Anomalies in string-inspired nonlocal extensions of QED

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We investigate anomalies in the class of nonlocal field theories that have been proposed as an ultraviolet completion of four-dimensional quantum field theory with generalizing the kinetic energy operators to an infinite series of higher derivatives inspired by the string field theory and ghost-free nonlocal approaches to quantum gravity. We explicitly calculate the vector and chiral anomalies in a string-inspired nonlocal extension of QED. We show that the vector anomaly vanishes as required by gauge invariance and the Ward identity. On the other hand, although the chiral anomaly vanishes to the leading order with massless fermions, it nonetheless does not vanish with the massive fermions and we calculate it to the leading order in the scale of nonlocality. We also calculate the nonlocal vector and axial currents explicitly and present an illustrative example by applying our results to the decay of $\pi^0 \to \gamma\gamma$.

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I. INTRODUCTION

It is a well-known fact that strings, being nonlocal objects by their nature, are free from ultraviolet (UV) divergences [1–13]. This fact inspired many physicists into trying to mimic this good UV behavior by formulating nonlocal quantum field theories (QFTs) as extensions of local QFTs, where nonlocality is introduced to eliminate any UV divergences that could exist in the local case. The general prescription for transforming local QFTs to nonlocal ones is to introduce nonlocality to the kinetic term via an entire function with infinite derivatives. For instance, in the scalar sector one writes

$$S_{\rm NL} = \int d^4x \left[\frac{1}{2}\phi\mathcal{K}(\Box)(\Box + m^2)\phi - V(\phi)\right], \qquad (1)$$

and the form factor $\mathcal{K}$ has the function of smearing the interaction vertex, such that it becomes spatially finite in size, rather than being pointlike, thereby making the interaction nonlocal. Apart from being an entire function of the $\Box$ operator with infinite derivatives so that no new poles are introduced to the theory, there are no conditions on the form of $\mathcal{K}(\Box)$, and any function that has the required properties is acceptable. However, in order for the UV behavior of loop amplitudes to be finite and avoid divergences, a common choice is to use a simple exponential function

$$\mathcal{K}(\Box) \equiv \exp\left(\frac{\Box + m^2}{\Lambda^2}\right), \qquad (2)$$

where m is the mass of the particle and Λ is the scale of nonlocality. With this choice of form factors, it is easy to see that at high energies, loop amplitudes behave like $\sim e^{-\frac{s}{\Lambda^2}}$, which is suppressed when $s > \Lambda^2$ and is thus free from UV divergences. However, the construction in Eqs. (1) and (2) is an ansatz not derived from first principles and should be treated as an effective field theory of yet another UV completion above the scale of nonlocality. Furthermore, the form factor in Eq. (2) will render the theory acausal, albeit at a level suppressed by the scale of nonlocality (which should be high). This is the same issue that plagues the Lee-Wick theory [14,15].[1] Causality violation in such theories was discussed in [16], and the authors of [17] described how such a causality violation could be measured in colliders. In spite of all of this, nonlocality introduced this way can still be used to calculate observables.

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[1]Actually, the Lee-Wick theory emerges as the leading order in the expansion of the form factor in Eq. (2).

The form factor in Eq. (2) is reminiscent of the star product in noncommutative geometries (see, for instance, [1,9,18]), which is defined as

$$(\phi_1 \star \phi_2)(x) = e^{\frac{i}{2}\Theta^{\mu\nu}\partial_\mu^y \partial_\nu^z} \phi_1(y)\phi_2(z)|_{x=y=z}, \tag{3}$$

where the matrix $\Theta^{\mu\nu}$ is a real antisymmetric matrix that defines the algebra of the noncomutative geometry,

$$[x^\mu, x^\nu] = i\Theta^{\mu\nu}. \tag{4}$$

However, there are subtle differences between the two. For example, while the form factor in Eq. (2) is symmetric and Lorentz invariant, the star product in Eq. (3) is antisymmetric and is not Lorentz invariant. Nonetheless, the noncommutative geometry encapsulated in the exponent of the star product regulates UV divergences as does the scale of nonlocality, and one can think of the scale of nonlocality as an emergent scale from the scale of noncommutativity. An example of how the two can be related can be found in [18].

The first serious step toward constructing a realistic nonlocal QFT was taken in [19], where the nonlocal version of the Abelian gauge theory was formulated and the corresponding LHC phenomenology was studied. The formulation of nonlocal QED makes it possible to investigate the effects of the putative nonlocality in this sector, such as the possible enhancement/suppression of scattering processes in colliders, the possible effect on electroweak precision observables, and its impact on gauge anomalies.

Local gauge anomalies were first explained in [20–22], and it is now understood that the anomaly associated with the vector current vanishes as a direct result of gauge invariance and the Ward identity, whereas the chiral anomaly associated with the axial current is nonvanishing since the axial current is global and cannot be gauged, implying that it cannot be conserved. The first (and to the best of our knowledge, only) study that attempted to investigate the $U(1)$ gauge anomalies in nonlocal QED was [23], where the authors utilized a novel formalism dubbed the "shadow field formalism" to show that introducing nonlocality does not affect the conservation of the vector current, nor does it remove the chiral anomaly. In the present paper, we attempt to extend a similar treatment to the nonlocal QED version formulated in [19]. In particular, we will try to show that the vector anomaly vanishes and that the Ward identity is respected, and we derive the nonlocal chiral anomaly and the associated nonlocal Noether currents. We show that our results through explicit calculation using the nonlocal QED formulation in [19] agree with the results obtained in [23].

This paper is organized as follows: In Sec. II, we review the nonlocal QED theory introduced in [19]. In Sec. III we explicitly calculate the vector and chiral anomalies in nonlocal QED, and we derive the associated Noether current. We relegate some technical detail to Appendix A, and then we compare our results with [23] and show that they agree. In Sec. IV we apply our findings to the decay process of $\pi^0 \to \gamma\gamma$ and use the result to set an experimental bound on the scale on nonlocality, and finally we present our conclusions in Sec. V.

II. REVIEW OF NONLOCAL QED

We begin by providing a quick overview of the nonlocal extension of QED that was derived in [19]. The basic idea behind obtaining the nonlocal version of QED is to start with the local version, and then introduce the nonlocality factor represented by the exponential of an entire function of derivatives, such that the action remains gauge invariant. With this prescription in mind, the nonlocal version of QED can be written as

$$\mathcal{L}_{\text{NL}} = -\frac{1}{4} F_{\mu\nu} e^{\frac{\Box}{\Lambda_g}} F^{\mu\nu} + \frac{1}{2}[i\bar{\Psi} e^{-\frac{\nabla^2}{\Lambda_f^2}} (\nabla\!\!\!/ + m)\Psi + \text{H.c.}], \tag{5}$$

where $\nabla_\mu = \partial_\mu + ieA_\mu$, which implies

$$\nabla^2 = \Box + ie(\partial \cdot A + A \cdot \partial) - e^2 A^2. \tag{6}$$

Here, we have accommodated for the fact that the scale of nonlocality for the fermions and photon could be different in principle. Notice that while we are using the ordinary derivative in the photon's kinetic term, the covariant derivative has to be used in the fermion sector to keep it gauge invariant. In calculating the nonlocal QED anomaly, one only needs the Feynman rules for the fermion propagator and the interaction vertices. The former is easily extracted to be

$$\Pi_f = \frac{ie^{\frac{p^2}{\Lambda_f^2}}(p\!\!\!/ + m)}{p^2 - m^2 + i\epsilon}. \tag{7}$$

It is easy to see that in the limit $\Lambda_f \to \infty$ one recovers the standard fermion propagator. On the other hand, extracting the interaction vertex is more subtle, as special care is needed to include the contribution from the covariant derivative in the exponent. To proceed, we expand the covariant derivative in the nonlocal factor, and then only keep the terms at linear order in A. The final result is given by

$$V(k_1, k_2) = -\frac{ie}{2}\left[(k_{1\mu} k\!\!\!/_2 + k_{2\mu} k\!\!\!/_1)\left(\frac{e^{\frac{k_1^2}{\Lambda_f^2}} - e^{\frac{k_2^2}{\Lambda_f^2}}}{k_1^2 - k_2^2}\right) + \left(e^{\frac{k_1^2}{\Lambda_f^2}} + e^{\frac{k_2^2}{\Lambda_f^2}}\right)\gamma_\mu\right], \tag{8}$$

where $k_{1,2}$ are the momenta of the fermions. In the limit $\Lambda_f \to \infty$ one recovers the local QED. We refer the interested reader to [19] for the detailed derivation.

III. ANOMALIES IN NONLOCAL QED

In this section, we will explicitly calculate the $U(1)$ vector and axial anomalies in the nonlocal extension of QED formulated in [19]. In our calculation, we follow the method presented in [24] based on calculating the triangle diagrams regularized via a Pauli-Villars regulator. However, unlike the case of local QED, no regulator is needed to calculate the loop diagrams in nonlocal QFTs, as they are already superrenormalizable due to the nonlocality form factor. Similar to the case of local QFTs, anomalies in nonlocal QED arise from triangle diagrams with charged fermions running in the loops, with two vector and one axial currents attached to the vertices as shown in the top row of Fig. 1. In nonlocal QED, there is an additional contribution from the bubble diagram shown in the bottom row of Fig. 1. One can see how this type of diagrams comes into play by inspecting Eqs. (5) and (6). We can see that when we expand the covariant derivative in the form factor, we obtain an infinite tower of nonrenormalizable effective vertices $\sim\bar{\Psi}\Psi A^n$, where we see that the bubble diagram arises from the vertex with $n = 2$. These interaction vertices are a direct consequence of the requirement of gauge invariance, which necessitated using the covariant derivative instead of the ordinary one in the nonlocality form factor. We present the detailed derivation of the Feynman rule associated with the $\bar{\Psi}\Psi A^2$ vertex in Appendix.

Before we proceed with calculating the anomalies, we point out that in general, calculating loop diagrams in nonlocal QFTs is not doable exactly due to the complex nature of the form factor that contains loop momenta to be integrated over. However, the calculation simplifies significantly if we assume that the scale of nonlocality is much larger than the external momenta, i.e., $\Lambda \gg p, q$. Given the lower bound on $\Lambda \sim 2.5$–3 TeV [19], the validity of this approximation is well justified, as was demonstrated in detail in [25]. In this limit, the form factors in the propagators and the interaction vertices are simplified and reduced to $e^{(k\pm p)^2/\Lambda^2} \simeq e^{(k\pm q)^2/\Lambda^2} \simeq e^{k^2/\Lambda^2}$, where k is the loop momentum to be integrated over.

A. Vector and chiral anomalies with massless fermions

We first investigate the case where the fermions in the loops are massless. We begin by calculating the bubble diagram. In the limit of small external momenta, the corresponding matrix element reads

$$\mathcal{M}_{\bigcirc}^{\mu\nu\rho} \simeq ie^2 \int \frac{d^4k}{(2\pi)^4} e^{\frac{4k^2}{\Lambda^2}} \mathrm{Tr}\left[\frac{\gamma^\mu\gamma^5(\not{k}+\not{p})V^{\nu\rho}(k+p,k-q,p,q)(\not{k}-\not{q})}{(k+p)^2(k-q)^2}\right], \tag{9}$$

where $V^{\nu\rho}$ is given by Eq. (A17). Using the explicit expression of $V^{\nu\rho}(k+p,k-q,p,q)$, we find that

$$\mathcal{M}_{\bigcirc}^{\mu\nu\rho} \sim \mathrm{Tr}[\gamma^\mu\gamma^5(\not{k}+\not{p})[(\not{k}-\not{q})(k+p)^\nu(k+p-q)^\rho - (\not{k}+\not{p})(k-q)^\nu k^\rho](\not{k}-\not{q})] = 0. \tag{10}$$

Therefore, the bubble diagram does not contribute to either the vector or the chiral anomalies. On the other hand, the triangle diagrams are given by

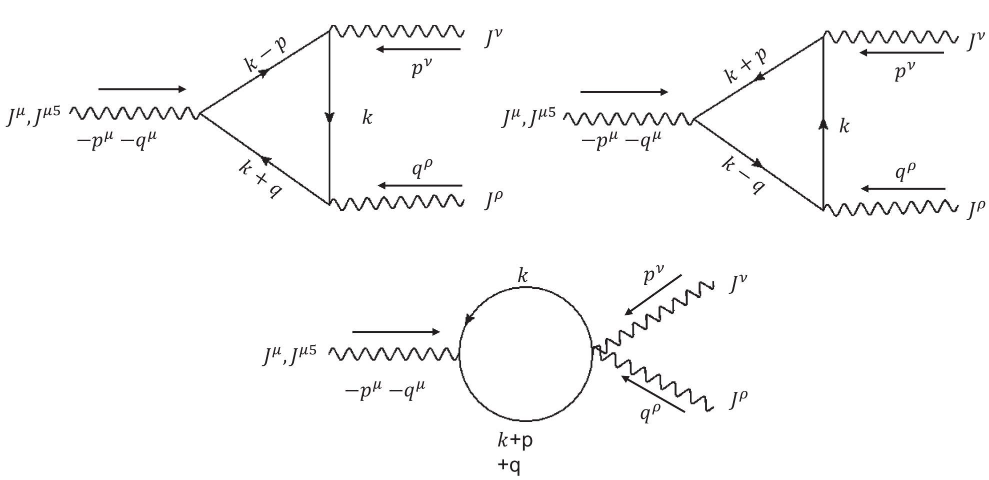

FIG. 1. Triangle (top) and bubble (bottom) diagrams contributing to the anomalies in nonlocal QED.

$$\mathcal{M}_{\triangle}^{\mu\nu\rho} \simeq -ie^2 \int \frac{d^4k}{(2\pi)^4} e^{\frac{6k^2}{\Lambda^2}} \mathrm{Tr}\left[\frac{\gamma^5\gamma^\mu(\not{k}+\not{p})\gamma^\nu\not{k}\gamma^\rho(\not{k}-\not{p})}{(k+p)^2k^2(k-q)^2}\right] + \begin{pmatrix} p \leftrightarrow q \\ \nu \leftrightarrow \rho \end{pmatrix}, \quad (11)$$

in the limit of small external momenta. Notice that this is identical to the local case multiplied by the nonlocality factor. The factor of 6 arises from 3 nonlocal vertices and 3 nonlocal propagators.

We begin by calculating the vector anomaly. Our aim is to verify that the vector anomaly indeed vanishes in the nonlocal QED and that the Ward identity is preserved. *Prima facie*, this should be the case, since the nonlocal QED action is gauge invariant by construction. To this avail, it is convenient to calculate $p_\nu M_5^{\mu\nu\rho}$. Using $\not{p} = \not{p} + \not{k} - \not{k}$, the trace in Eq. (11) simplifies to

$$\frac{1}{k^2(k-q)^2}\mathrm{Tr}[\gamma^5\gamma^\mu\not{k}\gamma^\rho(\not{k}-\not{q})] - \frac{1}{(k+p)^2(k-q)^2}\mathrm{Tr}[\gamma^5\gamma^\mu(\not{k}+\not{p})\gamma^\rho(\not{k}-\not{q})]. \quad (12)$$

It is a simple exercise to evaluate the traces. The first trace yields $-4ik_\nu q_\sigma\epsilon^{\mu\nu\rho\sigma}$, whereas the second trace evaluates to $-4i(k_\nu p_\sigma + k_\nu q_\sigma + p_\nu q_\sigma)\epsilon^{\mu\nu\rho\sigma}$. Thus, Eeq. (11) becomes

$$p_\nu\mathcal{M}_{\triangle}^{\mu\nu\rho} \simeq -4e^2\epsilon^{\mu\nu\rho\sigma} \int \frac{d^4k}{(2\pi)^4} e^{\frac{6k^2}{\Lambda^2}}\left[\frac{k_\mu q_\sigma}{k^2(k-q)^2} + \frac{k_\nu(p+q)_\sigma + p_\nu q_\sigma}{(k-q)^2(k+p)^2}\right] + \begin{pmatrix} p \leftrightarrow q \\ \nu \leftrightarrow \rho \end{pmatrix}. \quad (13)$$

It is sufficient to evaluate the first term. Focusing on the first part of the first term, we notice that the only external momentum it contains is q_σ, which means that after integrating over k_μ, Lorentz invariance implies that the result will be proportional to $q_\mu q_\sigma$, which vanishes upon contraction with $\epsilon^{\mu\nu\rho\sigma}$. This leaves us with the second integral to perform. Such loop integrals are fairly simple to evaluate and are UV finite due to the nonlocality form factor. Details on how to calculate these nonlocal momentum integrals are provided in [25]. Upon evaluating the momentum integral in Eq. (13), we find

$$p_\nu\mathcal{M}_{\triangle}^{\mu\nu\rho} \sim (p_\nu p_\sigma - q_\nu q_\sigma - p_\nu q_\sigma - q_\nu p_\sigma)\epsilon^{\mu\nu\rho\sigma}, \quad (14)$$

and we can see that $p_\nu p_\sigma$ and $q_\nu q_\sigma$ vanish upon contraction with $\epsilon^{\mu\nu\rho\sigma}$. This leaves $(p_\nu q_\sigma + q_\nu p_\sigma)\epsilon^{\mu\nu\rho\sigma}$, and it is easy to see that after relabeling $\nu \leftrightarrow \sigma$ in the second term and using the antisymmetry of $\epsilon^{\mu\nu\rho\sigma}$, the whole term vanishes. The same argument holds for $q_\nu\mathcal{M}_{\triangle}^{\mu\nu\rho}$ since p and q are symmetric. Thus, we can see that vector anomaly vanishes in nonlocal QED, as it should.

Turning our attention to the chiral anomaly, we need to calculate $(p+q)_\mu\mathcal{M}_{\triangle}^{\mu\nu\rho}$. Using

$$\gamma^5(\not{p}+\not{q}) = \gamma^5(\not{p}+\not{k}-\not{k}+\not{q}) = \gamma^5(\not{k}+\not{p}) + (\not{k}-\not{q})\gamma^5, \quad (15)$$

the trace in (11) simplifies to

$$\frac{1}{k^2(k-q)^2}\mathrm{Tr}[\gamma^5\gamma^\nu\not{k}\gamma^\rho(\not{k}-\not{q})] + \frac{1}{k^2(k+p)^2}\mathrm{Tr}[\gamma^5(\not{k}+\not{p})\gamma^\nu\not{k}\gamma^\rho]. \quad (16)$$

Notice that the first term is identical to the first term in (12) with $\mu \to \nu$, and therefore it vanishes as we saw above. On the other hand, the second traces yields $-4i\epsilon^{\mu\nu\rho\sigma}k_\mu p_\sigma$. Therefore, the chiral anomaly reads

$$-(p+q)_\mu\mathcal{M}_{\triangle}^{\mu\nu\rho} \simeq 4e^2\epsilon^{\mu\nu\rho\sigma}\int\frac{d^4k}{(2\pi)^4}e^{\frac{6k^2}{\Lambda^2}}\left[\frac{k_\mu p_\sigma}{k^2(k+p)^2}\right] + \begin{pmatrix} p \leftrightarrow q \\ \nu \leftrightarrow \rho \end{pmatrix}, \quad (17)$$

and we see that the first term contains p only, which means that after integrating over k, the result will be $\sim p_\mu p_\sigma$, which vanishes upon contraction with $\epsilon^{\mu\nu\rho\sigma}$; i.e., the chiral anomaly seems to vanish in non-local QED! This result is counter-intuitive, as the chiral anomaly in local QED is nonvanishing, and one would expect the same to carry on to the nonlocal case. The reason behind this apparent contradiction lies in our approximations. We limited our calculation to the leading order in the expansion of $p, q/\Lambda$, and assumed massless fermions. However, this situation does not hold once we include the NLO expansion in external momenta and/or we use massive fermions, and the chiral anomaly no longer vanishes. In Sec. III B below, we shall redo our calculation with massive fermions and show that chiral anomaly indeed persists. We will limit our calculation to the leading order (LO) in $p, q/\Lambda$ for simplicity.

B. Vector and chiral anomalies with massive fermions

Here we show the effect of including fermion masses on both the vector and chiral anomalies. First, let us focus on the bubble diagram. Including the fermion masses in Eq. (9) to simplify it, Eq. (10) becomes

$$\mathcal{M}_{\bigcirc}^{\mu\nu\rho} \simeq ie^2 \int \frac{d^4k}{(2\pi)^4} e^{\frac{4k^2}{\Lambda^2}} \mathrm{Tr}\left[\frac{\gamma^\mu\gamma^5(\not{k}+\not{p}+m)V^{\nu\rho}(k+p,k-q,p,q)(\not{k}-\not{q}+m)}{[(k+p)^2-m^2][(k-q)^2-m^2]}\right], \tag{18}$$

with $V^{\nu\rho}(k+p, k-q, p, q)$, which is unchanged compared to the massless case. Here, too, we find that the trace vanishes, and hence the bubble diagram does not contribute. On the other hand, the contribution of the triangle diagrams in Eq. (11) becomes

$$\mathcal{M}_{\triangle}^{\mu\nu\rho} \simeq -ie^2 \int \frac{d^4k}{(2\pi)^4} e^{\frac{6k^2}{\Lambda^2}} \mathrm{Tr}\left[\frac{\gamma^5\gamma^\mu(\not{k}+\not{p}+m)\gamma^\nu(\not{k}+m)\gamma^\rho(\not{k}-\not{p}+m)}{[(k+p)^2-m^2][k^2-m^2][(k-q)^2-m^2]}\right] + \begin{pmatrix} p \leftrightarrow q \\ \nu \leftrightarrow \rho \end{pmatrix}. \tag{19}$$

First, we investigate the vector anomaly by calculating $p_\nu \mathcal{M}_{\triangle}^{\mu\nu\rho}$. Simplifying the expression by writing $\not{p} = (\not{p}+\not{k}-m) - (\not{k}-m)$ and then evaluating the traces explicitly, it's not hard to see that the result is identical to Eq. (13) with the denominators being those of massive fermions. Therefore, the result in Eq. (14) continues to hold, and the vanishing of the vector anomaly remains unaffected, as is expected.

Turning our attention to the chiral anomaly by considering $-(p+q)_\mu \mathcal{M}_{\triangle}^{\mu\nu\rho}$ in the massive case, we first simplify the matrix element by using

$$\begin{aligned}\gamma^5(\not{p}+\not{q}) &= \gamma^5(\not{k}+\not{p}-m) + \gamma^5(\not{q}-\not{k}-m) + 2m\gamma^5 \\ &= \gamma^5(\not{k}+\not{p}-m) + (\not{k}-\not{q}-m)\gamma^5 + 2m\gamma^5,\end{aligned} \tag{20}$$

which simplifies the trace in Eq. (19) to

$$\begin{aligned}&= \mathrm{Tr}\left[\frac{\gamma^5\gamma^\nu(\not{k}+m)\gamma^\rho(\not{k}-\not{q}+m)}{[k^2-m^2][(k-q)^2-m^2]}\right] \\ &\quad + \mathrm{Tr}\left[\frac{\gamma^5(\not{k}+\not{p}+m)\gamma^\nu(\not{k}+m)\gamma^\rho}{[(k+p)^2-m^2][k^2-m^2]}\right] \\ &\quad + 2m\mathrm{Tr}\left[\frac{\gamma^5(\not{k}+\not{p}+m)\gamma^\nu(\not{k}+m)\gamma^\rho(\not{k}-\not{q}+m)}{[(k-q)^2-m^2][(k+p)^2-m^2][k^2-m^2]}\right].\end{aligned} \tag{21}$$

Focusing on first and second terms, it is a simple exercise to show that they yield identical results to Eq. (16) with the mass added in the denominators, and therefore, they vanish after integrating over k and contracting with $\epsilon^{\mu\nu\rho\sigma}$. The last term, on the other hand, is proportional to the mass and does not yield a vanishing contribution. The trace yields the factor $4imp_\mu q_\sigma \epsilon^{\mu\nu\rho\sigma}$, and thus Eq. (19) becomes

$$-(p+q)_\mu \mathcal{M}_{\triangle}^{\mu\nu\rho} \simeq \int \frac{d^4k}{(2\pi)^4} e^{\frac{6k^2}{\Lambda^2}} \frac{-8e^2m^2 p_\mu q_\sigma \epsilon^{\mu\nu\rho\sigma}}{[(k-q)^2-m^2][(k+p)^2-m^2][k^2-m^2]} + \begin{pmatrix} p \leftrightarrow q \\ \nu \leftrightarrow \rho \end{pmatrix}. \tag{22}$$

Evaluating the integral is fairly straightforward, and the result in terms of the Feynman parameters reads

$$-(p+q)_\mu \mathcal{M}_{\triangle}^{\mu\nu\rho} \simeq \frac{ie^2}{\pi^2} p_\mu q_\sigma \epsilon^{\mu\nu\rho\sigma} \int_0^1 dxdy \left[\frac{1}{1-xy\frac{Q^2}{m^2}} + \frac{6m^2}{\Lambda^2} + \frac{12m^2}{\Lambda^2}\mathrm{Ei}\left(\frac{6(xyQ^2-m^2)}{\Lambda^2}\right)\right], \tag{23}$$

where $Q^2 \equiv (p+q)^2$, and the exponential integral function $\mathrm{Ei}(x)$ is defined as

$$\mathrm{Ei}(x) = -\int_{-x}^{\infty} dt \frac{e^{-t}}{t}. \tag{24}$$

Linking Eq. (23) to the massless case is straightforward and can be done simply by taking the limit $m \to 0$, which leads to the vanishing of the anomaly at LO in the expansion of the external momenta, in a manner consistent with what we found in Sec. III A. On the other hand, the link to the local case is more subtle. Here one expects that the local case should be obtained by taking the limit $\Lambda \to \infty$; however, this turns out to be insufficient. The reason behind this can be best understood by calculating the local anomaly following the method in [24], where it is shown that the chiral anomaly in the local case arises purely from the regulator. However, a regulator is absent in the nonlocal case since it is already finite. Therefore, simply taking $\Lambda \to \infty$ will not render the regularized local result. Instead, we use the following prescription to remedy the situation: We assume that $m^2 \gg Q^2$, which *corresponds to the mass itself acting as regulator*. In the limit $\Lambda \gg m^2 \gg Q^2$, Eq. (23) becomes

$$\begin{aligned}&-(p+q)_\mu \mathcal{M}_{\triangle}^{\mu\nu\rho} \\ &\quad \simeq \frac{ie^2}{2\pi^2} p_\mu q_\sigma \epsilon^{\mu\nu\rho\sigma} \left[1 + \frac{6m^2}{\Lambda^2} + \frac{12m^2}{\Lambda^2}\mathrm{Ei}\left(\frac{-6m^2}{\Lambda^2}\right)\right],\end{aligned} \tag{25}$$

and it is easy to see that upon taking $\Lambda \to \infty$, the local case is retrieved.

C. Noether currents

Finally, here we derive the nonlocal Noether vector and axial currents. Notice that the action in Eq. (5) is invariant under the global transformations

$$\Psi \to e^{i\alpha}\Psi, \qquad \Psi \to e^{i\beta\gamma^5}\Psi. \tag{26}$$

To derive the corresponding Noether currents, we follow the usual prescription of demanding that the Lagrangian be invariant under the infinitesimal local transformations

$$\Psi \to (1 + i\alpha(x))\Psi, \qquad \Psi \to (1 + i\beta(x)\gamma^5)\Psi, \tag{27}$$

which leads to the current

$$J^\mu(x) = \frac{\delta\mathcal{L}}{\delta(\partial_\mu\Psi)}\Delta\Psi. \tag{28}$$

To derive the nonlocal QED Noether currents, we start with the Lagrangian

$$\mathcal{L} = \frac{i}{2}\bar{\Psi}\exp\left(\frac{-\Box - ie(\partial\cdot A + A\cdot\partial) - e^2A^2}{\Lambda^2}\right)(\partial\!\!\!/\Psi + ie A\!\!\!/\Psi) + \text{H.c.} \tag{29}$$

Notice that in order to evaluate the variation of the Lagrangian with respect to $\partial\Psi$, we need to pay special attention to the derivatives in the exponent. To this avail, we use the following prescription: First, we expand the derivative operators in the exponents, and then we act the derivatives on the associated field leaving only terms $\sim\partial\Psi$. Finally, we exponentiate the results and restore the operator form in the currents. Let us first focus on the second term in the parentheses in Eq. (29). We assume that the photon is on-shell, such that $\Box(A\!\!\!/\Psi) = A\!\!\!/\Box\Psi = -k_1^2 A\!\!\!/\Psi$. Therefore, we have

$$\begin{aligned}\exp\left(-\frac{\Box}{\Lambda^2}\right)(A\!\!\!/\Psi) &= \sum_{n=0}^{\infty}\left[\frac{(-i)^n\Box^n}{\Lambda^{2n}n!}\right](A\!\!\!/\Psi)\\ &= \sum_{n=0}^{\infty}\left[\frac{(k_1^2)^n}{\Lambda^{2n}n!}\right](A\!\!\!/\Psi)\\ &= \exp\left(\frac{k_1^2}{\Lambda^2}\right)(A\!\!\!/\Psi).\end{aligned} \tag{30}$$

On the other hand, the remaining derivative acting on $A\!\!\!/\Psi$ can be evaluated as follows:

$$\begin{aligned}&\exp\left(\frac{-ieA\cdot\partial}{\Lambda^2}\right)(ieA\!\!\!/\Psi)\\ &= ie\sum_{n=0}^{\infty}\frac{(-ieA\cdot\partial)^n}{\Lambda^{2n}n!}(A\!\!\!/\Psi)\\ &= ie\sum_{n=0}^{\infty}\frac{(-ieA^\mu)^n}{\Lambda^{2n}n!}\sum_{k=0}^{n}\binom{n}{k}\left(\partial_\mu^{n-k}A\!\!\!/\right)(\partial_\mu^k\Psi)\\ &= ieA\!\!\!/A\cdot\partial\Psi\sum_{n=0}^{\infty}\frac{(-ieA^\mu)^n}{\Lambda^{2n}n!}\sum_{k=0}^{n}\binom{n}{k}\left(iq\cdot A\right)^{n-k}(-ik_1\cdot A)^{k-1}\\ &= -\frac{e^2A\!\!\!/A\cdot\partial\Psi}{k_1\cdot A}\exp\left(\frac{eA\cdot k_2}{\Lambda^2}\right),\end{aligned} \tag{31}$$

where we have used conservation on momentum to eliminate the momentum of the photon. The Hermitian conjugate yields identical results with $k_1 \leftrightarrow k_2$. Thus, after restoring the operators, the second term in Eq. (29) becomes

$$\mathcal{L}_2 = -\frac{ie^2}{2}\bar{\Psi}\exp\left(\frac{-\Box - ieA\cdot\partial - e^2A^2}{\Lambda^2}\right) \times\left(\frac{1}{k_1\cdot A} + \frac{1}{k_2\cdot A}\right)A\!\!\!/A\cdot\partial\Psi. \tag{32}$$

Notice that when the photon is assumed to be on-shell, we have

$$\frac{1}{k_1\cdot A} + \frac{1}{k_2\cdot A} = \frac{(k_1+k_2)\cdot A}{(k_1\cdot A)(k_2\cdot A)} = \frac{q\cdot A}{(k_1\cdot A)(k_2\cdot A)} = 0, \tag{33}$$

which implies that the second term in eq. (29) does not contribute to the nonlocal Noether currents. On the other hand, the first term will give a nonvanishing contribution. Following the same procedure, we obtain

$$\mathcal{L}_1 = \frac{i}{2}\bar{\Psi}\exp\left(\frac{k_1^2 + eA\cdot k_2 - e^2A^2}{\Lambda^2}\right)\partial\!\!\!/\Psi + (1\leftrightarrow 2). \tag{34}$$

Using Eq. (34) in Eq. (28), and then restoring the operators in the exponents, we obtain the Noether currents

$$J^\mu(x) = \bar{\Psi}\gamma^\mu\Psi\exp\left(\frac{-\Box - ieA\cdot\partial - e^2A^2}{\Lambda^2}\right), \tag{35}$$

$$J^{\mu5}(x) = \bar{\Psi}\gamma^\mu\gamma^5\Psi\exp\left(\frac{-\Box - ieA\cdot\partial - e^2A^2}{\Lambda^2}\right). \tag{36}$$

Notice that taking the limit $\Lambda \to \infty$, the local limit is retrieved, i.e., $J^\mu \to \bar{\Psi}\gamma^\mu\Psi$, and $J^{\mu5} \to \bar{\Psi}\gamma^\mu\gamma^5\Psi$.

Before we conclude this section, there is an important point that we need to clarify. As is well-known, local anomalies are obtained by evaluating the expectation of the Noether currents. Thus, we should be able to obtain the nonlocal anomalies by evaluating

$$\int d^4xd^4yd^4ze^{-ip.x}e^{iq_1.y}e^{iq_2.z}\langle J^{\mu 5}(x)J^{\nu}(y)J^{\rho}(z)\rangle, \quad (37)$$

with the currents given by Eqs. (35) and (36). However, given the field A in the exponents of the vector and axial currents, we see that the expansion in A actually corresponds to the sum of all insertions of the vector current in the fermion loop; i.e., the quantity in Eq. (37) actually encodes all higher-order anomalies that correspond to an arbitrary number of the gauge field A inserted into a fermion loop (in addition to the insertions of vector and axial fields from the local piece). These anomalies in general might not be vanishing; however, we are only interested in the triangle anomalies. Triangle anomalies can be obtained by keeping the leading order in A, i.e.,

$$\exp\left(\frac{-\Box - ieA\cdot\partial - e^2A^2}{\Lambda^2}\right) \simeq \exp\left(-\frac{\Box}{\Lambda^2}\right) + O(A). \quad (38)$$

Thus we can see at this order that Eq. (37) leads to the same results we obtained above.

D. Summary of the results

In this section we summarize the results that we obtained in this paper:

(i) Vector anomalies in nonlocal QED vanish exactly, whether the fermions in the loops are massless or massive, and the Ward identity is respected. It is also not hard to show that the vanishing of the vector anomaly holds to all orders in the expansion of $p, q/\Lambda$. This is expected, since the nonlocal QED action in Eq. (5) is gauge invariant by construction,

(ii) Although in nonlocal QED with massless fermions, the chiral anomaly appears to vanish at the LO in $p, q/\Lambda$; one can show that it no longer holds once higher-order corrections are included. In addition, for nonlocal QED with massive fermions at LO, we find that the chiral anomaly persists and that it has the expected form. We found that while obtaining the massless limit is straightforward, the local limit is more subtle and cannot be obtained by simply taking $\Lambda \to \infty$. Instead, one needs to assume that the mass of the fermions is much larger than the other momentum scales in order to act as a regulator itself in the local limit. Using this prescription, the correct local limit is obtained,

(iii) The nonlocal vector and axial currents encode anomalies that correspond to all insertions of the gauge field in the fermion loop, with the triangle anomalies obtained from the LO expansion in the gauge field. This is a direct consequence of gauge invariance, which leads to rich structures in nonlocal QED that merit further investigation in the future.

(iv) Our results are consistent with those found in Ref. [23] using the shadow field formalism.

IV. APPLICATION: $\pi^0 \to \gamma\gamma$ DECAY

We present an application to anomalies in nonlocal QED by studying the decay process of $\pi^0 \to \gamma\gamma$. This decay proceeds through triangle diagrams such as the ones shown in Fig. 1, with the axial current replaced with a pseudoscalar and with protons running in the loops. The interaction Lagrangian is given by

$$\mathcal{L}_{\text{int}} = -i\lambda\pi\bar{\Psi}\gamma^5\Psi. \quad (39)$$

The matrix element can be written as $-\lambda e^2\epsilon^*_{1\mu}\epsilon^*_{2\nu}\mathcal{M}^{\mu\nu}$, where at LO in $q_{1,2}/\Lambda$ we have

$$\mathcal{M}^{\mu\nu} \simeq \int \frac{d^4k}{(2\pi)^4} e^{\frac{5k^2}{\Lambda^2}} \text{Tr}\left[\gamma^{\mu}\frac{i(\not{k}-\not{q}_1+m)}{(k-q_1)^2-m^2}\gamma^5\frac{i(\not{k}+\not{q}_2+m)}{(k+q_2)^2-m^2}\gamma^{\nu}\frac{i(\not{k}+m)}{k^2-m^2}\right] + \begin{pmatrix} 1\leftrightarrow 2 \\ \nu\leftrightarrow\rho \end{pmatrix}, \quad (40)$$

where m is the mass of the proton. $\mathcal{M}^{\mu\nu}$ can be evaluated following the procedure illustrated in Sec. III, and in the limit $m \gg m_{\pi}$, the decay width reads

$$\Gamma_{\text{NL}}(\pi^0 \to \gamma\gamma) \simeq \Gamma_0 \times \left[1 + \frac{5m^2}{\Lambda^2} + \frac{10m^2}{\Lambda^2}\text{Ei}\left(-\frac{5m^2}{\Lambda^2}\right)\right]^2, \quad (41)$$

where

$$\Gamma_0 = \frac{\alpha^2}{64\pi^3}\frac{m_{\pi}^3}{f_{\pi}^2} \quad (42)$$

is the decay width in the local case and f_{π} is the pion decay constant. We can use Eq. (41) to set a lower limit on the scale of nonlocality. The most recent measurement of the decay width of $\pi^0 \to \gamma\gamma$ comes from the PrimEx-II experiment:

$$\Gamma_{\text{Exp}}(\pi^0 \to \gamma\gamma) = 7.802 \pm 0.052(\text{stat}) \pm 0.105(\text{syst})\ \text{eV}, \quad (43)$$

which can be used to set a 2σ limit on the scale of nonlocality,

$$\Lambda \gtrsim 57\ \text{GeV}. \quad (44)$$

This bound is not very stringent and cannot compete with the collider bound of $\Lambda \gtrsim 2.5$–3 TeV [19,26].

V. CONCLUSION AND OUTLOOK

In this paper, we investigated the vector and chiral anomalies in the nonlocal QED formulated in [19]. We found that the vanishing of the vector anomaly remains unaffected and that the Ward identity continues to hold in the nonlocal case as well. This is to be expected since nonlocal QED is gauge invariant by construction.

We also found that at leading order the chiral anomaly vanishes in the massless case, while it does not vanish in the massive case. Also, the anomaly continues to exist at next to leading order in the massless case. Naively, one might speculate that since nonlocal QED lacks a regulator as it is already regularized, and that since the chiral anomaly in the local case arises purely from the regulator, the chiral anomaly in the nonlocal case would vanish. Nonetheless, this turned out not to be the case, and the chiral anomaly is nonvanishing at next to leading order for the massless case, and can be expressed in terms of the local anomaly plus corrections suppressed by the scale of nonlocality. We found that obtaining the local limit from the nonlocal case would require special care, and we found that with the correct prescription, the local limit is obtained when $\Lambda \to \infty$. Our results are consistent with the results found in [23] by using the shadow field formalism. We also found the corresponding vector and axial Noether currents in the nonlocal case and found that they encode all higher-order anomalies, with the triangle anomalies obtained from the LO expansion in the gauge field. We also showed that in the limit $\Lambda \to \infty$, the local currents are obtained.

As a simple application of our results, we calculated the corrections to the decay width $\pi^0 \to \gamma\gamma$ due to nonlocality and found that the constraint corresponding to the current experimental measurement is weak compared to the limit obtained from the LHC.

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APPENDIX: DERIVATION OF THE NONLOCAL $\bar{\Psi}\Psi\gamma\gamma$ VERTEX

Here we show how to derive the Feynman rule for the $\bar{\Psi}\Psi\gamma\gamma$ vertex in nonlocal QED. The Feynman rule for the $\bar{\Psi}\Psi\gamma$ vertex was derived in [19] and is shown in Eq. (8). The full Feynman rule of the $\bar{\Psi}\Psi\gamma\gamma$ vertex is rather complex; therefore, we simplify by assuming that the photons are *on-shell*, which is the case we are interested in for calculating the anomalies, and we only keep the leading terms in $1/\Lambda^2$. We start with the fermion part of the nonlocal QED action in Eq. (5):

$$S_{\rm NL} = \frac{1}{2}\int d^4x [i\bar{\Psi} e^{-\frac{\nabla^2}{\Lambda^2}}(\not{\nabla} + m)\Psi + \text{H.c.}]. \tag{A1}$$

We first expand the nonlocal form factor in powers of $1/\Lambda^2$ and write the covariant derivative explicitly as shown in Eq. (6):

$$S_{\rm NL} = \frac{1}{2}\int d^4x \bigg\{ i\bar{\Psi}\sum_{n=1}^{\infty}\frac{(-1)^n}{\Lambda^{2n} n!}[\Box + ie(\partial\cdot A + A\cdot\partial) - e^2A^2]^n[\partial + ie\not{A}]\Psi + \text{H.c.}\bigg\}. \tag{A2}$$

To obtain the $\bar{\Psi}\Psi\gamma\gamma$ vertex, we only keep terms that are proportional to A^2, i.e., the terms $\sim O(e^2)$. Inspecting Eq. (A2), we can see that we can obtain terms at $O(A^2)$ through three different ways: (1) For $n = 1$, we can have the A^2 term in the first bracket multiplied by the $\partial\Psi$ term in the second bracket. (2) For $n = 1$, we can have the $(\partial.A + A.\partial)$ term from the first bracket multiplied by the $\not{A}$ term in the second bracket. (3) For $n = 2$, we can have the $(\partial\cdot A + A\cdot\partial)^2$ term from the first bracket multiplied by the $\partial\Psi$ term in the second term. Explicitly, we have

$$S_{\rm NL} \supset -\frac{ie^2}{2}\sum_{n=0}^{\infty}\frac{(-1)^n}{\Lambda^{2n} n!}\int d^4x \bigg\{\sum_{m=0}^{n-1}(\Box^m\bar{\Psi})[A^2\Box^{n-m-1}(\partial\Psi) + (\partial\cdot A + A\cdot\partial)\Box^{n-m-1}(\not{A}\Psi)] + \sum_{m=0}^{n-2}\sum_{l=0}^{n-m-2}(\Box^m\bar{\Psi})(\partial\cdot A + A\cdot\partial)\Box^l(\partial\cdot A + A\cdot\partial)\Box^{n-m-l-2}(\partial\Psi) + \text{H.c.}\bigg\}, \tag{A3}$$

where we have integrated $\bar{\Psi}\Box^m$ by parts to obtain $\Box^m\bar{\Psi}$. We treat each of the three terms separately. Starting with the first term, notice that each $\Box$ operator will pull down a factor of $-k_{1,2}^2$, with $k_{1,2}$ being the 4-momentum of $\bar{\Psi}$ and Ψ, respectively. On the other hand, the $\partial\Psi$ will pull a factor of $-ik_2$, whereas the Hermitian conjugate will give a factor of $-ik_1$, thereby symmetrizing the result between k_1 and k_2. Thus, the first term yields

$$S_1 = \frac{e^2}{2}\sum_{n=0}^{\infty}\frac{1}{\Lambda^{2n}n!}\sum_{m=0}^{n-1}\int d^4x(\not{k}_1+\not{k}_2)(k_1^{2m}k_2^{2(n-m-1)})\bar{\Psi}\Psi A^2, \tag{A4}$$

and the sums can be evaluated as follows:

$$\sum_{n=0}^{\infty}\frac{1}{\Lambda^{2n}n!}\sum_{m=0}^{n-1}k_1^{2m}k_2^{2(n-m-1)} = \sum_{n=0}^{\infty}\frac{k_2^{2n-2}}{\Lambda^{2n}n!}\left[\frac{1-(k_1^2/k_2^2)^n}{1-(k_1^2/k_2^2)}\right] = \frac{e^{\frac{k_2^2}{\Lambda^2}}-e^{\frac{k_1^2}{\Lambda^2}}}{k_2^2-k_1^2}, \tag{A5}$$

which, together with Eq. (A4), implies that the contribution of the first term is given by

$$V_{1\mu\nu}(k_1,k_2,q_1,q_2) = ie^2(\not{k}_1+\not{k}_2)\left(\frac{e^{\frac{k_2^2}{\Lambda^2}}-e^{\frac{k_1^2}{\Lambda^2}}}{k_2^2-k_1^2}\right)g_{\mu\nu}, \tag{A6}$$

where $q_{1,2}$ are the momenta of the photons, which will be relevant for the remaining contributions. Turning to the second term in Eq. (A3), we have

$$S_{\mathrm{NL},2} = \frac{ie^2}{2}\sum_{n=0}^{\infty}\frac{1}{\Lambda^{2n}n!}\sum_{m=0}^{n-1}\int d^4x\Big[(k_1^{2m}k_2^{2(n-m-1)}) \times\bar{\Psi}(\partial\cdot A + A\cdot\partial)(\not{A}\Psi) + \mathrm{H.c.}\Big], \tag{A7}$$

where we have acted with the $\Box$ operators on the respective fields and assumed that the photon is on-shell, such that $\Box A = -q_1^2 A = 0$. Notice that the sums are identical to Eq. (A5). Therefore, writing the Hermitian conjugate explicitly, Eq. (A7) reads

$$S_2 = \frac{ie^2}{2}\left(\frac{e^{\frac{k_2^2}{\Lambda^2}}-e^{\frac{k_1^2}{\Lambda^2}}}{k_2^2-k_1^2}\right)\int d^4x[\bar{\Psi}(\partial\cdot A + A\cdot\partial)(\not{A}\Psi) + (\partial\cdot A + A\cdot\partial)(\bar{\Psi}\not{A})\Psi]. \tag{A8}$$

Notice that the second operator acts only on $\bar{\Psi}\not{A}$. Acting with the partial derivative on the fermions, the photon will pull down the momentum of the respective field, and one can eliminate the momentum of the photon in favor of the momenta of the two fermions, such that Eq. (A8) becomes

$$S_2 = \frac{ie^2}{2}(k_{1\mu}+k_{2\mu})\left(\frac{e^{\frac{k_2^2}{\Lambda^2}}-e^{\frac{k_1^2}{\Lambda^2}}}{k_2^2-k_1^2}\right)\int d^4x\bar{\Psi}\Psi A^\mu\not{A}, \tag{A9}$$

which implies that the Feynman rule corresponding to the second vertex is given by

$$V_{2\mu\nu}(k_1,k_2,q_1,q_2) = -e^2(k_{1\mu}+k_{2\mu})\gamma_\nu\left(\frac{e^{\frac{k_2^2}{\Lambda^2}}-e^{\frac{k_1^2}{\Lambda^2}}}{k_2^2-k_1^2}\right). \tag{A10}$$

Finally, we turn our attention to the last term given in the second line of Eq. (A3). This part is quite complex, so we resort to some approximations to evaluate it. First, we notice that

$$(\partial\cdot A + A\cdot\partial)\Box^l(\partial\cdot A + A\cdot\partial)\Box^{n-m-l-2}(\partial\Psi) = ik_{1\nu}\not{k}_2(q_{1\mu}-k_{2\mu})(-k_2^2)^{n-m-2}A^\mu A^\nu\Psi, \tag{A11}$$

where $q_{1\mu}$ is the momentum of one of the photons, and we have assumed that the photons are on-shell and utilized conservation of momentum to eliminate the momenta of the photons in favor of the momenta of the fermions whenever possible. Therefore, the third term in (A3) reads

$$S_3 = \frac{e^2}{2}\sum_{n=0}^{\infty}\frac{1}{\Lambda^{2n}n!}\sum_{m=0}^{n-2}\sum_{l=0}^{n-m-2}\int d^4x\Big[(k_1^{2m}k_2^{2(n-m-2)})k_{1\nu}\not{k}_2(q_{1\mu}-k_{2\mu})A^\mu A^\nu\bar{\Psi}\Psi + \mathrm{H.c.}\Big]. \tag{A12}$$

We need to evaluate the sums over l, m, and n. First, notice that the sum over l is trivial and just leads to a factor of $n-m-2$. Therefore, the sum over m becomes

$$\sum_{m=0}^{n-2}(n-m-2)(k_1^{2m}k_2^{2(n-m-2)}) = (n-2)\left[\frac{(k_2^2)^{n-1}-(k_1^2)^{n-1}}{k_2^2-k_1^2}\right] - (k_2^{2(n-2)})\left(\frac{k_1^2}{k_2^2}\right)\left[\frac{1-(n-1)(k_1^2/k_2^2)^{n-2}+(n-2)(k_1^2/k_2^2)^{n-1}}{(1-k_1^2/k_2^2)^2}\right], \tag{A13}$$

and we can now plug this into Eq. (A12) to evaluate the sum over n. The first term in the sum over n yields

$$\sum_{n=0}^{\infty}\frac{(n-2)}{\Lambda^{2n}n!}\left(\frac{k_2^{2(n-1)}-k_1^{2(n-1)}}{k_2^2-k_1^2}\right) = \frac{1}{\Lambda^2(k_2^2-k_1^2)}\left[\left(1-\frac{2\Lambda^2}{k_2^2}\right)e^{\frac{k_2^2}{\Lambda^2}} - \left(1-\frac{2\Lambda^2}{k_1^2}\right)e^{\frac{k_1^2}{\Lambda^2}}\right], \tag{A14}$$

whereas the second term yields

$$\sum_{n=0}^{\infty}\frac{1}{\Lambda^{2n}n!}(k_2^{2(n-2)})\left(\frac{k_1^2}{k_2^2}\right)\left[\frac{1-(n-1)(k_1^2/k_2^2)^{n-2}+(n-2)(k_1^2/k_2^2)^{n-1}}{(1-k_1^2/k_2^2)^2}\right]=\frac{1}{(k_2^2-k_1^2)^2}\left[\left(\frac{k_1^2}{k_2^2}\right)e^{\frac{k_2^2}{\Lambda^2}}+\left(\frac{k_1^2}{\Lambda^2}-\frac{k_2^2}{\Lambda^2}+\frac{k_2^2}{k_1^2}-2\right)e^{\frac{k_1^2}{\Lambda^2}}\right]. \tag{A15}$$

We simplify our results by keeping only the leading order in Λ, so we drop terms $\sim O(1/\Lambda^2)$. We plug Eqs. (A14) and (A15) into Eq. (A12) and then evaluate the Hermitian conjugate, which can simply be obtained from the first part by interchanging $k_1 \leftrightarrow k_2$. Finally, we arrive at the third contribution to the Feynman rule

$$V_{3\mu\nu}(k_1,k_2,q_1,q_2)\simeq ie^2 k_{1\nu}\not{k}_2(q_{1\mu}-k_{2\mu})\left\{\frac{2}{k_2^2-k_1^2}\left(\frac{e^{\frac{k_1^2}{\Lambda^2}}}{k_1^2}-\frac{e^{\frac{k_2^2}{\Lambda^2}}}{k_2^2}\right)+\frac{1}{(k_2^2-k_1^2)^2}\left[\left(\frac{k_1^2}{k_2^2}\right)e^{\frac{k_2^2}{\Lambda^2}}+\left(\frac{k_2^2}{k_1^2}-2\right)e^{\frac{k_1^2}{\Lambda^2}}\right]\right\}+(k_1\leftrightarrow k_2). \tag{A16}$$

Putting all the pieces together from Eqs. (A6), (A10), and (A16), we arrive at the final result

$$\begin{aligned}V_{\mu\nu}(k_1,k_2,q_1,q_2)\simeq{}& ie^2[(\not{k}_1+\not{k}_2)g_{\mu\nu}+i(k_{1\mu}+k_{2\mu})\gamma_\nu]\left(\frac{e^{\frac{k_2^2}{\Lambda^2}}-e^{\frac{k_1^2}{\Lambda^2}}}{k_2^2-k_1^2}\right)\\ &+k_{1\nu}\not{k}_2(q_{1\mu}-k_{2\mu})\left\{\frac{2}{k_2^2-k_1^2}\left(\frac{e^{\frac{k_1^2}{\Lambda^2}}}{k_1^2}-\frac{e^{\frac{k_2^2}{\Lambda^2}}}{k_2^2}\right)+\frac{1}{(k_2^2-k_1^2)^2}\left[\left(\frac{k_1^2}{k_2^2}\right)e^{\frac{k_2^2}{\Lambda^2}}+\left(\frac{k_2^2}{k_1^2}-2\right)e^{\frac{k_1^2}{\Lambda^2}}\right]\right\}\\ &+k_{2\nu}\not{k}_1(q_{1\mu}-k_{1\mu})\left\{\frac{2}{k_1^2-k_2^2}\left(\frac{e^{\frac{k_2^2}{\Lambda^2}}}{k_2^2}-\frac{e^{\frac{k_1^2}{\Lambda^2}}}{k_1^2}\right)+\frac{1}{(k_1^2-k_2^2)^2}\left[\left(\frac{k_2^2}{k_1^2}\right)e^{\frac{k_1^2}{\Lambda^2}}+\left(\frac{k_1^2}{k_2^2}-2\right)e^{\frac{k_2^2}{\Lambda^2}}\right]\right\}.\end{aligned} \tag{A17}$$

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