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Addendum

Addendum: A dispersive analysis of $\eta' \to \pi^+\pi^-\gamma$ and $\eta' \to \ell^+\ell^-\gamma$

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Abstract In this addendum to Ref. [1] we show that the mismatch between the ρ–ω mixing parameter $\epsilon_{\rho\omega}$ as extracted from $\eta' \to \pi^+\pi^-\gamma$ and $e^+e^- \to \pi^+\pi^-$ can be resolved by including higher orders in the expansion in e^2 in the description of the $\eta' \to \pi^+\pi^-\gamma$ decay. We repeat the analysis in this extended framework and update the numerical results accordingly.

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1 Extended formalism

Following the notation from Ref. [1] throughout, the spectrum for $P \to \pi^+\pi^-\gamma$ can be expressed as

$$\frac{\mathrm{d}\Gamma(P \to \pi^+\pi^-\gamma)}{\mathrm{d}s} = 16\pi\alpha\Gamma_0|F_\pi^V(s)|^2\bigg|P(s)\big(1+\Pi_\pi(s)\big) - \frac{e^2F_{P\gamma\gamma}}{s} - \frac{g_{P\omega\gamma}}{g_{\omega\gamma}}\frac{\epsilon_{\rho\omega}-e^2g_{\omega\gamma}^2}{M_\omega^2 - s - iM_\omega\Gamma_\omega}\bigg|^2, \tag{1.1}$$

generalizing Eq. (D.14) in Ref. [1] by the next order in the expansion in e^2 (the sign convention is such that $g_{P\omega\gamma} < 0$). The most important change, numerically, concerns $\epsilon_{\rho\omega} \to \epsilon_{\rho\omega} - e^2g_{\omega\gamma}^2$ in the numerator of the ω propagator, corresponding to the photon contribution in $\epsilon_{\rho\omega}$ as defined in resonance chiral perturbation theory [2–4]. In our formalism, $\epsilon_{\rho\omega}$, determined from a fit to the bare cross section for $e^+e^- \to \pi^+\pi^-$, does not include this VP effect, in line with the definition in Ref. [5] (numerically, it evaluates to $e^2g_{\omega\gamma}^2 = 0.34(1)\times 10^{-3}$). This shift removes the tension observed between $\eta' \to \pi^+\pi^-\gamma$ and $e^+e^- \to \pi^+\pi^-$ in Ref. [1].

The coefficients appearing in Eq. (3.9) of Ref. [1] are generalized according to Eq. (1.1):

$$\begin{aligned}
\mathcal{A}_2 &= -\Gamma(\eta' \to \pi^+\pi^-\gamma) + 16\pi\alpha\int_{4M_\pi^2}^{M_{\eta'}^2}\mathrm{d}s\;\Gamma_0|F_\pi^V(s)|^2 \\
&\quad\times\left|\frac{g_{\eta'\omega\gamma}}{g_{\omega\gamma}}\frac{\epsilon_{\rho\omega}-e^2g_{\omega\gamma}^2}{M_\omega^2-s-iM_\omega\Gamma_\omega} + \frac{e^2F_{\eta'\gamma\gamma}}{s}\right|^2, \\
\mathcal{A}_1 &= 32\pi\alpha\int_{4M_\pi^2}^{M_{\eta'}^2}\mathrm{d}s\;\Gamma_0|F_\pi^V(s)|^2\,\mathrm{Re}\bigg[P_{\mathrm{ev}}(s)\big(1+\Pi_\pi^*(s)\big) \\
&\quad\times\left(\frac{g_{\eta'\omega\gamma}}{g_{\omega\gamma}}\frac{e^2g_{\omega\gamma}^2-\epsilon_{\rho\omega}}{M_\omega^2-s-iM_\omega\Gamma_\omega} - \frac{e^2F_{\eta'\gamma\gamma}}{s}\right)\bigg], \\
\mathcal{A}_0 &= 16\pi\alpha\int_{4M_\pi^2}^{M_{\eta'}^2}\mathrm{d}s\;\Gamma_0|F_\pi^V(s)|^2P_{\mathrm{ev}}^2(s)\big|1+\Pi_\pi(s)\big|^2.
\end{aligned} \tag{1.2}$$

In the following, we provide the updated numerical results when including the additional e^2 effects as given in Eq. (1.1), implemented in the fit via Eq. (1.2).

2 Numerical results

The updated fit parameters are collected in Table 1, Fig. 1, and Table 2. The main difference to the results presented in Ref. [1] is that the shift $\epsilon_{\rho\omega} \to \epsilon_{\rho\omega} - e^2g_{\omega\gamma}^2$ removes the tension between $e^+e^- \to \pi^+\pi^-$ and the $\eta' \to \pi^+\pi^-\gamma$ spectrum, markedly improving the quality of the combined fit.

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Table 1 Comparison of the fit outcome of the differential decay width in Eq. (1.1) to the BESIII $\eta' \to \pi^+\pi^-\gamma$ spectrum [6] of the binned maximum likelihood and minimum χ^2 strategies. The χ^2/dof is 1.30 and 1.31, respectively, with the one of the Likelihood method extracted by means of the approximation described in App. C of Ref. [7]

Quantity	Likelihood	χ^2
A [GeV^{-3}]	17.12(35)	17.09(32)
β [GeV^{-2}]	0.714(55)	0.723(45)
γ [GeV^{-4}]	−0.412(55)	−0.420(45)
$\epsilon_{\rho\omega} \times 10^3$	1.998(67)	1.997(54)
M_ω [MeV]	782.99(33)	783.00(27)

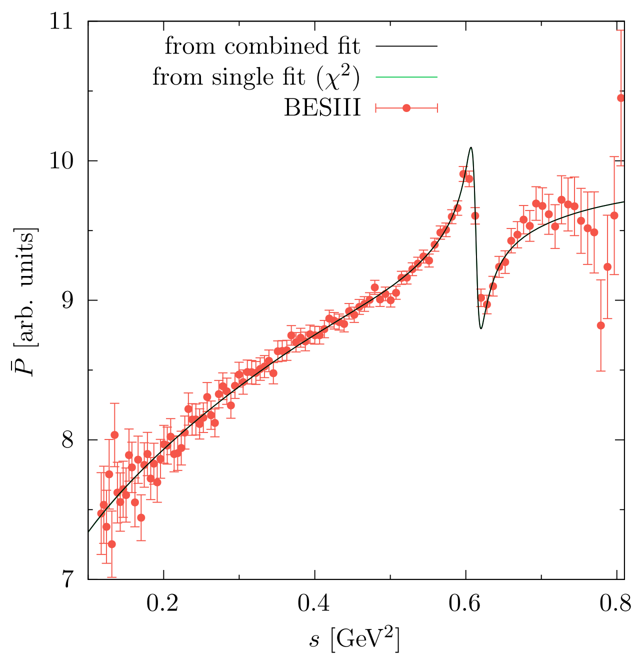

Fig. 1 Fit to the differential decay rate of $\eta' \to \pi^+\pi^-\gamma$ (individually or combined with the VFF). To highlight potential differences in the ρ–ω region, we show the associated function $\bar{P}$, as defined in Eq. (3.11) of Ref. [1], compared to the experimental data from BESIII [6]. The two fits cannot be distinguished on this scale

The updated results for the TFF are shown in Fig. 2 and Table 3. In particular, the prediction for the slope parameter

$$b_{\eta'} = 1.431(23)\,\text{GeV}^{-2} \tag{2.1}$$

is reduced by about 1σ, which traces back not to the change in $\epsilon_{\rho\omega}$ (which is marginal given the fact that the fit is dominated by $e^+e^- \to \pi^+\pi^-$), but to a stronger curvature in the polynomial $P(s)$ (the coefficient γ of the quadratic term increases by a factor 3).

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Table 2 Combined fit to several pion VFF data sets (BaBar, KLOE, CMD-2, SND) and $\eta' \to \pi^+\pi^-\gamma$ spectrum (BESIII) with overall χ^2/dof = 1.46. In the row for KLOE, the three values for M_ω refer to the combinations of the global KLOE ω mass and the corresponding mass shifts of the three underlying data sets from 2008, 2010, 2012, respectively

	χ^2/dof	M_ω [MeV]	A [GeV^{-3}]	β [GeV^{-2}]	γ [GeV^{-4}]	$\alpha_\pi \times 10^2$ [GeV^{-2}]	$\epsilon_{\rho\omega} \times 10^3$
BaBar	1.26	781.875(82)					
KLOE	1.61	781.65(12)					
		782.10(17)					
		781.84(27)				5.74(14)	2.007(10)
CMD-2	2.18	782.131(68)					
SND	2.16	781.457(97)					
BESIII	1.31	783.00(28)	17.10(32)	0.720(46)	−0.418(46)		

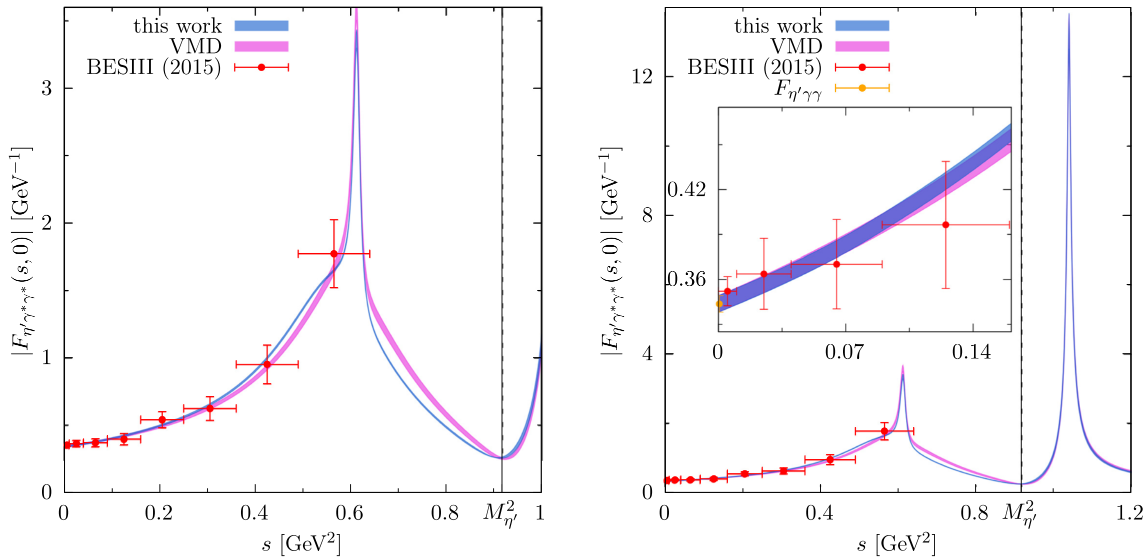

Fig. 2 Determination of the η' TFF in comparison to data from BESIII [8] (statistical and systematic errors added in quadrature) scaled with $F_{\eta'\gamma\gamma}$ and the VMD model from Ref. [1] for the ϕ resonance; for the kinematic range accessible in η' decays (left) and a larger time-like region including the ϕ resonance with inset magnifying the low-s region (right)

Table 3 Contributions from the various components of the TFF to the sum rules of the normalization and the slope parameter

	$(I=1)_{\epsilon_{\rho\omega}=0}$	$\Delta(I=1)_{\epsilon_{\rho\omega}\neq 0}$	$(I=0)^{\omega}_{\epsilon_{\rho\omega}=0}$	$\Delta(I=0)^{\omega}_{\epsilon_{\rho\omega}\neq 0}$	$(I=0)^{\phi}$	Total
Norm [%]	69.18(86)	−0.1388(19)	7.06(22)	−0.1397(47)	15.85(61)	91.9(1.1)
$b_{\eta'}$ [GeV^{-2}]	1.160(23)	0	0.1176(32)	0	0.1526(53)	1.431(23)

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