Dark radiation and the Hagedorn phase

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(Received 22 August 2021; accepted 28 February 2022; published 18 March 2022)

We point out that if the sector associated with the Standard Model degrees of freedom entered an open string Hagedorn phase in the early universe while the dark radiation sector was not part of this plasma, then this can lead to low values of the observable ΔN_{eff} (effective number of additional neutrinolike species). For explicit analysis, we focus on warped string compactifications with the Standard Model degrees of freedom at the bottom of a warped throat. If the Hubble scale during inflation is above the warped string scale associated with the throat, then the Standard Model sector will enter the Hagedorn phase. In this scenario, bulk axions are no longer dangerous from the point of view of dark radiation. While this article focuses on warped compactifications, the basic idea can be relevant to any scenario where the early universe entered a Hagedorn phase.

DOI: 10.1103/PhysRevD.105.066007

I. INTRODUCTION

The hot big bang model is highly successful in addressing most cosmological observations. Yet, some interesting puzzles remain. One of these is the low value of ΔN_{eff}—the energy density of new light degrees of freedom at the time of neutrino decoupling measured as an effective number of additional neutrinolike species[1]—i.e., the absence of dark radiation [2]. Theoretically, no reasons for a low value of ΔN_{eff} have been found; phenomenologically interesting models with new light degrees of freedom including axions and dark photons can generate non-negligible ΔN_{eff}. Furthermore, the anthropic arguments for a small value are not very strong (for a recent discussion see [3]). The purpose of this paper is to point out that if the visible sector of our universe entered an open string Hagedorn phase [4–6] after reheating, while the dark radiation was not part of the thermal soup, this can provide a mechanism of low ΔN_{eff}. Our analysis will be in the context of warped string compactifications [7], specifically in the setting of [8], although the basic features of the arguments could well be relevant for cosmologies where the early universe went through a Hagedorn phase.

Apart from containing the basic ingredients of the Standard Model (SM) along with gravity, string compactifications typically predict various additional species [9]. Many of these particles can be light. These light degrees of freedom often reside in hidden gauge sectors which are required for consistency of the compactification (see, e.g., [10–13]). Generically, one also expects a multitude of axions with a wide range of masses and decay constants [14] (see, e.g., [15–18] for explicit statistical analysis). Again, a large number of these can be light. As we describe below, these light degrees of freedom can lead to large value of ΔN_{eff}.

ΔN_{eff} is sensitive to all (relativistic) species that contribute to the energy density at the time of neutrino decoupling. It is independent of how such matter couples to the Standard Model.[2] The ratio of the energy density in dark radiation to ρ_{vis}, the energy density in the SM, at the time of neutrino decoupling gives ΔN_{eff}:

$$\Delta N_{\mathrm{eff}} = \frac{43}{7}\frac{\rho_{\mathrm{dr}}(t_\nu)}{\rho_{\mathrm{vis}}(t_\nu)}. \tag{1}$$

In scenarios where the early universe undergoes an inflationary epoch and the constituents of the universe are produced from the decay of the inflaton, one can obtain a rather simple formula relating ΔN_{eff} to the branching ratios of the inflaton decay process[3] [19,20]

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[2]This makes N_{eff} a powerful probe of additional hidden species.

[3]This assumes that the energy density in the dark radiation scales as $a^{-4}(t)$ throughout its entire evolution; we will also discuss generalizations to cases when this is not true.

$$\Delta N_{\rm eff} = \frac{43}{7}\frac{B_{\rm dr}}{B_{\rm vis}}\left(\frac{g(T_\nu)}{g(T_{\rm rh})}\right)^{1/3}, \qquad (2)$$

where $B_{\rm dr}$ is the branching ratio for decay of the inflaton to dark radiation, $B_{\rm vis}$ is the branching ratio to the visible sector, $g(T_\nu)$ and $g(T_{\rm rh})$ the g-factors at the time of neutrino decoupling and the reheating epoch. A similar formula also holds in the case that the universe undergoes an epoch of reheating as a result of vacuum misalignment of moduli fields [21], with the branching ratios replaced by those associated with the decay of the light modulus (although the discussion in the present article will not be relevant for this case). Note that if the ratio of the branching ratios is even of the order of one over five, the prediction for $\Delta N_{\rm eff}$ is order one (if the ratio of the g-factors is order one); conversely, if the ratio of branching ratios is order one, limiting to $\Delta N_{\rm eff} \lesssim 1$ requires $g(T_{\rm rh}) \sim 2500$, far above the high temperature SM value $g \sim 107$. As emphasized in [22], as the topology of the compactification manifold becomes rich, one can expect the number of light degrees of freedom to become large, so correspondingly $B_{\rm dr} \sim B_{\rm vis}$ (or greater), which results in a large value of $\Delta N_{\rm eff}$. On the other hand, the result of observations is very different. The latest results from the Planck collaboration [2] give $\Delta N_{\rm eff} < 0.3$, and big bang nucleosynthesis studies give similar constraints [23]. This tension has been termed as the "dark radiation problem in string theory." Various explicit studies have confirmed this tension and proposed possible resolutions [19,20,22,24,25], although it is fair to say that there is still no satisfactory solution. Furthermore, CMB stage 4 experiments [26] should be able to probe $\Delta N_{\rm eff} \approx 0.03$. Thus developing an understanding of scenarios with low values is of importance from the point for view of future observations.

As mentioned earlier, the goal of this paper is to revisit the problem in a setting where the SM sector goes through a Hagedorn phase in the early universe. We will do this in the context of warped compactifications making use of the setting in [8]. Here, the SM is supported on branes localized at the tip of a warped throat (or other highly warped region), and the warp factor provides a large hierarchy through the Randall-Sundrum mechanism [27]. This warps the 4D effective string length at the tip of a warped throat to the SM scale $\ell_{\rm SM} \sim 1/M_{\rm SM}$ as opposed to the 10D string length $\sqrt{\alpha'}$ (note, though, that $M_{\rm SM}$ may not be the electroweak scale if warping does not provide the full SM hierarchy). However, during inflation, the large Hubble scale $H \gg M_{\rm SM}$ means that the 4D effective theory would break down due to unsuppressed string corrections in the warped SM throat *if the SM throat remained in its vacuum state during inflation.* Instead, [8] argued that the SM throat modulus is far from its true vacuum value during inflation due to its coupling to the inflationary sector, leading to $\ell_{\rm SM} \sim 1/H$ during inflation.[4] During reheating, the modulus relaxes to its vacuum state, and the warped string scale relaxes to the SM scale with $\ell_{\rm SM} \sim 1/H(t)$ and 4D effective field theory is just valid throughout.

However, the total energy density available to the SM sector during reheating is $\rho_{\rm vis} \sim B_{\rm vis} M_P^2 H^2$ (where M_P is the reduced Planck mass); a naive estimate in field theory assuming rapid thermalization then gives a reheat temperature of order $\sqrt{H M_P} \gg H \sim 1/\ell_{\rm SM}$, so the reheating process must actually involve strings in the SM throat in a fundamental way. Assuming that the SM and other sectors exchange energy slowly compared to the SM sector thermalization time, [8] gave an argument based on entropy considerations, which we review below, that the SM throat reheats to a gas of long open strings extending along the SM branes and penetrating about a 10D string length into the warped transverse dimensions. This Hagedorn gas has a very large effective number of degrees of freedom $g(T_{\rm rh})$ and subsequently decays at equilibrium primarily by emitting SM radiation from the string endpoints [8].

We will find that this high value of $g(T_{\rm rh})$ can lead to lower values of $\Delta N_{\rm eff}$. The key point is that Eq. (2) implies $\Delta N_{\rm eff}$ decreases with an increase in $g(T_{\rm rh})$. Our analysis will show that this can significantly suppress the contributions to $\Delta N_{\rm eff}$ from bulk light degrees of freedom (light degrees of freedom whose wave function is supported in the bulk of the compactification) and also from any sector where the effective number of degrees of freedom does not undergo a significant change during the history of the universe. Highly warped throats (other than that containing the SM) with light degrees of freedom can be potentially dangerous. However, the problem can be ameliorated if the number of D-branes in such throats is small or if the wave function of the inflation has small support in such throats. Before closing our introductory remarks, we note that highly warped regions are expected to be generic in string compactifications [29]. Thus, the article also takes an important step toward developing a full understanding of the nature of predictions for dark radiation in string compactifications.

II. THE HAGEDORN PHASE OF OPEN STRINGS

We suppose that the SM is supported on a set of D-branes in a highly warped throat. The prototypical examples consist of either D7-branes embedded in or D3-branes at the tip of a warped conifold throat in a conformally Calabi-Yau manifold (with some additional structure such as an orbifold to generate the SM gauge group), but details are unimportant for our general argument. After inflation, the SM sector reheats to an energy density of about $\rho_{\rm vis} \lesssim M_p^2 H^2$, which is considerably above the effective string scale. Following [8],

[4] See also [28] for related discussions.

we argue that this high energy density leads to a Hagedorn gas phase of long open strings at the end of reheating.[5] Our specific formulae below assume that the SM is supported on D3-branes at the tip of a warped throat, but generalizations to other configurations are straightforward.

Consider a pure gas of open strings stretching between D3-branes. The entropy of the open string gas is [4–6]

$$S_o(E) = \beta_H E + \sqrt{\frac{8N_D^2 V_\parallel E}{m\mu^2 V_\perp}}, \tag{3}$$

where $\mu = 1/2\pi\alpha'$ is the string tension, m is an order one constant given by the probabilities for splitting and joining of long strings, $\beta_H = 2\pi\sqrt{2\alpha'}$ is the inverse Hagedorn temperature,[6] N_D is the number of D-branes, and $V_\parallel, V_\perp$ are respectively the volumes along and transverse to the D-branes. For comparison, the entropy of a black 3-brane (assuming for simplicity a brane charge $\sim N_D$) is $S_{bb} = A\sqrt{N_D}V^{1/4}E^{3/4}$, where A is a constant of order unity. Therefore, at energy densities $\rho_{\rm vis} \gtrsim N_D^2\mu^2$, the Hagedorn phase dominates the microcanonical ensemble. It is also important to note that the energy density of the open strings is much greater than that in the closed strings in the Hagedorn phase [4].

The above comparison of entropies is of course appropriate for the microcanonical ensemble. With the SM localized in a warped throat region, energy exchange between the SM and other sectors is slow due to the strong warping, so energy deposited into the SM sector from the inflaton is approximately conserved, meaning the SM thermalizes within the microcanonical ensemble. That said, the open string Hagedorn gas is also favored in the canonical ensemble at temperatures near but below the Hagedorn temperature.

Now we can compute the inverse temperature

$$\beta\left(=\frac{1}{T}\right) \equiv \frac{\partial S_o}{\partial E} \Rightarrow \beta = \beta_H + \sqrt{\frac{2N_D^2}{m\mu^2 V_\perp \rho}}, \tag{4}$$

where $\rho = E/V_\parallel$ is the energy density along the D-branes. The volume transverse to the branes is not the full compactification volume; rather, the warp factor acts as a potential preventing the strings from climbing the warped throat. Approximating this potential as a worldsheet mass term for transverse oscillations, [8] estimated

$$V_\perp = v(4\pi^2\alpha')^3. \tag{5}$$

In principle, the order 1 constant v is calculable with a rigorous derivation of string thermodynamics in warping, but it likely varies somewhat with the background, so it parametrizes our ignorance of the precise compactification. We note that, while we have derived (3) and (4) in 10D units, the open strings are all localized at the tip of the warped throat, so conversion to 4D units is simple scaling by the warp factor at the tip. In the following, we therefore replace $\alpha' \to \ell_{\rm SM}^2$, where $\ell_{\rm SM}$ is the warped string scale in the SM sector, and use 4D units throughout.

Therefore, we have

$$\beta = \beta_H + \sqrt{\frac{N_D^2}{8\pi^4 m v \ell_{\rm SM}^2 \rho}} \Rightarrow \rho = \frac{N_D^2}{8\pi^4 m v \ell_{\rm SM}^2 (\beta - \beta_H)^2}. \tag{6}$$

Defining $g_E(T)$ as usual by $\rho \equiv \pi^2 g_E(T)T^4/30$, we find

$$g_E(T) = \frac{15N_D^2}{4\pi^6 m v \ell_{\rm SM}^2}\frac{\beta^4}{(\beta-\beta_H)^2} = \frac{15N_D^2}{4\pi^6 m v}\frac{1/(T^2\ell_{\rm SM}^2)}{(1-T/T_H)^2}. \tag{7}$$

Near the Hagedorn temperature,

$$g_E(T) \approx \frac{15N_D^2}{4\pi^6 m v}\frac{1/(T_H^2\ell_{\rm SM}^2)}{(1-T/T_H)^2} = \frac{30N_D^2}{\pi^4 m v}\frac{1}{(1-T/T_H)^2}. \tag{8}$$

The effective number of degrees of freedom can therefore be very large near the Hagedorn temperature.

Of course, we may also measure degrees of freedom via the entropy. At high energy densities, the entropy density is

$$s_o \equiv \frac{S_o}{V_\parallel} \approx \beta_H \rho = \frac{\pi^2}{30} g_E(T)\beta_H T^4. \tag{9}$$

We define $g_s(T)$ by

$$s_o \equiv \frac{2\pi^2}{45} g_s(T) T^3, \tag{10}$$

so $g_s(T) = 3g_E(T)/4$ near the Hagedorn temperature.

We can also estimate reasonable values of the reheating temperature. Parametrize the energy density of inflation as $\rho_{\rm inf} = aM_G^4$ in terms of the unification scale $M_G \sim 2\times 10^{16}$ GeV, where a is an order 1 (or smaller for lower-scale inflation) numerical constant, since inflation is often taken to occur at this scale, which is also a reasonable proxy for the compactification and string scales and therefore likely a maximum value for the scale of inflation. Following the arguments of [8] that the modulus of the SM throat relaxes in such a way that effective field

[5]At high enough energy density, there could be a gas of black holes (as opposed to black branes) and strings, which has a lower temperature and larger number of effective degrees of freedom [6]. Our estimate is therefore conservative, and the highest temperature phase is still a Hagedorn gas.

[6]Note that the ratio $\beta_H/\sqrt{\alpha'}$ differs for different string theories; this is the value for type II strings [30,31].

theory is just valid through reheating, the Hubble parameter is $H = fT_H$ for $f \lesssim 1$. Since the energy density in the SM sector at reheating is $\rho^{\rm SM} = 3B_{\rm vis}M_P^2H^2 \approx \pi^2 g_E T_H^4/30$, where $B_{\rm vis} \lesssim 1$, we find

$$\left(1-\frac{T_{\rm rh}}{T_H}\right)^2 \approx \frac{1}{3\pi^2}\frac{N_D^2}{B_{\rm vis}mvf^4}\frac{H^2}{M_P^2}. \tag{11}$$

But $\rho^{\rm SM} \lesssim \rho_{\rm inf}$, so $H^2 \lesssim aM_G^4/3M_P^2$, meaning

$$\left(1-\frac{T_{\rm rh}}{T_H}\right)^2 \lesssim \frac{1}{9\pi^2}\frac{aN_D^2}{B_{\rm vis}mvf^4}\frac{M_G^4}{M_P^4} \Rightarrow$$
$$1-\frac{T_{\rm rh}}{T_H} \lesssim (7\times 10^{-6})\frac{N_D}{f^2}\sqrt{\frac{a}{B_{\rm vis}mv}}. \tag{12}$$

With $N_D = 10$ for a typical D-brane embedding of the SM sector into our string compactification, $f = 1/10$, and $a = B_{\rm vis} = m = v = 1$, $T_{\rm rh}/T_H \gtrsim 0.993$. For a lower scale of inflation ($a \ll 1$), this ratio is even larger.

III. LOW $\Delta N_{\rm eff}$ FROM A HAGEDORN PHASE

A. The simplest setting

Assuming that entropy is conserved for the visible sector[7] between the time of reheating and the decoupling of neutrinos, we have

$$\left(\frac{T_\nu}{T_{\rm rh}}\right)^3 = \frac{a^3(t_{\rm rh})g_s(T_{\rm rh})}{a^3(t_\nu)g_s(T_\nu)}, \tag{13}$$

independent of the equation of state (g_s as well as g_E below refer to the visible sector). Therefore, for the visible sector

$$\rho_{\rm vis}(t_\nu) = \rho_{\rm vis}^{\rm rh}\frac{g_E(T_\nu)T_\nu^4}{g_E(T_{\rm rh})T_{\rm rh}^4} = \rho_{\rm vis}^{\rm rh}\frac{g_E(T_\nu)}{g_E(T_{\rm rh})}\frac{a^4(t_{\rm rh})g_s^{\frac{4}{3}}(T_{\rm rh})}{a^4(t_\nu)g_s^{\frac{4}{3}}(T_\nu)}. \tag{14}$$

In the simplest models, dark radiation is highly noninteracting and the associated effective number of degrees of freedom do not change throughout the history of the universe.[8] The energy density falls as $a^{-4}(t)$ throughout the history of the universe, or

$$\rho_{\rm dr}(t_\nu) = \rho_{\rm dr}^{\rm rh}\frac{a^4(t_{\rm rh})}{a^4(t_\nu)}. \tag{15}$$

Along the lines of [19,25], $\Delta N_{\rm eff}$ from (1) becomes

$$\Delta N_{\rm eff} = \frac{43}{7}\frac{\rho_{\rm dr}^{\rm rh}}{\rho_{\rm vis}^{\rm rh}}\frac{g_E(T_{\rm rh})}{g_E(T_\nu)}\frac{g_s^{\frac{4}{3}}(T_\nu)}{g_s^{\frac{4}{3}}(T_{\rm rh})} = \frac{43}{7}\frac{B_{\rm dr}}{B_{\rm vis}}g_E^{\frac{1}{3}}(T_\nu)\frac{g_E(T_{\rm rh})}{g_s^{\frac{4}{3}}(T_{\rm rh})}, \tag{16}$$

where we use $B_{\rm dr}/B_{\rm vis} = \rho_{\rm dr}^{\rm rh}/\rho_{\rm vis}^{\rm rh}$ (by definition), and $g_E = g_s$ at the time of neutrino decoupling. If $g_E = g_s$ also at reheating, we find (2). On the other hand, with $T \to T_H$ in a Hagedorn phase, $g_s \approx \frac{3}{4}g_E$, so we can write

$$\Delta N_{\rm eff} \approx \frac{43}{7}\frac{B_{\rm dr}}{B_{\rm vis}}\left(\frac{4}{3}\right)^{4/3}\left(\frac{g_E(T_\nu)}{g_E(T_{\rm rh})}\right)^{1/3}. \tag{17}$$

With $T_{\rm rh}$ as in Eq. (12), the parameters allow for significant suppression through the ratio of g-factors; even large branching ratios $B_{\rm dr}/B_{\rm vis}$ can be accommodated with intermediate or low-scale inflation, so the tension with present observations can be addressed. Specifically, Eq. (17) is consistent with a given constraint $\Delta N_{\rm eff} \lesssim \overline{\Delta N}$ for

$$1-\frac{T_{\rm rh}}{T_H} \lesssim 6\times 10^{-3}N_D\sqrt{\frac{\overline{\Delta N}^3(B_{\rm vis}/B_{\rm dr})^3}{mv}}. \tag{18}$$

This is similar to the estimated reheating temperature (12) for $\overline{\Delta N} \sim 0.3$ and a ratio of branching ratios near unity even for high-scale inflation.

B. Beyond the simplest setting

1. Entropy production

A key input for the above calculation was entropy conservation in the visible sector from the time that the universe was at temperature $T_{\rm rh}$ to T_ν. Another possibility is that there *are* entropy generating processes in the visible sector in this period (particularly early). If entropy is produced then Eq. (13) becomes

$$a^3(t_\nu)g_s(T_\nu)T_\nu^3 = (1+\eta)a^3(t_{\rm rh})g_s(T_{\rm rh})T_{\rm rh}^3, \tag{19}$$

where $\eta > 0$, i.e.,

$$\left(\frac{T_\nu}{T_{\rm rh}}\right)^3 = \frac{(1+\eta)a^3(t_{\rm rh})g_s(T_{\rm rh})}{a^3(t_\nu)g_s(T_\nu)}. \tag{20}$$

Note that the effect of entropy production is thus captured by defining an effective $g_s^{\rm eff}(T_{\rm rh}) \equiv (1+\eta)g_s(T_{\rm rh})$. Now making use of this in (16), in the case there is entropy production, we obtain the analog of (17) to be

$$\Delta N_{\rm eff} \approx \frac{43}{7}\frac{B_{\rm dr}}{B_{\rm vis}}\left(\frac{4/3}{1+\eta}\right)^{4/3}\left(\frac{g_E(T_\nu)}{g_E(T_{\rm rh})}\right)^{1/3}. \tag{21}$$

Since $\eta > 0$, the effect of entropy production is to decrease $\Delta N_{\rm eff}$, i.e., it helps in the issue we want to address.

[7] We consider entropy conservation separately for visible and dark radiation sectors.

[8] This is certainly valid for very weakly interacting bulk axions.

We discuss entropy production in the dark radiation sector below.

2. Nontrivial dark radiation sectors

Let us consider the case that the dark radiation sector has nontrivial dynamics. As a result, Eq. (15) need not be valid. As we have done for the SM sector, we can write

$$\rho_{\rm dr}(t_\nu) = \rho_{\rm dr}^{\rm rh} \frac{g_{dr-E}(t_\nu) T_\nu^4}{g_{dr-E}(t_{\rm rh}) T_{\rm rh}^4} = \rho_{\rm dr}^{\rm rh} \frac{g_{dr-E}(t_\nu)}{g_{dr-E}(t_{\rm rh})} \frac{a^4(t_{\rm rh}) g_{dr-s}^{\frac{4}{3}}(t_{\rm rh})}{a^4(t_\nu) g_{dr-s}^{\frac{4}{3}}(t_\nu)}, \tag{22}$$

where the $g_{\rm dr}$ factors are for the dark radiation sector. We have adopted notation where the arguments of the dark sector g factors are the times at which they are to be evaluated (since dark sector can in principle be in a different temperature from the SM). Combining with (14) we get

$$\Delta N_{\rm eff} = \frac{43}{7} \frac{B_{\rm dr}}{B_{\rm vis}} \left(\frac{g_E(T_{\rm rh})}{g_E(T_\nu)} \frac{g_s^{\frac{4}{3}}(T_\nu)}{g_s^{\frac{4}{3}}(T_{\rm rh})} \right) \left(\frac{g_{dr-E}(t_\nu)}{g_{dr-E}(t_{\rm rh})} \frac{g_{dr-s}^{\frac{4}{3}}(t_{\rm rh})}{g_{dr-s}^{\frac{4}{3}}(t_\nu)} \right). \tag{23}$$

If the dark radiation sector never enters a Hagedorn phase then the factor in the last brackets will be order one. But, if the dark sector is localized in a warped throat and enters an open string Hagedorn phase, then one can get a large contribution from this factor.

For the case that both the visible sector and the dark sector are in "usual thermal baths" at $t = t_\nu$ and in open string Hagedorn phases at $t = t_{\rm rh}$, we obtain

$$\begin{aligned} \Delta N_{\rm eff} &= \frac{43}{7} \frac{B_{\rm dr}}{B_{\rm vis}} \left(\frac{g_{dr-E}(t_{\rm rh})}{g_E(T_{\rm rh})} \right)^{1/3} \left(\frac{g_E(T_\nu)}{g_{dr-E}(t_\nu)} \right)^{1/3} \\ &= \frac{43}{7} \frac{B_{\rm dr}}{B_{\rm vis}} \left(\frac{g_E(T_\nu)}{g_E(T_{\rm rh})} \right)^{1/3} \left(\frac{g_{dr-E}(t_{\rm rh})}{g_{dr-E}(t_\nu)} \right)^{1/3}. \end{aligned} \tag{24}$$

A large contribution can arise from the factor

$$(g_{dr-E}(t_{\rm rh}))^{1/3} \sim \frac{N_{\text{dr-D}}^{2/3}}{(1 - T_{\text{dr-rh}}/T_{\text{dr-}H})^{2/3}} \left(\frac{T_{\text{dr-rh}}}{T_{\rm rh}} \right)^{4/3}$$

where $T_{\text{dr-}H}$ is the Hagedorn temperature in the dark sector, $T_{\text{dr-rh}}$ the temperature of the dark sector at $t_{\rm rh}$ and $N_{\text{dr-D}}$ the number of D-branes in the dark sector. This is the case when the dark radiation is, for example, an unconfined gauge theory on D3-branes in a separate warped throat, although the contribution is suppressed if the number of D-branes in the dark throat is much less than that in the SM throat. Another possibility for the suppression of the contribution is that the inflaton branching ratios to throats which contain dark radiation candidates are low, which can happen if the inflaton wave function in the extra dimensions has limited support in the dark radiation throat.[9]

Specifically for axions, even if these are localized in a strongly warped region, we do not expect them to gain this sort of Hagedorn enhancement. If there are no D-branes present, a similar argument to that of Sec. II shows that the black 3-brane dominates the closed string Hagedorn gas in the microcanonical ensemble in the thermodynamic limit (of fixed energy density and infinite parallel volume). Since the black 3-brane has the same equation of state as radiation, we do not expect a parametric enhancement of degrees of freedom.

We can also consider entropy generation in the dark radiation sector. If the fractional entropy increase is η (that is, Eq. (20) applies for the dark radiation sector), that modifies Eq. (23) by inserting a factor of $(1+\eta)^{4/3}$ in the numerator, which increases $\Delta N_{\rm eff}$.

IV. CONCLUSION AND DISCUSSIONS

We have presented a proof of concept that an early open string Hagedorn phase can alleviate the dark radiation problem of string theory. We focused on the setting of [8], where the SM is localized in a warped throat, with the warped string scale below the Hubble scale during inflation. At the end of inflation, the SM sector enters a Hagedorn phase. The effective number of degrees of freedom in this phase is very high, diverging as $(T - T_H)^{-2}$ as $T \to T_H$. We found that for dark radiation which is not part of the thermal plasma (such as bulk axions) this large effective number of degrees of freedom leads to suppression in $\Delta N_{\rm eff}$. We also discussed the effects of entropy production and dark radiation sectors in other throats. In summary, the idea provides an attractive way to address the dark radiation problem in string compactifications. Furthermore, since warping is a generic feature of string compactifications, the work provides an important element for acquiring a complete understanding of the nature of predictions for dark radiation in string theory.

Our basic idea extends to many other models of early universe cosmology in string theory. For example, it seems likely that an open string Hagedorn phase appears in any model where the SM (or sufficiently strong intermediate) hierarchy is the result of warping or other inhomogeneity of a string compactification with the SM supported on D-branes. However, the Hagedorn phase of strings also appears intrinsically in the string gas model of cosmology [32,33] (or generalizations including black holes at the correspondence point with strings [34]), which is of current interest in light of conjectured constraints on inflation in the swampland program [35,36]. While the original string gas cosmology exclusively discusses closed string degrees of

[9]This is possible as throats occupy localized regions of the compact dimensions.

freedom, there is a natural extension involving branes [37]; since a dilute gas of D-branes converts closed strings to open strings efficiently, it is reasonable to expect that the energy density in a brane-supported SM would dominate over closed string dark radiation in a variation of the mechanism described here. Similar comments might apply to the models of [38] given their conceptual relationships to string gas cosmology. One difference with the key ideas presented here is important: we have considered a scenario in which the SM and dark radiation sectors are sequestered, but that is not the case in the string gas cosmologies studied so far (and may not be the case in more general warped compactifications). As a result, the dynamics of the transition between the open string Hagedorn and radiation phases, including coupling to the closed string sector, could play a more important role in a more general class of models.

Of course, there are several other interesting directions for future work. Let us list a few. First, one can try to embed the scenario in a concrete model of moduli stabilization. This will allow for explicit computations of various parameters and will help to arrive at explicit numerical values for predictions for ΔN_{eff}. Second, the present work relied on the understanding of the open string Hagedorn phase in warped throats as developed in [8]. There are many fronts in which this can be improved as described in detail in [8]. Finally, one can explore whether the Hagedorn phase has any other characteristic signatures for observations and how these correlate with ΔN_{eff}.

ACKNOWLEDGMENTS

R. M. is supported in part by the INFOSYS scholarship for senior students (HRI). A. M. is supported in part by the SERB, DST, Government of India by the Grant No. MTR/2019/000267. A. R. F. is supported by the Natural Sciences and Engineering Research Council of Canada Discovery Grant program, Grant No. 2020-00054.

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