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Using Unruh temperature and generalized chemical potential as alternative pathway to determine the Hubble parameter

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ABSTRACT

Hubble constant is found relating velocities and distances of galaxies. Actually Cepheid stars and cosmic background radiation present two distinct values for the Hubble constant. Theoretically such a Hubble parameter naturally emerges from the scale factor variations considered in cosmological models. On the other hand Unruh effect relates temperature to accelerated observers. Such a relation plots a search for physical constraints aiming to justify the referred effect. Here we establish a relation between the Unruh temperature and that one from ensemble fugacity, which can be obtained from an generalized chemical potential. Then, we find a dependence on the quantities involved in the generalized chemical potential for the cosmological constant. Finally, we determine a new relation for the Hubble parameter.

1. Introduction

Hubble's law states that the velocity of galaxies increases with distance, implying an accelerated universe. The Hubble constant (H_0), found out with a high degree of confidence, is the parameter that relates velocities and distances. Currently there are two distinct H_0 values for different physical components — either from the Cepheid stars [1,2] or from the cosmic background radiation [3,4]. Ultimately, H_0 is an extremely small percentage variation over time for the distance between two isotropic observers what, in some sense, should be expected. The Hubble parameter naturally emerges [5] from the scale factor variations considered in cosmological models.

In parallel, the thermal character of a quantum field emerges in accelerated observers, to such an extent that it is possible to establish a relation between temperature and acceleration — the Unruh effect. Regarding the origin of this phenomenon, several articles [6–10] have focused on the relation between accelerated particles and the Unruh effect; while others [11–13] have focused on the trajectories of these particles and the Unruh temperature. These works highlight the search for physical restrictions in order to justify that effect. Takagi [14], for instance, has shown that the pure state, which is the vacuum from the viewpoint of an inertial observer, is a canonical ensemble with a characteristic temperature proportional to the acceleration of an observer; he also has shown that accelerated detectors should observe Planckian spectrum. Furthermore, it is proved in [15] that the vacuum state induces the thermalization of an accelerated system. All these conjectures, corroborated by some authors [16–18], point to a relation between the fugacity of the ensemble and the Unruh effect, whose temperature is

$$T_{\mathrm{U}} = \frac{\hbar a}{2\pi c k_{\mathrm{B}}} \quad (1)$$

where $\hbar$, c and k_{B} are the reduced Planck constant, the speed of light and the Boltzmann constant, respectively.

Taking into account the bounds mentioned in the previous paragraph and considering the visible universe as a system of point particles, it is expected that the velocities inherent in Hubble's law could determine, in a thermodynamical rhumb line, the configurational temperature of that system. Although conjectural, the expectation can be carried out in order to establish *ad doc* a relation between the Unruh temperature and that one from the ensemble fugacity, which can be obtained from the generalized chemical po-

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tential proposed by [19]. Recent studies [20,21] have shown that it is possible to find other paths that lead to the understanding of cosmological models and its measurable parameters.

The present work proposes the Unruh temperature as the quantity relating thermal phenomena that occur in the universe and the Hubble parameter using the latter mentioned chemical potential. Below we briefly present the generalized chemical potential. Then we enumerate the mathematical development from the point of view of the physical framework involved in it: starting by relating the generalized chemical potential with the Unruh temperature; followed by relating it to the universe scale factor; ending by relating the Hubble parameter to the generalized chemical potential; and to accomplish a brief analysis on the H_0 values.

2. The generalized chemical potential

Recent paper [19] has used a generalized Saha equation based on a conformal-like fugacity in order to describe a binary chemical reaction by means an extended exponential function, which determines the phenomenological energy distribution of the particles involved in the reaction. In addition to provide an interdependent chemical potential and to make possible the study of chemical reactions out of balance, the generalization ranges over possibilities for their use — e.g., the study of the Hubble constant in connection with the Unruh effect.

Non-Gaussian thermostatistical considerations about the Saha equation have brought up a generalized chemical potential named q-potential

$$\mu_q = \mu_{\mathtt{std}} + (1-q)\frac{\mu_A\,\mu_B}{k_{\rm B}\,T} \tag{2}$$

where T is the ensemble temperature, $1-q$ is the deviation from standard statistics, and $\mu_{\mathtt{std}} = \mu_A + \mu_B - \mu_C$ is the standard chemical potential, with μ_A, μ_B and μ_C being the chemical potentials for the generic particles A, B and C, respectively, involved in a binary chemical reaction like

$$A + B \longrightarrow C + [\text{energy}].$$

Notice that the q-potential (Eq. (2)), μ_q, changes rapidly with q and the temperature (see Fig. 1 in [19]). For a fixed temperature, μ_q depends on $\mu_A\,\mu_B$. Besides, whenever $q \to 1$ or $k_{\rm B}\,T \gg \mu_A\mu_B$ then $\mu_q \to \mu_{\mathtt{std}}$. Moreover, whenever $q=1$ then $\mu_{\mathtt{std}} = 0$ recovering the classical statistics of equilibrium. In this way, if q represents the statistics of the particles involved in the reaction, $q=1$ represents the standard thermostatistics [19].

The q-potential opens new possibilities as to the application in new physical conjectures. One possible application, proposed here, occurs in the context of the Unruh effect.

3. Unruh temperature and the generalized chemical potential

Let consider that presumed particles in thermal equilibrium mentioned in the Unruh effect are generic A and B particles, the q-potential measured in the accelerated frame must be, ensuring $T_{\rm U} \geq 0$,

$$|(1-q)\,\bar{\mu}| = k_{\rm B}\,T_{\rm U} = \frac{\hbar\,a}{2\,\pi\,c} \tag{3}$$

with

$$\bar{\mu} \equiv \frac{\mu_A\,\mu_B}{\mu_q - \mu_{\mathtt{std}}} = \frac{k_{\rm B}\,T}{1-q} \tag{4}$$

the fraction of the reduced chemical potential.

Equation (3) implies that the gravity determines the q parameter and, therefore, the distribution profile of the particle energy states as well. In the opposite sense, the latter equation leads to more consequences, neglecting the temperature positiveness: i) negative acceleration for $q>1$ and $\bar{\mu}>0$; ii) positive acceleration otherwise; iii) acceleration vanishes when $q \to 1$; and iv) diverges when $\mu_{\mathtt{std}} \to \mu_q$.

4. Hubble parameter and the generalized chemical potential

Now consider a scale factor $\alpha(t)$ that parameterizes the distances as a function of the current distances r_0 in the universe. Then

$$r(t) = \alpha(t)r_0 \tag{5}$$

where $\alpha(t_0) = 1$ so that $r(t_0) = r_0$, and whose n-th derivative is

$$\frac{r^{(n)}}{r} = \frac{\alpha^{(n)}}{\alpha}\,.$$

Then we obtain a relationship between velocity and distance with a more straining Hubble law

$$v(t) = H(t)\,r(t) \tag{6}$$

with $H(t) = \dot{\alpha}(t)/\alpha(t)$. Hence that, the acceleration is

$$a(t) = \left[\dot{H}(t) + H^2(t)\right] r(t) \tag{7}$$

Neglecting the temperature positiveness and using equation (3) we obtain

$$\frac{a}{r} = \frac{\ddot{\alpha}}{\alpha} = \frac{2\,\pi\,c}{\hbar\,r_0}\,\frac{(1-q)\,\bar{\mu}}{\alpha} \tag{8}$$

a relation between the physical quantities of both analyses. From equations (7) and (8) we have

$$H^2 = \frac{2\,\pi\,c}{\hbar\,r_0}\,\frac{(1-q)\,\bar{\mu}}{\alpha} - \dot{H} \tag{9}$$

which constrains, for non-complex H, the limit

$$\dot{H} < \frac{2\,\pi\,c}{\hbar\,r_0}\,\frac{(1-q)\,\bar{\mu}}{\alpha} \tag{10}$$

As the Hubble parameter has two distinct values, the scale factor should have equally different values, which is possible with the equation (9) above.

5. Concerning the cosmological model

Lastly, consider the Robertson-Walker cosmological model for a homogeneous and isotropic universe for which

$$ds^2 = c^2\,dt^2 + \alpha^2(t)\left[\frac{dr^2}{1-\kappa\,r^2} + r^2(d\theta^2 + \sin^2\theta\,d\phi^2)\right] \tag{11}$$

with κ being the curvature of spacetime. This cosmological model in tune with Einstein field equations provides the Friedmann-Lemaître equations with the cosmological constant Λ in it

$$\frac{\dot{\alpha}^2}{\alpha^2} = \tfrac{8\pi G}{3}\rho - \tfrac{\kappa\,c^2}{\alpha^2} + \tfrac{\Lambda\,c^2}{3} \tag{12}$$

$$\frac{\ddot{\alpha}}{\alpha} = -\tfrac{4\pi G}{3}\rho - \tfrac{4\pi G}{c^2}\,p + \tfrac{\Lambda\,c^2}{3} \tag{13}$$

with (7) it gets

$$\frac{\ddot{\alpha}}{\alpha} - \frac{\dot{\alpha}^2}{\alpha^2} = \dot{H} = -4\pi\,G\left(\rho + p/c^2\right) + \frac{\kappa\,c^2}{\alpha^2} \tag{14}$$

This equation shows that the possible time variation from the Hubble parameter, $\dot{H}$, does not explicitly depend on Λ. It relates the evolution of Hubble parameter with the space-time ones $(\rho,\ p,\ \kappa,\ \alpha)$, and it also imposes that, since $4\pi G\left(\rho + p/c^2\right)$ is defined positive, $\dot{H} < 0$ whenever $\kappa < 0$, $\kappa = 0$ or $\alpha > \left[4\pi G\left(\rho/c^2 + p/c^4\right)\right]^{-1/2}$; and $\dot{H} > 0$ whenever $\kappa > 0$ and $\alpha < \left[4\pi G\left(\rho/c^2 + p/c^4\right)\right]^{-1/2}$.

Equations (10) and (14) impose

$$-\left(\rho/c^2 + p/c^4\right) + \frac{\kappa}{4\pi G\alpha^2} \leq \frac{(1-q)\,\bar{\mu}}{2G\hbar c r_0 \alpha} \tag{15}$$

as an upper limit for material parameters (ρ and p) and the curvature (κ) of space-time. Besides, equations (8) and (13) yield for the cosmological constant

$$\frac{\Lambda\, c^2}{3} = 4\pi G\left(\frac{\rho}{3} + \frac{p}{c^2}\right) + \frac{2\pi\, c}{\hbar\, r_0}\,\frac{(1-q)\,\bar{\mu}}{\alpha} \tag{16}$$

Hence, our proposal in establishing a relation between the Unruh temperature and the q-potential leads to achieve a cosmological constant dependent on the statistical parameter q.

Finally, from equations (12) and (16) in addition to $H(t) = \dot{\alpha}(t)/\alpha(t)$, we determine the Hubble parameter to be

$$H^2 = 4\pi G\left(\rho + \frac{p}{c^2}\right) + \frac{2\pi\, c}{\hbar\, r_0}\,\frac{(1-q)\,\bar{\mu}}{\alpha} - \frac{\kappa\, c^2}{\alpha^2\, r_0^2} \tag{17}$$

The reader should pay attention to the novel dependency of that parameter with $(1-q)\bar{\mu}$. Hubble parameter makes its dependence on the generalized chemical potential explicit.

By substituting $\hbar = 1.055 \times 10^{-34}\,\mathrm{J\,s}$, $G = 6.674 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$, $c = 299\,792\,458\,\mathrm{m\,s^{-1}}$ and $r_0 = 4.4 \times 10^{26}\,\mathrm{m}$ in the equation (17) we obtain

$$H^2 \approx 10^{-9}\rho + 10^{-26}p + 10^{16}\,\frac{(1-q)\bar{\mu}}{\alpha} - 10^{-36}\,\frac{\kappa}{\alpha^2} \tag{18}$$

Considering the magnitudes of the terms involved in the above equation, we notice that curvature is the least important parameter followed by pressure and density. It is also observed in such an equation that the most relevant parameter is definitely the one determined by the statistics.

6. Regarding the different H_0 values

The immediate question about this renewed form for the Hubble parameter should be about the steady discrepancy between the H_0 values obtained from the two different sources of determination — Cepheid variable stars and cosmic microwave background [1–4]. The above equation provides an alternative way to verify such a discrepancy. So we should do,

$$\Delta\left(H^2\right) \approx 10^{-9}\,\Delta\rho + 10^{-26}\,\Delta p + 10^{16}\,\Delta\left[\frac{(1-q)\bar{\mu}}{\alpha}\right] - 10^{-36}\,\Delta\left(\frac{\kappa}{\alpha^2}\right) \tag{19}$$

where Δ represents the difference between measurements obtained from those two sources. Assuming that contemporary measurements are made from different sources, we can neglect curvature, pressure and density due to the magnitude of the coefficients. In addition, the current scale factor is the unity as defined in equation (5). And whatever the spacetime curvature, it must be the same for both sources. All these considerations result

$$\Delta\left(H^2\right) \approx 10^{16}\,\Delta\left[(1-q)\bar{\mu}\right] \tag{20}$$

Using 74.03 km/s/Mpc and 67.4 km/s/Mpc, for Cepheids [1] and CMB [3] respectively, we compute $\Delta\left(H^2\right) = H^2_{\mathrm{Cepheids}} - H^2_{\mathrm{CMB}} = 937.7\,(\mathrm{km/s/Mpc})^2$, or $\Delta\left(H^2\right) \sim 10^{-36}\,\mathrm{s}^{-2}$. Therefore

$$\Delta\left[(1-q)\bar{\mu}\right] = \left[(1-q)\bar{\mu}\right]_{\mathrm{Cepheids}} - \left[(1-q)\bar{\mu}\right]_{\mathrm{CMB}} \sim 10^{-52}$$

implying that the statistics determining each source are practically the same but sufficiently different to provide the discrepancy in the Hubble constant values.

7. Conclusions

In summary, we use the function of temperature and acceleration presents in the Unruh effect in order to establish a relation between those physical quantities and the fugacity arising from the generalization of the Saha equation. The Friedmann-Lemaître equations provide $\ddot{\alpha}$ and H^2. Then, we find a dependence on the quantities involved in the q-potential for the cosmological constant. We determine a novel relation for the very Hubble parameter and we claim that the discrepancy between the current H_0 values may be due to imperceptible differences in the involved thermostatistics.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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