



Scotogenic $A_5 \rightarrow A_4$ Dirac neutrinos with freeze-in dark matter

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ABSTRACT

Radiative Dirac neutrino masses and their mixing are linked to dark matter through the non-Abelian discrete symmetry A_5 of the 4-dimensional pentatope, softly broken to A_4 of the 3-dimensional tetrahedron. This unifying understanding of neutrino family structure from dark matter is made possible through the interplay of gauge symmetry, renormalizable Lagrangian field theory, and softly broken discrete symmetries. Dark neutral fermions are produced through Higgs decay.

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1. Introduction

Two fundamental issues in particle physics and astroparticle physics are neutrinos [1] and dark matter [2]. An important development in recent years is the notion that they are intrinsically related, i.e. the origin of neutrino masses is the existence of dark matter. This is simply accomplished in the scotogenic model [3], where radiative Majorana neutrino masses are generated in one loop with the internal particles belonging to the dark sector. Numerous variations and studies of this basic premise have appeared in the past 16 years. More recently, the notion that neutrinos are Dirac fermions [4] has received more attention, and the scotogenic mechanism may also be applied [5].

Since neutrinos come in three families, their 3×3 mixing matrix with charged leptons could be a hint to a symmetry yet to be discovered among them. In the context of a renormalizable Lagrangian field theory subject to the $SU(3) \times SU(2) \times U(1)$ gauge symmetry of the Standard Model (SM), this poses a conundrum. How is it that a symmetry may govern mixing and yet all lepton masses are so different? This problem was solved 21 years ago [6] using the non-Abelian discrete symmetry A_4 and its breaking, which allow three arbitrary charged-lepton masses, and yet maintain a specific pattern for the neutrino mass matrix, which results in tribimaximal [7] or cobimaximal mixing [8,9], close to what is observed experimentally.

In the original scotogenic model for Majorana neutrinos, the dark sector is distinguished by a dark parity, which may be derived from lepton parity [10]. For Dirac neutrinos, lepton number L is considered instead. To prevent neutrinos from obtaining tree-level Dirac masses, a lepton family symmetry is used, specifically the non-Abelian discrete symmetry A_5 [11]. With the chosen fermion and scalar multiplets belonging to its irreducible representations, and the soft breaking of $A_5 \rightarrow A_4$, scotogenic Dirac neutrino masses are obtained, with dark number $D = L - (2j)_{[mod\ 2]}$ [5], where j is the intrinsic spin of the particle. Note that vector gauge bosons have $D = 0$.

2. Tetrahedron and pentatope

The tetrahedron is the simplest perfect geometric solid in three dimensions. In Cartesian coordinates, its four vertices may be simply put at the positions

$$(1, 0, 0), (0, 1, 0), (0, 0, 1), (1, 1, 1). \quad (1)$$

Each combination of three points forms an equilateral triangle with side $\sqrt{2}$, and there are four such triangles. The pentatope is the simplest perfect geometric solid in four dimensions. In Cartesian coordinates, its five vertices may be simply put at the positions

$$(2, 0, 0, 0), (0, 2, 0, 0), (0, 0, 2, 0), (0, 0, 0, 2), (\varphi, \varphi, \varphi, \varphi), \quad (2)$$

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Table 1
Fermions and scalars in the $A_5 \rightarrow A_4$ model.

Fermion/scalar	$SU(2)_L \times U(1)_Y$	A_5	A_4	L	$D = L - 2j$
$L_L = (\nu, l)_L$	$(2, -1/2)$	3	3	1	0
l_R	$(1, -1)$	3	3	1	0
ν_R	$(1, 0)$	$3'$	3	1	0
$E_{L,R}$	$(1, -1)$	3	3	1	0
$\Phi = (\phi^+, \phi^0)$	$(2, 1/2)$	1	1	0	0
$N_{L,R}$	$(1, 0)$	4	3, 1	0	-1
$\eta = (\eta^0, \eta^-)$	$(2, -1/2)$	$3'$	3	1	1
ζ^0	$(1, 0)$	3	3	1	1
ζ^-	$(1, -1)$	$3'$	3	1	1

where $\varphi = (1 + \sqrt{5})/2 = 1.618$ is the golden ratio. Each combination of four points forms a tetrahedron with side $2\sqrt{2}$, and there are five such tetrahedrons.

The tetrahedron is invariant under A_4 , the group of the even permutation of four objects. It has 12 elements and 4 irreducible representations, i.e.

$$\underline{1}, \underline{1}', \underline{1}'', \underline{3}. \quad (3)$$

The pentatope is invariant under A_5 , the group of the even permutation of five objects. It has 60 elements and 5 irreducible representations, i.e.

$$\underline{1}, \underline{3}, \underline{3}', \underline{4}, \underline{5}. \quad (4)$$

The multiplication rules of these A_5 representations are [11]

$$3 \times 3 = 1 + 3 + 5, \quad (5)$$

$$3' \times 3' = 1 + 3' + 5, \quad (6)$$

$$3 \times 3' = 4 + 5, \quad (7)$$

$$3 \times 4 = 3' + 4 + 5, \quad (8)$$

$$3' \times 4 = 3 + 4 + 5, \quad (9)$$

$$3 \times 5 = 3 + 3' + 4 + 5, \quad (10)$$

$$3' \times 5 = 3 + 3' + 4 + 5, \quad (11)$$

$$4 \times 4 = 1 + 3 + 3' + 4 + 5, \quad (12)$$

$$4 \times 5 = 3 + 3' + 4 + 5 + 5, \quad (13)$$

$$5 \times 5 = 1 + 3 + 3' + 4 + 4 + 5 + 5. \quad (14)$$

Since A_4 is a subgroup of A_5 , the decompositions of the latter representations to the former are [11]

$$1 \sim 1, \quad 3 \sim 3, \quad 3' \sim 3, \quad 4 \sim 3 + 1, \quad 5 \sim 3 + 1' + 1''. \quad (15)$$

With the choice of particle content of the model to be described under A_5 , the above properties of $A_5 \rightarrow A_4$ in the context of a renormalizable Lagrangian gauge theory will result in the radiative generation of neutrino masses with a realistic family structure.

3. Model

The idea of this model and its implementation are both very simple. The three copies of SM charged-lepton doublets $L_L = (\nu, l)_L$ and singlets l_R transform each as $\underline{3}$ under A_5 , whereas the three neutral singlets ν_R transform as $\underline{3}'$. There is one Higgs doublet (ϕ^+, ϕ^0) as in the SM, transforming as $\underline{1}$ under A_5 . The charged-lepton masses are obtained from heavy fermion singlets $E_{L,R} \sim \underline{3}$ in a seesaw manner. For radiative Dirac neutrino masses, four neutral Dirac fermion singlets $N \sim \underline{4}$ are added, with three scalar doublets $(\eta^0, \eta^-) \sim \underline{3}'$ and three scalar singlets $\zeta^0 \sim \underline{3}$, as shown in Table 1. There are also three charged scalar singlets $\zeta^- \sim \underline{3}'$ to allow the decay of the Higgs boson to $\bar{N}N$.

Whereas $\bar{l}_R \Phi^\dagger L_L$ and $\bar{E}_R \Phi^\dagger L_L$ terms ($3 \times 1 \times 3$) are allowed, $\bar{\nu}_R \tilde{\Phi}^\dagger L_L$ ($3' \times 1 \times 3$) is forbidden by A_5 and $\bar{N}_R \tilde{\Phi}^\dagger L_L$ ($4 \times 1 \times 3$) is forbidden by both A_5 and L . Hence the neutral singlet fermions ν_R are not tree-level Dirac mass partners to the SM doublet neutrinos ν_L even though they both have $L = 1$. On the other hand, $\bar{L}_L \eta N_R$ ($3 \times 3' \times 4$) and $\bar{\nu}_R \zeta^0 N_L$ ($3' \times 3 \times 4$) are allowed. As for the $\Phi \eta \zeta^0$ term, it transforms as $1 \times 3' \times 3$ which is not invariant under A_5 , but it is a dimension-three soft term, so it may be chosen to break $A_5 \rightarrow A_4$ in which case $1 \times 3 \times 3$ is invariant under A_4 . As a result, ν_L is linked to ν_R in one loop.

The structure of the 4×4 M_N mass matrix M_{ij} determines the 3×3 Dirac neutrino mass matrix. Assuming that N_4 is very heavy from the breaking of $A_5 \rightarrow A_4$, the resulting m_ν is proportional to

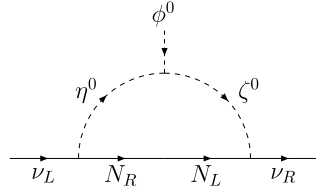


Fig. 1. One-loop scotogenic Dirac neutrino mass.

$$\tilde{\mathcal{M}} = \begin{pmatrix} \varphi^2 M_{12} & M_{11} + M_{33} & \varphi^{-2} M_{32} \\ M_{22} + M_{33} & \varphi^{-2} M_{21} & \varphi^2 M_{31} \\ \varphi^{-2} M_{13} & \varphi^2 M_{23} & M_{11} + M_{22} \end{pmatrix}.$$

The soft breaking of A_4 comes then from having $M_{ii} \neq M_{jj}$ and $M_{ij} \neq 0$ for $i \neq j$. Details will be provided in the following sections.

4. Charged leptons

Under A_5 , the 6×6 mass matrix linking $(l, E)_L$ to $(l, E)_R$ is given by

$$\mathcal{M} = \begin{pmatrix} 0 & M_{lE} \\ M_{El} & M_{EE} \end{pmatrix}, \quad (16)$$

where the 3×3 entries M_{lE} , M_{El} , and M_{EE} are all proportional to the identity, with M_{lE} coming from the vacuum expectation value v of the SM Higgs doublet Φ . Since the $E_L l_R$ coupling is a dimension-three soft term, it is assumed to break A_5 in a way compatible with A_4 using the procedure of Ref. [12], i.e.

$$M_{El} = \begin{pmatrix} h_1 v' & h_2 v'' & h_3 v'' \\ h_3 v'' & h_1 v' & h_2 v'' \\ h_2 v'' & h_3 v'' & h_1 v' \end{pmatrix}, \quad (17)$$

which is obtained from $E \sim 3$ and $l_R \sim 3$ and a gauge singlet flavon ~ 3 under A_4 . The magic of this matrix is that it decomposes to [12]

$$U_L \begin{pmatrix} h_1 v' + (h_2 + h_3) v'' & 0 & 0 \\ 0 & h_1 v' + (h_2 \omega + h_3 \omega^2) v'' & 0 \\ 0 & 0 & h_1 v' + (h_2 \omega^2 + h_3 \omega) v'' \end{pmatrix} U_R^\dagger, \quad (18)$$

where

$$U_L = U_R = U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \quad (19)$$

with $\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$, is the well-known transformation matrix studied in numerous A_4 models. Assuming that the singlet 3×3 M_{EE} masses to be much heavier than M_{lE} and M_{El} , the 3×3 charged-lepton mass matrix is given by

$$M_{ll} = M_{lE} M_{EE}^{-1} M_{El}, \quad (20)$$

which preserves the form of Eq. (18). Hence the charged-lepton mass matrix has three independent eigenvalues, which may be chosen to be m_e, m_μ, m_τ and yet a definite mixing matrix U_ω of Eq. (19) is obtained relative to the neutrino mass matrix, as in the original A_4 model [6].

5. Scotogenic neutrino masses

With the A_5 assignments of Table 1, Dirac neutrino masses are generated in one loop as shown in Fig. 1. To compute this diagram, the first step is to note that the $A_5 \rightarrow A_4$ breaking trilinear coupling $\eta^0 \phi^0 \bar{\zeta}^0$ with $\langle \phi^0 \rangle = v$ means that η^0 mixes with ζ^0 in a 2×2 mass-squared matrix. Hence the two mass eigenstates $\psi_{1,2}^0$ with $m_{1,2}$ are

$$\psi_1^0 = \eta^0 \cos \theta - \zeta^0 \sin \theta, \quad \psi_2^0 = \eta^0 \sin \theta + \zeta^0 \cos \theta. \quad (21)$$

The second step is to note that the $\bar{\nu}_L \eta^0 N_R$ and $\bar{\nu}_R \zeta^0 N_L$ couplings come from the decomposition of $3 \times 3' \rightarrow 4$. Let $(a_1, a_2, a_3) \sim 3$, $(b_1, b_2, b_3) \sim 3'$, then the four components of N are given by [11]

$$N = \frac{1}{\sqrt{3}} \begin{pmatrix} \varphi^{-1} a_3 b_2 - \varphi a_1 b_3 \\ \varphi a_3 b_1 + \varphi^{-1} a_2 b_3 \\ -\varphi^{-1} a_1 b_1 + \varphi a_2 b_2 \\ a_2 b_1 - a_1 b_2 + a_3 b_3 \end{pmatrix}. \quad (22)$$

In the A_5 limit, all components of N have the same mass, but since the 4×4 \mathcal{M}_N mass matrix is a soft term, it is allowed to break A_5 . The form of \mathcal{M}_v may then be chosen as in previous A_4 models to obtain cobimaximal mixing for example. This in turn would imply a family structure for the dark N fourplet.

The radiative Dirac neutrino mass matrix is

$$(\mathcal{M}_\nu)_{ij} = \frac{\sin \theta \cos \theta}{16\pi^2} \sum_{k,k',a} f_{ika}^L f_{k'k4}^\phi f_{k'ja}^R M_a [F(m_2^2, M_a^2) - F(m_1^2, M_a^2)], \quad (23)$$

where M_a ($a = 1, 2, 3, 4$) are the masses of the four N fermions. The function F is given by

$$F(x, y) = \frac{x \ln(x/y)}{x - y}. \quad (24)$$

The f_{ika} couplings are as defined by Eq. (22), with $i = 1, 2, 3$ from $\underline{3}$, $k = 1, 2, 3$ from $\underline{3}'$, and $a = 1, 2, 3, 4$ from $\underline{4}$. If all M_a are equal, all three Dirac neutrinos would have the same mass, as expected.

It is now assumed that the 4×4 M_N mass matrix is not invariant under A_5 . Note first that $\eta_{1,2,3}, \zeta_{1,2,3} \sim 3$, $N_{1,2,3} \sim 3$ and $N_4 \sim 1$ under A_4 . Assume then that N_4 is very much heavier than all other masses. As for $N_{1,2,3}$, they form a mass matrix M_{ab} which softly breaks A_4 with all entries much lighter than $m_{1,2}$. This reduces \mathcal{M}_ν to the form first recognized in Ref. [13], i.e.

$$(\mathcal{M}_\nu)_{ij} = \frac{\sin \theta \cos \theta \ln(m_2^2/m_1^2)}{16\pi^2} \sum_{k,k',a,b} f_{ika}^L f_{k'k4}^\phi f_{k'jb}^R M_{ab}, \quad (25)$$

where M_{ab} links N_R to N_L . Using Eq. (22), the above is then proportional to

$$\tilde{\mathcal{M}} = \begin{pmatrix} \varphi^2 M_{12} & M_{11} + M_{33} & \varphi^{-2} M_{32} \\ M_{22} + M_{33} & \varphi^{-2} M_{21} & \varphi^2 M_{31} \\ \varphi^{-2} M_{13} & \varphi^2 M_{23} & M_{11} + M_{22} \end{pmatrix}, \quad (26)$$

where ν_{L1} has been redefined with a minus sign. If $M_{11} = M_{22} = M_{33}$ and all $M_{ij} = 0$ with $i \neq j$, this reduces to the A_5 coupling of $3 \times 3'$ to the fourth component of $\underline{4}$, as expected.

To achieve cobimaximal mixing [14], i.e. $\theta_{13} \neq 0$, $\sin^2 \theta_{23} = 1/2$, and $\delta_{CP} = \pi/2, 3\pi/2$, the above mass matrix should be diagonalized by an orthogonal matrix \mathcal{O} on the left, so that the 3×3 neutrino mixing matrix becomes

$$U_{l\nu} = U_\omega^\dagger \mathcal{O}, \quad (27)$$

where U_ω comes from Eq. (19). It was shown 22 years ago [15] that this results automatically in cobimaximal mixing.

An equivalent formulation [9] for Majorana neutrinos is to consider the neutrino mass matrix in the basis of diagonal charged-lepton masses, i.e.

$$U_\omega^\dagger \mathcal{M}_\nu (U_\omega^\dagger)^T = \begin{pmatrix} A & C & E^* \\ C & D^* & B \\ E^* & B & F \end{pmatrix}. \quad (28)$$

The conditions for cobimaximal mixing are then [16]

$$E = C, \quad F = D, \quad A, B \text{ real}. \quad (29)$$

However for Dirac neutrinos, there are no unique conditions, because the 3×3 matrix diagonalizing \mathcal{M}_ν on the right is not constrained. Nevertheless, a suggestive form is [14]

$$\mathcal{M}_D = \begin{pmatrix} a & c & c^* \\ d & b & e \\ d^* & e^* & b^* \end{pmatrix}, \quad (30)$$

where a is real. Assuming \tilde{M} of Eq. (26) to be real, then $U_\omega^\dagger \tilde{M} U_\omega$ is automatically of this form.

6. Family structure of neutrinos and dark matter

Until 2012, the data were consistent with tribimaximal mixing, i.e. $\theta_{13} = 0$, $\sin^2 \theta_{23} = 1/2$, and $\sin^2 \theta_{12} = 1/3$. Now they are closer to cobimaximal mixing with $\delta_{CP} = 3\pi/2$. The present world averages are [17]

$$\sin^2 \theta_{13} = (2.20 \pm 0.07) \times 10^{-2}, \quad \sin^2 \theta_{12} = 0.307 \pm 0.013, \quad (31)$$

$$\sin^2 \theta_{23} = 0.546 \pm 0.021 \text{ (Normal order)}, \quad (32)$$

$$\sin^2 \theta_{23} = 0.539 \pm 0.022 \text{ (Inverted order)}, \quad (33)$$

$$\delta_{CP} = 1.36 (+0.20/-0.16) \pi, \quad \Delta m_{21}^2 = (7.53 \pm 0.18) \times 10^{-5} \text{ eV}^2, \quad (34)$$

$$\Delta m_{32}^2 = (2.453 \pm 0.033) \times 10^{-3} \text{ eV}^2 \text{ (Normal order)}, \quad (35)$$

$$\Delta m_{32}^2 = (-2.536 \pm 0.034) \times 10^{-3} \text{ eV}^2 \text{ (Inverted order)}. \quad (36)$$

With Eq. (26), a connection is predicted between neutrinos and dark matter. As an example, consider the case of cobimaximal mixing, with

$$U_{lv} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ (s_{12} - ic_{12}s_{13})/\sqrt{2} & (-c_{12} - is_{12}s_{13})/\sqrt{2} & ic_{13}/\sqrt{2} \\ (s_{12} + ic_{12}s_{13})/\sqrt{2} & (-c_{12} + is_{12}s_{13})/\sqrt{2} & -ic_{13}/\sqrt{2} \end{pmatrix}. \quad (37)$$

Using Eq. (27), the orthogonal matrix \mathcal{O} which diagonalizes Eq. (26) on the left is then

$$\mathcal{O} = \frac{1}{\sqrt{3}} \begin{pmatrix} c_{12}c_{13} + \sqrt{2}s_{12} & s_{12}c_{13} - \sqrt{2}c_{12} & s_{13} \\ c_{12}c_{13} - s_{12}/\sqrt{2} + \sqrt{3/2}c_{12}s_{13} & s_{12}c_{13} + c_{12}/\sqrt{2} + \sqrt{3/2}c_{12}s_{13} & s_{13} - \sqrt{3/2}c_{13} \\ c_{12}c_{13} - s_{12}/\sqrt{2} - \sqrt{3/2}c_{12}s_{13} & s_{12}c_{13} + c_{12}/\sqrt{2} - \sqrt{3/2}c_{12}s_{13} & s_{13} + \sqrt{3/2}c_{13} \end{pmatrix} \quad (38)$$

For the central values

$$s_{13} = 0.148, \quad c_{13} = 0.989, \quad s_{12} = 0.554, \quad c_{12} = 0.832, \quad (39)$$

the orthogonal matrix becomes

$$\mathcal{O} = \begin{pmatrix} 0.927 & -0.363 & 0.085 \\ 0.336 & 0.714 & -0.614 \\ 0.162 & 0.598 & 0.785 \end{pmatrix}, \quad (40)$$

which diagonalizes $\tilde{M}\tilde{M}^T$. In terms of the neutrino mass eigenvalues $m_{1,2,3}^2$, the M_{ij} entries of Eq. (26) are then related to \mathcal{O} .

There are nine parameters in \tilde{M} . Assuming the three conditions

$$\varphi^2 \begin{pmatrix} M_{12} \\ M_{31} \\ M_{23} \end{pmatrix} = \varphi^{-2} \begin{pmatrix} M_{21} \\ M_{13} \\ M_{32} \end{pmatrix}, \quad (41)$$

then \mathcal{M}_ν (which is proportional to \tilde{M}) becomes of the form

$$\mathcal{M}_\nu = \begin{pmatrix} y_1 & x_1 & y_3 \\ x_2 & y_1 & y_2 \\ y_2 & y_3 & x_3 \end{pmatrix}, \quad (42)$$

resulting in

$$\begin{pmatrix} x_1^2 + y_1^2 + y_3^2 \\ x_2^2 + y_1^2 + y_2^2 \\ x_3^2 + y_2^2 + y_3^2 \end{pmatrix} = \begin{pmatrix} 0.859 & 0.113 & 0.026 \\ 0.132 & 0.510 & 0.358 \\ 0.007 & 0.377 & 0.616 \end{pmatrix} \begin{pmatrix} m_1^2 \\ m_2^2 \\ m_3^2 \end{pmatrix}, \quad (43)$$

and

$$\begin{pmatrix} y_1(x_1 + x_2) + y_2y_3 \\ y_3(x_1 + x_3) + y_1y_2 \\ y_2(x_2 + x_3) + y_1y_3 \end{pmatrix} = \begin{pmatrix} -0.337 & 0.240 & 0.097 \\ 0.079 & -0.206 & 0.127 \\ -0.031 & -0.438 & 0.469 \end{pmatrix} \begin{pmatrix} m_1^2 \\ m_2^2 \\ m_3^2 \end{pmatrix}. \quad (44)$$

Assuming normal ordering of neutrino masses, the above six equations may be solved simply for $y_1 = 0$, resulting in

$$x_1 = 0.597 \times 10^{-2} \text{ eV}, \quad x_2 = 1.334 \times 10^{-2} \text{ eV}, \quad x_3 = 2.711 \times 10^{-2} \text{ eV}, \quad (45)$$

$$y_2 = 2.850 \times 10^{-2} \text{ eV}, \quad y_3 = 0.924 \times 10^{-2} \text{ eV}, \quad m_1 = 0.684 \times 10^{-2} \text{ eV}. \quad (46)$$

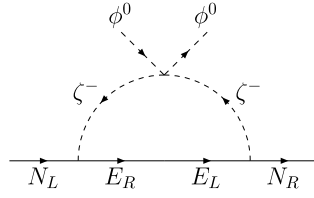
This implies $m_2 = 0.011 \text{ eV}$ and $m_3 = 0.051 \text{ eV}$, for a sum of three neutrino masses = 0.07 eV, comfortably below the astrophysical bound of 0.15 eV.

7. Production of dark matter

In Fig. 1, lepton number L is conserved with $L = 1$ for ν , η^0 , ζ^0 and $L = 0$ for N . Defining $D = L - 2j$ as shown in Table 1, dark number is also conserved with $D = 0$ for ν , $D = 1$ for η^0 , ζ^0 , and $D = -1$ for N . So far, the charged scalar gauge singlet $\zeta^- \sim 3'$ of Table 1 has not been used. It has $L = D = 1$ and allows N to be produced as freeze-in dark matter [18] through Higgs decay [19] as shown in Fig. 2. It is the analog of Fig. 1, but now L circulates in the loop, whereas D flows from N to ζ^- to N . Here A_5 is unbroken by all the couplings and the one-loop diagram is finite because it involves three propagators, two scalar and one fermion. It yields the effective Yukawa coupling f_h of the Higgs boson h to $\bar{N}N$.

$$f_h = \frac{\lambda_\zeta f_\zeta^L f_\zeta^R v m_E}{16\pi^2} \left[\frac{1}{m_\zeta^2 - m_E^2} - \frac{m_E^2 \ln(m_\zeta^2/m_E^2)}{(m_\zeta^2 - m_N^2)^2} \right]. \quad (47)$$

The decay rate of h to $\bar{N}N$ is

Fig. 2. Higgs decay to $\bar{N}N$.

$$\Gamma_h = \frac{f_h^2 m_h}{8\pi} \sqrt{1 - 4r^2(1 - 2r^2)}, \quad (48)$$

where $r = m_N/m_h$.

As shown in Ref. [20], if the reheat temperature T_R of the Universe after inflation is below the decoupling temperature of N but above m_h , say $T_R \sim 1 - 10$ TeV, then N is a feebly interacting massive particle (FIMP), which only production mechanism is freeze-in, through Higgs decay, before the latter decouples from the thermal bath. Typical values for this to happen here are $m_N \sim \text{GeV}$, $f_h \sim 10^{-11}$, $m_\zeta \sim 10^4$ GeV, and $m_E \sim 10^5$ GeV.

8. Concluding remarks

The non-Abelian discrete symmetry A_5 of the 4-dimensional pentatope is used to construct a radiative model of Dirac neutrinos through dark matter, with dark number D connected to lepton number L , i.e. $D = L - (2j)_{[mod\ 2]}$. Cobimaximal neutrino mixing is obtained from the soft breaking of A_5 to A_4 which is the symmetry of the 3-dimensional tetrahedron. The complete neutrino mass matrix is linked to the dark neutral fermions, with a realistic numerical example showing the normal ordering of neutrino masses. The dark fermions are produced by the freeze-in mechanism through the decay of the SM Higgs boson.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

- [1] See for example A. de Gouvea, *Annu. Rev. Nucl. Part. Sci.* 66 (2016) 197.
- [2] See for example B.-L. Young, *Front. Phys.* 12 (2017) 121201, <https://doi.org/10.1007/s11467-016-0583-4>.
- [3] E. Ma, *Phys. Rev. D* 73 (2006) 077301.
- [4] E. Ma, O. Popov, *Phys. Lett. B* 764 (2017) 142.
- [5] E. Ma, *Phys. Lett. B* 809 (2020) 135736.
- [6] E. Ma, G. Rajasekaran, *Phys. Rev. D* 64 (2001) 113012.
- [7] P.F. Harrison, D.H. Perkins, W.G. Scott, *Phys. Lett. B* 530 (2002) 167.
- [8] K.S. Babu, E. Ma, J.W.F. Valle, *Phys. Lett. B* 552 (2003) 207.
- [9] W. Grimus, L. Lavoura, *Phys. Lett. B* 579 (2004) 113.
- [10] E. Ma, *Phys. Rev. Lett.* 115 (2015) 011801.
- [11] L.L. Everett, A.J. Stuart, *Phys. Rev. D* 79 (2009) 085005.
- [12] E. Ma, *Mod. Phys. Lett. A* 21 (2006) 2931.
- [13] E. Ma, *Phys. Lett. B* 717 (2012) 235.
- [14] E. Ma, *Phys. Lett. B* 816 (2021) 136203.
- [15] K. Fukuda, T. Miura, E. Takasugi, M. Yoshimura, *Phys. Rev. D* 61 (2000) 073002.
- [16] E. Ma, *Phys. Lett. B* 752 (2016) 198.
- [17] P.A. Zyla, et al., Particle Data Group, *Prog. Theor. Exp. Phys.* 2020 (2020) 083C01.
- [18] L.J. Hall, K. Jedamzik, J. March-Russell, S.M. West, *J. High Energy Phys.* 1003 (2010) 080.
- [19] E. Ma, *Lett. High Energy Phys.* 2 (2019) 103, <https://doi.org/10.31526/LHEP.1.2019.103>.
- [20] E. Ma, V. De Romeri, *Phys. Rev. D* 104 (2021) 055004.