



# CDF W mass anomaly with a Stueckelberg-Higgs portal

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## ABSTRACT

We propose an explanation to the new W mass measurement recently reported by the CDF collaboration, which is larger than the standard model expectation by about 7 standard deviations. To alleviate the tensions that are imposed on the electroweak sector by the new W mass measurement, we carry out an analysis in the Stueckelberg extended standard model where a new neutral gauge boson appears which mixes with the two neutral gauge bosons in the electroweak sector both via the Stueckelberg mass terms and via the gauge invariant Stueckelberg-Higgs portal interaction and spoils the custodial symmetry at the tree level so that the simple relation between the W boson mass and the Z boson mass does not hold. We find that such an extension increases the W boson mass if the new gauge boson mass is larger than the Z boson mass. We further show that there exists a significant part of the parameter space in the extended model which includes the CDF mass anomaly and is consistent with the various observables at the Z pole and consistent with the ATLAS dilepton limits. The Stueckelberg  $Z'_{\text{st}}$  boson, which resolves the CDF W mass anomaly, should be searchable in future LHC experiments.

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## 1. Introduction

Recently, the W boson mass has been accurately measured by the CDF II detector with  $8.8 \text{ fb}^{-1}$  data accumulated at the Fermilab Tevatron collider with  $\sqrt{s} = 1.96 \text{ TeV}$  [1]. The new W mass measurement is

$$\begin{aligned} M_W^{\text{CDF}} &= 80,433.5 \pm 6.4 (\text{stat}) \pm 6.9 (\text{syst}) \text{ MeV} \\ &= 80,433.5 \pm 9.4 \text{ MeV}. \end{aligned} \quad (1)$$

The standard model (SM) expectation for the W mass is [1]

$$M_W^{\text{SM}} = 80,357 \pm 6 \text{ MeV}. \quad (2)$$

Thus the new CDF measurement on the W mass is larger than the SM expectation by the amount

$$\Delta M_W = +76 \text{ MeV}, \quad (3)$$

which corresponds to about 7 standard deviations, if the uncertainties are added in quadrature [1]. The W mass measurements

have a significant history with the most recent prior to the CDF II result being those from ATLAS [2] with W mass at  $80,370 \pm 19 \text{ MeV}$  and LHCb [3] with W mass at  $80,354 \pm 32 \text{ MeV}$ . It is then noted that the result of CDF II is in significant tension with previous measurements as well as with the SM prediction. This tension is discussed in the CDF II paper where they mention that the ‘LEP, Tevatron, and ATLAS measurements have not yet been combined, pending evaluation of uncertainty correlations’ [1]. The tension between the CDF II, ATLAS and other measurements will eventually be resolved by experimentalists. However, in the meantime it is appropriate for theorists to investigate the implications of anomalous results when they arise. The analysis here is in that spirit.

Early on a precise measurement of the  $W^\pm$  and Z masses has been viewed as an important testing ground for the standard model [4]. Thus in the SM, the W mass and the Z mass are related via (at tree level, or in the on-shell scheme)

$$M_W = M_Z \cos \theta_W, \quad (4)$$

where  $M_Z$  is the Z boson mass, and  $\theta_W$  is the weak mixing angle. The value of  $M_Z$  has been measured with great precision at LEP [5]

$$M_Z = 91.1876 \pm 0.0021 \text{ GeV}. \quad (5)$$

The weak mixing angle  $\theta_W$  is also well measured via various Z pole observables by LEP [6]. In the standard model where the gauge

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symmetry is broken by vacuum expectation value (VEV) of the Higgs doublet field, the so-called custodial symmetry holds where  $\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2 \theta_W} = 1$  holds at the tree level and  $\rho$  is also very close to 1 including radiative corrections. Thus, the new CDF W mass measurement with more than  $7\sigma$  deviation from the standard model hints at a violation of the custodial symmetry. To explain the CDF W mass anomaly, we carry out an analysis in the Stueckelberg extension of the standard model (StSM) [7–13] where the custodial symmetry does not hold at the tree level (see also for related early work in string theory [14–16] and the references therein). The StSM has an extended gauge symmetry  $SU(2)_L \times U(1)_Y \times U(1)_X$  where the new gauge boson associated with  $U(1)_X$  mixes with the hypercharge gauge boson of the SM via the Stueckelberg mass terms. As stated unlike SM, in StSM, the custodial symmetry no longer holds, because there are additional mass terms due to mass mixing with the new sector which leads to a mass correction to the Z boson. We find that the mass correction to the Z boson is negative if the new  $Z'_{\text{St}}$  boson is heavier than the Z boson. We further show that the negative contribution to the Z boson mass can provide an explanation to the CDF W mass anomaly. The various Z pole observables that have been precisely measured at LEP provide very stringent constraints to beyond-the-standard-model modifications to the electroweak sector. We carry out a detailed analysis by fitting various Z pole observables in the Stueckelberg extended model and show that the parameter space to interpret the CDF W mass anomaly can lead to a fit with a satisfactory total  $\chi^2$  which is nearly as good as the original LEP fit [6].

The rest of the paper is organized as follows. In section 2, we illustrate the implications of the new CDF W mass measurement to the electroweak sector and to the BSM models. In section 3, we present our model and provide an explanation to the W mass anomaly. In section 4, we carry out a detailed analysis by fitting various Z pole observables in the new model. In section 5, we present the results of our analysis and analyze constraints. In section 6, we discuss some related constraints and implications. In section 7, we summarize our results. Some further details of the model are given in Appendix A.

## 2. Implications of CDF W mass measurement

We discuss the implications of the CDF W mass anomaly to the electroweak sector of the SM. We illustrate the effects using tree level expressions in the SM. Although the expressions with the radiative corrections taken into account are more precise, they tend to be more complex and are often redefined with SM relations (which may or may not hold now). For the sake of clarity, we use the tree level expressions for the discussion in this section; the full analysis will result in small quantitative modifications which would not change the conclusions here.

One commonly used quantity in the electroweak precision fitting is the Fermi constant,

$$G_F = \frac{g_2^2}{4\sqrt{2}M_W^2}, \quad (6)$$

where  $g_2$  is the gauge coupling constant of the  $SU(2)_L$  gauge group. Experimentally, the Fermi constant is determined precisely via muon lifetime measurement [5]  $G_F = 1.1663787(6) \times 10^{-5} \text{ GeV}^{-2}$ . In the SM, the W and Z boson masses are

$$M_W = \frac{1}{2}g_2 v, \quad M_Z = \frac{1}{2}\sqrt{g_2^2 + g_Y^2} v, \quad (7)$$

where  $g_Y$  is the gauge coupling constant of the  $U(1)_Y$  gauge group, and  $v$  is the VEV of the SM Higgs potential. Thus, a larger  $M_W$  implies a larger  $g_2$ , if the relation given in Eq. (6) is unchanged. The tree level  $M_W$  expression also tells us that the Fermi

constant  $G_F$  is related to the Higgs VEV by  $G_F^{-\frac{1}{2}} = 2^{\frac{1}{4}}v$  which is then unmodified by the new W mass measurement. Since the Z boson mass is precisely measured in LEP, the larger  $g_2$  value then implies a smaller  $g_Y$  if we wish to maintain the  $M_Z$  relation of Eq. (7). At the tree level in the SM, the weak mixing angle  $\theta_W$  is given by

$$\sin^2 \theta_W = \frac{g_Y^2}{g_2^2 + g_Y^2} = 1 - \frac{M_W^2}{M_Z^2}. \quad (8)$$

Using the central values of  $M_W$  and  $M_Z$ , one finds that the deviation  $\Delta M_W = +76 \text{ MeV}$  leads to a shift

$$\Delta(\sin^2 \theta_W) = -0.00147. \quad (9)$$

Such a change on the weak mixing angle is severely constrained by the various Z pole observables; see the Z pole fit labeled “SM-CDF” in Table 1 in which the above shift on the weak angle has been made while the rest of the SM expressions are unchanged as in the “LEP” fit. The total  $\chi^2$  has increased from the LEP fit  $\sim 17$  to  $\sim 98$ , which is in ballpark agreement with the  $7\sigma$  deviation [1].<sup>1</sup>

If we wish to hold the weak angle fixed, we require that  $g_Y$  increase proportionately which implies an increase in the mass of the Z-boson above the current SM prediction by an amount

$$\Delta M_Z^{\text{exp}} = \frac{\Delta M_W}{\cos \theta_W}. \quad (10)$$

However, such an increase would be in severe conflict with the LEP measurement of the Z-boson mass given by Eq. (5). This problem is resolved in the Stueckelberg extension of the standard model where we show that the Stueckelberg sector generates a negative contribution to the Z-boson mass which cancels the contribution Eq. (10) in a significant region of the parameter space restoring consistency with the CDF data.

## 3. The model

In this section we consider an extended SM model along with a hidden sector which possesses a  $U(1)_X$  gauge symmetry. We consider a portal to the hidden sector consisting of two parts: one is the conventional Stueckelberg portal where there is mass mixing between the hypercharge field  $B_\mu$  and the Stueckelberg field  $C_\mu$ . In addition we consider a second Stueckelberg-Higgs portal where a  $SU(2)_L \times U(1)_Y$  gauge covariant derivative of the Higgs doublet couples to  $C_\mu$ . Thus we write the total Lagrangian so that  $\mathcal{L} = \mathcal{L}_{\text{SM}} + \Delta\mathcal{L}_{\text{St}}$  where

$$\begin{aligned} \Delta\mathcal{L}_{\text{St}} = & -\frac{C_{\mu\nu}^2}{4} - \frac{1}{2}(M_1 \bar{C}_\mu + M_2 B_\mu)^2 + J_{\text{hid}}^\mu C_\mu \\ & + \left(i\frac{g_c}{2}\Phi^\dagger D^\mu \Phi \bar{C}_\mu + h.c.\right) + \frac{g_c^2}{4}\Phi^\dagger \Phi \bar{C}_\mu^2. \end{aligned} \quad (11)$$

Here  $\Phi$  is the SM Higgs doublet,  $D_\mu \Phi = \left[\partial_\mu - i\frac{\bar{g}_Y}{2}B_\mu - i\frac{\bar{g}_2}{2}\tau^a A_\mu^a\right]\Phi$  is the  $SU(2)_L \times U(1)_Y$  covariant derivative of  $\Phi$ ,  $\tau_a$  is the Pauli matrix,  $\bar{C}_\mu = C_\mu + \partial_\mu \sigma / M_1$  is the combination of  $C$  and the axion field  $\sigma$  which is gauge invariant under the  $U(1)_X$  gauge transformation, where  $C_\mu$  couples to the hidden sector source  $J_{\text{hid}}^\mu$ ,  $M_1$

<sup>1</sup> In our “SM-CDF” fit, we first fit the new CDF W mass value, and then investigate the implications on the Z pole observables; we find that this leads to large deviations to some Z pole observables, such as  $A_{\text{FB}}^{(0,b)}$  and  $A_e$ , as shown in Table 1. The strategy in our “SM-CDF” fit is different from some other analyses, such as refs. [32,36,44]. For example, the “CDF” fit in ref. [32] to various electroweak observables shows small pulls on  $A_{\text{FB}}^{(0,b)}$  and on  $A_e$ , but a large pull on the W mass. However, the total SM  $\chi^2$  fit to both the W mass and other electroweak observables in our analysis are consistent with refs. [32].

**Table 1**

Fits to 19 LEP Z pole observables [17] for three cases: (1) LEP fit, (2) SM-CDF, and (3) the Stueckelberg-Higgs (St-Higgs) case. The LEP fit is taken from [6]. The SM-CDF fit is the same as the LEP fit except that the weak mixing angle  $\theta_W$  is modified as in Eq. (9) due to the new CDF  $M_W$  measurement [1]. The St-Higgs case is described in the text and the parameters in the benchmark model used here are:  $M_1 = 725$  GeV,  $g_c = 0.243$ ,  $\bar{g}_Y = g_Y(1 + 0.047\%)$ .

	$O^{\text{exp}} \pm \delta O$	LEP fit		SM-CDF		St-Higgs	
		$O^{\text{th}}$	$\chi$	$O^{\text{th}}$	$\chi$	$O^{\text{th}}$	$\chi$
$\Gamma_Z$ [GeV]	$2.4955 \pm 0.0023$	2.4960	-0.20	2.5000	-1.96	2.4999	-1.92
$\sigma_{\text{had}}$ [nb]	$41.481 \pm 0.033$	41.470	0.34	41.465	0.50	41.471	0.30
$R_e$	$20.804 \pm 0.05$	20.752	1.05	20.778	0.53	20.751	1.05
$R_\mu$	$20.784 \pm 0.034$	20.752	0.95	20.778	0.18	20.751	0.96
$R_\tau$	$20.764 \pm 0.045$	20.799	-0.77	20.825	-1.35	20.798	-0.76
$R_b$	$0.21629 \pm 0.00066$	0.21584	0.68	0.21578	0.77	0.21584	0.68
$R_c$	$0.1721 \pm 0.003$	0.1711	0.33	0.1712	0.30	0.1711	0.33
$A_{\text{FB}}^{(0,e)}$	$0.0145 \pm 0.0025$	0.0163	-0.71	0.0190	-1.81	0.0162	-0.70
$A_{\text{FB}}^{(0,\mu)}$	$0.0169 \pm 0.0013$	0.0163	0.48	0.0190	-1.64	0.0162	0.51
$A_{\text{FB}}^{(0,\tau)}$	$0.0188 \pm 0.0017$	0.0163	1.48	0.0190	-0.13	0.0162	1.50
$A_{\text{FB}}^{(0,b)}$	$0.0996 \pm 0.0016$	0.1033	-2.29	0.1118	-7.61	0.1032	-2.22
$A_{\text{FB}}^{(0,c)}$	$0.0707 \pm 0.0035$	0.0738	-0.89	0.0804	-2.78	0.0737	-0.86
$A_{\text{FB}}^{(0,s)}$	$0.0976 \pm 0.0114$	0.1034	-0.51	0.1119	-1.25	0.1033	-0.50
$A_e$	$0.15138 \pm 0.00216$	0.14733	1.88	0.15928	-3.66	0.14716	1.95
$A_\mu$	$0.142 \pm 0.015$	0.147	-0.36	0.159	-1.15	0.147	-0.34
$A_\tau$	$0.136 \pm 0.015$	0.147	-0.76	0.159	-1.55	0.147	-0.74
$A_b$	$0.923 \pm 0.02$	0.935	-0.58	0.936	-0.63	0.935	-0.58
$A_c$	$0.67 \pm 0.027$	0.67	0.07	0.67	-0.12	0.67	0.08
$A_s$	$0.895 \pm 0.091$	0.936	-0.45	0.937	-0.46	0.936	-0.45
$\chi^2$			17.3		97.8		20.9

and  $M_2$  are the Stueckelberg mass parameters, and  $g_c$  is a dimensionless parameter. We note here that the first set of terms in Eq. (11) without  $g_c$  are the standard Stueckelberg terms. The terms in Eq. (11) where  $g_c$  appears are the new Stueckelberg-Higgs portal of the visible sector to the hidden sector. The coefficient  $g_c^2/4$  of the  $\bar{C}^2$  term is determined by the constraint that the neutral vector boson sector has a massless mode, i.e., the photon.<sup>2</sup> One may combine the Stueckelberg-Higgs portal terms in an extended covariant derivative of the Higgs field so that  $\mathcal{D}_\mu \Phi = (D_\mu - i \frac{g_c}{2} \bar{C}_\mu) \Phi$ .

Here we also note that the Lagrangian of Eq. (11) is not invariant under  $U(1)_Y$  unless either  $M_2 = 0$  or  $g_c = 0$  because invariance under hypercharge transformations of  $(M_1 C_\mu + M_2 B_\mu + \partial_\mu \sigma)$  and  $(C_\mu + \partial_\mu \sigma/M_1)$  cannot be simultaneously achieved. The case  $g_c = 0$  leads to the standard Stueckelberg model [7,8] while the case we consider in the analysis here is  $M_2 = 0$ . However, for the sake of generality we will keep both  $M_2$  and  $g_c$  before we discuss the model  $M_2 = 0$  in further detail later.

We note here that we have in general three different portals. First we have the kinetic mixing portal such as  $-\frac{\delta}{2} C_{\mu\nu} B^{\mu\nu}$  which is invariant under both  $U(1)_X$  and  $U(1)_Y$  transformations and is parametrized by  $\delta$ . The second is the mass mixing portal which arises from the term  $-\frac{1}{2} (M_1 C_\mu + M_2 B_\mu + \partial_\mu \sigma)^2$  which is invariant under the simultaneous transformations of the  $U(1)_X$  and  $U(1)_Y$  where  $\sigma$  transforms dually. The Stueckelberg-Higgs coupling which is parametrized by  $g_c$  provides a third portal to the hidden sector.

The three portals are distinct and each one has been discussed in isolation, e.g., kinetic mixing in [18], mass mixing in [7] and Stueckelberg-Higgs portal ( $g_c \neq 0$ ) in this work. Where allowed one can further discuss two portals together such as kinetic energy and mass mixing [11]. In this context we note that for the case  $M_2 = 0$  and  $g_c \neq 0$ , we could have added a kinetic mixing term such as  $-\frac{\delta}{2} \bar{C}_{\mu\nu} B^{\mu\nu}$  which is invariant under the  $U(1)_X$  and  $U(1)_Y$  transformations. However, in the absence of  $g_c$ , the kinetic mixing is found not sufficient to resolve the CDF-II anomaly,<sup>3</sup> while  $\delta = 0$  and  $g_c \neq 0$  is found sufficient. Thus in the analysis below we consider the case  $\delta = 0$ .

Note that we have used  $\bar{g}_2$  and  $\bar{g}_Y$  in Eq. (11), because the gauge couplings in the new model do not necessarily take the same values as in SM. Thus after spontaneous breaking of the electroweak symmetry and in the unitary gauge the neutral gauge boson mass matrix in the basis  $V = (C, B, A^3)$  is given by

$$M^2 = \begin{pmatrix} M_1^2 + \frac{v^2}{4} g_c^2 & M_1^2 \epsilon + \frac{v^2}{4} g_c \bar{g}_Y & -\frac{v^2}{4} g_c \bar{g}_2 \\ M_1^2 \epsilon + \frac{v^2}{4} g_c \bar{g}_Y & M_1^2 \epsilon^2 + \frac{v^2}{4} \bar{g}_Y^2 & -\frac{v^2}{4} \bar{g}_Y \bar{g}_2 \\ -\frac{v^2}{4} g_c \bar{g}_2 & -\frac{v^2}{4} \bar{g}_Y \bar{g}_2 & \frac{v^2}{4} \bar{g}_2^2 \end{pmatrix}, \quad (12)$$

<sup>2</sup> However, for the special case in which  $M_2 = 0$ , there is no constraint on the coefficient of the  $\bar{C}^2$  term from the existence of the massless mode in the neutral vector boson sector, since the determinant of the mass matrix of Eq. (12) vanishes for any value of the coefficient of the  $\bar{C}^2$  term in this case.

<sup>3</sup> We find that the best-fit model points in the Stueckelberg mass mixing models (i.e.,  $M_2 \neq 0$ ,  $\delta = 0$ ,  $g_c = 0$ ) and in the kinetic mixing models (i.e.,  $M_2 = 0$ ,  $\delta \neq 0$ ,  $g_c = 0$ ) yield similar  $\chi^2$  fits to the 19 Z pole observables, which is also similar to the SM fit if  $g_Y$  is allowed to vary; the best  $\chi^2$  fits from these three models are found to be in the range of  $\sim (74-77)$ . Thus neither the Stueckelberg mass mixing models nor the kinetic mixing models can resolve the CDF W mass anomaly.

where we have defined  $\epsilon \equiv M_2/M_1$ . We note in passing that a restricted form of the mass matrix of Eq. (12) has been discussed previously [19] in an ad hoc fashion since no mechanism was proposed for generation of the mass matrix. The mass matrix has a vanishing determinant which ensures the existence of a massless photon mode. The mass matrix can be diagonalized via an orthogonal transformation  $\mathcal{O}$  such that  $E_i = \mathcal{O}_{ji} V_j$  and  $\mathcal{O}^T M^2 \mathcal{O} = \text{diag}(M_Z^2, M_Z^2, 0)$ . The mass eigenstates  $E_i = (Z', Z, A_\gamma)$  are the new  $Z'$ , the  $Z$ , and the photon. The interaction Lagrangian between the massive neutral gauge bosons and the SM fermion  $f$  is given by

$$\mathcal{L}_{NC} = \bar{f} \gamma^\mu \left[ (v_f - \gamma_5 a_f) Z_\mu + (v'_f - \gamma_5 a'_f) Z'_\mu \right] f + e \bar{f} \gamma^\mu Q_f A_\mu f, \quad (13)$$

where

$$a_f = \sqrt{\rho_f} (\bar{g}_Y \mathcal{O}_{22} - \bar{g}_2 \mathcal{O}_{32}) T_f^3 / 2, \quad (14)$$

$$v_f = a_f - \sqrt{\rho_f} \kappa_f \bar{g}_Y \mathcal{O}_{22} Q_f, \quad (15)$$

$$a'_f = (\bar{g}_Y \mathcal{O}_{21} - \bar{g}_2 \mathcal{O}_{31}) T_f^3 / 2, \quad (16)$$

$$v'_f = a'_f - \bar{g}_Y \mathcal{O}_{21} Q_f. \quad (17)$$

Here  $Q_f$  and  $T_f^3$  are the electric charge and  $SU(2)_L$  quantum number of the fermion  $f$ . For the  $Z$  couplings, we have incorporated the  $\rho_f$  and  $\kappa_f$  factors that contain radiative corrections from propagator self energies and flavor specific vertex corrections and are given in Ref. [20,6]. For the tree level expressions,  $\rho_f \rightarrow 1$  and  $\kappa_f \rightarrow 1$ .

In this new model the  $W$  mass is predicted as  $M_W = \bar{g}_2 v / 2$ . To explain the larger  $W$  mass measured by CDF, we increase the  $SU(2)_L$  gauge coupling

$$\bar{g}_2 = g_2 \left( 1 + \frac{\Delta M_W}{M_W} \right). \quad (18)$$

Since  $M_W = \bar{g}_2 v / 2$  appears in the (3,3) element in the neutral boson mass matrix, it tends to increase the eigenvalue  $M_Z$ . In the Stueckelberg extended standard model we compensate the increase of the  $Z$  mass due to the new CDF measurement on the  $W$  mass with a negative  $Z$  mass correction from the Stueckelberg mass shift of the  $Z$  boson arising from mixing of the neutral gauge bosons of the standard model with the Stueckelberg gauge boson.

As discussed earlier there are two model classes: (1)  $g_c = 0$ , and (2)  $M_2 = 0$ . In the former case, the model is the StSM [7,8] and here we find that the StSM has a total  $\chi^2$  for the  $Z$  pole fitting which is not better than the “SM-CDF” fit. Thus, in the present analysis we discuss (2) when  $M_2 = 0$ . In this case the Stueckelberg mass correction is given by

$$\Delta M_Z^{\text{th}} \simeq -\frac{1}{2} M_Z \frac{M_W^2}{M_1^2 - M_Z^2} \left( \frac{g_c}{g_2} \right)^2. \quad (19)$$

The Stueckelberg mass correction to the  $Z$ -boson is positive for  $M_1 < M_Z$  and negative for  $M_1 > M_Z$ . The branch favored by the CDF  $W$  mass measurement is  $M_1 > M_Z$  and here one can achieve a cancellation, i.e.,  $\Delta M_Z^{\text{exp}} + \Delta M_Z^{\text{th}} \simeq 0$ .

#### 4. Fitting $Z$ pole observables

To ensure that the modifications in the neutral sector respect the electroweak data, we perform a global analysis by fitting the precision electroweak LEP data [6] similar to the analysis done in Ref. [8]. There are two new parameters in the model:  $M_1$  and  $g_c$ . Unlike SM, the relation given in Eq. (8) no longer holds in the new

model. Thus, we also let  $\bar{g}_Y$  vary in our analysis. In the global fits we constrain the model by fitting the  $Z$  mass [5] and the other 19  $Z$  pole quantities [17], including the total  $Z$  decay width  $\Gamma_Z$ , the hadronic pole cross-section  $\sigma_{\text{had}}$ , ratios of branching ratios  $R$ , the various forward and backward asymmetries  $A_f$  and  $A_{\text{FB}}$ .

The partial decay width of the  $Z$  into a pair of fermions is given by [21,20,6,10]

$$\Gamma_{ff} = N_f^c \mathcal{R}_f \frac{M_Z}{12\pi} \sqrt{1 - 4\mu_f^2} \times \left[ |v_f|^2 (1 + 2\mu_f^2) + |a_f|^2 (1 - 4\mu_f^2) \right], \quad (20)$$

where  $N_f^c = (1, 3)$  for leptons and quarks,  $\mu_f = m_f / M_Z$ , and  $\mathcal{R}_f$  are the radiative corrections, which are given by

$$\mathcal{R}_f = \left( 1 + \delta_f^{\text{QED}} \right) \left( 1 + \frac{N_f^c - 1}{2} \delta_f^{\text{QCD}} \right), \quad (21)$$

$$\delta_f^{\text{QED}} = \frac{3\alpha}{4\pi} Q_f^2, \quad (22)$$

$$\delta_f^{\text{QCD}} = \frac{\alpha_s}{\pi} + a_1 \left[ \frac{\alpha_s}{\pi} \right]^2 + a_2 \left[ \frac{\alpha_s}{\pi} \right]^3 - Q_f^2 \frac{\alpha \alpha_s}{4\pi^2}, \quad (23)$$

where  $a_1 = 1.409$  and  $a_2 = -12.77$ . Here  $\alpha$  and  $\alpha_s$  are taken at the  $M_Z$  scale. Using the partial width, one can then compute the total  $Z$  decay width  $\Gamma_Z$ , and

$$\sigma_{\text{had}} \equiv \frac{12\pi}{M_Z^2} \frac{\Gamma_{ee}}{\Gamma_Z} \sum_{q \neq t} \frac{\Gamma_{qq}}{\Gamma_Z}, \quad R_\ell \equiv \frac{\Gamma_{\text{had}}}{\Gamma_{\ell\ell}}, \quad R_q \equiv \frac{\Gamma_{qq}}{\Gamma_{\text{had}}}. \quad (24)$$

The asymmetry parameters  $A_f$  and  $A_{\text{FB}}^{(0,f)}$  are defined in terms of the fermion couplings to the  $Z$  [6]

$$A_f = \frac{2v_f a_f}{v_f^2 + a_f^2}, \quad A_{\text{FB}}^{(0,f)} = \frac{3}{4} A_e A_f. \quad (25)$$

We compute  $\chi_i$  for each quantity via

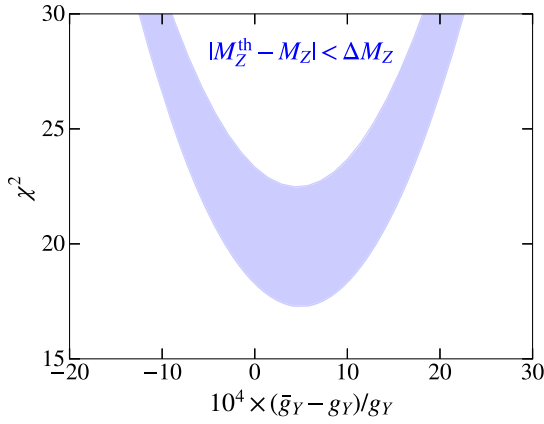
$$\chi_i = \frac{O_i^{\text{exp}} - O_i^{\text{th}}}{\delta O_i}, \quad (26)$$

where  $O_i^{\text{exp}}$ ,  $\delta O_i$  and  $O_i^{\text{th}}$  are the central value and uncertainty of LEP results, and the new model expectation in that order for the  $Z$ -pole quantities shown in the first column of Table 1. The total  $\chi^2$  is computed via  $\chi^2 = \sum_i \chi_i^2$ .

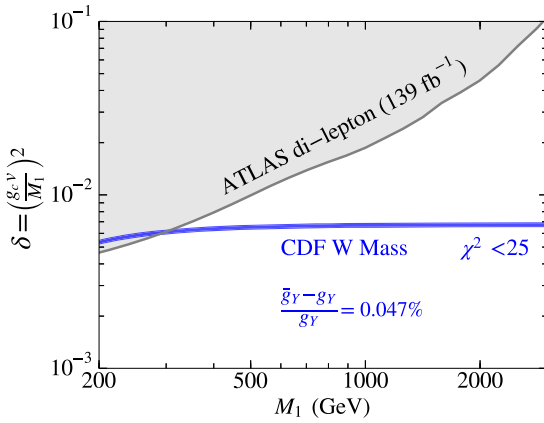
#### 5. Results

We carry out Monte Carlo scans in the parameter space spanned by  $M_1$ ,  $g_c$ , and  $\bar{g}_Y$ . In the calculation, we first fix the parameters  $g_2$ ,  $g_Y$ , and  $v$  to the SM values such that the canonical  $W$  and  $Z$  masses and  $G_F$  can be obtained using Eqs. (6)-(7). We then shift  $g_2$  to be  $\bar{g}_2$  according to Eq. (18). For each model point, we require that the mass matrix leads to the canonical  $Z$  mass within the uncertainty of  $\sim 2$  MeV [5], and compute the total  $\chi^2$  for the 19  $Z$  pole observables as shown in Table 1. We find that the total  $\chi^2$  has a strong dependence on  $\bar{g}_Y$ , as shown in Fig. 1. The model points with the least  $\chi^2$  are located in the region where  $\bar{g}_Y$  is about 0.047% larger than the SM value. We show one benchmark model point in Table 1 with  $\chi^2 \simeq 21$ .

Thus we fix  $\bar{g}_Y$  to be the value corresponding to the least  $\chi^2$ , which is  $\bar{g}_Y = g_Y (1 + 0.047\%)$ , and further carry out Monte Carlo scans in the 2D parameter space spanned by  $M_1$  and  $g_c$ . The results are given in Fig. 2 where the favored region for the CDF  $W$  mass measurement [1] and with  $\chi^2 < 25$  for fitting the  $Z$ -pole quantities and  $Z$  mass in the parameter space spanned by  $M_1$  and



**Fig. 1.** Total  $\chi^2$  of 19 Z pole observables versus  $\bar{g}_Y$  for the model points that reproduce the Z mass consistent with the experimental value within an error bar of 2.1 MeV [5].



**Fig. 2.** The favored region (blue) in the parameter space spanned by  $M_1$  and  $\delta = (g_c v / M_1)^2$  for the CDF W mass measurement [1] and  $\chi^2 < 25$  for fitting the 19 Z pole observables, where we fix the  $\bar{g}_Y = g_Y(1 + 0.047\%)$ . The constraints from di-lepton searches at ATLAS [22] are also shown.

$\delta \equiv (g_c v / M_1)^2$  are shown. It is found that the blue region can be well approximated by the following relation

$$\frac{1}{8} \left( \frac{g_c v}{M_1} \right)^2 \simeq \frac{\Delta M_W}{M_W}, \quad (27)$$

which is consistent with Eq. (10) and Eq. (19). Di-lepton final states which are from the heavy  $Z'$  boson at the LHC can be searched for by reconstructing their invariant mass. The excluded region by the di-lepton high mass resonance searches at ATLAS [22] is shown as the shaded region in Fig. 2. Thus the parameter space consistent with both the CDF W mass measurement and the ATLAS di-lepton constraints is  $M_1 \gtrsim 300$  GeV.

## 6. Discussion

Further, we note that the increase in the mass of the W boson would lead to a larger width for the W-boson from its current value of  $\Gamma_W = 2.085$  GeV to a new value of 2.091 GeV, which are consistent with the current uncertainty on the W width measurement,  $\Gamma_W = 2.085 \pm 0.042$  GeV [5]. On the other hand, the shift in the standard model electroweak correction to  $g_\mu - 2$  due to an increased value of the W-boson is totally negligible, i.e.,  $O(< 1) \times 10^{-11}$  compared to the standard model electroweak correction which is  $a_\mu^{\text{EW}}(1\text{-loop}) = 194.8 \times 10^{-11}$  [23]. One of the predictions of the model is the possibility of observation of the  $Z'$  boson consistent with CDF W mass anomaly which could be

searched for at the LHC. The electroweak data constrains the particle physics models including  $Z'$  physics; see e.g., [24–26]. We note in passing that in the usual standard Stueckelberg extension of the standard model [7,11,27–29] which here is the case  $g_c = 0$ , inclusion of matter in the  $U(1)_X$  hidden sector will make the hidden sector matter milli-charged. However, in the model we discussed here where  $M_2 = 0$ , the hidden sector matter does not develop a millicharge since the field  $C_\mu$  has no photonic content. This is due to the vanishing  $\mathcal{O}_{13}$  element in the rotation matrix in the  $M_2 = 0$  case, as shown in Appendix A.

## 7. Conclusion

In summary, it is shown that there exists a significant parameter space in the Stueckelberg extension of the standard model which can simultaneously explain the CDF W mass measurement, fit the LEP electroweak quantities well (with  $\chi^2$  as low as  $\chi^2 \simeq 20$ ) and avoid the ATLAS di-lepton search constraint [22]. The model is testable at the LHC because of the predicted existence of the Stueckelberg  $Z'_{St}$  boson.

### Note added

We note that prior to this work several papers have appeared since the new CDF W mass measurement became public. These are listed here [30–83].

### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

### Data availability

No data was used for the research described in the article.

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### Appendix A. The rotation matrix $\mathcal{O}$

The orthogonal matrix  $\mathcal{O}$  that diagonalizes the mass matrix of Eq. (12) when  $M_2 = 0$  can be parameterized by two angles as follows

$$\mathcal{O} = \begin{pmatrix} \cos \phi & \sin \phi & 0 \\ \sin \theta \cos \phi & -\sin \theta \cos \phi & \cos \theta \\ -\cos \theta \sin \phi & \cos \theta \cos \phi & \sin \theta \end{pmatrix}, \quad (A.1)$$

where

$$\cos \theta = \frac{\bar{g}_2}{\sqrt{\bar{g}_2^2 + \bar{g}_Y^2}}, \quad \sin \theta = \frac{\bar{g}_Y}{\sqrt{\bar{g}_2^2 + \bar{g}_Y^2}}, \quad (A.2)$$

and

$$\tan(2\phi) = \frac{2M_c M_{\bar{Z}}}{M_1^2 + M_c^2 - M_{\bar{Z}}^2}, \quad (A.3)$$

with

$$M_c = \frac{1}{2} g_c v, \quad M_{\bar{Z}} = \frac{1}{2} \sqrt{\bar{g}_2^2 + \bar{g}_Y^2} v. \quad (A.4)$$



The ratio of  $\mathcal{O}_{22}$  to  $\mathcal{O}_{32}$  is given by

$$\frac{\mathcal{O}_{22}}{\mathcal{O}_{32}} = -\tan\theta = -\frac{\bar{g}_Y}{g_2}. \quad (\text{A.5})$$

The angle  $\theta$  is effectively the weak mixing angle measured by the various forward-backward symmetries at LEP I. One may compare the result above with that in SM where the Z boson is given by  $Z = \cos\theta_W A^3 - \sin\theta_W B$ , and where  $\tan\theta_W = \frac{g_Y}{g_2}$ . Finally we note that since  $\mathcal{O}_{13} = 0$ ,  $C_\mu$  has no component that connects it to the photon, and thus the hidden sector matter if it exists would not receive a milli-charge as happens in the conventional StSM model.

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