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Page curve and island in EGB gravity

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Abstract

In this paper, the information paradox on lower-dimensional Gauss–Bonnet gravity, known as a three-dimensional EGB black hole, is studied using the quantum extremal island approach. For that, we connect an auxiliary flat bath system to this timelike singularity spacetime and estimate the entropy of hawking radiation in its asymptotic regions when gravity is weak. The addition of island areas to the entanglement wedge of radiation causes its entropy to obey the Page curve, as shown. The quantum extremal surface, or island boundary, is outside the horizon. This yields a time-independent equation for Hawking radiation fine-grained entropy compatible with the appropriate Page curve. We also discover the modifications to this entropy and Page time.

1. Introduction

From the perspective of quantum gravity theory, investigating the information loss paradox has been one of the essential challenges [1]. Hawking radiation has the same properties as thermal radiation, implying that the entanglement entropy outside the black hole is increasing monotonically. Quantum mechanics, however, mandates that the entanglement entropy at the end of the evaporation be zero because it must be the pure state. The so-called Page curve [2,3] describes the evolution of entanglement entropy over time. As a result, the information loss paradox is transformed into a challenge of reproducing the Page curve for Hawking radiation's entanglement entropy. By incorporating a time scale known as the Page time t_p, the Page curve effectively an-

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swers the problem of information loss paradox. The information loss paradox can be explained as follows regarding the Page curve. The von Neumann entropy of quantum fields on the area R outside the black hole identifies the fine-grained entropy of Hawking radiation. Assuming that the state on the whole Cauchy slice is pure, the fine-grained entropy of radiation S(R) = S(R_c), where S(R_c) is the fine-grained entropy of the black hole subsystem. Because the overall system consists of the black hole Plus radiation, the region R_c should correspond to the fine-grained entropy of the black hole if the region R corresponds to the fine-grained radiation entropy. This implies that S(R) should always satisfy the property S(R)≤S_{BH}, where S_{BH} is the black hole subsystem's coarse-grained entropy. However, it has been discovered that $S(R) > S_{BH}$ immediately after the Page time t_p, resulting in the contradiction.

The Page curve has recently been postulated to develop from the action of islands [4–7]. The density matrix of R is generally determined by taking the partial trace across the states in R_c, which is the complementary area of R when considering the state of the Hawking radiation as that in a region R outside the black hole. Recent research [8–27] has revealed that specific supplementary regions known as islands contribute to the entropy of Hawking radiation, with their borders defined as surfaces known as quantum extremal surfaces (QES). This indicates that some non-trivial QES arises in spacetime at Page time, canceling out S(R)'s time-dependency and resulting in a saturated fine-grained entropy of Hawking radiation. The fine-grained entropy of Hawking radiation, including the island contributions, is [5]

$$S(R) = \min \quad \text{ext}\left[\frac{Area(\partial I)}{4G_N} + S_{\text{matter}}(I \cup R)\right] . \tag{1}$$

The island rule was first postulated due to a conjectured quantum extremal surface prescription, and it was recently deduced using the replica approach for the gravitational path integral. When the replica trick [28–30] is used to gravitational theories, only the boundary conditions of the replica geometries can be fixed, and new saddles, where bulk wormholes join distinct copies of spacetime, must be considered. As these new saddles are known, replica wormholes lead to islands [31][8]. The partition function of the geometry is dominated by the one with the lowest entanglement entropy in the semi-classical gravity limit when with replicas. The replica trick for gravitational theories yields the same formula (1) as the quantum extremal surface prescription in this fashion.

Higher-curvature gravity theories continue to pique interest, partly because most approaches to quantum gravity suggest that such corrections modify the Einstein–Hilbert action and partly because they provide a new arena for testing our understanding of classical gravity in strong gravitational fields. Lovelock theories [32], which are the most general theories formed from the Riemann curvature tensor and preserve second-order equations of motion for the metric, are the most promising and most investigated options. For D < 5, however, they have the feature that their additional contributions to the action are either topological or zero. The cosmological and Einstein–Hilbert factors are combined in this theory, which includes new corrections for each odd spacetime dimension above four. The Gauss-Bonnet term is the first new correction of this type

$$\mathcal{G} = R_{\alpha\beta\gamma\tau}R^{\alpha\beta\gamma\tau} - 4R_{\alpha\beta}R^{\alpha\beta} + R^2 , \tag{2}$$

has five or more dimensions of activity. Significantly, for D < 5, these new contributions are either topological or identically zero, establishing Einstein's theory as the most general second-order metric theory of gravity in four dimensions. Recently, a new idea [33] for bypassing this limitation for Gauss–Bonnet gravity (the most basic of the Lovelock theories) has piqued interest. Constructing D = 4 and D = 3 variants of this theory is feasible by treating the spacetime dimension as a theory parameter and rescaling the Gauss–Bonnet coupling (α). Even though the initial approach involved taking limits of solutions to the field equations, which raised several consistency concerns [34–36], consistent actions in D < 5 can be obtained without making any assumptions about real solutions or extra dimensions [37,38]. This method is a generalization of one that was used a long time ago to reach a D → 2 limit of general relativity [39], and it is consistent with a dimensional reduction strategy recently proposed [40,41], as long as the internal space is flat. The resulting scalar-tensor theory of gravity is as follows

$$S = \int d^D x \sqrt{-g} \left[R - 2\Lambda + \alpha \left(\phi \mathcal{G} + 4 G^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi - 4(\partial\phi)^2 \Box\phi + 2((\nabla\phi)^2)^2 \right) \right] , \quad (3)$$

where $\mathcal{G}$ is Gauss-Bonet term given in equation (2) and $G_{\alpha\beta}$ is the Einstein tensor. In D < 4 dimensions, the Gauss–Bonnet term vanishes identically. There is no scalar propagating degree of freedom for D = 3 [42]. Also, at least one of Lovelock's Theorem assumptions, such as metricity [40] or temporal diffeomorphisms [43], must be disregarded to produce a local action in $D = 3$. If not, relying on the same fiducial dimensions-permeating symmetries as those used in [33] is necessary. If one considers spacetime conformally flat, it is demonstrated in [34] that most of the action will have a clearly defined lower D limit. The repercussions of lower-dimensional Gauss–Bonnet gravity, including black holes, star-like solutions, radiating and collapsing solutions, cosmic solutions, black hole thermodynamics, and other physical implications, have been studied extensively. So far, all black hole solutions have been for that are spherically symmetric metrics.

In general, the entropy formula for higher derivative gravity may be written as

$$S_{\text{total}} = S_{\text{gravity}} + S_{\text{matter}} , \quad (4)$$

where S_{gravity} and S_{matter} are the gravitational and matter contributions to total entanglement entropy, respectively. In higher derivative gravity theories, the first component in equation (4) may be determined using the Dong formula [44] while the S_{matter} term can be calculated using the Cardy formula [45,46]. Then we must extremize the overall entanglement entropy concerning the location of the island surfaces. If there are several surfaces, we must select the surface with the smallest area from among those surfaces. The authors of [19] constructed Page curves of Schwarzschild black holes in the presence of higher derivative components that are $\mathcal{O}(R^2)$. Following [19], [21] calculates the Page curves of an eternal Reissner-Nordström black hole in four dimensions in the presence of $\mathcal{O}(R^2)$ terms as discussed in [19] and in Einstein-Gauss-Bonnet gravity [47]. Throughout the paper, we set $l = 1$.

2. Preliminaries

2.1. BTZ black hole

The metric for BTZ black hole is

$$dS^2 = -\left(r^2 - r_+^2\right) dt^2 + \left(\frac{dr^2}{r^2 - r_+^2}\right) + r^2 d\phi^2 , \quad (5)$$

where r_+ is the location of the horizon. The thermodynamics of BTZ well studied in literature and can be written as

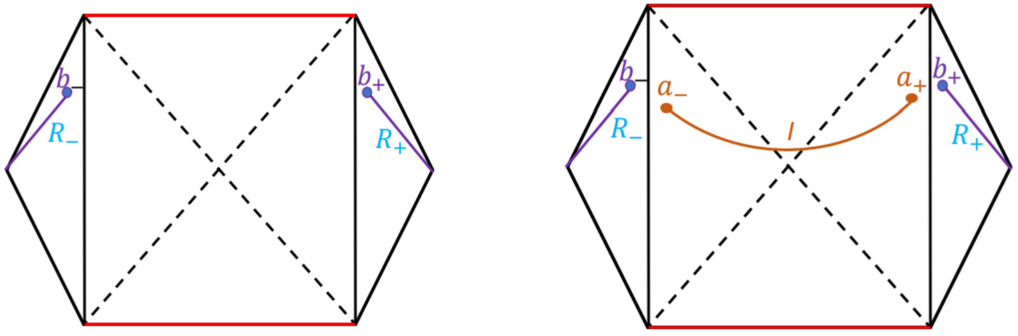

Fig. 1. Penrose diagrams of the BTZ black hole (left) and one with an island I (right). The Hawking radiation-identified area R is divided into two portions, R_+ and R_-, in the right and left wedges, respectively. R_+ and R_- boundary surfaces are denoted by b_+ and b_-, respectively. The island is located between the right and left wedges. At a_+ and a_-, I's boundaries are situated.

$$M = \frac{m}{8} = \frac{r_+^2}{8}\,, \qquad T = \frac{f'}{4\pi} = \frac{r_+}{2\pi}\,, \qquad \Omega = 0\,,$$
$$S = \frac{\pi r_+}{2}\,, \qquad P = \frac{1}{8\pi}\,, \qquad V = \pi r_+^2\,. \tag{6}$$

The Kruskal extension of BTZ spacetime with identification as

Right Wedge: $U = -e^{-k(t-r_*)}$ and $V = e^{k(t-r_*)}$
Left Wedge: $U = e^{-k(t-r_*)}$ and $V = -e^{k(t-r_*)}$

where $k = \frac{2\pi}{\beta}$ is the surface gravity, and r_* denotes the tortoise coordinate and given by

$$r_* = \int \frac{dr}{f(r)} = \frac{1}{2r_+}\left[\log\left(\frac{|r-r_+|}{r+r_+}\right)\right]\,. \tag{7}$$

Using the linking auxiliary baths [49][22], we can now make the BTZ spacetime boundary transparent. We can treat the bath as a flat Minkowski spacetime devoid of gravitational effects and suppose it is in thermal equilibrium with the black hole. In terms of Kruskal coordinate, the metric can be written as

$$dS^2 = \Omega^{-2} dU dV\,, \tag{8}$$

with $\Omega = \frac{r_+}{r+r_+}$. If we ignore the island's contribution, eq. (1) takes the typical form $S(R) = S_{gen}(R)$, implying that we must compute the von Neumann entropy of quantum fields on $R = R_+ \cup R_-$.

2.2. *Island and the information recovery*

To begin, we calculate the entanglement entropy of radiation in the absence of islands. In this case, the area term does not affect the fine-grained gravitational entropy (1). Only the right and left wedges of the Penrose diagram have Hawking radiation zones, and the borders of these entanglement regions are marked by b_+ and b_-, respectively (Fig. 1). As a result, the generalized entanglement entropy is made up entirely of the finite quantum matter von Neumann entropy provided by

$$S_{\rm gen} = \frac{c}{3}{\rm Log}[d(b_+, b_-)] \, . \tag{9}$$

The expression for the distances is already calculated in [20]. So we have

$$S_{\rm gen} = \frac{c}{6}{\rm Log}\left[\frac{4(b^2 - r_+^2)}{r_+^2}{\rm Cosh}^2(2r_+ t_b)\right] , \tag{10}$$

c is the central charge. At late time i.e., $t \to \infty$ then ${\rm Cosh}(r_+ t) \sim e^{r_+ t}$, the above equation can be approximated as

$$S(R) \sim \frac{c}{3} r_+ t \, . \tag{11}$$

We can see from equation (11) that in the absence of an island surface, the entanglement entropy of the Hawking radiation increases linearly with time and becomes infinite at late times, resulting in the BTZ black hole's information paradox. Next, we'll demonstrate that during late periods, after the Page time, an island appears, and the Hawking radiation's entanglement entropy remains constant and dominates in the presence of an island surface.

To check for the presence of the island in the BTZ+bath system, we assume that the distance between the right and left wedges is large enough that the s-wave approximation is valid and the von Neumann entropy of quantum matter in the overall region for the union of the radiation and island is large enough.

$$S(R \cup I) = \frac{c}{6}\log\left[\frac{d(a_+, a_-)d(b_+, b_-)d(a_+, b_+)d(a_-, b_-)}{d(a_+, b_-)d(a_-, b_+}\right] , \tag{12}$$

where $a_\pm = (\pm t_a, a)$. Because large distances between two wedges are assumed, it follows that

$$d(a_+, a_-) \simeq d(b_+, b_-) \simeq d(a_\pm, b_\mp) \gg d(a_\pm, b_\pm) \, . \tag{13}$$

As a result, the sum of the entanglement entropies between the $R \cup I$ in both left and right wedges can be used to estimate the entanglement entropy of the entire system.

The expression of the generalized entropy will be

$$\begin{aligned} S_{\rm gen} &= \frac{\pi a}{G_N} + \frac{c}{3}\log[d(a_+, b_+)] , \\ &= \frac{\pi a}{G_N} + \frac{c}{6}\log\left[\frac{(a+r_+)(b+r_+)}{r_+}\left[\left(\frac{a-r_+}{a+r_+}\right) + \left(\frac{b-r_+}{b+r_+}\right)\right.\right. \\ &\quad \left.\left. -2\left(\sqrt{\left(\frac{b-r_+}{b+r_+}\right)}\sqrt{\left(\frac{a-r_+}{a+r_+}\right)}{\rm Cosh}(r_+(t_a - t_b)\right)\right] . \end{aligned} \tag{14}$$

Now, extremizing the $S_{\rm gen}$ (14) with respect to t_a and equating to zero results in $t_a = t_b$. Substituting this value to the equation (14) and then extremizing with respect to a i.e., $\frac{\partial S_{\rm gen}}{\partial a} = 0$ reveals that

$$a = r_+ + \frac{1}{2r_+}\left(\frac{cG_N}{6\pi}\right)^2 + \dots \, . \tag{15}$$

Substituting this value in $S_{\rm gen}$ we get

$$S_{\rm gen} = 2S_{BH} - \frac{2c}{3}\log(S_{BH}) + \left(\frac{c}{6}\right)^2 \frac{1}{4S_{BH}} + \cdots \, . \tag{16}$$

[1] The above phrase contains a few distinguishing features. The statement includes universal corrections to the black hole's Hawking entropy and the leading item S_{BH}. Because these adjustments are quantum gravity signals, this is a pleasant surprise. The mutual information interpretation of the fact that the above result generates the proper Page curve and therefore answers the information loss problem is stunning.

2.3. Page curve

Entanglement entropy increases linearly with time in the absence of an island surface, according to equation (11), and is constant at late times, according to equation (16). The Page time island surface forms, saturating the linear expansion of entanglement entropy and reaching a constant value, i.e., double the Bekenstein-Hawking entropy of the BTZ black hole, and we have the Page curve of the BTZ black hole.

- **Page time** (t_p)**:** Page time is defined as the point at which the Hawking radiation's entanglement entropy begins to decline to zero for an evaporating black hole and achieves a constant value for an eternal black hole,

$$t_p = \frac{6S_{BH}}{cr_+} + \cdots = \left(\frac{6\beta}{c\pi}\right) S_{BH} - \cdots \tag{17}$$

  The familiar Page time t_p is the leading component in the above equation. Sub-leading corrections make up the rest of the words.
- **Scrambling time:** $(t_{\mathbf{scr}})$**:** The time interval during which we retrieve the information sent into the black hole in the form of Hawking radiation is known as scrambling time. It's also time for information to reach the island's surface. If we wish to transfer data from the cutoff surface $r = b$ to the black hole, the time it takes to get to the island surface $r = a$ is:

$$\begin{aligned} t_{\text{scr}} &\equiv |r_*(b) - r_*(a)| \\ &= \frac{1}{2r_+}\log\left(\frac{b-r_+}{b+r_+}\frac{a+r_+}{a-r_+}\right) \\ &\simeq \frac{\beta}{2\pi}\log\left(\frac{r_+}{cG_N}\right) + \cdots . \end{aligned}$$

  In the above calculation, we have set the order of b is same as that of r_+, fix the island location as in equation (15), and by assuming the $S_{\text{BH}} \simeq \frac{r_+}{G_N}$. We have an expression for Scrambling time as

$$t_{\text{scr}} \simeq \frac{\beta}{2\pi}\log\left(S_{\text{BH}}\right) \tag{18}$$

3. EGB gravity in lower dimensions

Let's start with the action that describes the Gauss-Bonnet (GB) gravity in the lower dimension from eq. (3) and is given by

[1] While approximating (16) we have used $b >> r_+$.

$$S=\int d^3x\sqrt{-g}\left[R-2\Lambda+\alpha\left(\phi\mathcal{G}+4G^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi-4(\partial\phi)^2\Box\phi+2((\nabla\phi)^2)^2\right)\right], \quad (19)$$

with $\mathcal{G}=R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}-R_{\alpha\beta}R^{\alpha\beta}+R^2$, in addition having extra scalar field ϕ and the GB coupling α. Following [38,48], some extra term has to be added in action to reduce the complications,[2] and the term is

$$S_\lambda=-2\int d^3x\sqrt{-g}\left[\lambda e^{-2\phi}(R+6(\partial\phi)^2)+3\lambda^2e^{-4\phi}\right], \quad (20)$$

where the interior space's curvature is represented by the λ. We investigate the equations of motion resulting from the action (19) along with the extra term (20) and the line element for deriving BH solutions to the GB gravity; we start with the line element

$$ds^2=-f(r)dt^2+\frac{1}{h(r)f(r)}dr^2+r^2\left(d\phi^2-\frac{J^2}{2r^2}dt\right)^2, \quad (21)$$

where J is some constant. We have three radial coordinate dependence functions $f(r), h(r)$ and the scalar field $\phi(r)$. So, varying the action (eq. (19) along with eq. (20)), we have three different equations of motion. We are interested in BTZ-like solutions, so we set $h(r)=1$. From [38,48],

1. For the case "ϕ=constant" and "$\lambda=0$":

   This choice results in

$$f(r)=-m+\frac{r^2}{l^2}+\frac{J^2}{4r^2} \qquad \Lambda=-\frac{1}{l^2}, \quad (22)$$

   where m is BH mass and J is the angular momentum of the BH. This situation is nothing but the BTZ case.
2. For the case "$\phi(r)=\ln\left(\frac{r}{l}\right)$" and "$\lambda=0$":

   This situation results in

$$f_\pm=-\frac{r^2}{2\alpha}\left(1\pm\sqrt{1+\frac{4\alpha}{r^2}\left[\frac{r^2}{l^2}-m\right]}\right). \quad (23)$$

   As the GB coupling constant approaches 0, the $f(r)_+$ of the solution (23) lacks a well-defined limit, i.e., in the limit $\alpha\to 0$, $f(r)_+$ blows up to infinity. So, only the $f(r)_-$ has a well-defined limit as $\alpha\to 0$, and it gives the BTZ metric. The $f(r)_-$ in this limit is

$$f(r)_-=\frac{r^2}{l^2}-m-\frac{\alpha}{r^2}\left(\frac{r^2}{l^2}-m\right)^2+\mathcal{O}\left(\alpha^2\right). \quad (24)$$

[2] As stated in [38], there are at least two distinct ways to retrieve the GB portion $\mathcal{G}$ in (19). These include the generalization of the Ross-Mann method to obtain the $D\to 2$ limit of general relativity [39] through a conformal transformation, and the Kaluza-Klein (KK) dimensional reduction of a D-dimensional theory compactified on an internal maximally symmetric space that results in a $D=3$ GB gravity [40]. As long as the maximally symmetric space employed in the KK technique is flat, both procedures lead to the action (19); otherwise, extra terms are generated [38] and that term is given in (20).

Now, the authors of [38] discussed the entropy expression based on the Iyer-Wald method [50][51], expression for entropy is

$$S = \frac{\pi r_+}{2}\left\{1 - 2\lambda\alpha e^{-2\phi}\right\} . \tag{25}$$

As we are working in $\lambda = 0$ and $\phi(r) = \ln\left(\frac{r}{l}\right)$. The thermodynamics of such BHs can be written as

$$M = \frac{m}{8} = \frac{r_+^2}{8l^2} , \quad T = \frac{f'}{4\pi} = \frac{r_+}{2\pi l^2} , \quad \Phi = 0 ,$$
$$S = \frac{\pi r_+}{2} , \quad P = \frac{1}{8\pi l^2} , \quad V = \pi r_+^2 . \tag{26}$$

It is the same as the well-known BTZ black hole in Einstein's gravity. The first law of BH thermodynamics states that $\delta M = T\delta S + V\delta P + \Phi_\alpha\delta$, it is to verify that it is verified with the above parameters. Also the Smarr formula $0 = TS - 2PV + 2\Phi_\alpha\alpha$, is also verified. This is interesting because, regardless of the value of α, the thermodynamic parameters remain the same even if the curvature is not constant.

The metric is

$$dS^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\phi^2 , \tag{27}$$

where $f(r) = \frac{r^2}{l^2} - m - \frac{\alpha}{r^2}\left(\frac{r^2}{l^2} - m\right)^2 + \mathcal{O}(\alpha^2) = \frac{r^2 - r_+^2}{l^2} - \frac{\alpha}{r^2}\left(\frac{r^2 - r_+^2}{l^2}\right)^2 + \mathcal{O}(\alpha^2)$. In the limit of small α, it reproduces the typical BTZ solution. r_+ is the location of the horizon, and it matches with the BTZ horizon of the same mass.

Working with the two-dimensional surface with ϕ = constant, the metric can be written as

$$dS^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 . \tag{28}$$

Furthermore, the Kruskal coordinate can be defined to achieve the maximal BTZ spacetime extension.

Right Wedge: $U = -e^{-k(t-r_*)}$ and $V = e^{k(t-r_*)}$

Left Wedge: $U = e^{-k(t-r_*)}$ and $V = -e^{k(t-r_*)}$

with the expression of surface gravity is $k = \frac{2\pi}{\beta}$ and r_* is the tortoise coordinate and given by

$$r_* = \int \frac{dr}{f(r)} = \frac{1}{2r_+}\left[\log\left(\frac{|r - r_+|}{r + r_+}\right) - \gamma \log\left(\frac{|r - \gamma r_+|}{r + \gamma r_+}\right)\right] , \tag{29}$$

where $\gamma = \sqrt{\frac{\alpha}{1-\alpha}}$. As we discussed above, the procedure of linking flat space. In terms of Kruskal coordinate, the metric can be written as

$$dS^2 = \Omega^{-2} dU dV , \tag{30}$$

with $\Omega = \frac{r+r_+}{rr_+}\sqrt{\left(\left[\frac{|r-\gamma r_+|}{r+\gamma r_+}\right]^\gamma [r^2 + (r^2-1)\gamma^2]\right)}$. R_- and R_+ are the left and right wedges of the radiation regions, b_- and b_+ are the boundaries of the R_- and R_+, and a_- and a_+ are the boundaries of the island surface in the left and right wedges for an eternal EGB black hole.

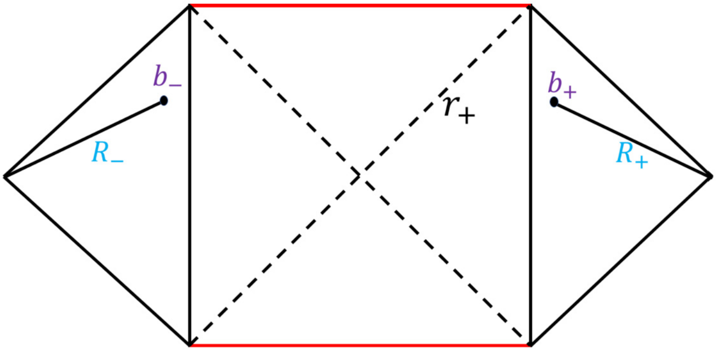

Fig. 2. The Penrose diagram of an EGB BH when there is no island.

4. Island and the information recovery

4.1. Without island

In this subsection, we discuss the calculation of EE without an island (Fig. 2).

Because there is no island surface initially, Hawking radiation's EE can be estimated using the formula below

$$S = \frac{c}{3}\ell(b_+, b_-) \,, \tag{31}$$

the points $b_\pm = (\pm t_b, b)$ are the radiation region limits in the right and left wedges of the EGB black hole. Defining β as inverse Hawking temperature as $\beta = \frac{2\pi}{k}$. Between the boundary points b_+ and b_-, the geodesics distance is

$$\ell(b_+, b_-) = \sqrt{\frac{4(b^2 - r_+^2)[b^2 + (b^2 - 1)\gamma^2]}{(1+\gamma^2)b^2 r_+^2}\mathrm{Cosh}^2(r_+ t_b)} \,, \tag{32}$$

putting eq. (32) in eq. (31) we get

$$S(R) = \left(\frac{c}{6}\right)\log\left[\frac{4(b^2 - r_+^2)[b^2 + (b^2 - 1)\gamma^2]}{(1+\gamma^2)b^2 r_+^2}\mathrm{Cosh}^2(r_+ t)\right] \,, \tag{33}$$

c is the central charge. At late time i.e., $t \to \infty$ then $\mathrm{Cosh}(r_+ t) \sim e^{r_+ t}$, the above equation can be approximated as

$$S(R) \sim \frac{c}{3} r_+ t \,. \tag{34}$$

As a result of equation (1), we can see that the EE of the Hawking radiation in the absence of an island surface increases linearly with time and becomes infinite at late times, resulting in the EGB black hole's information paradox. Next, we'll show that at late periods, an island appears, and the Hawking radiation's EE in the presence of an island surface remains constant and dominates after the Page time. The Page curve is obtained by combining the two contributions.

4.2. Entanglement entropy with island

The assumptions for the verification of the presence of the island in the EGB BH-Bath System are (Fig. 3):

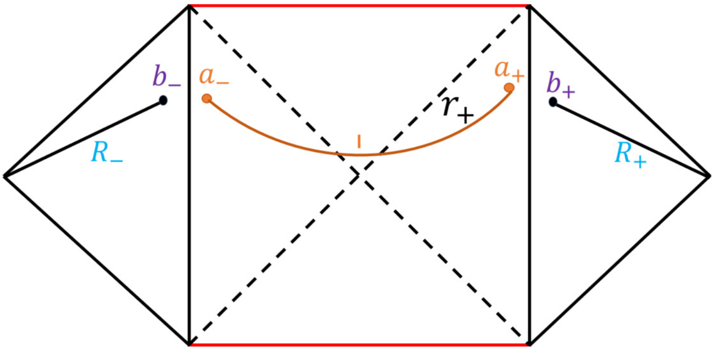

Fig. 3. The Penrose diagram of EGB BH when there is an island I indicated in orange with endpoints a_- and a_+.

- The right wedge is far apart from the left, so the s-wave approximation can be applied.
- The quantum matter's von Neumann entropy throughout the entire region for the $R \cup I$ is sufficiently large.

Using eq. (39), one may determine the von Neumann entropy; one can get the expression for $S_{\text{generalized}}$. Extremizing the $S_{\text{generalized}}$ with respect to t_a yields i.e., $\frac{\partial S_{\text{generalized}}}{\partial t_a} = 0$ results in $t_a = t_b$. By executing this result, one can easily conclude that

$$
\begin{aligned}
&S_{\text{generalized}} \\
&= \frac{\pi a}{G_N} + \frac{c}{6}\log\Bigg[\frac{1}{r_+^2}\sqrt{\frac{(b^2-r_+^2)[b^2+(b^2-1)\gamma^2]}{b^2(1+\gamma^2)}}\sqrt{\frac{(a^2-r_+^2)[a^2+(a^2-1)\gamma^2]}{a^2(1+\gamma^2)}} \\
&\quad \Bigg(\frac{1}{(b-r_+)(f-r_+)}\Bigg\{\sqrt{\frac{(b-r_+)}{(b+r_+)}\left[\frac{(b+\gamma r_+)}{(b-\gamma r_+)}\right]^{\gamma}}\sqrt{\frac{(a-r_+)}{(a+r_+)}\left[\frac{(a+\gamma r_+)}{(a-\gamma r_+)}\right]^{\gamma}} \\
&\quad \Bigg((f-r_+)(b+r_+)\left[\frac{b-\gamma r_+}{b+\gamma r_+}\right]^{\gamma} + (b-r_+)(f+r_+)\left[\frac{f-\gamma r_+}{f+\gamma r_+}\right]^{\gamma}\Bigg) - 2\Bigg\}\Bigg)\Bigg].
\end{aligned}
\tag{35}
$$

The location of QES can be obtained by extremizing the above equation w.r.t. a

$$
a = r_+ + \frac{1}{2r_+}\left(\frac{cG}{6\pi}\right)^2\left(\frac{b+r_+}{b-r_+}\right)\left(\frac{1+\gamma}{1-\gamma}\right)^{\gamma}\left(\frac{b-\gamma r_+}{b+\gamma r_+}\right)^{\gamma}. \tag{36}
$$

Substituting this value in $S_{generalized}$, we have

$$
\begin{aligned}
S_{generalized} &\simeq \frac{\pi r_+}{G_N} - \frac{2c}{3}\log(S_{BH}) + \frac{c^2 G_N}{72\pi r_+}\left[\frac{1+\gamma}{1-\gamma}\right]^{\gamma} + \mathcal{O}\left(G_N^2\right) \\
&\simeq 2S_{BH} - \frac{2c}{3}\log(S_{BH}) + \left(\frac{c}{6}\right)^2\frac{1}{S_{BH}}\left[\frac{1+\gamma}{1-\gamma}\right]^{\gamma} + \mathcal{O}\left(G_N^2\right).
\end{aligned}
\tag{37}
$$

The above expression has some notable characteristics. Aside from the first item S_{BH}, the phrase includes universal adjustments to the Hawking entropy of a BH. So, these corrections can be considered quantum gravity signals; this is a surprise. The information loss paradox is solved by the above conclusion yielding the proper Page curve and has a lovely mutual information interpretation.

5. Page curve

- **Page time (t_{page}):** Page time is when Hawking radiation's EE reaches its maximum. The entropy of an eternal black hole does not alter after it. By comparing (34) and (37)

$$t_{\text{page}} = \frac{6}{cr_+} S^{BH} + \frac{3}{cr_+}\frac{c^2 G_N}{72\pi^2}\left[\frac{1+\gamma}{1-\gamma}\right]^{\gamma} = \frac{3\beta}{2c\pi^2} S_{BH} + \frac{(cG_N)}{96\pi^4}\left[\frac{1+\gamma}{1-\gamma}\right]^{\gamma}. \tag{38}$$

In the above expression we have used the expression $\beta = \frac{2\pi}{T} = \frac{4\pi^2}{r_+}$.

- **Scrambling time (t_{scr}):** the Hayden-Preskill experiment [52] establishes the scrambling time is the shortest time during which information can be retrieved from Hawking radiation. Due to connections between the island's degree of freedom and radiation regions, the scrambling time is also given by the time for information to enter the island in the entanglement wedge reconstruction proposal [4]. One can compute the Scrambling time by

$$\begin{aligned} t_{\text{scr}} &\equiv |r_*(b) - r_*(a)| \\ &= \frac{1}{2r_+}\log\left[\left(\frac{b-r_+}{b+r_+}\frac{a+r_+}{a-r_+}\right)\left(\frac{\Gamma b+r_+}{\Gamma b-r_+}\frac{\Gamma a+r_+}{\Gamma a+r_+}\right)^{1/\Gamma}\right] \\ &\simeq \frac{\beta}{2\pi}\log\left(\frac{r_+}{cG_N}\right) + \cdots . \end{aligned}$$

The higher derivative terms do not affect scrambling time, similar to the BTZ BH's scrambling time.

6. Results and discussion

In this paper, we have calculated the holographic EE to calculate the Page curves of an evaporating EGB BH in higher derivative terms. In this paper, we have considered $\mathcal{O}(R^2)$ terms in gravitational action, and using that; we have plotted the Page curves 4. As a result, when we consider the $\mathcal{O}(R^2)$ terms, initially due to the absence of an island, we obtain that there is linear dependence of time in the EE, resulting in an information paradox for the EGB BH. Late in the game, an island appears, and the Hawking radiation's EE achieves a constant value equal to double the BH's Bekenstein-Hawking entropy, yielding Page curves for fixed values of the Gauss-Bonnet coupling (α). We have also plotted the EE (S_{EE}) with respect to time (t) with fixed parameter ($c = 1, G_N = 1, r_+ = 1$) for BTZ black hole and then $\alpha = 0.1$ and 0.2 for EGB BH. From the Fig. 4, it is clear that the Page curves change towards later times when Gauss-Bonnet coupling (α) rises in this example. We have also studied the page time and the scrambling time in BTZ and the EGB case. For consistency check, for $\alpha \to 0$, we get a similar expression as in the BTZ case from the EGB case.

As discussed in the section, we have considered some specific choices for $\phi(r)$ and $\lambda = 0$. It is interesting to investigate the expression for the function $f(r)$ if we consider the case with $\lambda \neq 0$. We will come back to these issues later on.

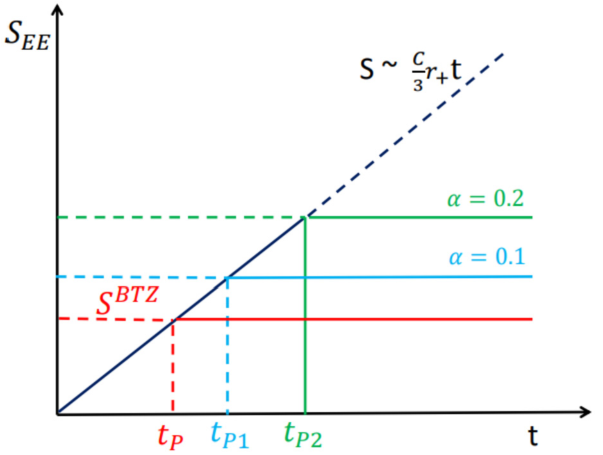

Fig. 4. Page curves of an eternal BTZ black hole and for EGB BH for various values of Gauss-Bonnet coupling (α).

CRediT authorship contribution statement

Prasanta K. Tripathy: For a careful manuscript reading. **Ashis Saha:** Some calculation help over email.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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Appendix A. Expressions for distances

The Expressions for distances are

$$\ell(a_+, b_+) = \Bigg[\frac{1}{r_+^2} \sqrt{\frac{(b^2 - r_+^2)[b^2 + (b^2-1)\gamma^2]}{b^2(1+\gamma^2)}} \sqrt{\frac{(a^2 - r_+^2)[a^2 + (a^2-1)\gamma^2]}{a^2(1+\gamma^2)}}$$

$$\Bigg(\frac{1}{(b-r_+)(f-r_+)} \Bigg\{ \sqrt{\frac{(b-r_+)}{(b+r_+)} \left[\frac{(b+\gamma r_+)}{(b-\gamma r_+)}\right]^{\gamma}} \sqrt{\frac{(a-r_+)}{(a+r_+)} \left[\frac{(a+\gamma r_+)}{(a-\gamma r_+)}\right]^{\gamma}}$$

$$\left((f-r_+)(b+r_+)\left[\frac{b-\gamma r_+}{b+\gamma r_+}\right]^{\gamma}+(b-r_+)(f+r_+)\left[\frac{f-\gamma r_+}{f+\gamma r_+}\right]^{\gamma}\right)$$
$$\left.\left.-2\mathrm{Cosh}[r_+(t_a-t_b)]\right\}\right]^{\frac{1}{2}}, \tag{39}$$

$$\ell(a_+,b_-)=\left[\frac{1}{r_+^2}\sqrt{\frac{(b^2-r_+^2)[b^2+(b^2-1)\gamma^2]}{b^2(1+\gamma^2)}}\sqrt{\frac{(a^2-r_+^2)[a^2+(a^2-1)\gamma^2]}{a^2(1+\gamma^2)}}\right.$$
$$\left(\frac{1}{(b-r_+)(f-r_+)}\left\{\sqrt{\frac{(b-r_+)}{(b+r_+)}}\left[\frac{(b+\gamma r_+)}{(b-\gamma r_+)}\right]^{\gamma}\sqrt{\frac{(a-r_+)}{(a+r_+)}}\left[\frac{(a+\gamma r_+)}{(a-\gamma r_+)}\right]^{\gamma}\right.\right.$$
$$\left((f-r_+)(b+r_+)\left[\frac{b-\gamma r_+}{b+\gamma r_+}\right]^{\gamma}+(b-r_+)(f+r_+)\left[\frac{f-\gamma r_+}{f+\gamma r_+}\right]^{\gamma}\right)$$
$$\left.\left.-2\mathrm{Cosh}[r_+(t_a+t_b)]\right\}\right]^{\frac{1}{2}}=\ell(a_-,b_-)\,, \tag{40}$$

$$\ell(b_+,b_-)=\sqrt{\frac{4(b^2-r_+^2)[b^2+(b^2-1)\gamma^2]}{(1+\gamma^2)b^2r_+^2}\mathrm{Cosh}^2(r_+t_b)}\,. \tag{41}$$

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