

Aspects of the Domain-Wall Standard Model

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In the “Domain-Wall Standard Model,” all the Standard Model (SM) fields are localized in certain domains in a non-compact 5D space-time. While the SM is realized as a 4D effective theory at low energies, the model involves Kaluza–Klein (KK) modes of the SM particles. In this paper we introduce two simple solvable examples which lead to domain-wall configurations for the SM particles and their KK-modes. Based on the examples, we address a variety of phenomenologies of the Domain-Wall SM, such as the KK-mode gauge boson phenomenology at the Large Hadron Collider (LHC), the effect of the KK-mode SM fermions on Higgs boson phenomenology, and the KK-mode fermion search at the LHC with its decay into a corresponding SM fermion and a Nambu–Goldstone boson associated with a spontaneous breaking of the translational invariance in the 5th dimension. We also propose a simple unified picture of localizing all the SM fields.
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1. Introduction

An idea that our world consists of more than 4D space-time has always been fascinating us. After the discovery of the D-brane in string theories [1], the brane-world scenario has been intensively studied as new physics beyond the Standard Model (SM). In extra-dimensional models, a new property, “geometry,” comes into play in phenomenology and provides us with a new possibility of understanding mysteries in the SM. One well-known brane-world scenario is the so-called large extra-dimension model [2], [3], which offers a solution to the gauge hierarchy problem with the extra-dimensional Planck mass being at the TeV scale while reproducing the 4D Planck mass by a large extra-dimensional volume. Another well-known scenario is the warped extra-dimension model [4] in 5D, where the Planck scale is warped down to the TeV scale in the presence of the anti-de Sitter (AdS) curvature in the 5th dimension.

Usually in extra-dimensional models, extra dimensions are compactified on some manifolds or orbifolds, and our three spatial dimensions and extra dimensions are treated differently. We may think it more natural if all spatial dimensions are non-compact and the SM is realized as a 4D effective theory. This picture requires that all the SM fields as well as the 4D graviton are localized in certain three-spatial dimensional domains in the bulk space. The so-called RS-2 scenario proposed in Ref. [5] provides a simple realization of this picture for the 4D graviton.

In the scenario, due to the 5D AdS curvature, the 4D graviton is localized around a point in the non-compact 5th dimension and the 4D Einstein gravity is reproduced at low energies.

Recently, the authors of the present paper have proposed a framework to construct the “Domain-Wall Standard Model” in a non-compact 5D space-time [6], where all the SM fields are localized in certain domains of the 5th dimension.¹ This model is a field-theoretical realization of a “3-brane” on which all the SM fields are confined. However, the finite widths of the “3-branes” corresponding to the SM fields lead to rich particle physics phenomenologies. Based on a simple setup for localizing the SM fields, we have obtained the SM as a 4D effective theory. The localization mechanism predicts the Kaluza–Klein (KK) modes of the SM particles, and we have investigated the phenomenology of the KK-mode gauge boson at the Large Hadron Collider (LHC) experiment.

In this paper, we investigate various aspects of the Domain-Wall SM in detail. For concreteness, we introduce two solvable examples to localize all the SM fields in certain domains of the 5th dimension, and provide the explicit forms of the KK-mode mass spectrum and eigenfunctions. We then derive the 4D effective Lagrangian involving the KK-mode SM particles. Among a variety of possible phenomenologies of the Domain-Wall SM, we address the LHC phenomenology of the KK-mode gauge bosons, the Higgs boson phenomenology in the presence of the KK-mode SM fermions, and the KK-mode fermion search at the LHC.

We begin with the localization of the gauge field in the next section, where the basic formalism to maintain the 4D gauge invariance is presented. For detailed analysis, we introduce two solvable examples and obtain the explicit form for the KK-mode mass spectrum and the KK-mode eigenfunctions. In Sect. 3, we consider the Higgs sector and apply the same procedure taken for the gauge field to localize the Higgs boson and its vacuum expectation value (VEV). We derive the mass spectrum for the zero-mode and KK-modes of the gauge boson after breaking of the gauge symmetry. In Sect. 4, we consider the localization mechanism of the SM chiral fermions and derive the SM Lagrangian in the 4D effective theory, which also involves the KK-mode gauge bosons and Higgs bosons. We investigate a variety of phenomenologies for the Domain-Wall SM in Sect. 5. The last section is devoted to conclusions and discussions.

2. The gauge sector

Let us first consider the gauge sector of the Domain-Wall SM, where a gauge field is localized around a point in the 5th dimension while keeping the 4D gauge invariance. Since the essential mechanism for localizing a gauge field is independent of the gauge group structure, we address the gauge field localization based on a U(1) gauge theory. For localizing the gauge field, we adopt a simple way proposed in Ref. [12]² and introduce the following Lagrangian for the U(1) gauge field in 5D flat Minkowski space:

$$\mathcal{L}_5 = -\frac{1}{4}s(y)F_{MN}F^{MN}, \quad (1)$$

where F_{MN} is the gauge field strength, and $M, N = 0, 1, 2, 3, y$ with y being the index for the 5th coordinate. Our convention for the metric is $\eta_{MN} = \text{diag}(1, -1, -1, -1, -1)$, and in general we suppress coordinate dependence of the fields unless emphasis is needed. In the original Lagrangian, we identify $s(y) = 1/\bar{g}^2$ with \bar{g} being the 5D gauge coupling, and this y -dependence

¹See also Refs. [7], [8], [9], [10] and [11] for previous proposals of a Domain-Wall SM/Grand Unified Theory.

²We can find that the same idea was discussed elsewhere before Ref. [12]. See e.g. Ref. [13].

is the key to localizing the gauge field. In the defined Lagrangian, the gauge field and $s(y)$ have mass dimension 1.

Decomposing the field strength into its components yields the following expression (up to total derivative terms):

$$\begin{aligned} \mathcal{L}_5 = & \frac{1}{2} s A^\mu (g_{\mu\nu} \square_4 - \partial_\mu \partial_\nu) A^\nu - \frac{1}{2} s A_y \square_4 A_y \\ & - \frac{1}{2} A_\mu \partial_y (s \partial_y A^\mu) - (\partial_\mu A^\mu) \partial_y (s A_y), \end{aligned} \quad (2)$$

where A_μ ($\mu = 0, 1, 2, 3$) and A_y are a gauge field and a scalar field in 4D space-time, and the first line in the right-hand side denotes the 4D kinetic terms for these fields. Since the original Lagrangian formally maintains the 5D gauge invariance, we may use the gauge degrees of freedom to eliminate 2 components from A_M . Although the so-called ‘‘axial gauge’’ ($A_y = 0$) is sometimes employed in the literature, we treat A_y as a dynamical field and eliminate 2 degrees of freedom from A_μ as we usually do in 4D gauge field theories. As we will show in the following, the zero-mode of A_y vanishes because of the breakdown of the gauge invariance due to 5th coordinate-dependence of the gauge coupling $s(y)$. Hence, the gauge choice of $A_y = 0$ may make the gauge structure of the theory unclear. The last term in Eq. (2) contains a mixing between A_μ and A_y . Note that this structure is analogous to that in spontaneously broken gauge theories, and suggests to us to eliminate it by adding a gauge fixing term, which is a 5D analog to the R_ξ gauge [6,]:³

$$\mathcal{L}_{\text{GF}} = -\frac{s}{2\xi} \left(\partial_\mu A^\mu - \frac{\xi}{s} \partial_y (s A_y) \right)^2, \quad (3)$$

where ξ is a gauge parameter. The total Lagrangian now reads $\mathcal{L} = \mathcal{L}_5 + \mathcal{L}_{\text{GF}} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{scalar}}$, where

$$\begin{aligned} \mathcal{L}_{\text{gauge}} = & \frac{1}{2} s A^\mu \left(g_{\mu\nu} \square_4 - \left(1 - \frac{1}{\xi} \right) \partial_\mu \partial_\nu \right) A^\nu - \frac{1}{2} A_\mu \partial_y (s \partial_y A^\mu), \\ \mathcal{L}_{\text{scalar}} = & -\frac{1}{2} s A_y \square_4 A_y + \frac{1}{2} s \xi A_y \partial_y \left(\frac{1}{s} \partial_y (s A_y) \right). \end{aligned} \quad (4)$$

Using the KK-mode decomposition of the gauge and scalar fields,

$$A_\mu(x, y) = \sum_{n=0}^{\infty} A_\mu^{(n)}(x) \chi^{(n)}(y), \quad A_y(x, y) = \sum_{n=0}^{\infty} \eta^{(n)}(x) \psi^{(n)}(y), \quad (5)$$

we can rewrite the Lagrangians in Eq. (4) as

$$\begin{aligned} \mathcal{L}_{\text{gauge}} = & \sum_{n=0}^{\infty} \frac{1}{2} s \left(\chi^{(n)} \right)^2 \left[A_\mu^{(n)} \left(g^{\mu\nu} (\square_4 + m_n^2) - \left(1 - \frac{1}{\xi} \right) \partial^\mu \partial^\nu \right) A_\nu^{(n)} \right], \\ \mathcal{L}_{\text{scalar}} = & -\sum_{n=0}^{\infty} \frac{1}{2} s \left(\psi^{(n)} \right)^2 \left[\eta^{(n)} (\square_4 + \xi m_n^2) \eta^{(n)} \right], \end{aligned} \quad (6)$$

³During the completion of our manuscript (the first version), we have learned that another group is considering essentially the same procedure for localized gauge fields. Their paper came out in arXiv on the same day as the first version of our paper: M. Arai, F. Blaschke, M. Eto, N. Sakai, Prog. Theor. Exp. Phys. 2018, 063B02 (2018) [arXiv:1801.02498 [hep-th]] [Search inSPIRE]. See also their previous work: M. Arai, F. Blaschke, M. Eto, N. Sakai, Prog. Theor. Exp. Phys. 2013, 013B05 (2013) [arXiv:1208.6219 [hep-th]] [Search inSPIRE]; Prog. Theor. Exp. Phys. 2013, 093B01 (2013) [arXiv:1303.5212 [hep-th]] [Search inSPIRE]; Prog. Theor. Exp. Phys. 2017, 053B01 (2017) [arXiv:1703.00427 [hep-th]] [Search inSPIRE].

where $\chi^{(n)}$ and $\eta^{(n)}$ are the solutions of the KK-mode equations:

$$\frac{d}{dy} \left(s \frac{d}{dy} \chi^{(n)} \right) + s m_n^2 \chi^{(n)} = 0, \quad \frac{d}{dy} \left(\frac{1}{s} \frac{d}{dy} (s \psi^{(n)}) \right) + m_n^2 \psi^{(n)} = 0. \quad (7)$$

Here we have expected the KK-modes of $\eta^{(n)}$ are would-be Nambu–Goldstone (NG) modes eaten by the KK-modes of $A_\mu^{(n)}$ and enjoy the degrees of freedom for their longitudinal modes. This picture is consistent only if the two equations in Eq. (7) have solutions with common m_n eigenvalues for a given function $s(y)$. In the following, we will discuss a couple of solvable examples for $s(y)$ and show this consistency explicitly. However, we emphasize that the pairing of the KK-mode mass spectrum between $\chi^{(n)}$ and $\psi^{(n)}$ is held only for massive modes while the zero-mode $\psi^{(0)}$ vanishes.

Even for a general function of $s(y)$, we can easily find zero-mode solutions ($m_0 = 0$) for Eq. (7) such that

$$\chi^{(0)} = \tilde{c}_\chi + c_\chi \int^y \frac{dy'}{s(y')}, \quad \psi^{(0)} = \frac{\tilde{c}_\psi}{s(y)} + \frac{c_\psi}{s(y)} \int^y dy' s(y'), \quad (8)$$

where \tilde{c}_χ , c_χ , \tilde{c}_ψ , and c_ψ are constants. In order to localize the gauge field in the finite domain, we impose $s(y) \rightarrow 0$ as $|y| \rightarrow \infty$. In addition, the gauge and scalar fields in the 4D effective theory must be normalizable in the sense that

$$\int_{-\infty}^{\infty} dy s(y) \chi^{(0)}(y) \chi^{(0)}(y) < \infty, \quad \int_{-\infty}^{\infty} dy s(y) \eta^{(0)}(y) \eta^{(0)}(y) < \infty. \quad (9)$$

Considering the zero-mode solution for the gauge field, these constraints require $c_\chi = 0$, resulting in the zero-mode for the gauge boson having a constant configuration in the 5th dimension. Note that this unique solution leads to the universal gauge coupling, in other words, 4D gauge invariance in the 4D effective theory, independently of configurations of matter fields in the bulk. On the other hand, the solution of $\psi^{(0)}$ cannot satisfy the requirement given in Eq. (9) unless $c_\psi = \tilde{c}_\psi = 0$, and hence $\psi^{(0)} = 0$ is the only appropriate choice for the zero-mode of the scalar. Therefore, no (normalizable) zero-mode exists for the scalar component. We may consider a special setup where $s(y)$ is independent of y . This is a trivial case that the 5D gauge invariance is manifest and a constant $\psi^{(0)}$ is a solution of the KK-mode equation, although the gauge field is not localized. Therefore, the absence of the zero-mode scalar originates from the explicit breaking of the 5D gauge invariance due to the y -dependence of the gauge coupling $s(y)$.

The Domain-Wall SM has rich phenomenological aspects as we will discuss below. For our phenomenology discussions, we need a solvable system that provides us with explicit forms for the gauge boson KK-mode spectrum and eigenfunctions. A simple example is discussed in our previous work [6]. In the following, we introduce two solvable examples, which are much more nontrivial than the example in Ref. [6].

In solving the KK-mode equations in Eq. (7), it is convenient to rewrite the equations with a function $f(y)$ defined as $s(y) = f(y)^2$ and introducing new variables,

$$\tilde{\chi}^{(n)}(y) = f(y) \chi^{(n)}(y), \quad \tilde{\psi}^{(n)}(x) = f(y) \psi^{(n)}(x), \quad (10)$$

corresponding to the gauge and the scalar fields. We then have the KK-mode equations which have the form of the Schrödinger equation:

$$\begin{aligned} \left[-\partial_y^2 - G(y)' + G(y)^2 \right] \tilde{\chi}^{(n)} &= m_n^2 \tilde{\chi}^{(n)}, \\ \left[-\partial_y^2 + G(y)' + G(y)^2 \right] \tilde{\psi}^{(n)} &= m_n^2 \tilde{\psi}^{(n)}, \end{aligned} \quad (11)$$

where ' denotes d/dy , and $G(y) = -f(y)/f'(y)$. It is easy to find the zero-mode solution as $\tilde{\chi}^{(0)}(y) \propto f(y)$. This result implies that if we have a solvable 1D Quantum Mechanical system resulting in bound states, we adopt this system as our solvable example by identifying $f(y)$ with the ground-state eigenfunction.

Solvable example 1

We now consider the first solvable example which is a Gaussian-type function,

$$s(y) = f(y)^2 = M \exp[-(m_V y)^2], \quad (12)$$

where M and m_V are (positive) mass parameters. Substituting it into the KK-mode equations, we obtain

$$\begin{aligned} [-\partial_y^2 + m_V^4 y^2] \tilde{\chi}^{(n)} &= (m_n^2 + m_V^2) \tilde{\chi}^{(n)}, \\ [-\partial_y^2 + m_V^4 y^2] \tilde{\psi}^{(n)} &= (m_n^2 - m_V^2) \tilde{\psi}^{(n)}, \end{aligned} \quad (13)$$

which are nothing but the Schrödinger equation for the 1D harmonic oscillator,

$$H\Psi^{(n)} = \omega \left(a^\dagger a + \frac{1}{2} \right) \Psi^{(n)} = E_n \Psi^{(n)}, \quad (14)$$

with the identifications of the frequency $\omega = 2m_V^2$ and the annihilation/creation operator,

$$a = \frac{1}{\sqrt{2m_V^2}} \left(\frac{d}{dy} + m_V^2 y \right), \quad a^\dagger = \frac{1}{\sqrt{2m_V^2}} \left(-\frac{d}{dy} + m_V^2 y \right). \quad (15)$$

Thus, using the energy eigenvalues given by $E_n = 2m_V^2 (n + \frac{1}{2})$ ($n = 0, 1, 2, \dots$) for the harmonic oscillator, we find the KK-mode spectra for the gauge bosons and the would-be NG modes as $m_n^2 = 2n m_V^2$ and $m_n^2 = 2(n+1)m_V^2$ for $n = 0, 1, 2, \dots$, respectively. Note that no zero-mode exists for the scalar field. Shifting $n \rightarrow n+1$ for the scalar mode, we can see the pairing of the massive modes of the gauge bosons and would-be NG modes ($2nm_V^2$ and $\xi 2nm_V^2$ for $n = 1, 2, \dots$). This is nothing but what we expected.

Using the harmonic oscillator algebra, $[a, a^\dagger] = 1$, it is straightforward to obtain the KK-mode functions. For example, the zero-mode function $\tilde{\chi}^{(0)}$ is obtained as a solution of $a\tilde{\chi}^{(0)} = 0$, and the n -th KK-mode function is generated as $\tilde{\chi}^{(n)} \propto (a^\dagger)^n \tilde{\chi}^{(0)}$ by using the zero-mode function. After integrating out the 5th-dimensional degrees of freedom for the Lagrangian of Eq. (6), we obtain the 4D effective Lagrangians with the canonically normalized kinetic terms as

$$\begin{aligned} \mathcal{L}_{\text{gauge}}^4 &= \sum_{n=0}^{\infty} \frac{1}{2} \left[\int_{-\infty}^{\infty} s \left(\chi^{(n)} \right)^2 \right] \left[A_\mu^{(n)} \left(g^{\mu\nu} (\square_4 + m_n^2) - \left(1 - \frac{1}{\xi} \right) \partial^\mu \partial^\nu \right) A_\nu^{(n)} \right] \\ &= \sum_{n=0}^{\infty} \frac{1}{2} \left[A_\mu^{(n)} \left(g^{\mu\nu} (\square_4 + m_n^2) - \left(1 - \frac{1}{\xi} \right) \partial^\mu \partial^\nu \right) A_\nu^{(n)} \right] \\ \mathcal{L}_{\text{scalar}}^4 &= - \sum_{n=1}^{\infty} \frac{1}{2} \left[\int_{-\infty}^{\infty} s \left(\psi^{(n)} \right)^2 \right] \left[\eta^{(n)} (\square_4 + \xi m_n^2) \eta^{(n)} \right], \\ &= - \sum_{n=1}^{\infty} \frac{1}{2} \left[\eta^{(n)} (\square_4 + \xi m_n^2) \eta^{(n)} \right], \end{aligned} \quad (16)$$

for the KK-mode decomposition,

$$A_\mu(x, y) = \sum_{n=0}^{\infty} A_\mu^{(n)}(x)\chi^{(n)}(y), \quad A_y(x, y) = \sum_{n=1}^{\infty} \eta^{(n)}(x)\chi^{(n-1)}(y). \quad (17)$$

Here, we have changed the label n for the KK-mode decomposition of A_y by $n \rightarrow n + 1$, and $\psi^{(n)}$ is given by $\psi^{(n)} = \chi^{(n-1)}$ for $n = 1, 2, \dots$. The explicit forms of the first three KK-mode functions are

$$\begin{aligned} \chi^{(0)}(y) &= g, \\ \chi^{(1)}(y) &= \sqrt{2} g (m_V y), \\ \chi^{(2)}(y) &= \frac{g}{\sqrt{2}} (1 - 2(m_V y)^2), \\ \chi^{(3)}(y) &= \frac{g}{\sqrt{3}} (m_V y) (3 - 2(m_V y)^2), \end{aligned} \quad (18)$$

where g is the U(1) gauge coupling in the 4D effective theory defined by $g = \pi^{-1/4} \sqrt{m_V/M}$.

Solvable example 2

As the second example, we consider

$$s(y) = f(y)^2 = \frac{M}{[\cosh(m_V y)]^{2\gamma}}, \quad (19)$$

where M and m_V are positive mass parameters, and γ is a positive constant. We then express Eq. (11) as

$$\begin{aligned} \left[-\partial_y^2 - \frac{\gamma(\gamma + 1)m_V^2}{\cosh^2(m_V y)} \right] \tilde{\chi}^{(n)} &= (m_n^2 - \gamma^2 m_V^2) \tilde{\chi}^{(n)}, \\ \left[-\partial_y^2 - \frac{\gamma(\gamma - 1)m_V^2}{\cosh^2(m_V y)} \right] \tilde{\psi}^{(n)} &= (m_n^2 - \gamma^2 m_V^2) \tilde{\psi}^{(n)}. \end{aligned} \quad (20)$$

These equations have the form of the Schrödinger equation, $(-\partial_y^2 + V)\Psi^{(n)} = E_n\Psi^{(n)}$. Since the potential corresponds to $V \propto -1/\cosh^2(m_V y) < 0$, we expect the existence of a bound state with $E_n < 0$.

We are interested in the localization of the gauge field, namely, bound states from the Schrödinger equation satisfying the following boundary conditions: $|\tilde{\chi}^{(n)}(0)| < \infty$ for $y \rightarrow 0$, and $\tilde{\chi}^{(n)}(y) \rightarrow 0$ for $|y| \rightarrow \infty$. Such solutions are described by using the hypergeometric function $F[a, b; c; y]$ [14]. We find the eigenvalues for $\tilde{\chi}^{(n)}$ as

$$m_n^2 = n(2\gamma - n)m_V^2 \quad (n = 0, 1, 2, \dots < \gamma). \quad (21)$$

The number of (localized) KK-modes is terminated by a condition $E_n = m_n^2 - \gamma^2 m_V^2 < 0$, and thus a massive mode exists for $\gamma > 1$. The eigenfunctions for even numbers of $n = 2n'$ ($n' = 0, 1, 2, \dots$) and odd numbers of $n = 2n'' + 1$ ($n'' = 0, 1, 2, \dots$) are given by (up to the normalization factor):

$$\tilde{\chi}^{(n)}(y) = [\cosh(m_V y)]^{-\gamma} F\left[-n', -\gamma + n'; 1/2; 1 - \cosh^2(m_V y)\right], \quad (22)$$

and

$$\tilde{\chi}^{(n)}(y) = \sinh(m_V y) [\cosh(m_V y)]^{-\gamma} F\left[-n'', -\gamma + n'' + 1; 3/2; 1 - \cosh^2(m_V y)\right], \quad (23)$$

respectively.

Similarly, for the scalar field we impose the boundary conditions: $|\tilde{\psi}^{(n)}(0)| < \infty$ for $y \rightarrow 0$, and $\tilde{\psi}^{(n)}(y) \rightarrow 0$ for $|y| \rightarrow \infty$. We can easily find the eigenfunctions of $\tilde{\psi}^{(n)}$ as follows. Substituting $\gamma = \bar{\gamma} + 1$ and $m_n^2 = \bar{m}_n^2 + (2\bar{\gamma} + 1)m_V^2$ into the second equations, we obtain

$$\left[-\partial_y^2 - \frac{\bar{\gamma}(\bar{\gamma} + 1)m_V^2}{\cosh^2(m_V y)} \right] \tilde{\psi}^{(n)} = (\bar{m}_n^2 - \bar{\gamma}^2 m_V^2) \tilde{\psi}^{(n)}, \quad (24)$$

which is identical to the equation for $\tilde{\chi}^{(n)}$. Thus, the eigenvalues for $\tilde{\psi}^{(n)}$ are given by

$$m_n^2 = (n + 1)(2\gamma - (n + 1))m_V^2 \quad (n = 0, 1, 2, \dots < \gamma - 1). \quad (25)$$

As for the first example, no zero-mode exists for the scalar mode, and we can see the pairing of the KK-mode mass spectrum between the gauge fields and the corresponding would-be NG modes. The eigenfunctions for even numbers of $n = 2n'$ ($n' = 0, 1, 2, \dots$) and odd numbers of $n = 2n'' + 1$ ($n'' = 0, 1, 2, \dots$) are given by (up to the normalization factor):

$$\tilde{\psi}^{(n')}(y) = [\cosh(m_V y)]^{-\gamma+1} F \left[-n', -\gamma + n' + 1; 1/2; 1 - \cosh^2(m_V y) \right], \quad (26)$$

and

$$\tilde{\psi}^{(n'')}(y) = \sinh(m_V y) [\cosh(m_V y)]^{-\gamma+1} F \left[-n'', -\gamma + n'' + 2; 3/2; 1 - \cosh^2(m_V y) \right], \quad (27)$$

respectively.

Unlike the first example, we have a finite number of the localized KK-modes in the second example. For concreteness, let us fix $\gamma = 2$. In this case, we have only one KK-mode gauge boson with the mass eigenvalue $m_1^2 = 3m_V^2$. In the 4D effective theory with the canonically normalized kinetic terms like Eq. (16), the explicit form of the KK-mode expansions is given by

$$\begin{aligned} A_\mu(x, y) &= g A_\mu^{(0)}(x) + \sqrt{2} g \sinh(m_V y) A_\mu^{(1)}(x), \\ A_y(x, y) &= \sqrt{\frac{2}{3}} g \cosh(m_V y) \eta^{(1)}(x), \end{aligned} \quad (28)$$

where the gauge coupling in the 4D effective theory is defined as $g = \sqrt{\frac{3m}{4M}}$.

It is straightforward to extend the U(1) gauge theory to the SM case. For the SM gauge group of $SU(3)_c \times SU(2)_L \times U(1)_Y$, we introduce three y -dependent gauge couplings in the original 5D Lagrangian. Let us call them $s_i(y)$ for $i = 1, 2, 3$ corresponding to the three gauge groups. For simplicity, we set $s_1 \propto s_2$ in this paper, so that the KK-mode spectrum for the $SU(2)_L \times U(1)_Y$ gauge bosons is the same. As we will see in the next section, this choice simplifies our calculation of the KK-mode mass spectrum after the electroweak symmetry breaking.

3. The Higgs sector

Next we consider the 5D Higgs sector of the Abelian Higgs model, corresponding to the previous section on the localized U(1) gauge field. It is straightforward to extend our discussion to the SM Higgs doublet case. In a non-compact 5th dimension, we need to consider a localization mechanism for not only the Higgs field but also its vacuum expectation value (VEV). For this purpose, we apply the same procedure as was taken for the gauge field in the previous section. We thus consider the Lagrangian for the Higgs sector of the form,

$$\mathcal{L}_5^H = s_H(y) \left[(\mathcal{D}^M H)^\dagger (\mathcal{D}_M H) - \frac{1}{2} \lambda_H \left(H^\dagger H - \frac{v^2}{2} \right)^2 \right], \quad (29)$$

where H is the Higgs field, v is its VEV, λ_H is a Higgs quartic coupling, and the covariant derivative is given by $\mathcal{D}_M = \partial_M - iQ_H A_M$ with a U(1) charge Q_H for the Higgs field. Here, we

have introduced a y -dependent kinetic term $s_H(y)$. In our convention, the Higgs field and s_H have mass dimension 1.

Expanding about the vacuum $H = (v + h + i\phi)/\sqrt{2}$ and neglecting the interaction terms, we obtain (up to total derivative terms):

$$\begin{aligned}\mathcal{L}_5^H &\supset \frac{1}{2}s_H [(\partial^M h)(\partial_M h) - m_h^2 h^2] + \frac{1}{2}s_H(\partial^M \phi)(\partial_M \phi) \\ &= -\frac{1}{2}s_H h(\square_4 + m_h^2)h + \frac{1}{2}h \partial_y (s_H \partial_y h) - \frac{1}{2}s_H \phi \square_4 \phi + \frac{1}{2}\phi \partial_y (s_H \partial_y \phi),\end{aligned}\quad (30)$$

where $m_h^2 = \lambda_H v^2$ is the physical Higgs boson mass. Applying the KK-mode decomposition to these fields,

$$h(x, y) = \sum_{n=0}^{\infty} h^{(n)}(x) \chi_h^{(n)}(y), \quad \phi(x, y) = \sum_{n=0}^{\infty} \phi^{(n)}(x) \chi_\phi^{(n)}(y),\quad (31)$$

we see that the KK-mode equations for $\chi_h^{(n)}$ and $\chi_\phi^{(n)}$ are identical to those of the gauge boson in Eq. (7) with the replacement by $s(y) \rightarrow s_H(y)$. Since the zero-mode $\phi^{(0)}$ is the would-be NG mode eaten by $A_\mu^{(0)}$, the theoretical consistency here requires the configurations of $\phi^{(0)}$ and $A_\mu^{(0)}$ to be identical. With the solutions of the KK-mode equations, the free Lagrangian for the scalar fields in the 4D effective theory is described as

$$\mathcal{L}_4^H \supset -\frac{1}{2} \sum_{n=0}^{\infty} \left[\int_{-\infty}^{\infty} dy s_H \left(\chi_h^{(n)} \right)^2 \right] \left[h^{(n)} (\square_4 + (m_h^2 + m_n^2)) h^{(n)} + \phi^{(n)} (\square_4 + m_n^2) \phi^{(n)} \right],\quad (32)$$

where we have used $\chi_h^{(n)}(y) = \chi_\phi^{(n)}(y)$ since their equations are identical. After canonically normalizing the kinetic terms, the KK-mode decomposition with respect to physical Higgs bosons is formally given by $h(x, y) = h^{(0)}(x) + \chi_h^{(n)}(y) h^{(n)}(x)$, where we have scaled the KK-mode functions so as to satisfy

$$\int_{-\infty}^{\infty} dy s_H \left(\chi_h^{(n)} \right)^2 = 1 \quad (n = 0, 1, 2, \dots).\quad (33)$$

Recall that $\chi_h^{(0)}$ is a constant. The U(1) gauge symmetry is broken by $\langle H \rangle = v/\sqrt{2}$, from which the masses for the U(1) gauge bosons in the 4D effective theory are generated. Their mass terms are given by

$$\mathcal{L}_4^H \supset \frac{1}{2} Q_H^2 v^2 \sum_{n,m=0}^{\infty} \left[\int_{-\infty}^{\infty} dy s_H(y) \chi^{(n)}(y) \chi^{(m)}(y) \right] \eta^{\mu\nu} A_\mu^{(n)}(x) A_\nu^{(m)}(x).\quad (34)$$

This formula indicates that the KK-mode gauge bosons, $A_\mu^{(n)}$ and $A_\mu^{(m)}$ ($n \neq m$), have a mixing mass in general, and the analysis of the gauge boson mass spectrum is complicated. For simplicity, let us take $s_H(y) \propto s(y)$ in this paper, so that no mixing mass is generated because of the orthogonal condition,

$$\int_{-\infty}^{\infty} dy s_H(y) \chi^{(n)}(y) \chi^{(m)}(y) \propto \int_{-\infty}^{\infty} dy s(y) \chi^{(n)}(y) \chi^{(m)}(y) = 0,\quad (35)$$

for $n \neq m$. After normalizing the kinetic terms for the zero-mode and KK-mode gauge bosons, we find the mass spectrum as

$$m_A^{(0)} = Q_H g v, \quad m_A^{(n)} = \sqrt{m_n^2 + \left(m_A^{(0)} \right)^2},\quad (36)$$

for the zero-mode and the KK-modes ($n = 1, 2, \dots$), respectively. Here, we have obtained the zero-mode gauge boson mass of the same form as the one in the Abelian Higgs model in 4D.

We now extend the model to the SM case. As we mentioned in the previous section, we set $s_1 \propto s_2$, so that the KK-mode spectrum of the $SU(2)_L \times U(1)_Y$ gauge bosons is the same. The Higgs field in the Abelian Higgs model is extended to the SM Higgs doublet field. After the electroweak symmetry breaking we have the gauge boson (photon (γ), W boson, and Z boson) mass spectrum: for the zero-modes,

$$m_\gamma = 0, \quad m_W = \frac{1}{2}g_2 v, \quad m_Z = \frac{1}{2}g_Z v, \quad (37)$$

where $g_Z = \sqrt{g_2^2 + g_Y^2}$ with g_2 and g_Y being the $SU(2)_L$ and $U(1)_Y$ gauge couplings, respectively, and $v_h = 246$ GeV is the Higgs doublet VEV, while their KK-mode mass spectrum is given by

$$m_\gamma^{(n)} = m_n, \quad m_W^{(n)} = \sqrt{m_n^2 + m_W^2}, \quad m_Z^{(n)} = \sqrt{m_n^2 + m_Z^2}. \quad (38)$$

4. Domain-wall fermions

In this section, we consider localized chiral fermions, whose zero-modes are identified with the SM chiral fermions. Again, we consider the $U(1)$ gauge theory to simplify our discussion, which can be easily extended to the 5D SM case. We follow a mechanism in Ref. [15] to generate the domain-wall fermion in 5D space-time and first introduce a real scalar field ($\varphi(x, y)$) in the 5D bulk:

$$\mathcal{L}_{(5)} = \frac{1}{2} (\partial_M \varphi) (\partial^M \varphi) - V(\varphi), \quad (39)$$

where the scalar potential is given by

$$V(\varphi) = \frac{m_\varphi^4}{2\lambda} - m_\varphi^2 \varphi^2 + \frac{\lambda}{2} \varphi^4. \quad (40)$$

It is well known that there is a nontrivial background configuration $\varphi_0(y)$ as a solution of the equation of motion, namely, the so-called kink solution,

$$\varphi_{\text{kink}}(y) = \frac{m_\varphi}{\sqrt{\lambda}} \tanh[m_\varphi y]. \quad (41)$$

Here, we have chosen the kink center at $y = 0$, for simplicity. Expanding the scalar around the kink background, $\varphi(x, y) = \varphi_{\text{kink}}(y) + U_\varphi(y) \tilde{\varphi}(x)$, we can solve the linearized equation of motion. It is easy to notice that this system is the same as the second solvable example in Sect. 2 with $\gamma = 2$, so that we have the solution [16]:

$$\varphi(x, y) = \varphi_{\text{kink}}(y) + \frac{\sqrt{3m_\varphi}}{2} \left[\frac{1}{\cosh^2(m_\varphi y)} \right] \varphi^{(0)}(x) + \sqrt{\frac{3m_\varphi}{2}} \left[\frac{\sinh(m_\varphi y)}{\cosh^2(m_\varphi y)} \right] \varphi^{(1)}(x), \quad (42)$$

where $\varphi^{(0)}(x)$ is a massless NG mode corresponding to the spontaneous breaking of the translational invariance in the 5th dimension, and $\varphi^{(1)}(x)$ is a massive mode with a mass $m_\varphi^{(1)} = \sqrt{3}m_\varphi$. Here, the kinetic terms for the eigenstates are canonically normalized.

Following Ref. [15], we now introduce the Lagrangian for a bulk fermion coupling with φ ,

$$\begin{aligned} \mathcal{L} &= i\bar{\psi} [\gamma^\mu D_\mu + i\gamma^5 D_y] \psi + Y\varphi\bar{\psi}\psi \\ &= i\bar{\psi}_L \gamma^\mu D_\mu \psi_L + i\bar{\psi}_R \gamma^\mu D_\mu \psi_R \\ &\quad - \bar{\psi}_L D_y \psi_R + \bar{\psi}_R D_y \psi_L + Y\varphi (\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L), \end{aligned} \quad (43)$$

where we decompose the Dirac fermion ψ into its chiral components, $\psi = \psi_L + \psi_R$, the covariant derivative is given by $D_M = \partial_M - iQ_f A_M$ with a $U(1)$ charge Q_f for ψ , and Y is a positive constant. Neglecting the gauge interaction and replacing φ by the kink solution, the

equations of motion are given by

$$\begin{aligned} i\gamma^\mu \partial_\mu \psi_L - \partial_y \psi_R + Y\varphi_0 \psi_R &= 0, \\ i\gamma^\mu \partial_\mu \psi_R + \partial_y \psi_L + Y\varphi_0 \psi_L &= 0. \end{aligned} \quad (44)$$

Using the KK-mode decompositions,

$$\psi_L(x, y) = \sum_{n=0}^{\infty} \psi_L^{(n)}(x) \chi_L^{(n)}(y), \quad \psi_R(x, y) = \sum_{n=0}^{\infty} \psi_R^{(n)}(x) \chi_R^{(n)}(y), \quad (45)$$

we have the KK-mode equations as

$$\begin{aligned} \left[-\partial_y^2 - (Y\varphi_0)' + (Y\varphi_0)^2 \right] \chi_L^{(n)} &= m_n^2 \chi_L^{(n)}, \\ \left[-\partial_y^2 + (Y\varphi_0)' + (Y\varphi_0)^2 \right] \chi_R^{(n)} &= m_n^2 \chi_R^{(n)}. \end{aligned} \quad (46)$$

We can easily show that these equations are equivalent to the two equations in Eq. (20) by the identifications, $m_\varphi \rightarrow m_V$, $Y/\sqrt{\lambda} \rightarrow \gamma$, and $\chi_L^{(n)}, \chi_R^{(n)} \rightarrow \tilde{\chi}^{(n)}, \tilde{\psi}^{(n)}$. Hence, the mass eigenvalues and eigenfunctions are given by Eqs. (21)–(27). Note that we have only one (left) chiral fermion in the 4D effective theory, which is identified with an SM fermion in the extension of the SM case.

For our phenomenology discussion in the next section, let us set $Y/\sqrt{\lambda} = 2$. In this case, we have only one KK-mode Dirac fermion in the 4D effective theory with mass $\sqrt{3}m_\varphi$. The KK-mode expansion is explicitly described as

$$\begin{aligned} \psi_L(x, y) &= \frac{\sqrt{3m_\varphi}}{2} \left[\frac{1}{\cosh^2(m_\varphi(y - y_0))} \right] \psi_L^{(0)}(x) + \sqrt{\frac{3m_\varphi}{2}} \left[\frac{\sinh(m_\varphi(y - y_0))}{\cosh^2(m_\varphi(y - y_0))} \right] \psi_L^{(1)}(x), \\ \psi_R(x, y) &= \sqrt{\frac{m_\varphi}{2}} \left[\frac{1}{\cosh(m_\varphi(y - y_0))} \right] \psi_R^{(1)}(x), \end{aligned} \quad (47)$$

for which the kinetic terms are canonically normalized. Here, we have generalized the system and set the kink center at y_0 . The KK-mode functions for ψ_L are the same as those for the scalar φ shown in Eq. (42).

Let us now describe the Lagrangian for the chiral fermion in the 4D effective theory as

$$\mathcal{L}_4 \supset \overline{\psi_L^{(0)}} i\gamma^\mu \left(\partial_\mu - iQ_f g A_\mu^{(0)} \right) \psi_L^{(0)} + \sum_{n=1}^{\infty} Q_f g_{\text{eff}}^{(n)} A_\mu^{(n)} \left[\overline{\psi_L^{(0)}} \gamma^\mu \psi_L^{(0)} \right], \quad (48)$$

where the 4D effective gauge coupling between the chiral fermion and the n -th KK-mode gauge boson is given by

$$g_{\text{eff}}^{(n)} = \int_{-\infty}^{\infty} dy \left(\chi_L^{(0)} \right)^2 \chi^{(n)}. \quad (49)$$

For simplicity, we consider the solvable examples in Sect. 2 and take $m_V = m_\varphi$. For the first and second examples, the effective gauge couplings of the 1st KK-mode gauge boson are, respec-

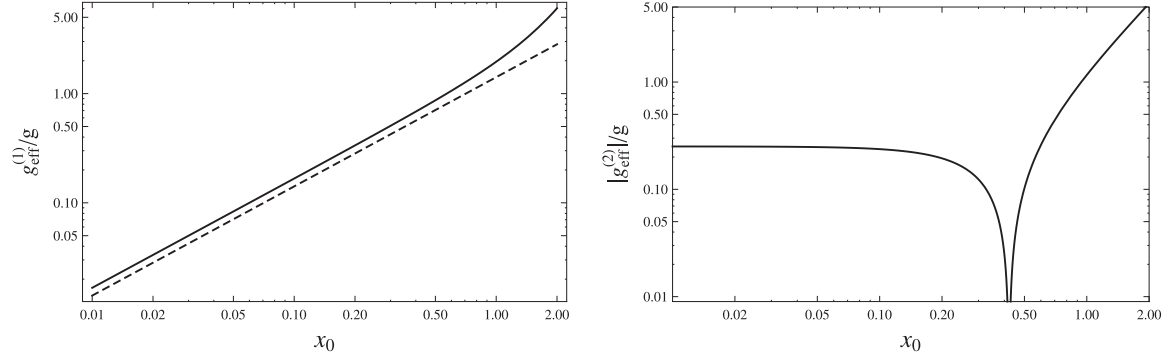


Fig. 1. Left: The effective coupling ($g_{\text{eff}}^{(1)}$) between the 1st KK-mode gauge boson and the chiral fermion as a function of x_0 for the first (solid) and second (dashed) examples in Sect. 2. The gauge coupling of the zero-mode gauge boson is denoted as g . Right: The effective coupling ($g_{\text{eff}}^{(2)}$) of the 2nd KK-mode gauge boson with the chiral fermion as a function of x_0 for the second example in Sect. 2. For $x_0 > 0.421$, we find $g_{\text{eff}}^{(2)} < 0$.

tively, given by

$$\begin{aligned}
 \text{Example 1 : } \frac{g_{\text{eff}}^{(1)}}{g} &= \frac{3\sqrt{2}}{4} m_\varphi^2 \int_{-\infty}^{\infty} dy \frac{y}{\cosh^4(m_\varphi(y - y_0))} \\
 &= \frac{3\sqrt{2}}{4} \int_{-\infty}^{\infty} dx \frac{x}{\cosh^4(x - x_0)}, \\
 \text{Example 2 : } \frac{g_{\text{eff}}^{(1)}}{g} &= \frac{3\sqrt{2}}{4} m_\varphi \int_{-\infty}^{\infty} dy \frac{\sinh(m_\varphi y)}{\cosh^4(m_\varphi(y - y_0))} \\
 &= \frac{3\sqrt{2}}{4} \int_{-\infty}^{\infty} dx \frac{\sinh(x)}{\cosh^4(x - x_0)}, \tag{50}
 \end{aligned}$$

where $x_0 = m_\varphi y_0$. Since the eigenfunction of the 1st KK-mode gauge boson is an odd-function of y , the effective coupling vanishes $g_{\text{eff}}^{(1)}/g \rightarrow 0$ for $x_0 \rightarrow 0$. In the first example, there is an infinite tower of KK-modes, and we also calculate the effective gauge coupling of the 2nd KK-mode gauge boson,

$$\frac{g_{\text{eff}}^{(2)}}{g} = \frac{3}{4\sqrt{2}} m_\varphi \int_{-\infty}^{\infty} dy \frac{1 - 2(m_\varphi y)^2}{\cosh^4(m_\varphi(y - y_0))} = \frac{3}{4\sqrt{2}} \int_{-\infty}^{\infty} dx \frac{1 - 2x^2}{\cosh^4(x - x_0)}, \tag{51}$$

which approaches $g_{\text{eff}}^{(2)}/g \rightarrow 0.251$ for $x_0 \rightarrow 0$.

In Fig. 1, we show the effective gauge couplings between the 1st KK-mode gauge boson and the chiral fermion for the first (solid) and second (dashed) examples (left panel), and the effective gauge coupling of the 2nd KK-mode gauge boson with the chiral fermion (right panel). In the left panel, the gauge couplings vanish for $x_0 \rightarrow 0$, while the gauge coupling of the 2nd KK-mode approaches a constant value, $g_{\text{eff}}^{(2)}/g \rightarrow 0.251$. We find $g_{\text{eff}}^{(2)} < 0$ for $x_0 > 0.421$. When applied to the SM, the gauge coupling g corresponds to one of the SM gauge couplings and the chiral fermion is identified with an SM fermion. We will discuss implications of this coupling behavior for LHC phenomenology in the next section.

Finally, let us extend our system to the SM case: we introduce the Yukawa coupling of the SM fermions in 5D as

$$\mathcal{L}_Y = -Y_f \bar{D} H S + \text{H.c.} = -Y_f \bar{D}_L H S_R - Y_f \bar{D}_R H S_L + \text{H.c.}, \tag{52}$$

where D and S are 5D fermions of the SM SU(2) doublet and singlet, respectively, we have decomposed them into their chiral components $D = D_L + D_R$ and $S = S_L + S_R$, and H is the 5D Higgs doublet. With the kink background, zero-modes of D_L and S_R are identified with left-handed SM doublet and right-handed singlet fermions. For simplicity, suppose the KK-mode expansions for D and S (we exchange the chiralities for S) are given by Eq. (47). For the Higgs doublet field in 5D, let us take, for simplicity,

$$s_H(y) = \frac{3}{4} \frac{m_\phi}{\cosh^4(m_\phi y)} \quad (53)$$

as in Sect. 2, so that the KK-mode decomposition of the physical Higgs boson after the electroweak symmetry breaking is given by (see Eq. (28)):

$$h(x, y) = h^{(0)}(x) + \sqrt{2} \sinh(m_\phi y) h^{(1)}(x). \quad (54)$$

When we identify Eq. (52) with top Yukawa coupling in 5D, we obtain a 4D effective Yukawa coupling for the top quark as

$$\mathcal{L}_Y^4 \supset -m_t \left(1 + \frac{h^{(0)}}{v}\right) \bar{t}_L t_R - \left(\frac{m_t}{v}\right) C_{\text{eff}} h^{(1)} \bar{t}_L t_R, \quad (55)$$

where $m_t = Y_f v / \sqrt{2}$, and C_{eff} is given by Eq. (50). The top quark mass formula is the same as the SM one in 4D, while the model involves a KK-mode Higgs boson with the Yukawa coupling $m_t C_{\text{eff}} / v$ and its mass $\sqrt{3m_\phi^2 + m_h^2}$ with the SM Higgs boson mass m_h .

5. KK-mode phenomenology

Prediction of the KK-modes in the 4D effective theory is a common property of extra-dimensional models and we can investigate the phenomenology involving the KK-modes. However, in the Domain-Wall SM, the KK-mode spectra and the coupling manner of each KK-mode with the SM particles depend on the localization mechanism. This property is in sharp contrast to, e.g. the Universal Extra-Dimension model [17], where the KK-mode eigenfunctions are uniquely determined by boundary conditions associated with the compactification of the 5th dimension. The Domain-Wall SM offers more variety of the KK-mode phenomenologies than usual compactified extra-dimensional models, thanks to rich geometrical structures of the localized SM particles and their KK-modes. In this section, we address a few directions for interesting KK-mode phenomenologies.

5.1. Phenomenology of KK-mode gauge bosons

The ATLAS and the CMS collaborations have been searching for a new gauge boson resonance with a variety of final states at the LHC Run-2. The so-called sequential SM Z' and W' bosons, which have the same properties as the SM Z and W bosons except for their masses, have been examined as a benchmark model. According to the LHC Run-2 final report by the ATLAS collaboration on their search results with $\sqrt{s} = 13$ TeV and an integrated luminosity of 139 fb^{-1} , the lower mass bound on the sequential SM Z' boson is $m_{Z'} \geq 5.1$ TeV, which was obtained by the search with dilepton final states [18] (a similar bound, $m_{Z'} \geq 5.15$ TeV, was obtained by the CMS collaboration [19]). A more severe constraint, $m_{W'} \geq 6.0$ TeV, was obtained for the sequential SM W' boson mass from the search with its decay mode $W' \rightarrow l\nu$ [20] (a similar bound, $m_{W'} \geq 5.7$ TeV, was obtained by the CMS collaboration [21]). In this subsection, we interpret these results as constraints on the KK-mode gauge bosons in the Domain-Wall SM.

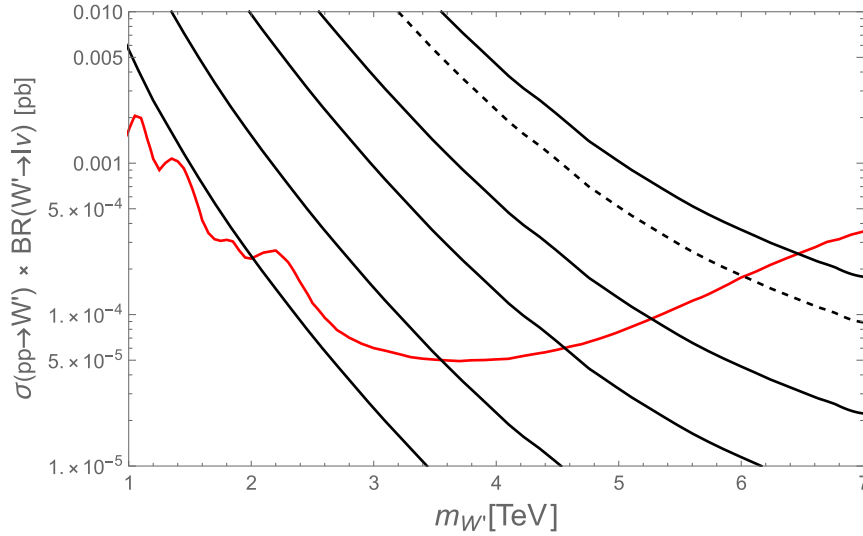


Fig. 2. The cross section $\sigma(pp \rightarrow W^{(1)} \rightarrow l\nu)$ as a function of $m_{W'} = m_{W'}^{(1)}$ for $g_{\text{eff}}^{(1)}/g = 0.04, 0.1, 0.251, 0.5,$ and $\sqrt{2}$ (solid lines) from left to right, along with the upper bound on the cross section from the ATLAS results (red horizontal curve) and the theoretical prediction of $\sigma(pp \rightarrow W' \rightarrow l\nu)$ for the sequential SM W' boson (dashed line). For these $g_{\text{eff}}^{(1)}/g$ values, we find the lower bounds on the 1st KK-mode W boson as $m_{W'}^{(1)}[\text{TeV}] \geq 2.0, 3.5, 4.6, 5.3,$ and 6.5 , respectively. The result for $g_{\text{eff}}^{(1)}/g = 0.251$ can be identified as the result for the 2nd KK-mode W boson in the limit of $x_0 \rightarrow 0$.

In Sect. 2, we have chosen $s_1 \propto s_2$, so that the mass spectrum of the KK-mode Z and W bosons is the same, neglecting the mass terms generated by the electroweak symmetry breaking. Thus, we consider the most severe constraint from the W' boson search. Since the total decay width of the sequential W' boson is about 3% of its mass for $m_{W'} \gtrsim 1$ TeV, we employ the narrow-width approximation in evaluating the parton-level cross section of the process,

$$\hat{\sigma}(q\bar{q}' \rightarrow W') \propto \Gamma_{W'}(W' \rightarrow q\bar{q}') \delta(M_{\text{inv}}^2 - m_{W'}^2) \propto g^2, \quad (56)$$

where M_{inv}^2 is the invariant mass of the initial partons, $\Gamma_{W'}(W' \rightarrow q\bar{q}')$ is the partial decay width into $q\bar{q}'$, and g is the SM SU(2) gauge coupling. The difference of the sequential W' boson and the KK-mode W boson in the Domain-Wall SM is only the coupling constant. Because of the nontrivial eigenfunctions of the SM fermions and the KK-mode W boson, the effective gauge coupling constant is not the same as the SM gauge coupling constant, as shown in Fig. 1 for our example. Hence, we have a relation between the production cross sections of the sequential W' and the 1st KK-mode W bosons in the narrow-width approximation:

$$\sigma(pp \rightarrow W^{(1)} \rightarrow l\nu) = \left(\frac{g_{\text{eff}}^{(1)}}{g} \right)^2 \sigma(pp \rightarrow W' \rightarrow l\nu), \quad (57)$$

by which we can interpret the current ATLAS constraints as those on the 1st KK-mode W boson ($W^{(1)}$).

In Fig. 2, we show the cross section $\sigma(pp \rightarrow W^{(1)} \rightarrow l\nu)$ as a function of $m_{W'} = m_{W'}^{(1)}$ for various values of $g_{\text{eff}}^{(1)}/g$, along with the upper bound on the cross section from the ATLAS results [20] (horizontal curve (in red)) and the theoretical prediction of $\sigma(pp \rightarrow W' \rightarrow l\nu)$ for the sequential SM W' boson (dashed line). The solid diagonal lines from left to right depict the theoretical predictions of the cross section $\sigma(pp \rightarrow W^{(1)} \rightarrow l\nu)$ for $g_{\text{eff}}^{(1)}/g = 0.04, 0.1, 0.251, 0.5,$ and $\sqrt{2}$, respectively, corresponding to $x_0 = 0.024, 0.060, 0.15, 0.30,$ and 0.77 ($x_0 = 0.028,$

0.071, 0.18, 0.35, and 1.0) from the solid (dashed) line in the left panel of Fig. 1. For these $g_{\text{eff}}^{(1)}/g$ values, we read off the lower bounds on the 1st KK-mode mass as $m_{W'}^{(1)}[\text{TeV}] \geq 2.0, 3.5, 4.6, 5.3, \text{ and } 6.5$, respectively. The result for $g_{\text{eff}}^{(1)}/g = 0.251$ is identified with the result for the 2nd KK-mode gauge boson for $x_0 \rightarrow 0$ from the right panel of Fig. 1.

The structure of the KK-modes depends on a localization mechanism, in particular, the shape of the y -dependent gauge couplings. We have introduced two solvable examples in Sect. 2: the first one predicts an infinite tower of KK-modes, whereas the KK-mode expansion is terminated in the second example. The second example is in sharp contrast to extra-dimensional models with compactified extra dimensions, which predict the infinite tower of the KK-mode spectrum. Once the 1st KK-mode state is discovered, the search for higher KK-mode states at high-energy colliders can test if an extra dimension is compactified or not.

5.2. Higgs boson phenomenology

Since the KK-modes of the SM particles have couplings with the SM Higgs boson, the presence of the KK-modes affects Higgs boson phenomenology. In particular, the Higgs boson properties measured at the LHC [22] are altered from the SM predictions. In this section, we consider the implications of the KK-modes of the top quark and W boson for the Higgs boson phenomenology. See, e.g. Ref. [23] for a pioneering work in this direction.

The SM Higgs boson has effective couplings with the digluon and diphoton of the form,

$$\mathcal{L}_{\text{Higgs-gauge}} = C_{gg} h G_{\mu\nu}^A G^{A\mu\nu} + C_{\gamma\gamma} h F_{\mu\nu} F^{\mu\nu}, \quad (58)$$

where $G_{\mu\nu}^A$ $A = 1, 2, \dots, 8$ and $F_{\mu\nu}$ are the field strengths of the gluon and photon. In the SM, these effective operators are induced dominantly through 1-loop corrections of the top quark and W -boson (and associated would-be NG bosons and ghosts) [24]. The effective coupling with the digluon from top quark loop corrections is calculated to be

$$C_{gg}^{\text{SM}} = \frac{\alpha_s}{16\pi v} F_{1/2}(\tau_t), \quad (59)$$

where α_s is the quantum chromodynamics (QCD) coupling, and $\tau_t = 4m_t^2/m_h^2$ with top quark mass m_t and Higgs boson mass m_h . The effective coupling with the diphoton is from 1-loop corrections with the top quark and W -boson, and we have

$$C_{\gamma\gamma}^{\text{SM}} = \frac{\alpha_{\text{em}}}{8\pi v} \left(\frac{4}{3} F_{1/2}(\tau_t) + F_1(\tau_W) \right), \quad (60)$$

where $\tau_W = 4m_W^2/m_h^2$ with the W -boson mass m_W . The explicit formulas of the loop functions are given by

$$F_{1/2}(\tau) = 2\tau [1 + (1 - \tau)f(\tau)], \quad F_1(\tau) = -[2 + 3\tau + 3\tau(2 - \tau)f(\tau)], \quad (61)$$

with $f(\tau) = [\sin^{-1}(1/\sqrt{\tau})]^2$ ($\tau > 1$). For $m_t = 172.69$ GeV, $m_W = 80.377$, and $m_h = 125.25$ GeV [22], we find $F_{1/2}(\tau_t) \simeq 1.38$ and $F_1(\tau_W) \simeq -8.33$.

In the presence of the KK-mode top quarks and W bosons, the effective Higgs couplings receive 1-loop corrections with the KK-modes. Again, we consider the KK-mode expansions in Eq. (47) for top quarks of the SM SU(2) doublet component and the SU(2) singlet. After the electroweak symmetry breaking, it is easy to derive the mass matrix for the 1st KK-mode top quarks as

$$\mathcal{M}_t = \begin{bmatrix} -m_t^{(1)} & m_t \\ m_t & m_t^{(1)} \end{bmatrix}, \quad (62)$$

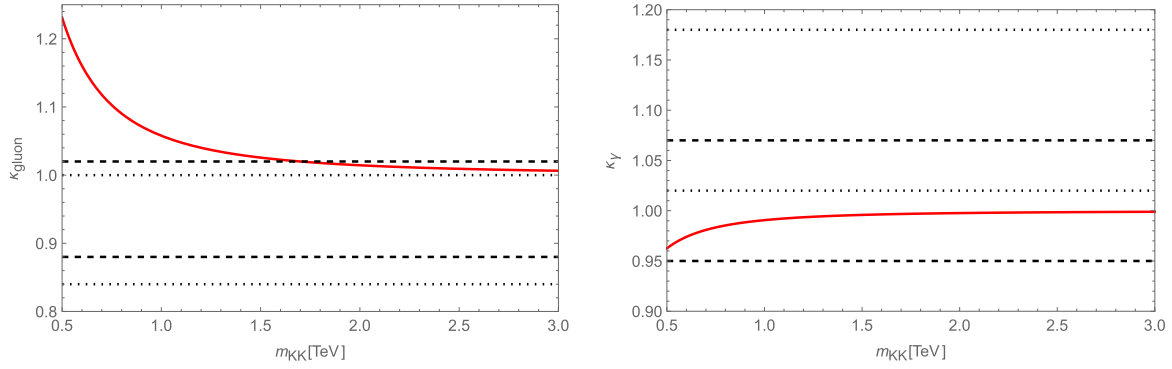


Fig. 3. Left: The ratio of the effective Higgs boson coupling with the digluon to the SM one as a function of m_{KK} (red solid line), along with the ATLAS and CMS results. Right: The ratio of the effective Higgs boson coupling with the diphoton to the SM one as a function of m_{KK} (red solid line), along with the ATLAS and CMS results.

where $m_t^{(1)} = \sqrt{3}m_\phi$ is the KK-mode mass. We then have degenerate mass eigenvalues, $m_{\text{KK}} = \sqrt{(m_t^{(1)})^2 + m_t^2}$. For $m_{\text{KK}}^2 \gg m_t^2$, we can easily calculate the contribution of the KK-mode top quarks to C_{gg} by using the Higgs low-energy theorem [24] as

$$C_{gg}^{\text{KK-top}} \simeq \frac{\alpha_s}{8\pi v} b'_3 \frac{\partial}{\partial \log v} \log(m_{\text{KK}}) \times 2 \simeq \frac{\alpha_s}{6\pi v} \left(\frac{m_t}{m_{\text{KK}}} \right)^2, \quad (63)$$

where $b'_3 = 2/3$ is the top quark contribution to the beta function coefficient of QCD. Similarly, the contribution to $C_{\gamma\gamma}$ from the KK-mode top quarks is given by

$$C_{\gamma\gamma}^{\text{KK-top}} \simeq \frac{\alpha_{\text{em}}}{6\pi v} b'_1 \frac{\partial}{\partial \log v} \log(m_{\text{KK}}) \times 2 \simeq \frac{4\alpha_{\text{em}}}{9\pi v} \left(\frac{m_t}{m_{\text{KK}}} \right)^2, \quad (64)$$

where $b'_1 = 4/3$ is the top quark contribution to the quantum electrodynamics (QED) beta function coefficient. In calculating the contribution from the KK-mode W boson, we consider the expansion in Eq. (28) with $m_V = m_\phi$ to simplify our analysis. In this case, the 1st KK-mode W boson has mass $\sqrt{(m_W^{(1)})^2 + m_W^2} \simeq m_{\text{KK}}$, and we find the contribution to $C_{\gamma\gamma}$ from the KK-mode W boson as

$$C_{\gamma\gamma}^{\text{KK-W}} \simeq \frac{\alpha_{\text{em}}}{8\pi v} b_1^W \frac{\partial}{\partial \log v} \log \left(\sqrt{(m_W^{(1)})^2 + m_W^2} \right) \simeq -\frac{7\alpha_{\text{em}}}{8\pi v} \left(\frac{m_W}{m_{\text{KK}}} \right)^2, \quad (65)$$

where $b_1^W = -7$ is the W -boson contribution to the QED beta function coefficient.

At the LHC, the Higgs bosons are produced through various production processes and various Higgs boson decay channels have been detected. The progress on precision measurements of the Higgs boson's properties since the first discovery in 2012 has been reported by the ATLAS collaboration [25] and the CMS collaboration [26]. Their results show that the measured Higgs boson properties are consistent with the SM predictions, constraining the new physics contributions.

Let us first evaluate the ratio of the effective Higgs boson coupling with the digluon to the SM one as

$$\kappa_{\text{gluon}} = 1 + \frac{C_{gg}^{\text{KK-top}}}{C_{gg}^{\text{SM}}}. \quad (66)$$

This ratio as a function of m_{KK} is shown in Fig. 3 (left panel). In the presence of the KK-

mode top quarks, κ_{gluon} is deviating from $\kappa_g = 1$ as m_{KK} is lowered. In the figure, we also show the ATLAS result [25] of $0.88 \leq \kappa_{\text{gluon}} \leq 1.02$ at the 1σ confidence level (region between two dashed lines) and the CMS result [26] of $0.84 \leq \kappa_{\text{gluon}} \leq 1.00$ at the 1σ confidence level (region between two dotted lines). We find the lower bound on $m_{\text{KK}} \geq 1.7$ TeV to be consistent with the ATLAS result, while our result is not consistent with the CMS result at the 1σ confidence level. The KK-mode mass of $m_{\text{KK}} \gtrsim 0.5$ TeV is consistent with both of the ATLAS and CMS results when we adopt the 2σ confidence level.

We also evaluate the ratio of the effective Higgs boson coupling with the diphoton to the SM one as

$$\kappa_\gamma = 1 + \frac{C_{gg}^{\text{KK-top}} + C_{gg}^{\text{KK-W}}}{C_{gg}^{\text{SM}}}, \quad (67)$$

which is shown in Fig. 3 (right panel), along with the ATLAS result [25] of $0.95 \leq \kappa_g \leq 1.07$ at the 1σ confidence level (region between two dashed lines) and the CMS result [26] of $1.02 \leq \kappa_g \leq 1.18$ at the 1σ confidence level (region between two dotted lines). We find our result for $m_{\text{KK}} \gtrsim 0.5$ TeV is consistent with the ATLAS result, while our result is inconsistent with the CMS result at the 1σ confidence level. When we adopt the CMS result at the 2σ confidence level, our result becomes consistent.

The SM Higgs boson is accompanied by a KK-mode Higgs boson which couple with a top quark pair, as we have seen in Eqs. (54) and (55). Hence, the KK-mode Higgs boson can be produced at the LHC through gluon fusion. The coupling of the KK-mode Higgs boson to top quarks can be enhanced for a large x_0 (see the left panel of Fig. 1). It would be worth investigating the LHC phenomenology for the KK-mode Higgs boson.

5.3. Phenomenology of KK-mode fermions

Let us consider interactions between the SM fermions and their KK-modes. Since the eigenfunctions of the SM gauge bosons, which are the zero-modes, are independent of y , an interaction among one SM fermion, one KK-mode fermion, and one SM gauge boson vanishes by the orthogonal condition for the eigenfunctions. This is also true for a Yukawa interaction among one SM fermion, one KK-mode fermion, and one SM Higgs boson, if the SM SU(2) doublet and singlet fermions are decomposed by the same KK-mode eigenfunctions, as we have considered in this paper to simplify our discussions. However, there is a unique interaction between an SM fermion and its KK-mode derived from the Yukawa interaction with the kink scalar in Eq. (43),

$$\begin{aligned} \mathcal{L} &\supset Y\varphi (\overline{\psi}_L\psi_R + \overline{\psi}_R\psi_L) \\ &\supset Y \frac{3m_\varphi}{2} \sqrt{\frac{m_\varphi}{2}} \left[\frac{1}{\cosh^5(m_\varphi(y-y_0))} \right] \varphi^{(0)}(x) \overline{\psi}_L^{(0)}(x) \psi_R^{(1)}(x) + \text{H.c.}, \end{aligned} \quad (68)$$

where we have used the KK-mode expansions in Eqs. (42) and (47) with the shift of the kink center to $y = 0 \rightarrow y_0$. Integrating out the 5th-dimensional degrees of freedom, we obtain a 4D effective interaction,

$$\mathcal{L}_{\text{eff}} \supset y_{\text{eff}} \varphi^{(0)} \overline{\psi}_L^{(0)} \psi_R^{(1)} + \text{H.c.}, \quad (69)$$

where

$$y_{\text{eff}} = \frac{3m_\varphi}{2} \sqrt{\frac{m_\varphi}{2}} \int_{-\infty}^{\infty} \frac{dy}{\cosh^5(m_\varphi(y-y_0))} = Y \frac{9\pi}{16} \sqrt{\frac{m_\varphi}{2}}. \quad (70)$$

Therefore, the KK-mode fermion decays to the zero-mode fermion and the massless scalar, the NG boson associated with the breakdown of 5th-dimensional translational invariance.

In the extension to the SM case, we identify the fermions in Eq. (69) with an SM fermion and its “KK-mode partner.” This Yukawa interaction leads to an interesting phenomenology for the KK-mode fermions. At the LHC, a pair of KK-mode fermions can be produced through gauge interactions. For example, we may consider a pair of KK-mode quarks produced through gluon fusion. Once a KK-mode fermion is produced, it decays into an SM fermion and the SM-singlet NG boson $\varphi^{(0)}$. Hence, a characteristic signature of the process at the LHC is a final state with two SM fermion jets and a missing energy from $\varphi^{(0)}$ s. Note that this is analogous to a signature of superparticle pair production in the simplified supersymmetric models [27], where a superparticle produced in a pair decays to its partner SM particle and a stable neutralino. In order to obtain the current LHC constraints on KK-mode quarks, e.g. we may apply the ATLAS and CMS results from the search for squarks with a process $pp \rightarrow \tilde{q}\tilde{q}$, followed by $\tilde{q} \rightarrow q\tilde{\chi}_1^0$. A massless limit for the neutralino corresponds to our case. Although the production cross section for KK-mode quarks is a few times larger than that for squarks, we roughly obtain $m_{\text{KK}} \gtrsim 2$ TeV for the KK-mode quarks from the current LHC results [28].

6. Conclusions and discussions

In Ref. [6], the authors of the present paper have proposed a framework to construct the Domain-Wall SM which is defined in a non-compact 5D space-time. Considering localization mechanisms for the gauge field, the Higgs field and its VEV, and the chiral fermion in 5D flat Minkowski space, we have derived the SM as the 4D effective theory at low energies. The model predicts the KK-modes for the SM particles, and we have briefly addressed LHC phenomenology for KK-mode gauge bosons.

In this paper, we have investigated aspects of the Domain-Wall SM in detail. For concreteness, we have introduced two solvable examples to localize all the SM particles in certain domains of the 5th dimension, and have explicitly shown the KK-mode mass spectrum and eigenfunctions. One example predicts an infinite tower of KK-modes of the SM gauge bosons, while the number of KK-modes is finite in the other example. With explicit forms of the KK-mode eigenfunctions, we have derived the 4D effective Lagrangian involving the KK-mode SM particles. The Domain-Wall SM offers a variety of interesting phenomenologies. Among others, we have addressed, in this paper, the LHC phenomenology of the KK-mode gauge boson, the effect of the KK-mode SM fermions on Higgs boson phenomenology, and the KK-mode fermion search at the LHC with its decay into a corresponding SM fermion and a NG boson associated with a spontaneous breaking of the translational invariance in the 5th dimension.

In our solvable examples, we have introduced a special function $s(y)$ to localize the 5D gauge field, as well as the 5D Higgs field and its VEV. In the theoretical point of view, we may seek a possible origin of $s(y)$. The second example of Eq. (19) is particularly interesting, since it is expressed in terms of the kink solution when $m_V = m_\varphi$,

$$s(y) = \frac{M}{[\cosh(m_\varphi y)]^{2\gamma}} = M \left(1 - \frac{\sqrt{\lambda}}{m_\varphi} \varphi_{\text{kink}}(y)^2 \right)^\gamma. \quad (71)$$

In the normal field theory sense, the parameter γ is an integer. Since a KK-mode exists for $\gamma > 1$, we are interested in the choice of $\gamma \geq 2$. Now we propose a unified picture of localizing

all the SM fields by

$$s(y) = M_G \left(1 - \frac{\sqrt{\lambda}}{m_\varphi} \varphi_{\text{kink}}(y)^2 \right)^{\gamma_G}, \quad s_H(y) = M_H \left(1 - \frac{\sqrt{\lambda}}{m_\varphi} \varphi_{\text{kink}}(y)^2 \right)^{\gamma_H}, \quad (72)$$

where $M_{G,H}$ are positive mass parameters, and $\gamma_{G,H}$ are integers. Note that this picture also introduces interactions between the physical modes in φ and the gauge bosons. This phenomenology is worth considering.

In our analysis, we have implicitly assumed that all the SM fermions have the same domain-wall configuration. However, in general, SM chiral fermions can be localized around different points. Such a generalization opens up a possibility to solve the fermion mass hierarchy problem in the SM from the wave-function overlapping, leading to an exponentially suppressed effective Yukawa coupling, as proposed in Ref. [29]. Configurations of the domain-wall fermions reflect their effective gauge couplings with the KK-mode gauge bosons. Therefore, this ‘‘geometry’’ relating to the fermion mass hierarchy can be tested at a future LHC experiment, once a KK-mode gauge boson is discovered and its branching ratios into final state fermions are measured.

Since the graviton resides in the bulk, we also need to consider a localization of the graviton to make the Domain-Wall SM phenomenologically viable. For this purpose, we may combine our model with the RS-2 scenario [5] with the Planck brane at $y = 0$. Here we may identify the Planck brane as a domain-wall with the zero-width limit. The mass spectrum of the KK-modes of the SM fields is controlled by the width of the domain-walls, and the current LHC results constrain it to be $\lesssim (1 \text{ TeV})^{-1}$. On the other hand, the width of the 4D graviton is controlled by the AdS curvature κ in the RS-2 scenario and its experimental constraint is quite weak, $\kappa \gtrsim 10^{-3} \text{ eV}$ [5]. Therefore, we can take $\kappa \ll 1 \text{ TeV}$ and neglect the warped background geometry in our setup of the Domain-Wall SM. The energy density from the SM domain-walls can affect the RS-2 background geometry. However, we expect this energy density is of $\mathcal{O}(\Lambda^4)$ with $\Lambda = \mathcal{O}(1 \text{ TeV})$, while the energy density of the Planck brane in the RS-2 scenario is given by $\mathcal{O}(M_P^2 \kappa^2)$ with the reduced Planck mass of $M_P \simeq 2.4 \times 10^{18} \text{ GeV}$. Therefore, we choose the AdS curvature in the range of $10^{-3} \text{ eV} \ll \kappa \ll 1 \text{ TeV}$ for the theoretical consistency of our scenario.

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