Published for SISSA by Springer

Received: *November 17, 2022*
Accepted: *November 17, 2022*
Published: *November 29, 2022*

Erratum 2: Analytic thin wall false vacuum decay rate

Aleksandar Ivanov,[a,1] **Marco Matteini,**[b,a,2] **Miha Nemevšek**[b,a,3] **and Lorenzo Ubaldi**[c,d,4]

[a] *Faculty of Mathematics and Physics, University of Ljubljana, Jadranska 19, 1000 Ljubljana, Slovenia*
[b] *Jožef Stefan Institute, Jamova 39, 1000 Ljubljana, Slovenia*
[c] *INFN Sezione di Trieste, Via Bonomea 265, 34136 Trieste, Italy*
[d] *Institute for Fundamental Physics of the Universe, Via Beirut 2, 34014 Trieste, Italy*

E-mail: ace.aleksandar0@gmail.com, marco.matteini@ijs.si, miha.nemevsek@ijs.si, ubaldi.physics@gmail.com

Erratum to: JHEP03(2022)209

ArXiv ePrint: 2202.04498

JHEP11(2022)157

We correct the result in $D = 4$, including a contribution to the functional determinant missed in the first version. The additional term is present inside $\tilde{I}_2$, introduced in eq. (6.12) of [1], and contributes to the complete renormalized determinant. The $\tilde{I}_2$ integral is defined in the first line of eq. (6.14):

$$\tilde{I}_2 = \int_0^\infty \mathrm{d}\rho\, \rho^3 \left(V^{(2)2} - V_{\mathrm{FV}}^{(2)2}\right) \left(\frac{1}{\epsilon} + \gamma_E + 1 + \ln\left(\frac{\mu\rho}{2}\right)\right) . \tag{1}$$

It can be written as

$$\tilde{I}_2 = I_2 \left(\frac{1}{\epsilon} + \gamma_E + 1 + \ln\left(\frac{\mu R}{2}\right)\right) + \int_0^\infty \mathrm{d}\rho\, \rho^3 \left(V^{(2)2} - V_{\mathrm{FV}}^{(2)2}\right) \ln\frac{\rho}{R} . \tag{2}$$

Here, I_2 is the UV integral given in eq. (6.13) of [1], where $R = r/(\sqrt{\lambda}v) \simeq r_0/(\sqrt{\lambda}v\Delta)$ is the bubble radius, with $r_0 = 1$ in $D = 4$. The first term in (2) corresponds to the second

[1] https://orcid.org/0000-0002-6198-8118.
[2] https://orcid.org/0000-0001-6481-3025.
[3] https://orcid.org/0000-0003-1110-342X.
[4] https://orcid.org/0000-0002-9567-9719.

https://doi.org/10.1007/JHEP11(2022)157

line in eq. (6.14) in [1]. The second term was dropped in [1], however it also contains a contribution that scales as $1/\Delta^3$, and must therefore be counted as leading order.

To perform the integral with $\ln \rho/R$, we switch to the dimensionless variables used throughout [1],

$$r^3 \int_{-r}^{\infty} \mathrm{d}z \left(1+\frac{z}{r}\right)^3 \left(\tilde{V}^{(2)2} - \tilde{V}_{\rm FV}^{(2)2}\right) \ln\left(1+\frac{z}{r}\right) , \tag{3}$$

and expand the log term $\ln\left(1+\frac{z}{r}\right) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}\left(\frac{z}{r}\right)^n$. It is also useful to expand the second derivative of the potential squared in powers of Δ

$$\left(\tilde{V}^{(2)2} - \tilde{V}_{\rm FV}^{(2)2}\right) = \left(\tilde{V}^{(2)2} - \tilde{V}_{\rm FV}^{(2)2}\right)_0 + \Delta\left(\tilde{V}^{(2)2} - \tilde{V}_{\rm FV}^{(2)2}\right)_1 + \mathcal{O}(\Delta^2) , \tag{4}$$

where

$$\left(\tilde{V}^{(2)2} - \tilde{V}_{\rm FV}^{(2)2}\right)_0 = \frac{3}{4}\left(3\tanh^4\left(\frac{z}{2}\right) - 2\tanh^2\left(\frac{z}{2}\right) - 1\right) , \tag{5}$$

$$\left(\tilde{V}^{(2)2} - \tilde{V}_{\rm FV}^{(2)2}\right)_1 = -9\tanh^3\left(\frac{z}{2}\right) + 3\tanh\left(\frac{z}{2}\right) + 6 \equiv h_1(z) . \tag{6}$$

The zeroth order in (5), is an even function of z which vanishes exponentially at $z \to \pm\infty$. In that case we can then extend the lower limit of integration to $-\infty$ in (3), and the final contribution is Δ^2 suppressed.

The first order in (6) vanishes exponentially at $z \to \infty$, but goes to a constant at $z \to -\infty$:

$$h_1(z \to -\infty) = 12 . \tag{7}$$

This gives a contribution at the lower boundary of integration in (3). To compute it we integrate by parts:

$$\Delta r^3 \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \int_{-r}^{\infty} \mathrm{d}z \left(\frac{z}{r}\right)^n \left(1+\frac{z}{r}\right)^3 h_1(z) \tag{8}$$

$$= \Delta r^3 \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \left[z\left(\frac{z}{r}\right)^n \left(\frac{1}{n+1} + 3\frac{z}{r}\frac{1}{n+2} + 3\frac{z^2}{r^2}\frac{1}{n+3} + \frac{z^3}{r^3}\frac{1}{n+4}\right) h_1\Big|_{-r}^{\infty} \right. \tag{9}$$

$$\left. - \int_{-\infty}^{\infty} \mathrm{d}z\, z\left(\frac{z}{r}\right)^n \left(\frac{1}{n+1} + 3\frac{z}{r}\frac{1}{n+2} + 3\frac{z^2}{r^2}\frac{1}{n+3} + \frac{z^3}{r^3}\frac{1}{n+4}\right) \frac{\mathrm{d}h_1}{\mathrm{d}z} \right] . \tag{10}$$

In (10) we can extend the lower limit of integration to $-\infty$, as $\mathrm{d}h_1/\mathrm{d}z$ vanishes exponentially at $z \to \pm\infty$. We can drop the contribution from that line, as it is also Δ^2 suppressed. In (9), the contribution at ∞ vanishes, because of $h_1(z)$, while at $-r$ we have $h_1(-r) \simeq h_1(z \to -\infty) = 12$, so we have

$$\Delta r^3 \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \left[-12(-r)(-1)^n \left(\frac{1}{n+1} - 3\frac{1}{n+2} + 3\frac{1}{n+3} - \frac{1}{n+4}\right)\right] \tag{11}$$

$$= -12\Delta r^4 \sum_{n=1}^{\infty} \frac{1}{n}\left(\frac{1}{n+1} - 3\frac{1}{n+2} + 3\frac{1}{n+3} - \frac{1}{n+4}\right) = -\frac{3}{4} r_0 \left(\frac{r_0}{\Delta}\right)^3 . \tag{12}$$

To summarize, the leading order contribution to the second term in (2) is

$$\int_0^\infty \mathrm{d}\rho\, \rho^3 \left(V^{(2)2} - V_{\rm FV}^{(2)2}\right) \ln\frac{\rho}{R} = -\frac{3}{4} r_0 \left(\frac{r_0}{\Delta}\right)^3 (1 + \mathcal{O}(\Delta)) \,. \tag{13}$$

In $D = 4$, where $r_0 = 1$, this equals to $I_2/4$, so in the end we get

$$\tilde{I}_2 = I_2 \left(\frac{1}{\epsilon} + \gamma_E + \frac{5}{4} + \ln\left(\frac{\mu}{2\sqrt{\lambda} v \Delta}\right)\right) \,. \qquad (D = 4) \tag{14}$$

This corrects the second line of eq. (6.14) in [1]: the +1 is replaced by +5/4. In turn, this changes the last term of eq. (6.21) in [1] from $1/\Delta^3((45 - 4\pi\sqrt{3})/192)$ to $1/\Delta^3((27 - 2\pi\sqrt{3})/96)$. The final result in eq. (2.2) in [1] for $D = 4$, changes accordingly. The corrected one reads

$$\frac{\Gamma}{\mathcal{V}} \simeq \left(\left(\frac{S}{2\pi}\right) \frac{12}{e^{D-1}} \lambda v^2\right)^{D/2} \exp\left(-S - \frac{1}{\Delta^{D-1}} \frac{27 - 2\pi\sqrt{3}}{96}\right) , \qquad D = 4 \,. \tag{15}$$

The $D = 3$ result is unchanged.

Acknowledgments

We thank Andreas Ekstedt, Oliver Gould, and Joonas Hirvonen, for pointing out the contribution to the $\tilde{I}_2$ UV integral we had originally missed. MN was supported by the Slovenian Research Agency under the research core funding No. P1-0035 and in part by the research grants J1-3013, N1-0253 and J1-4389. MM was supported by the Slovenian Research Agency's young researcher program under grant No. PR-11241. LU and MN acknowledge the support of the COST action CA16201 — "Unraveling new physics at the LHC through the precision frontier", in particular for sponsoring the two short-term scientific missions at the IJS. LU was supported by the MIUR grant PRIN 2017FNJFMW.

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