $\frac{M'}{M} = 10^{-3}$ are illustrated.

of neutrino and antineutrino channels can not provide, at present, a sensitive measurement of δ_{CP} phase. The CPV phase can therefore be measured by the long-baseline accelerator experiments T2K and NOνA, and also by Super-Kamiokande atmospheric neutrino data. Similarly, for $\frac{M'}{M} = 10^{-2}$, we obtain the best fit value of $\delta_{CP} \sim 0.8\pi$ in our present analysis corresponding to $\alpha \sim 310°$. The predictions made in this analysis can also be tested in currently running and upcoming neutrino oscillation experiments. The predictions made by our model to electron neutrino appearance probability oscillation experiments are displayed in Fig. 4. This conjecture is for the T2K setup, neglecting matter effects, as an approximation. Clearly, the allowed range of electron neutrino appearance probability at T2K is significantly constrained with respect to the generic expectation. One finds from the right panel in Fig. 5, that NOνA preference of $\delta_{CP} \sim 0.8\pi$ propounds the parameter of perturbation in neutrino sector $\frac{1}{\rho}$ to be around 5×10^{-4} and 2×10^{-3}.



(a) (b)

Fig. 6. The plot of sine squared values of mixing angles for maximal δ_{CP} through a $Z_2 \times Z_2$ invariant perturbations in neutrino sector are presented in Fig. 6(a) and Fig. 6(b) which corresponds to $\frac{M'}{M} = 10^{-2}$ and $\frac{M'}{M} = 10^{-3}$ respectively.

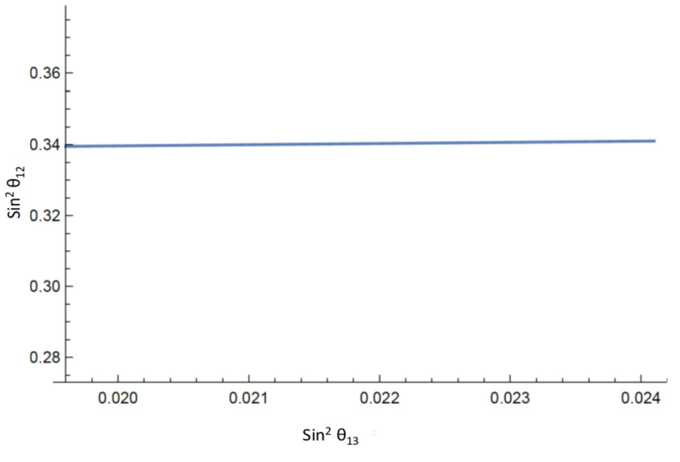
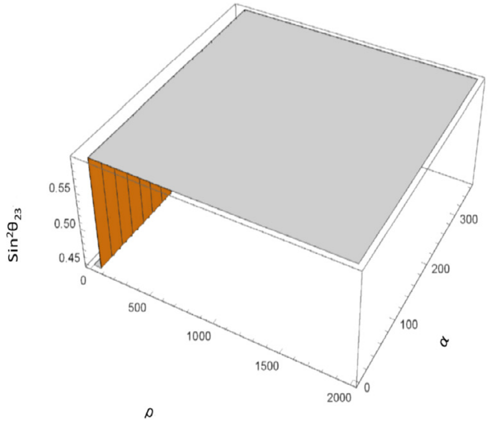


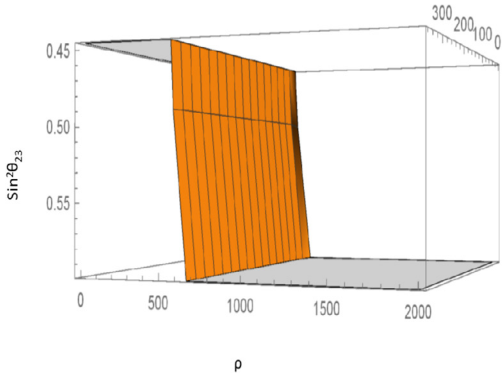
Fig. 7. In Fig. 7 the plot of sine squared values of mixing angles for maximal δ_{CP} through a $Z_2 \times Z_2$ invariant perturbations in neutrino sector is presented.

From the left panel it is found that preference of $\delta_{CP} \sim 0.8\pi$ constrains the parameter of perturbation in neutrino sector $\frac{1}{\rho}$ to limit itself around 4 $\times 10^{-43}$. The inclusion of reactor data can help to improve the determination of δ_{CP} phase, owing to the existing correlation between the CP phase and θ_{13}. From the results presented in this work, we obtain the best fit value for the CP phase at $\delta_{CP} \sim 0.8\pi$ for NO. The CP conserving value $\delta = 0$ is favoured with $\frac{1}{\rho}$ in the region, 1×10^{-3} to 5×10^{-4} in NO as can be seen from the left panel in Fig. 6.

Next, we discuss the perturbations in determining the atmospheric mixing angle, $Sin^2\theta_{23}$. Accelerator and atmospheric oscillation experiments estimate the disappearance of muon (anti)neutrinos and are mostly sensitive to $Sin^2 2\theta_{23}$. Thus, one cannot resolve the octant [50–53] of the angle. In other words, one cannot decide whether $Sin^2 2\theta_{23} > 0.5$ or $Sin^2 2\theta_{23} < 0.5$. Nonetheless, on account of matter effects in the neutrino trajectories inside the Earth, this degeneracy is slightly broken for atmospheric neutrino oscillation experiment. The quantity $Sin^2\theta_{23}$ finds itself in the expressions for appearance channels of these probability experiments. Examining the data



(a)



(b)

Fig. 8. Fig. 8(a) demonstrates the region in $\rho - \alpha$ space which is consistent with the 3σ constraints on $Sin^2\theta_{23}$ for $\frac{M'}{M} = 10^{-2}$. Fig. 8(b) illustrates the region in $Sin^2\theta_{23} - \rho$ space corresponding to the 3σ bounds on $Sin^2\theta_{23}$ for $\frac{M'}{M} = 10^{-3}$.

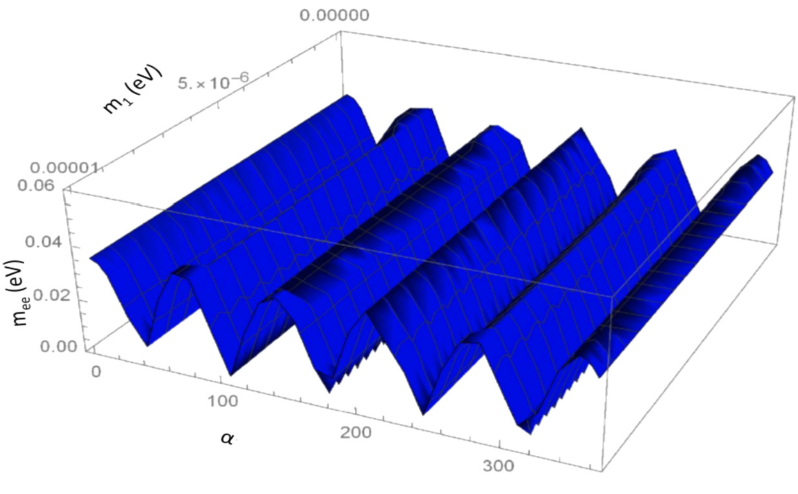


Fig. 9. $|m_{ee}|$ prediction for lightest neutrino mass m_1 (eV) and α space in our model.

from long-baseline accelerators, one finds two essentially degenerate solutions for $Sin^2 2\theta_{23}$ for both mass orderings, The best fit is obtained for $Sin^2\theta_{23} = 0.46$, and a local minimum appears at $Sin^2 2\theta_{23} = 0.57$ with $\Delta\chi^2 = 0.3(0.7)$ for normal mass (inverted mass) ordering. In the present analysis the best fit experimental value of $Sin^2\theta_{23} = 0.57$ is 8.59% larger than the TBM value of 0.5. To procure this deviation, one needs $\frac{1}{\rho}$ to be 0.0111 and $\frac{1}{\rho}$ to be 1.54×10^{-3} for $\frac{M'}{M} = 10^{-2}$ and $\frac{M'}{M} = 10^{-3}$ respectively as seen from Fig. 8. The above values of $\frac{1}{\rho}$ lead to small values of κ and hence of δ_{CP} phase. Thus there is a strain in obtaining large values of δ_{CP} phase and best fit experimental value of $Sin^2\theta_{23}$. For $\kappa > 2Sin\,\alpha$ we have large CPV phase, and that keeps the value of $Sin^2\theta_{23}$ close to the TBM value of 0.5 as is evident from Eq. (35).

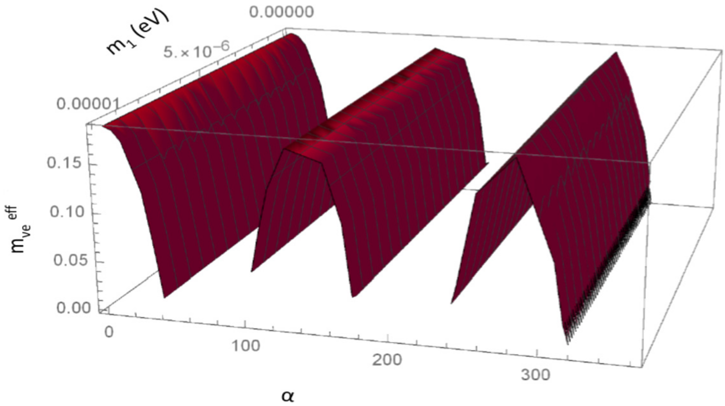


Fig. 10. $|m^{eff}_{\nu_e}|$ prediction for lightest neutrino mass m_1 (eV) and α space in our model.

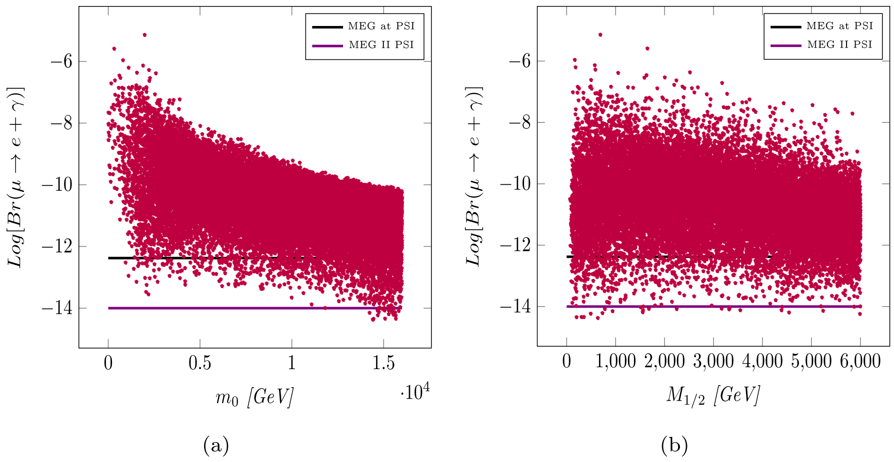


Fig. 11. In Figs. 11a, 11b, different horizontal lines black and violet represent the present MEG bound at PSI and future MEG II PSI bounds for BR($\mu \to e + \gamma$) respectively.

Owing to the constrained limited nature of the mixing angles and perturbing factors like $\frac{1}{\rho}$ and α in our model, one also gets predictions for $|m_{ee}|$ and $|m^{eff}_{\nu_e}|$, as shown in Fig. 9 and Fig. 10 respectively. To test various CLFV processes, like $\mu \to e\gamma$, $\tau \to e\gamma$, $\tau \to \mu\gamma$, the parameters we use in our model are scalar masses m_0, trilinear coupling, A_0 and unified gaugino mass $M_{1/2}$. There is also the Higgs potential parameter μ and the undetermined ratio of the Higgs VEVs, Tanβ. The present MEG bound at PSI i.e., $< 4.2 \times 10^{-13}$ together with a non zero values of θ_{13} [49] puts notable constraints on SUSY parameter space. As can be seen from Fig. 11a, only small part of the heavy m_0 space around 15000 GeV survives for Tan $\beta = 10$ in our model restricted by future MEG II bound at PSI for BR($\mu \to e\gamma$) $\sim 10^{-14}$. Fig. 11b reveals that the parameter space $M_{1/2} \geq 1$ TeV is allowed by present MEG PSI bounds on BR($\mu \to e\gamma$), while future MEG limit BR($\mu \to e\gamma$)$\sim 10^{-14}$ excludes almost whole of $M_{1/2}$ space. Very few points around ∼1–2 TeV are favourable. In Fig. 12a, 12b, we show the correlation among the branching

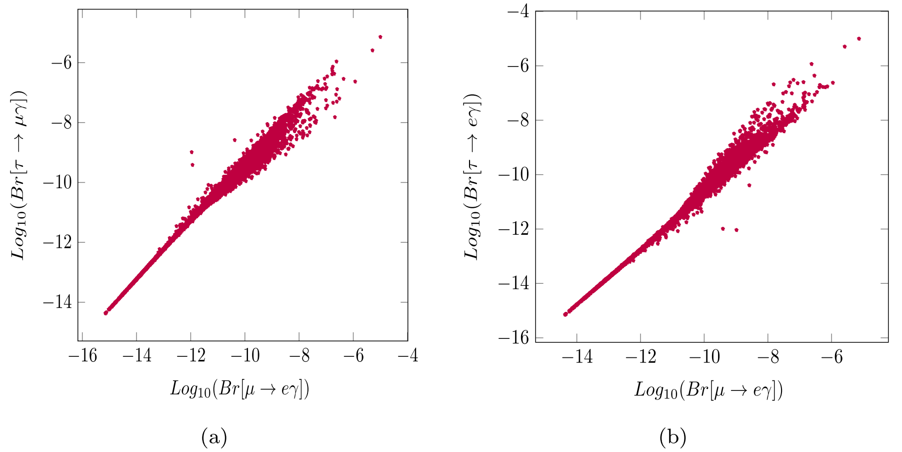


Fig. 12. In Figs. 12a, 12b correlations between different LFV decays, BR($\tau \to \mu + \gamma$) versus BR($\mu \to e + \gamma$) and BR($\tau \to e + \gamma$) versus BR($\mu \to e + \gamma$) are shown respectively.

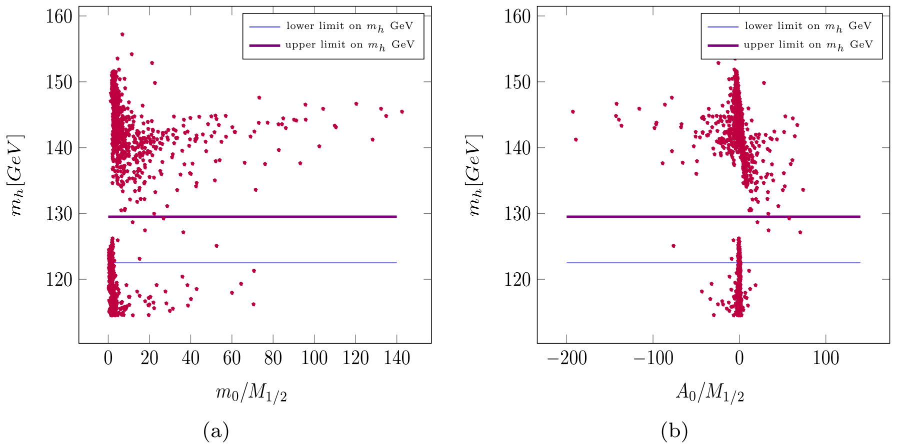


Fig. 13. In Figs. 13a, 13b we have shown feasible SUSY parameters space allowed by present bound at MEG at PSI. Different horizontal lines denote the range of Higgs mass as given by the data at LHC, i.e. 122.5 GeV $\leq m_h \leq$ 129.5 GeV [1].

ratios of BR($\tau \to \mu + \gamma$) versus BR($\mu \to e + \gamma$) and BR($\tau \to e + \gamma$) versus BR($\mu \to e + \gamma$) respectively.

The current upper limit on BR($\mu \to e + \gamma$) implies an upper limit BR($\tau \to \mu + \gamma$) $\sim 10^{-13}$ which is notably smaller than the sensitivity of current generation experiments. Thus, any signatures of CLFV decay $\tau \to \mu + \gamma$ may rule out the present favoured value of $\alpha \sim 60^\circ$. In Fig. 13a, 13b we plot the lightest Higgs mass m_h as a function of $m_0/M_{1/2}$, $A_0/M_{1/2}$ respectively. For the allowed range of Higgs mass as given by the data at LHC, i.e. 122.5 GeV $\leq m_h \leq$ 129.5 GeV, $m_0/M_{1/2}$ should be around 5 as allowed by present MEG PSI bounds on BR($\mu \to e\gamma$).

In Fig. 14a, for the allowed range of Higgs mass, i.e. 122.5 GeV $\leq m_h \leq$ 129.5 GeV, A_0/m_0 should be around -1 to $+1$. Asymmetry in the value of A_0, can be seen in Fig. 14a. The space $M_{1/2} \geq 4$ TeV is allowed as can be seen from Fig. 14b. In Fig. 15a we have presented results

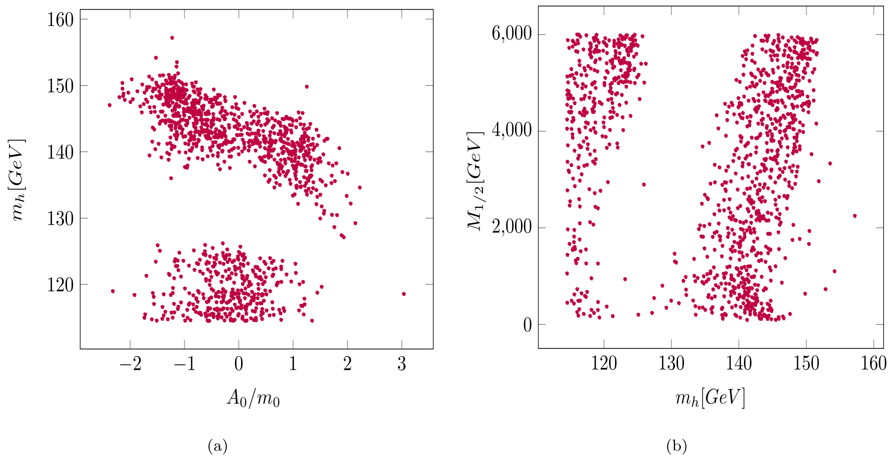


Fig. 14. In Figs. 14a, 14b we have shown feasible SUSY parameters space allowed by present bound at MEG at PSI. Different horizontal lines denote the range of Higgs mass as given by the data at LHC, i.e. 122.5 GeV $\leq m_h \leq$ 129.5 GeV [1].

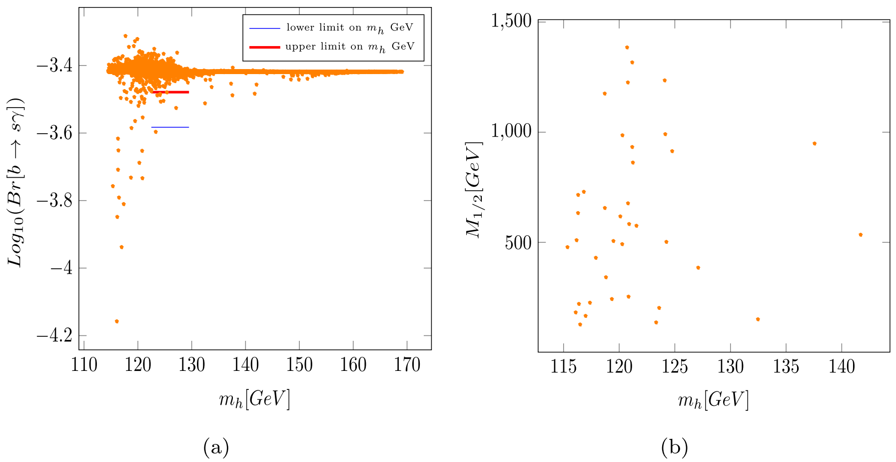


Fig. 15. In Fig. 15a, different horizontal lines represent the present and future bounds for hadronic flavour violation BR($b \to s + \gamma$). In Fig. 15b we have shown SUSY parameters space allowed by present bounds on BR($b \to s + \gamma$).

for the decay $b \to s\gamma$. In Fig. 15b we have shown SUSY parameters space allowed by present bounds on BR($b \to s + \gamma$).

4. Conclusion

We consider a model based on A_4 symmetry which gives the corrections to the TBM form for the leading order neutrino mixing matrix. We present here the phenomenology of a model with A_4 symmetry which envisage the tribimaximal form for the PMNS matrix. In this model, we have instigated a $Z_2 \times Z_2$ invariant perturbations in both the charged lepton in the form of $Sin\,\alpha$ and the neutrino sectors in the form of κ. We perceive that perturbations in the neutrino sector leads to allowable values of non zero θ_{13} varying within its 3σ range as indicated by current neutrino oscillation global fit [49] and maximal CP violation for $Sin\alpha = 0$. The desired value of the CP violating phase δ_{CP} lying within its 3σ range can be procured by choosing the fitting and pertinent values for $Sin\alpha$ term and for the perturbations in neutrino sectors. However, there is a strain in obtaining large values of δ_{CP} phase and best fit experimental value of $Sin^2\theta_{23}$. For $\kappa > 2Sin\alpha$ we have large CPV phase, and that keeps the value of $Sin^2\theta_{23}$ close to the TBM value of 0.5. Also, our analysis of δ_{CP} phase $\sim 144°$ in this model exactly coincides with the preferred value of $\delta_{CP} \sim 0.8\pi$ by the analysis of NoνA results [42].

We have considered leading order corrections in the form of $Z_2 \times Z_2$ invariant perturbations in neutrino sector after spontaneous breaking of A_4 symmetry. The neutrino mixing angles, thus obtained are found to be within the 3σ ranges of their experimental values. The CP violating phase δ_{CP} is around $\sim 144°$ in this model. We also studied the variation of the neutrino oscillation probability $P(\nu_\mu \rightarrow \nu_e)$, the effective Majorana mass $|m_{ee}|$ and $|m_{\nu e}^{eff}|$ with the lightest neutrino mass m_1 in the case of normal hierarchy and found its value to be lower than the experimental upper limit for all allowed values of $m_1 \in [0 \text{ eV}, 10^{-5} \text{ eV}]$.

We show that our predicted value of $\delta_{CP} \sim 144°$ corresponding to $\frac{M'}{M} = 10^{-3}$ and $\alpha = 60°$ indicates signatures of various charged LFV channels in a class of $G_{SM} \times A_4 \times U(1)_X$ model incorporating $Z_2 \times Z_2$ invariant perturbations in charged lepton and neutrino sector, which is the most interesting feature of our work. These ratios depend on the form of Dirac neutrino yukawa couplings as fixed by Dirac CP phase and on the details of soft SUSY breaking parameters and Tanβ. We have used the Higgs mass as measured at LHC, non zero reactor mixing angle θ_{13} for neutrinos, and latest present and future constraints on BR($\mu \rightarrow e\gamma$). We find that very heavy m_0 region is allowed by future MEG bound of BR($\mu \rightarrow e\gamma$) $\sim 10^{-14}$ and $M_{1/2}$ values greater than 1 TeV is allowed. We have also shown the predictions for neutrinoless double beta decay in terms of lightest neutrino mass for a mass range of $m_1 \in [0 \text{ eV}, 10^{-5} \text{ eV}]$, which we epitomize in Fig. 9, 10. Ultimately we have estimated the parameters of our model where the branching ratios of $\mu \rightarrow e\gamma$ of the order of $\sim 10^{-14}$ are calculated, which is well within the latest experimental constraints and is summarised in Fig. 11.

To conclude, we have proposed a pragmatic generalisation of the TBM ansatz, which in addition to explaining nonzero θ_{13}, also makes exact and certain testable predictions for the other parameters of the lepton mixing matrix, including CP violating and CP conserving phases. The $Z_2 \times Z_2$ invariant perturbations in neutrino sector are characterised in terms of three independent parameters, $\frac{1}{\rho}$, α and φ which determine all three mixing angles and CP violating phase, leading to several testable predictions. A more comprehensive version of the generalised CP methodology and its potential to produce other hypothetically and realistic ansatz forms for the lepton mixing matrix will be presented in our future work.

CRediT authorship contribution statement

Gayatri Ghosh: Conceptualization, Data curation, Formal analysis, Funding acquisition, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Visualization, Writing – original draft, Writing – review & editing.

Declaration of competing interest

The author declares that she has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Basics of A_4 group

A_4 is the smallest non Abelian group with an irreducible triplet representation. Alternating group A_4 is a group of even permutations of four objects, with a three one dimensional irreducible representation which makes it one of the most favoured group in neutrino mass models.

A_4 group is a non − Abelian, and it is not a direct product of cyclic groups. Group A_4 has twelve elements and it is isomorphic to tetrahedral T_d.

A_4 group consists of twelve elements which are written in terms of generators of the group $\boldsymbol{S}$ and T, the generators satisfy the relation,

$$\boldsymbol{S}^2 = (\boldsymbol{S}T)^3 = T^3 = 1. \tag{A.1}$$

There are three one dimensional irreducible representations of the A_4 group defined as

$$\begin{aligned} &1, \quad \boldsymbol{S} = 1, \quad T = 1 \\ &1', \quad \boldsymbol{S} = 1, \quad T = \omega^2 \\ &1'', \quad \boldsymbol{S} = 1, \quad T = \omega. \end{aligned}$$

The three dimensional unitary representations of T and S are,

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{bmatrix}, \tag{A.2}$$

and

$$\boldsymbol{S} = \frac{1}{3} \begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}. \tag{A.3}$$

The multiplication rules for the singlet and triplet representations of two generators $\boldsymbol{S}$ and T of A_4 are,

$$\begin{aligned} &1 \otimes 1 = 1, \quad 1'' \otimes 1'' = 1' \\ &1' \otimes 1'' = 1, \quad 1' \otimes 1' = 1'' \\ &3 \otimes 3 = 3_1 + 3_2 + 1 + 1' + 1''. \end{aligned}$$

A_4 is a symmetry group of tetrahedron. There are twelve independent transformations of the tetrahedron and hence there are twelve group elements as follows:

a. four rotations by 120° clockwise (as seen from the vertex) which are T-type,
b. four rotations by 120° anticlockwise (as seen from the vertex),
c. three rotations by 180° – $\boldsymbol{S}$ type,
d. 1 unit operator 1.

A_4 has four irreducible representations which are three singlets -1, $1'$, $1''$ and one triplet. The products of singlets are:

$$1 \otimes 1 = 1, \quad 1'' \otimes 1'' = 1'$$
$$1' \otimes 1'' = 1, \quad 1' \otimes 1' = 1''.$$

If we consider two triplets $a = (a_1, a_2, a_3)$, $b = (b_1, b_2, b_3)$ then one can write,

$$(ab)_1 = a_1 b_1 + a_2 b_2 + a_3 b_3, \tag{A.4}$$
$$(ab)_{1'} = a_1 b_1 + \omega^2 a_2 b_2 + \omega a_3 b_3, \tag{A.5}$$
$$(ab)_{1''} = a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3, \tag{A.6}$$
$$(ab)_{3_1} = a_2 b_3 + a_3 b_1 + a_1 b_2, \tag{A.7}$$
$$(ab)_{3_2} = a_3 b_2 + a_1 b_3 + a_2 b_1, \tag{A.8}$$
$$\omega^3 = 1. \tag{A.9}$$

In the basis of triplet representations

$$\boldsymbol{S} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \tag{A.10}$$
$$T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}. \tag{A.11}$$

One generates 12 real 3×3 matrix group elements, after multiplication of the matrices together in all possible ways, like

$$\boldsymbol{S}^2 = \frac{1}{9}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{A.12}$$

A_4 has four classes [12] denoted by,

$$a) \quad C_1 : \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{A.13}$$

which is the 3×3 matrix representation of A_4 elements in C_1. Likewise we have,

$$b) \quad C_2 : \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{A.14}$$
$$c) \quad C_2 : \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}. \tag{A.15}$$
$$d) \quad C_2 : \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \tag{A.16}$$

where Z_3 elements are

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

and $Z_2 \times Z_2$ elements in this classification are

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad \text{(A.17)}$$

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