

# Off-shell invariants of linearized $4D, \mathcal{N} = 2$ supergravity in the harmonic approach

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Using the harmonic superspace approach, we construct, at the linearized level,  $\mathcal{N} = 2$  supersymmetric curvatures generalizing scalar curvature, Ricci curvature and Weyl tensor. These supercurvatures are the building blocks of various linearized  $4D, \mathcal{N} = 2$  Einstein supergravity invariants. The supercurvatures involving the scalar and Ricci curvatures are analytic harmonic  $\mathcal{N} = 2$  superfields, while the Weyl supertensor is a chiral  $\mathcal{N} = 2$  superfield. As the basic distinguished feature of our construction, all these objects are expressed through the fundamental analytic gauge prepotentials  $h^{++M}$ ,  $M = (\alpha\dot{\alpha}, \alpha+, \dot{\alpha}+, 5)$ . The related characteristic features are the heavy use of harmonic derivatives and harmonic zero-curvature equations. On a number of instructive examples, we describe the component reduction of the superfield invariants constructed.

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## I. INTRODUCTION

Among various generalizations of Einstein's theory of gravity, supergravity and higher-curvature gravity are of most attention for long. Supergravity naturally unifies fields of different spins within a single theory and leads to essential improvements of UV behavior (see, e.g., [1,2]). Theories with higher curvatures also exhibit a number of nontrivial quantum properties: e.g., (1) modification of the Gilbert-Einstein action by terms quadratic in curvature yields a renormalizable theory [3]; (2) the  $R + R^2$  Starobinsky theory [4] is ghost free [5] and is exploited in models of cosmological inflation; and (3) the pure  $R^2$  gravity (without a  $R$  term) is also ghost free [6]. It is of significant interest to study the supergravity theories involving higher-curvature terms in the bosonic limit [7]. The invariants of the higher-derivative supergravities find applications as possible supergravity counterterms, are relevant to the black hole entropy problem, have applications in cosmology, and also provide information on the relevant conformal anomalies. Supersymmetric generalizations of the terms constructed from curvatures are also of interest on their own as they can shed more light on the intrinsic algebraic and geometric structure of diverse off-shell formulations of supergravity. An overview of

applications of the higher-order supergravity invariants and a detailed discussion of methods of their construction can be found, e.g., in Ref. [8].

The most efficient and convenient way to deal with supersymmetric theories is the superspace approach [9–11]. In this paper, we consider linearized higher-order invariants of  $\mathcal{N} = 2$  supergravity in the harmonic superspace [12]. Among all the covariant superspace approaches to  $\mathcal{N} = 2$  supergravity (for review, see [13]) the formulation in harmonic superspace has a number of advantages. The main merit of such a formulation is the manifest presence of  $\mathcal{N} = 2$  supergravity prepotentials, which have a natural geometric interpretation as vielbeins covariantizing the derivatives with respect to harmonic variables. This way, the harmonic formulation of  $\mathcal{N} = 2$  supergravities non-trivially generalizes the geometric Ogievetsky-Sokatchev approach to  $\mathcal{N} = 1$  supergravity [14].

The  $\mathcal{N} = 2$  supergravity unconstrained prepotentials appear as analytic vielbeins in the covariant harmonic derivative which preserves the notion of harmonic analyticity:

$$\mathfrak{D}^{++} = \mathcal{D}^{++} + h^{++\alpha\dot{\alpha}} \partial_{\alpha\dot{\alpha}} + h^{++\alpha+} \partial_{\alpha}^{-} + h^{++\dot{\alpha}+} \partial_{\dot{\alpha}}^{-} + h^{++5} \partial_5. \quad (1.1)$$

The prepotentials with a clear geometric origin remain “terra incognita” in all other formulations of  $\mathcal{N} = 2$  supergravity.<sup>1</sup> Meanwhile, the presence of prepotentials is a necessary

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<sup>1</sup>The basic difference from the Mezincescu-type [15] prepotentials is that the latter carry some gauge invariances of non-geometric character that admit no a direct generalization to the full nonlinear level (see, e.g., discussion in [16]).

ingredient for studying the equations of motion and quantizing the theory in a manifestly supersymmetric manner. Another striking feature of the harmonic superspace is the possibility to construct actions that do not admit formulations in the conventional superspaces.

Despite the active developments of superfield formulations of extended supergravity theories for recent years, not too many studies were specially devoted to the harmonic superspace formulation of  $\mathcal{N} = 2$  supergravity and so many relevant questions remained opened. In this regard, it makes sense to start by briefly listing the main works on the subject (for a comprehensive review, see [16,17]).

$\mathcal{N} = 2$  harmonic superspace was introduced in [12] as the most adequate superspace for description of general theories with manifest off-shell  $\mathcal{N} = 2$  supersymmetry. In [12],  $\mathcal{N} = 2$  Einstein supergravity analytic prepotentials and the corresponding gauge group were discovered. The invariant nonlinear action for  $\mathcal{N} = 2$  Einstein supergravity was built by Galperin *et al.* in [18]. In the accompanying work [19] there was constructed a multiplet of  $\mathcal{N} = 2$  conformal supergravity ( $\mathcal{N} = 2$  Weyl multiplet) in harmonic superspace and various versions of  $\mathcal{N} = 2$  Einstein supergravity were recovered through the appropriate superconformal harmonic compensators. In particular, the “principal” version of  $\mathcal{N} = 2$  Einstein supergravity with an infinite number of auxiliary fields was introduced. In this version, an off-shell hypermultiplet is used as a superconformal compensator (in addition to the  $\mathcal{N} = 2$  vector multiplet) and just this version permits the most general interaction Lagrangians of hypermultiplet matter in the  $\mathcal{N} = 2$  supergravity background. The harmonic superspace construction of general off-shell hypermultiplet couplings in  $\mathcal{N} = 2$  supergravity background and their relation to quaternionic-Kähler geometry were elaborated in a series of papers [20–23]. The linearized action for  $\mathcal{N} = 2$  Einstein supergravity was constructed by Zupnik [24] and recently it was generalized to  $\mathcal{N} = 2$  higher spin supergravities in [25]. The interrelations between  $\mathcal{N} = 2$  supercurrents and supergravity prepotentials were discussed in [26,27]. More recently [28], there was suggested a new covariant approach to the harmonic superspace as a symbiosis of the latter with conformal  $4D, \mathcal{N} = 2$  superspace [29]. Nevertheless, many issues related to the harmonic superspace formulation of  $\mathcal{N} = 2$  supergravity have never been addressed. In particular, this concerns the construction of off-shell higher-derivative  $\mathcal{N} = 2$  supergravity invariants in the harmonic approach.

On the other hand, quite a few remarkable results in this area were achieved in other superfield approaches to  $\mathcal{N} = 2$  supergravity. The  $\mathcal{N} = 2$  curvature-squared off-shell invariants were constructed in [30,31] (see also review [32]), based upon the formulation of  $\mathcal{N} = 2$  conformal supergravity in  $SU(2)$  superspace [33,34] and  $\mathcal{N} = 2$  conformal superspace [29]. The  $\mathcal{N} = 2$  supersymmetrizations of  $R^4$  invariants were considered in [35,36]. Using the

logarithm construction, proposed in [30] (for a review see Sec. 10.2 of [13]) one can set up some higher-order invariants. For example, higher-derivative super-Weyl  $\mathcal{N} = 2$  invariants, containing  $F^{2n}$ , with  $n = 3, 4, \dots$  were proposed in [37]. Another mechanism of generating higher-derivative couplings in  $\mathcal{N} = 2$  supergravity is the procedure of constructing composite reduced chiral superfields [38] (for a brief review see Sec. 8.4 of [13]). However, to the best of our knowledge, specific explicit examples of higher-derivative  $\mathcal{N} = 2$  supergravity invariants beyond  $R^2$  were not so far constructed in this approach.

Note that the construction of the on-shell  $\mathcal{N}$ -extended supergravity invariants in harmonic superspace have also been extensively discussed in a number of papers devoted to the on-shell extended supergravity counterterms. For example, in [39] some invariants of  $4D, \mathcal{N} = 8$  linearized supergravity were constructed; the on-shell harmonic superspace linearized counterterms  $[\partial^{2(\mathcal{N}-4)} R^4]$  for  $\mathcal{N} \geq 4$  supergravities were presented in [40]. A short summary of the results on such invariants can be found in a review [8]. For a recent discussion of these on-shell superinvariants at the nonlinear level, see, e.g., [41].

In this paper, we construct off-shell higher-derivative  $\mathcal{N} = 2$  supergravity invariants in harmonic superspace at the linearized level, consistently applying the whole machinery of the harmonic approach. The main distinction of this scheme from the more conventional superspace methods [30–32,35,36] is that its basic building blocks are directly related to the unconstrained analytic prepotentials underlying the structure of curved  $\mathcal{N} = 2$  harmonic superspace and encapsulating the off-shell supergravity multiplet in Wess-Zumino (WZ) gauge. This radically simplifies deducing the component forms of the invariants. Though the most difficult part of the road to components remains solving the so-called harmonic zero-curvature equations, it proves to be rather straightforward in WZ gauge.

As we shall show, the basic objects in constructing  $\mathcal{N} = 2$  higher-derivative supergravity invariants are linearized gauge-invariant supercurvatures that have a surprisingly simple structure:

$$\begin{aligned} \mathcal{F}^{++\alpha\dot{\alpha}} &= (\mathcal{D}^+)^4 G^{--\alpha\dot{\alpha}} \\ &= -8i\theta_\rho^+ \bar{\theta}_{\dot{\rho}}^+ \left( \mathcal{R}^{(\alpha\rho)(\dot{\alpha}\dot{\rho})} - \frac{1}{8} \epsilon^{\alpha\rho} \epsilon^{\dot{\alpha}\dot{\rho}} R \right) + \dots, \end{aligned} \quad (1.2)$$

$$\mathcal{F}^{++5} = (\mathcal{D}^+)^4 G^{--5} = \frac{i}{2} (\theta^+)^2 R - \frac{i}{2} (\bar{\theta}^+) R + \dots, \quad (1.3)$$

$$\begin{aligned} \mathcal{W}_{(\alpha\beta)} &= (\bar{\mathcal{D}}^+)^2 (\mathcal{D}_{(\alpha}^+ G_{\beta)}^{---} + \mathcal{D}_{(\alpha}^- G_{\beta)}^{--+} - \mathcal{D}_{(\alpha}^{\dot{\rho}} G_{\beta)\dot{\rho}}^{--}) \\ &= 32\theta^{-(\gamma\theta^{+\delta})} \mathcal{R}_{(\alpha\beta\gamma\delta)} + \dots \end{aligned} \quad (1.4)$$

The linearized  $\mathcal{N} = 2$  supercurvatures  $\mathcal{F}^{++\alpha\dot{\alpha}}$  and  $\mathcal{F}^{++5}$  are analytic superfields, while the linearized  $\mathcal{N} = 2$

Weyl tensor  $\mathcal{W}_{(\alpha\beta)}$  is a chiral superfield. All these supercurvatures are constructed out of the superfield potentials  $G^{--\alpha\dot{\alpha}}, G^{--5}, G^{--\alpha\pm}$  related to the basic analytic prepotentials via the well defined harmonic zero-curvature equations. In WZ gauge, these equations can be directly solved, giving rise to the explicit expressions for the  $G$  potentials in terms of the  $\mathcal{N} = 2$  supergravity multiplet fields and their derivatives. In their component expansion, the supercurvatures involve all possible irreducible linearized gravity curvatures quadratic in derivatives: the scalar curvature  $R$ , the irreducible part of Ricci tensor  $\mathcal{R}_{(\alpha\beta)(\dot{\alpha}\dot{\beta})}$  and the linearized Weyl tensor  $\mathcal{R}_{(\alpha\beta\gamma\delta)}$ . So these supercurvatures can be viewed as the building blocks of various linearized  $\mathcal{N} = 2$  supergravity invariants.

The paper is organized as follows. In Sec. II we introduce notations and shortly recall the structure of harmonic superspace. In Sec. III we describe prepotentials of  $\mathcal{N} = 2$  Einstein supergravity multiplet and the corresponding gauge group. Next we perform the linearization procedure and study in detail the component contents of the analytic prepotentials. In Sec. IV, the covariant  $\mathcal{N} = 2$  superfields  $G^{\pm\pm M}$  are introduced and the linearized  $\mathcal{N} = 2$  supersymmetric Einstein action is constructed. In Sec. V, the linearized supercurvatures are defined and all invariant quadratic actions in harmonic superspace are presented. In Sec. VI, we elaborate on the component reduction of the linearized  $\mathcal{N} = 2$  Einstein supergravity, obtain the linearized supercurvatures and the linearized quadratic invariants. In Sec. VII we discuss construction of more general linearized  $\mathcal{N} = 2$  supergravity invariants. Section VIII is for conclusions and outlook. In particular, we briefly sketch the higher-spin generalizations of the invariants constructed.

Three Appendixes collect some technical details. Appendix A provides the linearized gravity invariants in the spinor notations used throughout the paper. In Appendix B we introduce the linearized spin  $\frac{3}{2}$  invariants that are needed when performing the component reduction in the fermionic sector. In Appendix C we present the solutions of the harmonic zero-curvature equations, which, as was already mentioned, are a necessary part of passing to the component Lagrangians.

## II. HARMONIC SUPERSPACE

We proceed from  $\mathcal{N} = 2$  harmonic superspace in the analytic basis, with an extra  $x^5$  coordinate (for a pedagogical introduction, see [16])<sup>2</sup>:

$$\{x^{\alpha\dot{\alpha}}, \theta^{+\hat{\alpha}}, \theta^{-\hat{\alpha}}, u_i^{\pm}, x^5\}, \quad \hat{\alpha} = (\alpha, \dot{\alpha}). \quad (2.1)$$

<sup>2</sup>Here we use standard notations:  $x^{\alpha\dot{\alpha}} = x^m(\tilde{\sigma}_m)^{\alpha\dot{\alpha}}$ ,  $\partial_{\alpha\dot{\alpha}} = \frac{1}{2}\sigma_{\alpha\dot{\alpha}}^m\partial_m$  (so  $\partial_{\alpha\dot{\alpha}}x^{\beta\dot{\beta}} = \delta_{\alpha}^{\beta}\delta_{\dot{\alpha}}^{\dot{\beta}}$ ,  $\partial_{\alpha\dot{\beta}}\partial^{\beta\dot{\beta}} = \frac{1}{4}\delta_{\alpha}^{\beta}\delta_{\dot{\alpha}}^{\dot{\beta}}$ ),  $\hat{\alpha} = (\alpha, \dot{\alpha})$ ,  $\partial_{\hat{\alpha}} = \frac{\partial}{\partial\theta^{+\hat{\alpha}}}$ ,  $\partial_{\hat{\alpha}} = \frac{\partial}{\partial\theta^{-\hat{\alpha}}}$ ,  $(\psi\chi) = \psi^{\alpha}\chi_{\alpha}$ ,  $(\bar{\psi}\bar{\chi}) = \bar{\psi}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}}$ ,  $(\theta^+)^2 = \theta^{+\hat{\alpha}}\theta_{\hat{\alpha}}^+$  etc.

The harmonics  $u_i^{\pm}$  are auxiliary coordinates that parametrize an internal sphere  $S^2 \sim SU(2)/U(1)$  and satisfy the relation  $u^{+i}u_i^- = 1$ . The presence of harmonics  $u_i^{\pm}$  in the coordinate set inevitably leads to an infinite number of component fields in any unconstrained harmonic superfield. The additional coordinate  $x^5$  is needed to introduce the interaction of  $\mathcal{N} = 2$  Maxwell multiplet with hypermultiplet, as well as to describe massive hypermultiplet (with a mass equal to central charge). In this paper, all harmonic superfields are assumed to be  $x^5$  independent. The coordinate  $x^5$  will be merely used for the purpose to simplify handling  $\mathcal{N} = 2$  supergravity prepotentials in the linearized theory. In the analytic basis, rigid  $\mathcal{N} = 2$  supersymmetry transformations act on the coordinates (2.1) as

$$\begin{aligned} \delta_{\epsilon}x^{\alpha\dot{\alpha}} &= -4i(\epsilon^{-\alpha}\bar{\theta}^{+\dot{\alpha}} + \theta^{+\alpha}\bar{\epsilon}^{-\dot{\alpha}}), & \delta_{\epsilon}\theta^{\pm\hat{\alpha}} &= \epsilon^{\pm\hat{\alpha}}, \\ \delta_{\epsilon}u_i^{\pm} &= 0, & \delta_{\epsilon}x^5 &= 2i\epsilon^{-\hat{\alpha}}\theta_{\hat{\alpha}}^+. \end{aligned} \quad (2.2)$$

Here  $\epsilon^{\pm\hat{\alpha}} := \epsilon^{\hat{\alpha}i}u_i^{\pm}$  are relevant spinorial parameters. These transformations preserve an analytic subspace of harmonic superspace:

$$\zeta := \{x^{\alpha\dot{\alpha}}, \theta^{+\hat{\alpha}}, u_i^{\pm}\}. \quad (2.3)$$

This invariant subspace contains just half the original Grassmann variables.

An analogue of complex conjugation in harmonic superspace is the tilde conjugation. On complex functions, it acts like the ordinary complex conjugation. Its action on the harmonic superspace coordinates is as follows:

$$\begin{aligned} \widetilde{x^{\alpha\dot{\alpha}}} &= x^{\alpha\dot{\alpha}}, & \widetilde{\theta_{\hat{\alpha}}^{\pm}} &= \bar{\theta}_{\hat{\alpha}}^{\pm}, & \widetilde{\bar{\theta}_{\hat{\alpha}}^{\pm}} &= -\theta_{\hat{\alpha}}^{\pm}, \\ \widetilde{u^{\pm i}} &= -u_i^{\pm}, & \widetilde{u_i^{\pm}} &= u^{\pm i}, & \widetilde{x^5} &= x^5. \end{aligned} \quad (2.4)$$

The spinor derivatives which are covariant with respect to  $\mathcal{N} = 2$  supersymmetry (2.2) are defined as

$$\begin{aligned} \mathcal{D}_{\hat{\alpha}}^+ &:= \partial_{\hat{\alpha}}^+, \\ \mathcal{D}_{\hat{\alpha}}^- &:= -\partial_{\hat{\alpha}}^- + 4i\bar{\theta}^{-\hat{\alpha}}\partial_{\alpha\dot{\alpha}} - 2i\theta_{\hat{\alpha}}^-\partial_5, \\ \bar{\mathcal{D}}_{\hat{\alpha}}^- &:= -\partial_{\hat{\alpha}}^- - 4i\theta^{-\hat{\alpha}}\partial_{\alpha\dot{\alpha}} + 2i\bar{\theta}_{\hat{\alpha}}^-\partial_5. \end{aligned} \quad (2.5)$$

The covariant harmonic derivatives are given by

$$\begin{aligned} \mathcal{D}^{++} &= \partial^{++} - 4i\theta^{+\rho}\bar{\theta}^{+\dot{\rho}}\partial_{\rho\dot{\rho}} + \theta^{+\dot{\rho}}\partial_{\dot{\rho}}^+ + i\theta^{+\dot{\rho}}\theta_{\dot{\rho}}^+\partial_5, \\ \mathcal{D}^{--} &= \partial^{--} - 4i\theta^{-\rho}\bar{\theta}^{-\dot{\rho}}\partial_{\rho\dot{\rho}} + \theta^{-\dot{\rho}}\partial_{\dot{\rho}}^- + i\theta^{-\dot{\rho}}\bar{\theta}_{\dot{\rho}}^-\partial_5, \\ \mathcal{D}^0 &= \partial^0 + \theta^{+\dot{\rho}}\partial_{\dot{\rho}}^- - \theta^{-\dot{\rho}}\partial_{\dot{\rho}}^+. \end{aligned} \quad (2.6)$$

Here we used the following notations for the partial harmonic derivatives:

$$\begin{aligned} \partial^{++} &= u^{+i}\frac{\partial}{\partial u^{-i}}, & \partial^{--} &= u^{-i}\frac{\partial}{\partial u^{+i}}, \\ \partial^0 &= u^{+i}\frac{\partial}{\partial u^{+i}} - u^{-i}\frac{\partial}{\partial u^{-i}}. \end{aligned} \quad (2.7)$$

They satisfy these relations:

$$[\partial^{++}, \partial^{--}] = \partial^0, \quad [\partial^0, \partial^{\pm\pm}] = \pm 2\partial^{\pm\pm}. \quad (2.8)$$

The covariant harmonic derivatives play an important role in the formulation of  $\mathcal{N} = 2$  supergravity in harmonic superspace. They satisfy the  $SU(2)$ -algebra relations similar to (2.8):

$$[\mathcal{D}^{++}, \mathcal{D}^{--}] = \mathcal{D}^0, \quad [\mathcal{D}^0, \mathcal{D}^{\pm\pm}] = \pm 2\mathcal{D}^{\pm\pm}. \quad (2.9)$$

The spinor covariant derivatives (2.5) obey the following relations:

$$\begin{aligned} [\mathcal{D}^{--}, \mathcal{D}_{\hat{\alpha}}^+] &= \mathcal{D}_{\hat{\alpha}}^-, & [\mathcal{D}^{++}, \mathcal{D}_{\hat{\alpha}}^-] &= \mathcal{D}_{\hat{\alpha}}^+, \\ [\mathcal{D}^{++}, \mathcal{D}_{\hat{\alpha}}^+] &= 0, & [\mathcal{D}^{--}, \mathcal{D}_{\hat{\alpha}}^-] &= 0, \end{aligned} \quad (2.10a)$$

$$\begin{aligned} \{\mathcal{D}_{\hat{\alpha}}^+, \bar{\mathcal{D}}_{\hat{\alpha}}^-\} &= -4i\partial_{\alpha\dot{\alpha}}, & \{\bar{\mathcal{D}}_{\hat{\alpha}}^+, \mathcal{D}_{\hat{\alpha}}^-\} &= 4i\partial_{\alpha\dot{\alpha}}, \\ \{\mathcal{D}_{\hat{\alpha}}^+, \mathcal{D}_{\hat{\beta}}^-\} &= 2i\epsilon_{\alpha\beta}\partial_5, & \{\bar{\mathcal{D}}_{\hat{\alpha}}^+, \bar{\mathcal{D}}_{\hat{\beta}}^-\} &= -2i\epsilon_{\dot{\alpha}\dot{\beta}}\partial_5. \end{aligned} \quad (2.10b)$$

Harmonic superspace can be looked upon as the most general  $4D, \mathcal{N} = 2$  superspace [42,43], and it is indispensable for formulation and study of classical and quantum  $\mathcal{N} = 2$  theories [16]. The analytic superspace (2.3) plays the fundamental role as it underlies the geometric structure of all  $\mathcal{N} = 2$  theories (see, e.g., [44] for application to the construction of the hypermultiplet couplings to higher spins). In this paper,  $\mathcal{N} = 2$  harmonic analyticity will be shown to be a necessary ingredient also in constructing higher-derivative  $\mathcal{N} = 2$  supergravity invariants. In the next section, as an instructive example, we shall expound how the analytic unconstrained prepotentials of  $\mathcal{N} = 2$  Einstein supergravity naturally appear in the harmonic superspace framework. After that, we shall proceed to the analysis of the linearized theory.

### III. $\mathcal{N} = 2$ EINSTEIN SUPERGRAVITY MULTIPLER IN HARMONIC SUPERSPACE

This section is devoted to a detailed discussion of  $\mathcal{N} = 2$  Einstein supergravity multiplet and gauge freedom in both the full-fledged nonlinear case and its linearized approximation. We will also define the auxiliary fields redefinitions needed for performing the component reduction in Sec. VI.

#### A. $\mathcal{N} = 2$ supergravity multiplet: Nonlinear setup

The gauge group of  $\mathcal{N} = 2$  Einstein supergravity is constituted by the analyticity-preserving transformations of harmonic superspace [16,18,19]<sup>3</sup>

<sup>3</sup>They can be treated as an analytic gauging of rigid  $\mathcal{N} = 2$  supersymmetry transformations (2.2).

$$\begin{aligned} \delta_\lambda x^{\alpha\dot{\alpha}} &= \lambda^{\alpha\dot{\alpha}}(\zeta), & \delta_\lambda \theta^{+\hat{\alpha}} &= \lambda^{+\hat{\alpha}}(\zeta), & \delta_\lambda \theta^{-\hat{\alpha}} &= \lambda^{-\hat{\alpha}}(\zeta, \theta^-), \\ \delta_\lambda u_i^\pm &= 0, & \delta_\lambda x^5 &= \lambda^5(\zeta). \end{aligned} \quad (3.1)$$

Note that the only parameters which are not analytic are  $\lambda^{-\hat{\alpha}}$ . Another peculiarity is that the harmonic variables retain inert under gauge transformations.<sup>4</sup> It is useful to introduce a multi-index  $M := (\alpha\dot{\alpha}, +\hat{\alpha}, -\hat{\alpha}, 5)$  and denote the harmonic superspace coordinates as  $z^M := (x^{\alpha\dot{\alpha}}, \theta^{+\hat{\alpha}}, \theta^{-\hat{\alpha}}, x^5)$ . Then superdiffeomorphism transformations can be represented as the action of the differential operator:

$$\delta_\lambda z^M = [\hat{\Lambda}, z^M], \quad \hat{\Lambda} := \lambda^M \partial_M. \quad (3.2)$$

This is a very convenient notation, because, for instance, transformations of the superspace derivatives  $\partial_M := (\partial_{\alpha\dot{\alpha}}, \partial_{\hat{\alpha}}^-, \partial_{\hat{\alpha}}^+, \partial_5)$  can be succinctly written as

$$\delta_\lambda \partial_M = [\hat{\Lambda}, \partial_M]. \quad (3.3)$$

In particular, it implies that the harmonic derivative  $\mathcal{D}^{++}$  ceases to be covariant<sup>5</sup> under supertranslations (3.1):

$$\begin{aligned} \delta_\lambda \mathcal{D}^{++} &= [\hat{\Lambda}, \mathcal{D}^{++}] = -\mathcal{D}^{++} \lambda^M \partial_M - 4i\lambda^{+\rho} \bar{\theta}^{+\dot{\rho}} \partial_{\rho\dot{\rho}} \\ &\quad - 4i\theta^{+\rho} \bar{\lambda}^{+\dot{\rho}} \partial_{\rho\dot{\rho}} + \lambda^{+\hat{\rho}} \partial_{\hat{\rho}}^+ + 2(\lambda^{+\hat{\rho}} \theta^{+\hat{\rho}}) \partial_5. \end{aligned} \quad (3.4)$$

The unwanted terms can be canceled by introducing a set of gauge prepotentials:

$$\begin{aligned} \mathcal{D}^{++} \rightarrow \mathfrak{D}^{++} &= \mathcal{D}^{++} + h^{++\alpha\dot{\alpha}} \partial_{\alpha\dot{\alpha}} + h^{++\hat{\alpha}+} \partial_{\hat{\alpha}}^+ + h^{++\hat{\alpha}-} \partial_{\hat{\alpha}}^- \\ &\quad + h^{++5} \partial_5 \end{aligned} \quad (3.5)$$

with the gauge freedom:

$$\begin{cases} \delta_\lambda h^{++\alpha\dot{\alpha}} = \mathfrak{D}^{++} \lambda^{\alpha\dot{\alpha}} + 4i\lambda^{+\alpha} \bar{\theta}^{+\dot{\alpha}} + 4i\theta^{+\alpha} \bar{\lambda}^{+\dot{\alpha}} - \hat{\Lambda} h^{++\alpha\dot{\alpha}} \\ \delta_\lambda h^{++\hat{\alpha}+} = \mathfrak{D}^{++} \lambda^{+\hat{\alpha}} - \hat{\Lambda} h^{++\hat{\alpha}+} \\ \delta_\lambda h^{++\hat{\alpha}-} = \mathfrak{D}^{++} \lambda^{-\hat{\alpha}} - \lambda^{+\hat{\alpha}} - \hat{\Lambda} h^{++\hat{\alpha}-} \\ \delta_\lambda h^{++5} = \mathfrak{D}^{++} \lambda^5 - 2\lambda^{+\hat{\alpha}} \theta_{\hat{\alpha}}^+ - \hat{\Lambda} h^{++5} \end{cases} \quad (3.6)$$

The modified derivative  $\mathfrak{D}^{++}$  is covariant under (3.1), i.e., it satisfies

$$\delta_\lambda \mathfrak{D}^{++} = [\hat{\Lambda}, \mathfrak{D}^{++}] + \delta_\lambda h^{++M} \partial_M = 0. \quad (3.7)$$

Since the harmonic derivative  $\mathcal{D}^{++}$  is real with respect to the tilde conjugation, it is natural to require that  $\mathfrak{D}^{++}$  obeys the same property:

<sup>4</sup>It is the gauge group of  $\mathcal{N} = 2$  conformal supergravity that affects harmonics, see [16,17,19].

<sup>5</sup>The rigid  $\mathcal{N} = 2$  supersymmetry transformations can be defined as a subgroup of (3.1) distinguished by the condition  $\delta_\epsilon \mathcal{D}^{++} = 0$ .

$$\begin{aligned} \widetilde{h^{++\alpha\dot{\alpha}}} &= h^{++\alpha\dot{\alpha}}, \\ \widetilde{h^{++5}} &= h^{++5}, \\ \widetilde{h^{++\alpha+}} &= h^{++\alpha+}, \\ \widetilde{h^{++\dot{\alpha}+}} &= -h^{++\alpha+}. \end{aligned} \quad (3.8)$$

Note that the transformation law of  $h^{++\dot{\alpha}-}$  in (3.6) allows to impose the analytic gauge  $h^{++\dot{\alpha}-} = 0$ , which expresses  $\lambda^{-\dot{\alpha}}$  in terms of the analytic parameter  $\lambda^{+\dot{\alpha}}$ :

$$\mathfrak{D}^{++}\lambda^{-\dot{\alpha}} = \lambda^{+\dot{\alpha}}. \quad (3.9)$$

In the analytic gauge, the differential operator  $\mathfrak{D}^{++}$  becomes analytic; i.e., it satisfies the condition  $[\mathcal{D}_{\dot{\alpha}}^+, \mathfrak{D}^{++}] = 0$ . Hereafter, we always stick to this gauge.

Notably, the gauge freedom (3.6) allows one to gauge away infinitely many fields from the prepotentials and to end up with the off-shell  $\mathcal{N} = 2$  Einstein supergravity multiplet in the component expansion of  $h^{++M}$ . In this WZ gauge we have<sup>6</sup>

$$\begin{aligned} h_{WZ}^{++\alpha\dot{\alpha}} &= -4i\theta^{+\beta}\bar{\theta}^{+\dot{\beta}}\Phi_{\beta\dot{\beta}}^{\alpha\dot{\alpha}} + 16(\bar{\theta}^+)^2\theta^{+\beta}\psi_{\beta}^{\alpha\dot{\alpha}}u_i^- - 16(\theta^+)^2\bar{\theta}^{+\dot{\beta}}\bar{\psi}_{\dot{\beta}}^{\alpha\dot{\alpha}}u_i^- + (\theta^+)^4V^{\alpha\dot{\alpha}(ij)}u_i^-u_j^-, \\ h_{WZ}^{++5} &= -4i\theta^{+\beta}\bar{\theta}^{+\dot{\beta}}C_{\beta\dot{\beta}} + 8(\bar{\theta}^+)^2\theta^{+\beta}\rho_{\beta}^iu_i^- - 8(\theta^+)^2\bar{\theta}^{+\dot{\beta}}\bar{\rho}_{\dot{\beta}}^iu_i^- + (\theta^+)^4S^{(ij)}u_i^-u_j^-, \\ h_{WZ}^{++\alpha+} &= (\bar{\theta}^+)^2\theta_{\beta}^{+\alpha}T^{(\alpha\beta)} + (\bar{\theta}^+)^2\theta^{+\alpha}T + (\theta^+)^2\bar{\theta}_{\dot{\beta}}^{+\alpha}P^{\alpha\dot{\beta}} + (\theta^+)^4\chi^{\alpha i}u_i^-, \\ h_{WZ}^{++\dot{\alpha}+} &= (\theta^+)^2\bar{\theta}_{\dot{\beta}}^{+\dot{\alpha}}\bar{T}^{(\dot{\alpha}\dot{\beta})} + (\theta^+)^2\bar{\theta}^{+\dot{\alpha}}\bar{T} - (\bar{\theta}^+)^2\theta_{\beta}^{+\dot{\alpha}}\bar{P}^{\beta\dot{\alpha}} + (\theta^+)^4\bar{\chi}^{\dot{\alpha} i}u_i^-. \end{aligned} \quad (3.10)$$

The remaining fields are defined up to a residual gauge freedom. As usual, it is specified by the requirement of preservation of WZ gauge, i.e.,  $\delta_{\lambda}^{\text{res}}h^{++M} \sim h_{WZ}^{++M}$ . As a result, the set of residual parameters is obtained:

$$\left\{ \begin{aligned} \lambda_{WZ}^{\alpha\dot{\alpha}} &= a^{\alpha\dot{\alpha}}(x) - 4i\theta^{+\alpha}\bar{\epsilon}^{\dot{\alpha} i}(x)u_i^- - 4i\epsilon^{\alpha i}(x)u_i^-\bar{\theta}^{+\dot{\alpha}}, \\ \lambda_{WZ}^{+\alpha} &= \epsilon^{\alpha i}(x)u_i^+ + \theta^{+\beta}l_{(\beta}^{\alpha)}(x) + 4i\theta^{+\beta}\bar{\theta}^{+\dot{\beta}}(\delta_{\beta}^{\rho}\delta_{\dot{\beta}}^{\dot{\rho}} + \Phi_{\beta\dot{\beta}}^{\rho\dot{\rho}})\partial_{\rho\dot{\rho}}\epsilon^{\alpha i}(x)u_i^- \\ \lambda_{WZ}^{+\dot{\alpha}} &= \bar{\epsilon}^{\dot{\alpha} i}(x)u_i^+ + \bar{\theta}^{+\dot{\beta}}l_{(\dot{\beta}}^{\dot{\alpha})}(x) + 4i\theta^{+\beta}\bar{\theta}^{+\dot{\beta}}(\delta_{\beta}^{\rho}\delta_{\dot{\beta}}^{\dot{\rho}} + \Phi_{\beta\dot{\beta}}^{\rho\dot{\rho}})\partial_{\rho\dot{\rho}}\bar{\epsilon}^{\dot{\alpha} i}(x)u_i^- \\ \lambda_{WZ}^5 &= c(x) + 2i\epsilon^{\hat{\alpha} i}(x)u_i^-\theta_{\hat{\alpha}}^+ \end{aligned} \right. \quad (3.11)$$

The realization of these transformations on the WZ fields directly follows from (3.6). As an example, let us quote the transformations of the spin 2,  $\frac{3}{2}$ , 1 gauge fields:

$$\delta\Phi_{\beta\dot{\beta}}^{\alpha\dot{\alpha}} = (\delta_{\beta}^{\rho}\delta_{\dot{\beta}}^{\dot{\rho}} + \Phi_{\beta\dot{\beta}}^{\rho\dot{\rho}})\partial_{\rho\dot{\rho}}a^{\alpha\dot{\alpha}} - a^{\rho\dot{\rho}}\partial_{\rho\dot{\rho}}\Phi_{\beta\dot{\beta}}^{\alpha\dot{\alpha}} - (l_{(\beta}^{\alpha)}\delta_{\dot{\beta}}^{\dot{\rho}} + \delta_{\beta}^{\rho}\bar{l}_{(\dot{\beta}}^{\dot{\rho})})(\delta_{\rho}^{\alpha}\delta_{\dot{\rho}}^{\dot{\alpha}} + \Phi_{\rho\dot{\rho}}^{\alpha\dot{\alpha}}) - 2i\epsilon_{\beta i}\bar{\psi}_{\dot{\beta}}^{\alpha\dot{\alpha}} - 2i\bar{\epsilon}_{\dot{\beta} i}\psi_{\beta}^{\alpha\dot{\alpha}}, \quad (3.12a)$$

$$\delta\psi_{\beta}^{\alpha\dot{\alpha} i} = -\epsilon^{\dot{\alpha}\dot{\beta}}(\delta_{\beta}^{\rho}\delta_{\dot{\beta}}^{\dot{\rho}} + \Phi_{\beta\dot{\beta}}^{\rho\dot{\rho}})\partial_{\rho\dot{\rho}}\epsilon^{\alpha i} + i\frac{1}{2}T_{(\beta}^{\alpha)}\bar{\epsilon}^{\dot{\alpha} i} - i\frac{1}{2}T\delta_{\beta}^{\alpha}\bar{\epsilon}^{\dot{\alpha} i} - i\frac{1}{2}\bar{F}_{\beta}^{\dot{\alpha}}\epsilon^{\alpha i} + \frac{1}{6}\epsilon_{\beta j}V^{\alpha\dot{\alpha}(ij)} + \psi_{\beta}^{\rho\dot{\rho}}\partial_{\rho\dot{\rho}}a^{\alpha\dot{\alpha}} - a^{\rho\dot{\rho}}\partial_{\rho\dot{\rho}}\psi_{\beta}^{\alpha\dot{\alpha} i} - l_{(\beta}^{\rho)}\psi_{\rho}^{\alpha\dot{\alpha} i}, \quad (3.12b)$$

$$\delta C_{\alpha\dot{\alpha}} = (\delta_{\alpha}^{\rho}\delta_{\dot{\alpha}}^{\dot{\rho}} + \Phi_{\alpha\dot{\alpha}}^{\rho\dot{\rho}})\partial_{\rho\dot{\rho}}c - a^{\rho\dot{\rho}}\partial_{\rho\dot{\rho}}C_{\alpha\dot{\alpha}} - (l_{(\alpha}^{\beta)}\delta_{\dot{\alpha}}^{\dot{\rho}} + \delta_{\alpha}^{\rho}l_{(\dot{\alpha}}^{\dot{\rho})})C_{\beta\dot{\beta}} - i\epsilon_{\beta i}\bar{\rho}_{\dot{\beta}}^i - i\epsilon_{\dot{\beta} i}\rho_{\beta}^i. \quad (3.12c)$$

These transformations make it possible to link the harmonic superspace formulation to the component approach. For example, we see that the combination  $\delta_{\beta}^{\alpha}\delta_{\dot{\beta}}^{\dot{\alpha}} + \Phi_{\beta\dot{\beta}}^{\alpha\dot{\alpha}} := \frac{1}{2}\sigma_m^{\alpha\dot{\alpha}}\sigma_{\beta\dot{\beta}}^m e_a^m$  can be identified with the vierbein  $e_a^m$ , the parameter  $a^{\alpha\dot{\alpha}}$  stands for local translations and  $l^{(\alpha\beta)}, \bar{l}^{(\dot{\alpha}\dot{\beta})}$  for local Lorentz transformations, while  $\epsilon^{\alpha i}, \bar{\epsilon}^{\dot{\alpha} i}$  are the parameters of local supersymmetry. The metric tensor is then given by  $g^{mn} = e_a^m e_b^n \eta^{ab}$  and have the standard transformation law  $\delta g^{mn} = \partial_k a^m g^{kn} + \partial_k a^n g^{mk} - a^k \partial_k g^{mn}$ . The fermionic field  $\psi_{\beta}^{\alpha\dot{\alpha} i}$  describes the doublet of the spin  $\frac{3}{2}$  Rarita-Schwinger fields,  $C_{\alpha\dot{\alpha}}$  is the spin 1 gauge field. Other fields are auxiliary fields of the off-shell  $\mathcal{N} = 2$  Einstein supergravity multiplet<sup>7</sup>:

<sup>6</sup>The numerical coefficients are chosen so as to have the field transformation laws as simple as possible, see (3.19).

<sup>7</sup>Some auxiliary fields require redefinition, see the explicit redefinition in the linearized limit (3.26).

Physical sector:  $\Phi_{\beta\dot{\beta}}^{\alpha\dot{\alpha}}, \psi_{\beta}^{\alpha\dot{\alpha}}, C_{\beta\dot{\beta}};$   
 Auxiliary sector:  $V^{\alpha\dot{\alpha}(ij)}, \rho_{\beta}^i, S^{(ij)}, T, T^{(\alpha\beta)}, P^{\alpha\dot{\beta}}, \chi^{\alpha i}. \quad (3.13)$

Totally, the multiplet carries  $40_B + 40_F$  off-shell degrees of freedom. The dimensions of the involved fields can be directly determined from that of the prepotentials in WZ gauge (3.10). The above component WZ expansion corresponds to the “minimal” version of  $\mathcal{N} = 2$  supergravity obtained from the conformal one through the compensation by the so-called nonlinear multiplet (in our formulation there are in fact two hidden compensators:  $\mathcal{N} = 2$  vector multiplet and the nonlinear multiplet, see, e.g., [1, 16, 45]). The off-shell component content of this version of  $\mathcal{N} = 2$  Einstein supergravity was first found by Fradkin and Vasiliev [46, 47] and, independently, by de Wit and van Holten [48].

### B. $\mathcal{N} = 2$ supergravity multiplet: Linearization

Linearizing gauge transformations (3.6) yields

$$\begin{cases} \delta_{\lambda} h^{++\alpha\dot{\alpha}} = \mathcal{D}^{++}\lambda^{\alpha\dot{\alpha}} + 4i\lambda^{+\alpha}\bar{\theta}^{+\dot{\alpha}} + 4i\theta^{+\alpha}\bar{\lambda}^{+\dot{\alpha}} \\ \delta_{\lambda} h^{++\hat{\alpha}+} = \mathcal{D}^{++}\lambda^{+\hat{\alpha}} \\ \delta_{\lambda} h^{++\hat{\alpha}-} = \mathcal{D}^{++}\lambda^{-\hat{\alpha}} - \lambda^{+\hat{\alpha}} \\ \delta_{\lambda} h^{++5} = \mathcal{D}^{++}\lambda^5 - 2i\lambda^{+\hat{\alpha}}\theta_{\hat{\alpha}}^+ \end{cases}. \quad (3.14)$$

In the linearized case we also impose the analytic gauge  $h^{++\hat{\alpha}-} = 0$ , so  $\mathcal{D}^{++}\lambda^{-\hat{\alpha}} = \lambda^{+\hat{\alpha}}$ . In this gauge all prepotentials are analytic. The WZ gauge has the same form as in the nonlinear theory, Eq. (3.10).

Rigid supersymmetry transformations of the analytic prepotentials can be obtained by requiring invariance of the covariant derivative  $\mathfrak{D}^{++}$  under the rigid  $\mathcal{N} = 2$  supersymmetry:

$$\delta_{\epsilon}\mathfrak{D}^{++} = [\hat{\Lambda}_{\epsilon}, \mathcal{D}^{++}] + \delta_{\epsilon}h^{++M}\partial_M = 0. \quad (3.15)$$

As a solution of (3.15), all parameters  $\lambda_{\epsilon}^M$  span rigid supersymmetry:

$$\begin{aligned} \lambda_{\epsilon}^{\alpha\dot{\alpha}} &= -4i\theta^{+\alpha}\bar{\epsilon}^{-\dot{\alpha}} - 4i\epsilon^{-\alpha}\bar{\theta}^{+\dot{\alpha}}, & \lambda_{\epsilon}^{+\alpha} &= \epsilon^{+\alpha}, \\ \lambda_{\epsilon}^{+\hat{\alpha}} &= \bar{\epsilon}^{+\hat{\alpha}}, & \lambda_{\epsilon}^5 &= 2i\epsilon^{-\hat{\alpha}}\theta_{\hat{\alpha}}^+. \end{aligned} \quad (3.16)$$

Note that, in full agreement with (2.2),  $\delta_{\epsilon}z^M = \lambda_{\epsilon}^M$ .

From (3.15) we find that the rigid supersymmetry transformations of the prepotentials are given by

$$\begin{cases} \delta_{\epsilon}h^{++\alpha\dot{\alpha}} = -4ih^{++\alpha+}\bar{\epsilon}^{-\dot{\alpha}} - 4i\epsilon^{-\alpha}h^{++\hat{\alpha}+} - \delta_{\epsilon}z^M\partial_M h^{++\alpha\dot{\alpha}} \\ \delta_{\epsilon}h^{++\alpha+} = -\delta_{\epsilon}z^M\partial_M \delta_{\epsilon}h^{++\alpha+} \\ \delta_{\epsilon}h^{++\hat{\alpha}+} = -\delta_{\epsilon}z^M\partial_M h^{++\hat{\alpha}+} \\ \delta_{\epsilon}h^{++5} = 2ih^{++\hat{\alpha}+}\epsilon_{\hat{\alpha}}^+ - \delta_{\epsilon}z^M\partial_M h^{++5} \end{cases} \quad (3.17).$$

Note that the prepotentials  $h^{++\alpha\dot{\alpha}}$  and  $h^{++5}$  have nonstandard transformation laws. As usual, in WZ gauge, supersymmetry transformations must be accompanied by compensating gauge transformations that ensure the preservation of the gauge-fixed form of  $h_{WZ}^{++M}$ .

From (3.17) one can deduce supersymmetry transformations of the component fields. For example, the gravity field transforms as

$$\delta_{\epsilon}\Phi_{\beta\dot{\beta}}^{\alpha\dot{\alpha}} = 2i(\epsilon_{\beta}^i\bar{\psi}_{\dot{\beta}i}^{\alpha\dot{\alpha}} - \bar{\epsilon}_{\dot{\beta}}^i\psi_{\beta i}^{\alpha\dot{\alpha}}). \quad (3.18)$$

The complete set of linearized gauge transformation laws for the component fields can be deduced from (3.14) and the linearized version of the residual gauge parameters (3.11):

$$\delta_{\lambda}\Phi_{\beta\dot{\beta}}^{\alpha\dot{\alpha}} = \partial_{\beta\dot{\beta}}\lambda^{\alpha\dot{\alpha}} - l_{(\beta}^{\alpha)}\delta_{\dot{\beta}}^{\dot{\alpha}} - \delta_{\beta}^{\alpha}l_{(\dot{\beta}}^{\dot{\alpha})}, \quad (3.19a)$$

$$\delta_{\lambda}\psi_{\beta}^{\alpha\dot{\alpha}i} = -\partial_{\beta}^{\dot{\alpha}}\epsilon^{\alpha i}, \quad (3.19b)$$

$$\delta_{\lambda}C_{\alpha\dot{\alpha}} = \partial_{\alpha\dot{\alpha}}\lambda, \quad (3.19c)$$

$$\delta_{\lambda}P^{\alpha\dot{\alpha}} = -2i\partial^{\alpha\dot{\alpha}}l_{(\beta}^{\dot{\alpha})}, \quad (3.19d)$$

$$\delta_{\lambda}\chi^{\alpha i} = 2\Box\epsilon^{\alpha i}, \quad (3.19e)$$

$$\delta_{\lambda}\rho_{\alpha}^i = \partial_{\alpha\dot{\alpha}}\bar{\epsilon}^{\dot{\alpha}i}. \quad (3.19f)$$

Other fields in WZ gauge are invariant under gauge transformations. The transformations (3.19a)–(3.19c) are linearized gauge transformations for the spin 2, spin  $\frac{3}{2}$ , and spin 1 fields, respectively. The fields  $P^{\alpha\dot{\alpha}}$ ,  $\chi^{\alpha i}$ , and  $\rho_{\alpha}^i$  can be properly redefined with the help of the spin 2 and spin  $\frac{3}{2}$  gauge fields so as to ensure trivial gauge transformation laws for them. Such redefinitions play an important role in the component reduction, so we will explicitly present them here.

Spin 2 field has the following decomposition into the irreducible parts:

$$\Phi^{\alpha\dot{\alpha}\beta\dot{\beta}} = \Phi^{(\alpha\dot{\alpha})(\beta\dot{\beta})} + \epsilon^{\alpha\dot{\alpha}}\Phi^{(\dot{\alpha}\dot{\beta})} + \epsilon^{\dot{\alpha}\dot{\beta}}\Phi^{(\alpha\beta)} + \epsilon^{\alpha\beta}\epsilon^{\dot{\alpha}\dot{\beta}}\Phi. \quad (3.20)$$

Using local Lorentz transformations, one can fix the “symmetric” gauge:  $\Phi^{(\alpha\beta)} = 0$ ,  $\Phi^{(\dot{\alpha}\dot{\beta})} = 0$ , thus expressing local Lorentz transformations in terms of local translations  $a^{\alpha\dot{\alpha}}$ :

$$l_{(\beta}^{\alpha)} = \frac{1}{2}\partial_{(\beta\dot{\beta}}a^{\alpha)\dot{\beta}}, \quad l_{(\dot{\beta}}^{\dot{\alpha})} = \frac{1}{2}\partial_{\beta(\dot{\beta}}a^{\beta\dot{\alpha})}. \quad (3.21)$$

In this gauge, the remaining components undergo the transformation laws:

$$\delta\Phi^{(\alpha\beta)(\dot{\alpha}\dot{\beta})} = \partial^{(\alpha(\dot{\alpha}} a^{\beta)\dot{\beta})}, \quad \delta\Phi = \frac{1}{4} \partial_{\beta\dot{\beta}} a^{\beta\dot{\beta}}. \quad (3.22)$$

The field  $P^{\alpha\dot{\alpha}}$  possesses the nontrivial gauge transformation law (3.19d). Because of the relations (3.21), in the symmetric gauge we have

$$\partial^{\dot{\alpha}\beta} l_{(\beta}^{\alpha)} = \frac{1}{16} \square a^{\alpha\dot{\alpha}} - \frac{1}{4} \partial_{\beta}^{\alpha} \partial_{\dot{\beta}}^{\dot{\alpha}} a^{\beta\dot{\beta}} = \frac{1}{8} \square a^{\alpha\dot{\alpha}} - \frac{1}{4} \partial^{\alpha\dot{\alpha}} \partial^{\beta\dot{\beta}} a_{\beta\dot{\beta}}. \quad (3.23)$$

Then the variation (3.19d) can be canceled by adding to  $P^{\alpha\dot{\alpha}}$  the term  $iB^{\alpha\dot{\alpha}}$ , where

$$B^{\alpha\dot{\alpha}} = -\partial_{\beta\dot{\beta}} \Phi^{(\alpha\beta)(\dot{\alpha}\dot{\beta})} + 3\partial^{\alpha\dot{\alpha}} \Phi, \\ \delta B^{\alpha\dot{\alpha}} = -2\partial^{\dot{\alpha}\beta} l_{(\beta}^{\alpha)} = -\frac{1}{4} \square a^{\alpha\dot{\alpha}} + \frac{1}{2} \partial^{\alpha\dot{\alpha}} \partial^{\beta\dot{\beta}} a_{\beta\dot{\beta}}. \quad (3.24)$$

Note the important property,  $\partial_{\beta\dot{\beta}} B^{\beta\dot{\beta}} = -\frac{1}{4} R$ , where  $R$  is the linearized scalar curvature defined in (A5). It is invariant under the linearized gauge transformations (3.22).

The spin  $\frac{3}{2}$  field can also be decomposed into the irreducible pieces:

$$\psi^{\beta\alpha\dot{\alpha}i} = \psi^{(\beta\alpha)\dot{\alpha}i} + \epsilon^{\beta\alpha} \psi^{\dot{\alpha}i}, \quad \delta_{\lambda} \psi^{(\beta\alpha)\dot{\alpha}i} = -\partial^{(\beta\dot{\alpha}} \epsilon^{\alpha)i}, \\ \delta_{\lambda} \psi^{\dot{\alpha}i} = -\frac{1}{2} \partial_{\alpha}^{\dot{\alpha}} \epsilon^{\alpha i}. \quad (3.25)$$

The fields  $\chi^{\alpha i}$  and  $\rho_{\alpha}^i$  have nontrivial gauge transformation laws (3.19e) and (3.19f). So they need to be redefined in a similar way.

Finally, the complete set of the field redefinitions required amounts to

$$P^{\alpha\dot{\alpha}} \rightarrow P^{\alpha\dot{\alpha}} + iB^{\alpha\dot{\alpha}}, \quad \chi^{\alpha i} \rightarrow \chi^{\alpha i} + 4\partial_{\beta}^{\alpha} \psi_{\beta}^{\dot{\alpha}i}, \quad \rho_{\alpha}^i \rightarrow \rho_{\alpha}^i - 2\bar{\psi}_{\alpha}^i. \quad (3.26)$$

After these redefinitions, the fields  $P^{\alpha\dot{\alpha}}$ ,  $\chi^{\alpha i}$ , and  $\rho_{\alpha}^i$  become invariant under gauge transformations and so can be treated as genuine auxiliary fields.

#### IV. $\mathcal{N}=2$ SUPERGRAVITY ACTIONS: NONLINEAR AND LINEARIZED

This section is a review of the construction of the nonlinear  $\mathcal{N}=2$  supergravity action and its linearized version. An important ingredient of the construction is the nonanalytic negatively charged gauge potentials related to the fundamental analytic ones by the harmonic zero-curvature conditions.

#### A. Nonlinear theory

The action of  $\mathcal{N}=2$  Einstein supergravity in harmonic superspace was constructed by Galperin *et al.* [19]. Here we briefly recall its main features. The action describes  $\mathcal{N}=2$  Abelian vector multiplet in the  $\mathcal{N}=2$  supergravity background<sup>8</sup>:

$$S_{\text{full}} = - \int d^4x d^8\theta du E H^{++5} H^{--5}. \quad (4.1)$$

Here  $E$  is the  $\mathcal{N}=2$  full harmonic superspace integration measure with the transformation law:

$$\delta_{\lambda} E = -(\partial_{\alpha\dot{\alpha}} \lambda^{\alpha\dot{\alpha}} - \partial_{\dot{\alpha}}^{-} \lambda^{+\dot{\alpha}} - \partial_{\alpha}^{+} \lambda^{-\dot{\alpha}}) E. \quad (4.2)$$

The measure  $E$  can be constructed from supergravity prepotentials and it satisfies some useful properties. We shall not present the whole construction here, for details see [16,17,19]. It will be important that it is expressed in terms of the potentials  $H^{++M}$  and  $H^{--M}$  that covariantize the harmonic derivatives  $\mathcal{D}^{\pm\pm}$

$$\mathfrak{D}^{++} = \partial^{++} + H^{++\alpha\dot{\alpha}} \partial_{\alpha\dot{\alpha}} + H^{++\dot{\alpha}+} \partial_{\dot{\alpha}}^{-} + \theta^{+\dot{\alpha}} \partial_{\dot{\alpha}}^{+} + H^{++5} \partial_5, \quad (4.3a)$$

$$\mathfrak{D}^{--} = \partial^{--} + H^{--\alpha\dot{\alpha}} \partial_{\alpha\dot{\alpha}} + H^{--\dot{\alpha}+} \partial_{\dot{\alpha}}^{-} + H^{--\dot{\alpha}-} \partial_{\dot{\alpha}}^{+} + H^{--5} \partial_5. \quad (4.3b)$$

The nonanalytic potentials  $H^{--M}$  are uniquely determined in terms of  $H^{++M}$  as solutions of the harmonic zero-curvature condition:

$$[\mathfrak{D}^{++}, \mathfrak{D}^{--}] = D^0. \quad (4.4)$$

One could also consider another nonlinear action:

$$S_{\text{alt}} = \int d^4x d^8\theta du E H^{++\alpha\dot{\alpha}} H_{\alpha\dot{\alpha}}^{--}. \quad (4.5)$$

However, it vanishes due to the property (easily checkable in WZ gauge):

$$(\mathcal{D}^{+})^4 (E H^{--\alpha\dot{\alpha}}) = 0. \quad (4.6)$$

In this paper we limit our attention to studying invariants and their component reductions in the linear approximations, so we shall not discuss any higher-curvature generalization of (4.1) or (4.5).

<sup>8</sup>Integration over Grassmann variables is defined as  $d^8\theta = d^4\theta^{+} d^4\theta^{-} := (\mathcal{D}^{-})^4 (\mathcal{D}^{+})^4$ ,  $(\mathcal{D}^{\pm})^4 := \frac{1}{16} (\mathcal{D}^{+})^2 (\bar{\mathcal{D}}^{+})^2$ . Integral over harmonic variables is defined as  $\int du 1 = 1$  and zero otherwise.

### B. Linearized action and covariant superfields

To construct the linearized  $\mathcal{N} = 2$  supergravity action with manifest  $\mathcal{N} = 2$  supersymmetry it is natural to pass to the covariant superfields, since the analytic prepotentials defined above have nonstandard transformations laws (3.17) under rigid  $\mathcal{N} = 2$  supersymmetry. The most natural way to introduce  $\mathcal{N} = 2$  covariant superfields is to use the basis of covariant derivatives. To this end, one needs to split the covariant harmonic derivative into the flat and the supergravity parts:

$$\mathfrak{D}^{++} = \mathcal{D}^{++} + \hat{\mathcal{H}}^{++}, \quad \hat{\mathcal{H}}^{++} := h^{++M} \partial_M. \quad (4.7)$$

These parts are independently invariant under rigid supersymmetry:  $\delta_e \mathcal{D}^{++} = 0$ ,  $\delta_e \hat{\mathcal{H}}^{++} = 0$ . As the next step, one rewrites the supergravity part  $\hat{\mathcal{H}}^{++}$  in the basis of covariant derivatives (2.5):

$$\hat{\mathcal{H}}^{++} = G^{++\alpha\dot{\alpha}} \partial_{\alpha\dot{\alpha}} + G^{++\alpha+} \mathcal{D}_{\alpha}^{-} + G^{++\dot{\alpha}+} \bar{\mathcal{D}}_{\dot{\alpha}}^{-} + G^{++5} \partial_5. \quad (4.8)$$

Due to the invariance of both  $\hat{\mathcal{H}}^{++}$  and covariant derivatives under rigid  $\mathcal{N} = 2$  supersymmetry, the new vielbeins  $G^{++M}$  are  $\mathcal{N} = 2$  superfields with the standard rigid supersymmetry laws. However, most of the newly introduced superfields  $G^{++M}$  are nonanalytic:

$$\begin{aligned} G^{++\alpha\dot{\alpha}} &= h^{++\alpha\dot{\alpha}} + 4ih^{++\alpha+} \bar{\theta}^{-\dot{\alpha}} + 4i\theta^{-\alpha} h^{++\dot{\alpha}+}, \\ G^{++5} &= h^{++5} - 2ih^{++\hat{\alpha}+} \theta_{\hat{\alpha}}^{-}, \\ G^{++\hat{\alpha}+} &= -h^{++\hat{\alpha}+}, \\ G^{++\hat{\alpha}-} &= h^{++\hat{\alpha}-} = 0. \end{aligned} \quad (4.9)$$

The linearized gauge transformations of  $\hat{\mathcal{H}}^{++}$  are given by

$$\delta_{\lambda} \hat{\mathcal{H}}^{++} = [\mathcal{D}^{++}, \hat{\Lambda}], \quad \hat{\Lambda} := \lambda^M \partial_M = \Lambda^M \mathcal{D}_M, \quad (4.10)$$

where the parameters  $\Lambda^M$  are defined as

$$\begin{aligned} \Lambda^{\alpha\dot{\alpha}} &= \lambda^{\alpha\dot{\alpha}} + 4i\lambda^{+\alpha} \bar{\theta}^{-\dot{\alpha}} + 4i\theta^{-\alpha} \lambda^{+\dot{\alpha}}, \\ \Lambda^5 &= \lambda^5 - 2i\lambda^{+\hat{\alpha}} \theta_{\hat{\alpha}}^{-}, \\ \Lambda^{+\hat{\alpha}} &= -\lambda^{+\hat{\alpha}}, \\ \Lambda^{-\hat{\alpha}} &= \lambda^{-\hat{\alpha}}, \quad \mathcal{D}^{++} \Lambda^{-\hat{\alpha}} + \Lambda^{+\hat{\alpha}} = 0. \end{aligned} \quad (4.11)$$

The linearized gauge transformations of the covariant superfields  $G^{++M}$  are then obtained in the form:

$$\begin{aligned} \delta_{\lambda} G^{++\alpha\dot{\alpha}} &= \mathcal{D}^{++} \Lambda^{\alpha\dot{\alpha}}, \quad \delta_{\lambda} G^{++5} = \mathcal{D}^{++} \Lambda^5, \\ \delta_{\lambda} G^{++\hat{\alpha}+} &= \mathcal{D}^{++} \Lambda^{+\hat{\alpha}}. \end{aligned} \quad (4.12)$$

The linearized zero-curvature condition reads

$$[\mathcal{D}^{++}, \hat{\mathcal{H}}^{--}] = [\mathcal{D}^{--}, \hat{\mathcal{H}}^{++}]. \quad (4.13)$$

Here we have introduced the negatively charged differential operator  $\hat{\mathcal{H}}^{--}$ :

$$\begin{aligned} \mathfrak{D}^{--} &:= \mathcal{D}^{--} + \hat{\mathcal{H}}^{--}, \\ \hat{\mathcal{H}}^{--} &:= G^{--\alpha\dot{\alpha}} \partial_{\alpha\dot{\alpha}} + G^{--\hat{\alpha}+} \mathcal{D}_{\hat{\alpha}}^{-} + G^{--\hat{\alpha}-} \bar{\mathcal{D}}_{\hat{\alpha}}^{+} + G^{--5} \partial_5. \end{aligned} \quad (4.14)$$

By construction, it trivially transforms under rigid  $\mathcal{N} = 2$  supersymmetry,  $\delta_e \hat{\mathcal{H}}^{--} = 0$ .

In terms of the covariant superfields the zero-curvature condition (4.13) amounts to the set of equations:

$$\begin{cases} \mathcal{D}^{++} G^{--\alpha\dot{\alpha}} = \mathcal{D}^{--} G^{++\alpha\dot{\alpha}} \\ \mathcal{D}^{++} G^{--5} = \mathcal{D}^{--} G^{++5} \\ \mathcal{D}^{++} G^{--\hat{\alpha}+} = \mathcal{D}^{--} G^{++\hat{\alpha}+} \\ \mathcal{D}^{++} G^{--\hat{\alpha}-} + G^{--\hat{\alpha}+} = 0 \end{cases} \quad (4.15)$$

These are linear harmonic equations and they have a unique solution [16]. Some constituents of this solution are explicitly quoted in Appendix C.

The zero-curvature condition (4.13) is invariant under the linearized gauge transformations:

$$\delta_{\lambda} \hat{\mathcal{H}}^{++} = [\mathcal{D}^{++}, \hat{\Lambda}], \quad \delta_{\lambda} \hat{\mathcal{H}}^{--} = [\mathcal{D}^{--}, \hat{\Lambda}]. \quad (4.16)$$

Then the linearized gauge freedom of the potentials  $G^{--M}$  is given by

$$\begin{aligned} \delta_{\lambda} G^{--\alpha\dot{\alpha}} &= \mathcal{D}^{--} \Lambda^{\alpha\dot{\alpha}}, \quad \delta_{\lambda} G^{--5} = \mathcal{D}^{--} \Lambda^5, \\ \delta_{\lambda} G^{--\hat{\alpha}+} &= \mathcal{D}^{--} \Lambda^{+\hat{\alpha}} + \Lambda^{-\hat{\alpha}}, \quad \delta_{\lambda} G^{--\hat{\alpha}-} = \mathcal{D}^{--} \Lambda^{-\hat{\alpha}}. \end{aligned} \quad (4.17)$$

Now it is not difficult to verify that the supersymmetric and gauge invariant quadratic action has the form

$$S_{\text{lin}} = - \int d^4 x d^8 \theta du [G^{++\alpha\dot{\alpha}} G_{\alpha\dot{\alpha}}^{--} + 4G^{++5} G^{--5}]. \quad (4.18)$$

Indeed, the gauge transformation of this action is

$$\delta_{\lambda} S_{\text{lin}} = -2 \int d^4 x d^8 \theta du [G^{++\alpha\dot{\alpha}} \mathcal{D}^{--} \Lambda_{\alpha\dot{\alpha}} + 4G^{++5} \mathcal{D}^{--} \Lambda^5]. \quad (4.19)$$

Integrating over nonanalytic Grassmann variables  $d^4 \theta^- := (\mathcal{D}^+)^4 = \frac{1}{16} (\mathcal{D}^+)^2 (\bar{\mathcal{D}}^+)^2$  yields

$$(\mathcal{D}^+)^4(G^{++\alpha\dot{\alpha}}\mathcal{D}^{--}\Lambda_{\alpha\dot{\alpha}}) = 16ih^{++\alpha+}\partial_{\alpha\dot{\alpha}}\lambda^{+\dot{\alpha}} - 16ih^{++\dot{\alpha}+}\partial_{\alpha\dot{\alpha}}\lambda^{+\alpha}, \quad (4.20)$$

$$(\mathcal{D}^+)^4(G^{++5}\mathcal{D}^{--}\Lambda^4) = -4ih^{++\alpha+}\partial_{\alpha\dot{\alpha}}\lambda^{+\dot{\alpha}} + 4ih^{++\dot{\alpha}+}\partial_{\alpha\dot{\alpha}}\lambda^{+\alpha}. \quad (4.21)$$

We observe that these terms in (4.19) explicitly cancel each other.

In Sec. VI we shall perform a detailed analysis of the component structure of this action and explicitly demonstrate that it indeed describes the linearized off-shell  $\mathcal{N} = 2$  supergravity.

## V. LINEARIZED SUPERCURVATURES AND QUADRATIC INVARIANTS

In this section, using the covariant superfields  $G^{\pm\pm M}$  introduced in the previous section, we construct the linearized supercurvatures that contain linearized gravity invariants  $R, \mathcal{R}_{(\alpha\beta)(\dot{\alpha}\dot{\beta})}, \mathcal{R}_{(\alpha\beta\gamma\delta)}, \mathcal{R}_{(\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta})}$  (see Appendix A for notations). We shall present all the superfield actions that in components involve terms quadratic in the gravity curvatures.

Inspecting the gauge transformations of the negatively charged potentials (4.17) and generalizing the method that was exploited for construction of the linearized  $\mathcal{N} = 2$  supergravity action, we can construct two manifestly supersymmetric analytic objects:

$$\mathcal{F}^{++\alpha\dot{\alpha}} := (\mathcal{D}^+)^4 G^{--\alpha\dot{\alpha}}, \quad (5.1a)$$

$$\mathcal{F}^{++5} := (\mathcal{D}^+)^4 G^{--5}. \quad (5.1b)$$

These quantities are invariant under the linearized gauge transformations (4.17):

$$\delta_\lambda \mathcal{F}^{++\alpha\dot{\alpha}} = (\mathcal{D}^+)^4 \delta G^{--\alpha\dot{\alpha}} = (\mathcal{D}^+)^4 \mathcal{D}^{--}\Lambda^{\alpha\dot{\alpha}} = 0, \quad (5.2a)$$

$$\delta_\lambda \mathcal{F}^{++5} = (\mathcal{D}^+)^4 \delta G^{--5} = (\mathcal{D}^+)^4 \mathcal{D}^{--}\Lambda^5 = 0. \quad (5.2b)$$

It is then natural to treat these quantities as linearized  $\mathcal{N} = 2$  supergravity curvatures. Note that, as a consequence of the superfield equations of motion of  $\mathcal{N} = 2$  linearized Einstein supergravity (4.18) [49], these supercurvatures are zero on shell of the latter theory:

$$\mathcal{F}^{++\alpha\dot{\alpha}} = 0, \quad \mathcal{F}^{++5} = 0. \quad (5.3)$$

Using these linearized supercurvatures, one can build two  $\mathcal{N} = 2$  linearized supergravity invariants

$$I_1 := \int d\zeta^{(-4)} \mathcal{F}^{++\alpha\dot{\alpha}} \mathcal{F}_{\alpha\dot{\alpha}}^{++} = \int d^4x d^8\theta du G^{--\alpha\dot{\alpha}} (\mathcal{D}^+)^4 G_{\alpha\dot{\alpha}}^{--}, \quad (5.4)$$

$$I_2 := \int d\zeta^{(-4)} \mathcal{F}^{++5} \mathcal{F}^{++5} = \int d^4x d^8\theta du G^{--5} (\mathcal{D}^+)^4 G^{--5}. \quad (5.5)$$

In the second expressions the integration goes over the total harmonic superspace. The invariants (5.4) and (5.5) have the same form as the invariant constructed in [50] in six-dimensional renormalizable higher-derivative gauge theory and so can be considered as its spin 2 generalization. As we shall see in the next section, at the component level these invariants provide  $\mathcal{N} = 2$  supergravity extensions of the linearized invariants  $\frac{1}{4}R^{mn}R_{mn} = 16\mathcal{R}^{(\alpha\beta)(\dot{\alpha}\dot{\beta})}\mathcal{R}_{(\alpha\beta)(\dot{\alpha}\dot{\beta})} + R^2$  and  $R^2$ , respectively.

It is also possible to construct one more quadratic invariant, with a more complicated structure. First, we define the object:

$$\mathcal{H}_{\alpha\dot{\beta}}^{++} := \mathcal{D}_\alpha^- G_\beta^{+++} - \partial_\alpha^\beta G_{\beta\dot{\rho}}^{++}. \quad (5.6)$$

Since it is constructed out of the covariant superfields, it is manifestly supersymmetric. Under the linearized gauge transformation it behaves as

$$\delta_\lambda \mathcal{H}_{\alpha\dot{\beta}}^{++} = \mathcal{D}^{++}\Lambda_{\alpha\dot{\beta}}, \quad \Lambda_{\alpha\dot{\beta}} := \mathcal{D}_\alpha^- \Lambda_\beta^+ - \partial_\alpha^\beta \Lambda_{\beta\dot{\rho}}. \quad (5.7)$$

The newly introduced object is half-analytic:

$$\bar{\mathcal{D}}_\alpha^+ \mathcal{H}_{\alpha\dot{\beta}}^{++} = 0, \quad \bar{\mathcal{D}}_\alpha^+ \Lambda_{\alpha\dot{\beta}} = 0. \quad (5.8)$$

This property fixes the relative coefficient in (5.6). The superfield  $\mathcal{H}_{\alpha\dot{\beta}}^{++}$  can be decomposed into the irreducible parts as  $\mathcal{H}_{\alpha\dot{\beta}}^{++} = \mathcal{H}_{(\alpha\dot{\beta})}^{++} + \epsilon_{\alpha\dot{\beta}} \mathcal{H}^{++}$ . Using the zero-curvature equation, one can define its negatively charged counterpart  $\mathcal{H}_{\alpha\dot{\beta}}^{--}$ :

$$\mathcal{D}^{++}\mathcal{H}_{\alpha\dot{\beta}}^{--} = \mathcal{D}^{--}\mathcal{H}_{\alpha\dot{\beta}}^{++}. \quad (5.9)$$

The superfield  $\mathcal{H}_{\alpha\dot{\beta}}^{--}$  has the following linearized gauge transformation law:

$$\delta_\lambda \mathcal{H}_{\alpha\dot{\beta}}^{--} = \mathcal{D}^{--}\Lambda_{\alpha\dot{\beta}}. \quad (5.10)$$

Using the zero-curvature equations (4.15) one can explicitly express  $\mathcal{H}_{\alpha\dot{\beta}}^{--}$  in terms of the negatively charged potentials  $G^{--M}$ :

$$\mathcal{H}_{\alpha\dot{\beta}}^{--} = \mathcal{D}_\alpha^+ G_\beta^{---} + \mathcal{D}_\alpha^- G_\beta^{--+} - \partial_\alpha^\beta G_{\beta\dot{\rho}}^{--}. \quad (5.11)$$

With the help of the irreducible parts of  $\mathcal{H}_{\alpha\dot{\beta}}^{++}$  and  $\mathcal{H}_{\alpha\dot{\beta}}^{--}$  one can construct two invariant actions:

$$I_3 = \int d^4x d^8\theta du \mathcal{H}^{++(\alpha\beta)} \mathcal{H}_{(\alpha\beta)}^-, \quad (5.12)$$

$$I_4 = \int d^4x d^8\theta du \mathcal{H}^{++} \mathcal{H}^{--}. \quad (5.13)$$

The invariance of  $I_3$  and  $I_4$  directly follows from the half-analyticity condition (5.8). For example, the transformation of  $I_3$  is given by

$$\begin{aligned} \delta_\lambda I_3 &= 2 \int d^4x d^8\theta du \mathcal{H}^{++(\alpha\beta)} \mathcal{D}^{--} \Lambda_{(\alpha\beta)}, \\ &= \frac{1}{8} \int d^4x d^4\theta^+ (\mathcal{D}^+)^2 \mathcal{H}^{++(\alpha\beta)} (\bar{\mathcal{D}}^+)^2 \mathcal{D}^{--} \Lambda_{(\alpha\beta)}. \end{aligned} \quad (5.14)$$

Then we can use the half-analyticity property (5.8),

$$(\bar{\mathcal{D}}^+)^2 \mathcal{D}^{--} \Lambda_{(\alpha\beta)} = -\bar{\mathcal{D}}_\alpha^+ \bar{\mathcal{D}}^{-\dot{\alpha}} \Lambda_{(\alpha\beta)} = \bar{\mathcal{D}}^{-\dot{\alpha}} \bar{\mathcal{D}}_\alpha^+ \Lambda_{(\alpha\beta)} = 0. \quad (5.15)$$

The invariants  $I_3$  and  $I_4$  have the same structure as the action of  $\mathcal{N} = 2$  vector multiplet [16]. The gauge invariance of the latter action was ensured by the analyticity of both the prepotential  $V^{++}$  and the gauge parameter  $\lambda$ . In checking the invariance of  $I_3$  and  $I_4$  we used the weaker condition of half-analyticity.

There is yet another interesting way to represent these invariants. Let us define the gauge invariant superfield:

$$\mathcal{W}_{\alpha\beta} := (\bar{\mathcal{D}}^+)^2 \mathcal{H}_{\alpha\beta}^-, \quad \delta_\lambda \mathcal{W}_{\alpha\beta} = 0. \quad (5.16)$$

As we shall show in the next section, the tensor  $\mathcal{W}_{(\alpha\beta)}$  in components involves the linearized Weyl tensor  $\mathcal{R}_{(\alpha\beta\gamma\delta)}$ , so it is natural to dub  $\mathcal{W}_{(\alpha\beta)}$  as the linearized  $\mathcal{N} = 2$  super-Weyl tensor. Antisymmetric part  $\mathcal{W}$  does not involve gravitational curvatures and so (5.13) cannot lead to any gravitation quadratic invariant, as will be shown below.

The tensor (5.16) satisfies some useful properties:

$$\begin{aligned} \mathcal{D}^{++} \mathcal{W}_{\alpha\beta} &= 0, & \mathcal{D}^{--} \mathcal{W}_{\alpha\beta} &= 0, \\ \bar{\mathcal{D}}_\alpha^+ \mathcal{W}_{\alpha\beta} &= 0, & \bar{\mathcal{D}}_\alpha^- \mathcal{W}_{\alpha\beta} &= 0. \end{aligned} \quad (5.17)$$

The last property can be checked in the following way:

$$\begin{aligned} \mathcal{D}^{++} \bar{\mathcal{D}}_\alpha^- \mathcal{W}_{\alpha\beta} &= \bar{\mathcal{D}}_\alpha^+ \mathcal{W}_{\alpha\beta} + \bar{\mathcal{D}}_\alpha^- \mathcal{D}^{++} \mathcal{W}_{\alpha\beta} \\ &= 0 \Rightarrow \bar{\mathcal{D}}_\alpha^- \mathcal{W}_{\alpha\beta} = 0. \end{aligned} \quad (5.18)$$

Here we used the proposition that a negatively charged harmonic object annihilated by the harmonic derivative  $\mathcal{D}^{++}$  is zero. So we conclude that  $\mathcal{W}_{\alpha\beta}$  is a covariantly harmonic-independent chiral tensor.

Integrals over Grassmann variables can be represented as  $d^8\theta := d^4\theta d^4\bar{\theta} = \frac{1}{16} d^4\theta (\bar{\mathcal{D}}^+)^2 (\bar{\mathcal{D}}^-)^2$ , so due to the half-analyticity condition for  $\mathcal{H}^{++(\alpha\beta)}$ , the invariant  $I_3$  can be rewritten as

$$I_3 = \frac{1}{16} \int d^4x d^4\theta du (\bar{\mathcal{D}}^-)^2 \mathcal{H}^{++(\alpha\beta)} \mathcal{W}_{(\alpha\beta)}. \quad (5.19)$$

Using the relation  $\bar{\mathcal{D}}^{-\dot{\alpha}} = [\mathcal{D}^{--}, \bar{\mathcal{D}}^{\dot{\alpha}}]$  and the properties of the linearized  $\mathcal{N} = 2$  super-Weyl tensor (5.17), we obtain

$$I_3 = \frac{1}{16} \int d^4x d^4\theta du \bar{\mathcal{D}}_\alpha^- \mathcal{D}^{+\dot{\alpha}} \mathcal{D}^{--} \mathcal{H}^{++(\alpha\beta)} \mathcal{W}_{(\alpha\beta)}. \quad (5.20)$$

Finally, using the zero-curvature condition (5.9) and integrating by parts, we find

$$I_3 = -\frac{1}{16} \int d^4x d^4\theta \mathcal{W}^{(\alpha\beta)} \mathcal{W}_{(\alpha\beta)}. \quad (5.21)$$

Here we have also used the covariant harmonic-independence of  $\mathcal{W}_{(\alpha\beta)}$ . So we end up with the integral over  $\mathcal{N} = 2$  chiral superspace.

Using the same reasonings, the invariant  $I_4$  can also be represented in the chiral form:

$$I_4 = -\frac{1}{16} \int d^4x d^4\theta \mathcal{W}^2. \quad (5.22)$$

In such a form, the invariants  $I_3$  and  $I_4$  are manifestly gauge invariant.

Quite analogously, one can introduce the conjugated superfields  $\bar{\mathcal{H}}_{(\dot{\alpha}\dot{\beta})}^{++}$ ,  $\bar{\mathcal{H}}_{(\dot{\alpha}\dot{\beta})}^{--}$ ,  $\bar{\mathcal{W}}_{(\dot{\alpha}\dot{\beta})}$  and construct the conjugated invariants.

## VI. COMPONENT CONTENTS OF THE LINEARIZED INVARIANTS

Given the component expansion of the prepotentials in WZ gauge (3.10), it is straightforward to deduce the component expansion of the invariants constructed. Technically, the most involved part of the component reduction procedure is solving the harmonic zero-curvature equations (4.15). The solutions of these equations are collected in Appendix C. Using these results, in this section we present component expansions of all invariants constructed earlier. In some cases, if necessary, we present the full component structure, while otherwise quote only the relevant terms in the gravity sector.

### A. Einstein supergravity

We start with the component reduction of the linearized  $\mathcal{N} = 2$  supergravity action (4.18). It is convenient to consider the bosonic and fermionic parts of the action separately.

After substituting the solutions of the zero-curvature equations and integrating over Grassmann and harmonic coordinates, we obtain the linearized off-shell  $\mathcal{N} = 2$  Einstein supergravity action in the bosonic sector as

$$S_{\text{lin}}^{(\text{bos})} = 8 \int d^4x \left[ \Phi^{\alpha\dot{\alpha}\beta\dot{\beta}} \mathcal{G}_{\alpha\dot{\alpha}\beta\dot{\beta}} + 4C^{\rho\dot{\rho}} \left( \partial_{\rho\dot{\rho}} \partial^{\beta\dot{\beta}} C_{\beta\dot{\beta}} - \frac{1}{2} \square C_{\rho\dot{\rho}} \right) + 4iT^{(\alpha\beta)} \partial_{(\alpha}^{\dot{\beta}} C_{\beta\dot{\beta})} - 4i\bar{T}^{(\dot{\alpha}\dot{\beta})} \partial_{(\dot{\alpha}}^{\beta} C_{\beta\dot{\beta})} \right. \\ \left. + \frac{1}{2} T^{(\alpha\beta)} T_{(\alpha\beta)} + \frac{1}{2} T^{(\dot{\alpha}\dot{\beta})} T_{(\dot{\alpha}\dot{\beta})} - 2T\bar{T} - \frac{1}{2} P^{\alpha\dot{\alpha}} \bar{P}_{\alpha\dot{\alpha}} - \frac{3}{4} P^{\alpha\dot{\alpha}} P_{\alpha\dot{\alpha}} - \frac{3}{4} \bar{P}^{\alpha\dot{\alpha}} \bar{P}_{\alpha\dot{\alpha}} - \frac{1}{72} V^{\alpha\dot{\alpha}(ij)} V_{\alpha\dot{\alpha}(ij)} - \frac{1}{18} S^{ij} S_{ij} \right]. \quad (6.1)$$

It will be useful to decompose the electromagnetic tensor into the irreducible parts:

$$\mathcal{F}_{\rho\dot{\rho}\beta\dot{\beta}} = \partial_{\rho\dot{\rho}} C_{\beta\dot{\beta}} - \partial_{\beta\dot{\beta}} C_{\rho\dot{\rho}} = \epsilon_{\rho\dot{\rho}} \partial_{(\rho}^{\sigma} C_{\sigma\dot{\beta})} + \epsilon_{\dot{\rho}\beta} \partial_{(\rho}^{\sigma} C_{\sigma\dot{\beta})} := \epsilon_{\rho\dot{\rho}} \bar{\mathcal{F}}_{(\dot{\rho}\dot{\beta})} + \epsilon_{\dot{\rho}\beta} \mathcal{F}_{(\rho\beta)}. \quad (6.2)$$

The irreducible spin tensors  $\mathcal{F}_{(\rho\beta)}$  and  $\bar{\mathcal{F}}_{(\dot{\rho}\dot{\beta})}$  are gauge invariant and they will be used for finding out the component structure of invariants.

For the Maxwell part of the action (6.1) we find the following (up to integration by parts):

$$32C^{\rho\dot{\rho}} \left( \partial_{\rho\dot{\rho}} \partial^{\beta\dot{\beta}} C_{\beta\dot{\beta}} - \frac{1}{2} \square C_{\rho\dot{\rho}} \right) = 16(\partial^{\beta\dot{\beta}} C^{\rho\dot{\rho}} - \partial^{\rho\dot{\rho}} C^{\beta\dot{\beta}})(\partial_{\beta\dot{\beta}} C_{\rho\dot{\rho}} - \partial_{\rho\dot{\rho}} C_{\beta\dot{\beta}}) \\ = -32\bar{\mathcal{F}}^{(\dot{\beta}\dot{\rho})} \bar{\mathcal{F}}_{(\dot{\beta}\dot{\rho})} - 32\mathcal{F}^{(\beta\rho)} \mathcal{F}_{(\beta\rho)}. \quad (6.3)$$

This Maxwell action has the incorrect sign. This is because, within the compensating procedure of deriving the Einstein supergravity action from the conformal supergravity one, the above Maxwell field comes from the vector superconformal compensator action with a wrong sign. However, the situation can be remedied by taking into account the tensorial auxiliary fields. The equations of motion for them are algebraic:

$$T_{(\alpha\beta)} = -4i\mathcal{F}_{(\alpha\beta)}, \quad \bar{T}_{(\dot{\alpha}\dot{\beta})} = 4i\mathcal{F}_{(\dot{\alpha}\dot{\beta})}, \\ T = \bar{T} = 0, \quad P_{\alpha\dot{\alpha}} = \bar{P}_{\alpha\dot{\alpha}} = 0, \quad V^{\alpha\dot{\alpha}(ij)} = 0, \quad S^{(ij)} = 0 \quad (6.4)$$

and so these auxiliary fields can be eliminated from the action. As a result of this elimination, we arrive at the on-shell massless Pauli-Fierz and Maxwell actions with the correct signs:

$$S_{\text{lin|on-shell}}^{(\text{bos})} = 8 \int d^4x (\Phi^{\alpha\dot{\alpha}\beta\dot{\beta}} \mathcal{G}_{\alpha\dot{\alpha}\beta\dot{\beta}} + 4\bar{\mathcal{F}}^{(\dot{\beta}\dot{\rho})} \bar{\mathcal{F}}_{(\dot{\beta}\dot{\rho})} + 4\mathcal{F}^{(\beta\rho)} \mathcal{F}_{(\beta\rho)}), \quad (6.5)$$

thereby confirming the status of  $C_{\alpha\dot{\alpha}}$  as the physical “graviphoton” field.

In the fermionic sector we are left with the following off-shell action<sup>9</sup>:

$$S_{\text{lin}}^{(\text{ferm})} = 32i \int d^4x \left( \psi^{\beta\alpha\dot{\alpha}i} \bar{\mathcal{G}}_{\beta\alpha\dot{\alpha}i} + 2\rho^{\beta i} \partial_{\beta\dot{\beta}} \bar{\rho}_{\dot{\beta}i} + \frac{1}{16} \rho^{\alpha i} \chi_{\alpha i} + \text{c.c.} \right). \quad (6.6)$$

Here  $\bar{\mathcal{G}}_{\beta\alpha\dot{\alpha}i}$  is a complex spin  $\frac{3}{2}$  gauge field strength [see Eq. (B3)]. The auxiliary fields  $\chi^{\alpha i}$  and  $\rho^{\alpha i}$  have the algebraic equations of motion and are equal to zero on shell. So on shell we end up with the free action of the  $SU(2)$  doublet of the spin  $\frac{3}{2}$  Rarita-Schwinger gauge fields  $\psi^{\beta\alpha\dot{\alpha}i} = \psi^{(\beta\alpha)\dot{\alpha}i} + \epsilon^{\alpha\beta} \psi^{\dot{\alpha}i}$ .

Thus we have come to the component off-shell action of the linearized  $\mathcal{N} = 2$  supergravity and confirmed that on shell it is reduced to the sum of the free spin 2 action, the action of the doublet of spins  $\frac{3}{2}$  and the spin 1 action.

## B. $R^{mn}R_{mn}$ invariant

Using the relations of Appendix C, one can straightforwardly deduce the component contents of the linearized supercurvatures. The supercurvature (5.1a) in WZ gauge has the following component expansion:

<sup>9</sup>Note that, in the process of calculating the component action, many unexpected cancelations occur.

$$\begin{aligned}
\mathcal{F}^{++\alpha\dot{\alpha}} = & -8i\theta^+_{\dot{\rho}}\bar{\theta}^+_{\dot{\rho}}\left(\mathcal{R}^{(\alpha\rho)(\dot{\alpha}\dot{\rho})} - \frac{1}{8}\epsilon^{\alpha\rho}\epsilon^{\dot{\alpha}\dot{\rho}}R + i\epsilon^{(\dot{\alpha}\dot{\rho}}\partial^{\dot{\rho}}\rho P^{\alpha}_{\dot{\rho}} - i\epsilon^{(\alpha\rho}\partial^{\rho}\dot{\rho}\bar{P}^{\dot{\alpha}}_{\dot{\rho}}\right) + 4(\bar{\theta}^+)^2(\partial^{\dot{\alpha}}_{\dot{\rho}}T^{(\alpha\rho)} + \partial^{\alpha\dot{\alpha}}T) \\
& + 4(\theta^+)^2(\partial^{\dot{\alpha}}_{\dot{\rho}}\bar{T}^{(\dot{\alpha}\dot{\rho})} + \partial^{\alpha\dot{\alpha}}\bar{T}) + \frac{1}{3}V^{\alpha\dot{\alpha}(ij)}u_i^+u_j^+ + \frac{4}{3}i\theta^{+\rho}\bar{\theta}^{+\dot{\rho}}\partial_{\rho\dot{\rho}}V^{\alpha\dot{\alpha}(ij)}u_i^+u_j^- - \frac{2}{3}(\theta^+)^4\Box V^{\alpha\dot{\alpha}(ij)}u_i^-u_j^- \\
& - 16i\bar{\theta}^+_{\dot{\rho}}\mathcal{G}^{\dot{\rho}\dot{\alpha}ai}u_i^+ - 16i\theta^+_{\dot{\rho}}\bar{\mathcal{G}}^{\rho\alpha\dot{a}i}u_i^+ + 32(\bar{\theta}^+)^2\theta^{+\rho}\partial_{\rho\dot{\rho}}\mathcal{G}^{\dot{\rho}\dot{\alpha}ai}u_i^- - 32(\theta^+)^2\bar{\theta}^{+\dot{\rho}}\partial_{\rho\dot{\rho}}\bar{\mathcal{G}}^{\rho\alpha\dot{a}i}u_i^- \\
& + 2i\theta^{+\alpha}\bar{\chi}^{\dot{\alpha}i}u_i^+ - 2i\bar{\theta}^{+\dot{\alpha}}\chi^{\alpha i}u_i^+ - 4(\bar{\theta}^+)^2\theta^{+\rho}\partial_{\rho\dot{\rho}}\chi^{\alpha i}u_i^- - 4(\theta^+)^2\bar{\theta}^{+\dot{\rho}}\partial_{\rho\dot{\rho}}\bar{\chi}^{\dot{\alpha}i}u_i^-. \tag{6.7}
\end{aligned}$$

From this expression we conclude that the analytic superfield  $\mathcal{F}^{++\alpha\dot{\alpha}}$  is  $\mathcal{N} = 2$  generalization of the linearized Einstein tensor (A11).

After integration over Grassmann coordinates and harmonics, the linearized invariant (5.4) yields the following component Lagrangian:

$$\begin{aligned}
I_1 = & -16 \int d^4x \left[ \mathcal{R}^{(\alpha\beta)(\dot{\alpha}\dot{\beta})}\mathcal{R}_{(\alpha\beta)(\dot{\alpha}\dot{\beta})} + \frac{1}{16}R^2 + i\mathcal{R}^{(\alpha\beta)(\dot{\alpha}\dot{\beta})}\partial_{\alpha\dot{\alpha}}P_{\beta\dot{\beta}} - 16i\mathcal{R}^{(\alpha\beta)(\dot{\alpha}\dot{\beta})}\partial_{\alpha\dot{\alpha}}\bar{P}_{\beta\dot{\beta}} + \frac{3i}{8}\partial_{\beta\dot{\beta}}P^{\beta\dot{\beta}}R - \frac{3i}{8}\partial_{\beta\dot{\beta}}\bar{P}^{\beta\dot{\beta}}R \right. \\
& - 5P^{\alpha\dot{\alpha}}\Box P_{\alpha\dot{\alpha}} - 5\bar{P}^{\alpha\dot{\alpha}}\Box \bar{P}_{\alpha\dot{\alpha}} + 4P^{\alpha\dot{\alpha}}\partial_{\alpha\dot{\alpha}}\partial_{\beta\dot{\beta}}\bar{P}^{\beta\dot{\beta}} - 2T^{(\alpha\beta)}\partial_{\alpha\dot{\alpha}}\partial_{\beta\dot{\beta}}\bar{T}^{(\dot{\alpha}\dot{\beta})} + T\Box T + \frac{1}{8}\frac{1}{27}V^{\alpha\dot{\alpha}(ij)}\Box V_{\alpha\dot{\alpha}(ij)} + 16i\mathcal{G}^{\dot{\rho}\dot{\alpha}ai}\partial_{\rho\dot{\rho}}\bar{\mathcal{G}}_{\rho\dot{\rho}ai} \\
& \left. + 2i\partial_{\rho\dot{\rho}}\mathcal{G}^{\dot{\rho}\dot{\alpha}ai}\bar{\chi}_{\dot{a}i} - 2i\partial_{\rho\dot{\rho}}\bar{\mathcal{G}}^{\rho\alpha\dot{a}i}\chi_{ai} - \frac{i}{2}\bar{\chi}^{\dot{\alpha}i}\partial_{\alpha\dot{\alpha}}\chi^{\alpha i} \right]. \tag{6.8}
\end{aligned}$$

From this component structure we see that  $I_1$  is just  $\mathcal{N} = 2$  superextension of the square of the linearized Ricci curvature. The auxiliary fields in  $I_1$  have nonalgebraic equations of motions and so are dynamical, in contrast to those in the linearized  $\mathcal{N} = 2$  Einstein supergravity.<sup>10</sup>

### C. $R^2$ invariant

The component structure of the second linearized supercurvature (5.1b) is worked out in a similar way:

$$\begin{aligned}
\mathcal{F}^{++5} = & \frac{i}{2}(\theta^+)^2(R - 4i\partial_{\beta\dot{\beta}}P^{\beta\dot{\beta}}) - \frac{i}{2}(\bar{\theta}^+)(R + i\partial_{\beta\dot{\beta}}\bar{P}^{\beta\dot{\beta}}) + 8i\theta^+_{\dot{\alpha}}\bar{\theta}^{+\dot{\alpha}}\left(\partial^{\alpha\dot{\beta}}\bar{\mathcal{F}}_{(\dot{\alpha}\dot{\beta})} + \frac{i}{2}\partial_{\alpha\dot{\alpha}}T^{(\alpha\beta)}\right) \\
& - 8i\bar{\theta}^+_{\dot{\alpha}}\theta^{+\alpha}\left(\partial^{\dot{\alpha}\beta}\mathcal{F}_{(\alpha\beta)} - \frac{i}{2}\partial_{\alpha\dot{\alpha}}\bar{T}^{(\dot{\alpha}\dot{\beta})}\right) + \frac{1}{3}S^{(ij)}u_i^+u_j^+ + \frac{4}{3}i\theta^{+\rho}\bar{\theta}^{+\dot{\rho}}\partial_{\rho\dot{\rho}}S^{(ij)}u_i^+u_j^- - \frac{2}{3}(\theta^+)^4\Box S^{(ij)}u_i^-u_j^- \\
& + 4i\theta^+_{\dot{\alpha}}\mathcal{R}^{\alpha i}u_i^+ - 4i\bar{\theta}^+_{\dot{\alpha}}\bar{\mathcal{R}}^{\dot{\alpha}i}u_i^+ + 8(\theta^+)^2\bar{\theta}^{+\dot{\rho}}\partial_{\rho\dot{\rho}}\mathcal{R}^{\alpha i}u_i^- + 8(\bar{\theta}^+)^2\theta^{+\rho}\partial_{\rho\dot{\rho}}\bar{\mathcal{R}}^{\dot{\alpha}i}u_i^- \\
& + 8i\theta^{+\sigma}\partial_{\sigma\dot{\rho}}\bar{\rho}^{\dot{\rho}}u_i^+ + 8i\bar{\theta}^{+\dot{\sigma}}\partial_{\dot{\sigma}\rho}\rho^{\rho}u_i^+ - 4(\bar{\theta}^+)^2\theta^{+\beta}\Box\rho^{\beta}u_i^- + 4(\theta^+)^2\bar{\theta}^{+\dot{\beta}}\Box\bar{\rho}^{\dot{\beta}}u_i^- \\
& - i\theta^{+\alpha}\chi^{\alpha i}u_i^+ + i\bar{\theta}^{+\dot{\alpha}}\bar{\chi}^{\dot{\alpha}i}u_i^+ - 2(\theta^+)^2\bar{\theta}^{+\dot{\rho}}\partial_{\rho\dot{\rho}}\chi^{\alpha i}u_i^- - 2(\bar{\theta}^+)^2\theta^{+\rho}\partial_{\rho\dot{\rho}}\bar{\chi}^{\dot{\alpha}i}u_i^-. \tag{6.9}
\end{aligned}$$

Then the component content of the invariant (5.5) is given by the following expression:

$$\begin{aligned}
I_2 = & \int d^4x \left[ \frac{1}{4}R^2 - iR\partial_{\beta\dot{\beta}}\bar{P}^{\beta\dot{\beta}} + iR\partial_{\beta\dot{\beta}}P^{\beta\dot{\beta}} + 4(\partial_{\beta\dot{\beta}}P^{\beta\dot{\beta}})(\partial_{\rho\dot{\rho}}\bar{P}^{\rho\dot{\rho}}) + (2\bar{\mathcal{F}}^{(\dot{\alpha}\dot{\beta})} + i\bar{T}^{(\dot{\alpha}\dot{\beta})})\Box(2\bar{\mathcal{F}}_{(\dot{\alpha}\dot{\beta})} + i\bar{T}_{(\dot{\alpha}\dot{\beta})}) \right. \\
& \left. + (2\mathcal{F}^{(\alpha\beta)} - iT^{(\alpha\beta)})\Box(2\mathcal{F}_{(\alpha\beta)} - iT_{(\alpha\beta)}) - 8(2\bar{\mathcal{F}}^{(\dot{\alpha}\dot{\beta})} + i\bar{T}^{(\dot{\alpha}\dot{\beta})})\partial_{\alpha\dot{\alpha}}\partial_{\beta\dot{\beta}}(2\mathcal{F}^{(\alpha\beta)} - iT^{(\alpha\beta)}) + \dots \right]. \tag{6.10}
\end{aligned}$$

This is  $\mathcal{N} = 2$  generalization of the  $R^2$  invariant.

### D. Weyl-squared invariant

Though the component reduction of the linearized  $\mathcal{N} = 2$  Weyl tensor is technically more involved, it can be performed quite analogously to the previous supercurvatures. We confine our consideration merely to the lowest term in the supercurvature expansion and to the gravity sector. As a result, we obtain

<sup>10</sup>This property is typical for superinvariants with higher derivatives, see, e.g., [50].

$$\mathcal{W}_{(\alpha\beta)} = 32\theta^{-(\gamma\theta^{+\delta})}\mathcal{R}_{(\alpha\beta\gamma\delta)} + \dots, \quad (6.11)$$

$$\mathcal{W} = \theta^-\theta^+ \cdot 0 + \dots \quad (6.12)$$

From these formulas we observe that  $\mathcal{W}$  does not contribute to the pure gravity sector. The terms denoted by ellipses also contain at least three derivatives of the spin 2 field. So for the invariant  $I_3$  defined in (5.12) and (5.21) we have the following expression in the gravity sector:

$$I_3 \sim \int d^4x \mathcal{R}^{(\alpha\beta\gamma\delta)} \mathcal{R}_{(\alpha\beta\gamma\delta)}. \quad (6.13)$$

The conclusion is that the invariant  $I_3$  is an irreducible part of the linearized  $\mathcal{N} = 2$  Weyl-squared superinvariant. It is worth to note that, using  $I_3$ , its complex conjugate and the invariants  $I_1, I_2$ , one can also construct a supersymmetric generalization of the square of Riemann tensor.

To summarize, we have the complete set of the linearized off-shell  $\mathcal{N} = 2$  supergravity invariants in terms of analytic prepotentials. In components, they start with the

squares of gravity curvatures:  $I_1 \sim R^{mn}R_{mn}$ ,  $I_2 \sim R^2$ , and  $I_3 \sim \mathcal{R}^{(\alpha\beta\gamma\delta)}\mathcal{R}_{(\alpha\beta\gamma\delta)}$ .

## VII. LINEARIZED $\mathcal{N} = 2$ SUPERGRAVITY INVARIANTS BEYOND THE QUADRATIC ORDER

In harmonic superspace one can construct invariant actions as integrals over both analytic and full harmonic superspaces:

$$I_{\text{ASS}} = \int d\zeta^{(-4)} \mathcal{L}^{(+4)}, \quad I_{\text{HSS}} = \int d^4x d^8\theta du \mathcal{L}. \quad (7.1)$$

Using the gauge-invariant supercurvatures  $\mathcal{F}^{++a\dot{a}}$ ,  $\mathcal{F}^{++5}$ ,  $\mathcal{W}_{(\alpha\beta)}$ ,  $\bar{\mathcal{W}}_{(\dot{\alpha}\dot{\beta})}$  defined in (5.1a), (5.1b), (5.16) and covariant derivatives (2.5), (2.6), one can construct a family of such invariant actions. Being aware of the component structure of supercurvatures, one can generate linearized higher-order  $\mathcal{N} = 2$  supergravity invariants and deduce their component structure in the gravity sector.

As examples, we present some interesting invariants:

(1)  $R^4$  invariant:

$$\int d^4x d^8\theta du (\mathcal{D}^{--}\mathcal{F}^{++5})^4 \sim \int d^4x R^4 + \dots \quad (7.2)$$

(2)  $\square R \square R$  invariant:

$$\int d^4x d^8\theta du (\mathcal{D}^-)^2 \mathcal{F}^{++5} (\bar{\mathcal{D}}^-)^2 \mathcal{F}^{++5} \sim \int d^4x \square R \square R. \quad (7.3)$$

(3)  $R^2 \square R$  invariant:

$$\int d^4x d^8\theta du (\mathcal{D}^{--}\mathcal{F}^{++5})^2 (\mathcal{D}^-)^2 \mathcal{F}^{++5} \sim \int d^4x R^2 \square R. \quad (7.4)$$

(4)  $R^{4+n}$  invariant:

$$\int d^4x d^8\theta du (\mathcal{D}^{--}\mathcal{F}^{++5})^4 [(\mathcal{D}^-)^2 \mathcal{F}^{++5}]^n \sim \int d^4x R^{4+n} + \dots \quad (7.5)$$

(5)  $G^4$  invariant:

$$\begin{aligned} & \int d^4x d^8\theta du (\mathcal{D}^{--}\mathcal{F}^{++\alpha_1\dot{\alpha}_1})(\mathcal{D}^{--}\mathcal{F}^{++\alpha_2\dot{\alpha}_2})(\mathcal{D}^{--}\mathcal{F}^{++\alpha_3\dot{\alpha}_3})(\mathcal{D}^{--}\mathcal{F}^{++\alpha_4\dot{\alpha}_4}) \\ & \sim \int d^4x \mathcal{G}^{\alpha_1\beta_1\dot{\alpha}_1\dot{\beta}_1} \mathcal{G}^{\alpha_2\beta_2\dot{\alpha}_2\dot{\beta}_2} \mathcal{G}_{\beta_2\alpha_1\dot{\beta}_1\dot{\alpha}_2} \mathcal{G}_{\beta_1\alpha_2\dot{\beta}_2\dot{\alpha}_1} + \dots \end{aligned} \quad (7.6)$$

(6) Bel-Robinson tensor squared:

Bel-Robinson tensor in the spinor notations is defined as (see, e.g., [51])

$$\mathcal{R}_{(\alpha\beta\gamma\delta)} \bar{\mathcal{R}}_{(\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta})}. \quad (7.7)$$

Inspecting the component structure of supercurvatures, one immediately observes that the Bel-Robinson tensor is contained in the superfield

$$\mathcal{W}_{(\alpha\beta)}\bar{\mathcal{W}}_{(\dot{\alpha}\dot{\beta})} \sim \theta^{+\gamma}\theta^{-\delta}\bar{\theta}^{+\dot{\gamma}}\bar{\theta}^{-\dot{\delta}}\mathcal{R}_{(\alpha\beta\gamma\delta)}\bar{\mathcal{R}}_{(\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta})}. \quad (7.8)$$

Then one can construct  $\mathcal{N} = 2$  extension of the Bel-Robinson tensor-squared invariant:

$$\begin{aligned} & \int d^4x d^8\theta du \mathcal{W}^{(\alpha\beta)}\mathcal{W}_{(\alpha\beta)}\bar{\mathcal{W}}^{(\dot{\alpha}\dot{\beta})}\bar{\mathcal{W}}_{(\dot{\alpha}\dot{\beta})} \\ & \sim \int d^4x \mathcal{R}^{(\alpha\beta\gamma\delta)}\mathcal{R}_{(\alpha\beta\gamma\delta)}\bar{\mathcal{R}}^{(\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta})}\bar{\mathcal{R}}_{(\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta})}. \end{aligned} \quad (7.9)$$

(7)  $R^3$  invariants and two-loop finiteness of supergravity:

It is known that  $\mathcal{N} = 2$  supergravity is finite to two loops. This means the absence of the on-shell cubic supergravity invariants. It is interesting to analyze this issue using our off-shell approach. On dimensional grounds,  $[R^3] = 6$ , so such invariants can be constructed as an integral over the full harmonic superspace Lagrangian with the dimension  $[\mathcal{L}] = 2$  or as an integral over the analytic superspace with  $[\mathcal{L}^{(+4)}] = 4$ . The supercurvatures have dimensions  $[\mathcal{F}^{++\alpha\dot{\alpha}}] = 1$ ,  $[\mathcal{F}^{++5}] = 1$ , and  $[\mathcal{W}_{(\alpha\beta)}] = 1$ .

The only cubic invariant which does not vanish on shell is the product of three Riemann tensors. The product of three Riemann tensors in the spinor notation corresponds to the invariant

$$\mathcal{R}_{\alpha\beta}{}^{\gamma\delta}\mathcal{R}_{\gamma\delta}{}^{\rho\kappa}\mathcal{R}_{\rho\kappa}{}^{\alpha\beta}. \quad (7.10)$$

The  $\mathcal{N} = 2$  supersymmetric invariant extending such a product can naturally be constructed from three  $\mathcal{N} = 2$  super-Weyl tensors  $\mathcal{W}_{(\alpha\beta)}$ . Since  $\mathcal{W}_{(\alpha\beta)}$  is not analytic superfield, one needs to integrate over full harmonic superspace or chiral superspace. Using the component structure, it is easy to prove that the invariants containing (7.10) after passing to components, cannot be constructed in such a way. Indeed, though a superfield containing (7.10) can be constructed from the  $\mathcal{N} = 2$  Weyl tensor,

$$\begin{aligned} \mathcal{D}^{+\delta}\mathcal{W}_{\alpha}{}^{\beta}\mathcal{W}_{\beta}{}^{\gamma}\mathcal{D}_{\delta}^{-}\mathcal{W}_{\gamma}{}^{\alpha} & \sim (\theta^{+})^2(\theta^{-})^2\mathcal{R}_{\alpha\beta}{}^{\gamma\delta}\mathcal{R}_{\gamma\delta}{}^{\rho\kappa}\mathcal{R}_{\rho\kappa}{}^{\alpha\beta} \\ & + \dots, \end{aligned} \quad (7.11)$$

this superfield is not chiral and so does not contain the necessary invariant as a coefficient of the highest  $(\theta^{+})^4(\theta^{-})^4$  monomial.

This reasoning is similar to the  $\mathcal{N} = 1$  superspace arguments about absence of such an invariant in  $\mathcal{N} = 1$  supergravity [51]. So we conclude, that it is not possible to  $\mathcal{N} = 2$  supersymmetrize the invariant (7.10).

## VIII. CONCLUSIONS AND OUTLOOK

In this paper, using the prepotential formulation of  $\mathcal{N} = 2$  supergravity in harmonic superspace, we constructed the off-shell linearized  $\mathcal{N} = 2$  supercurvatures

$$\mathcal{F}^{++\alpha\dot{\alpha}}(\zeta), \quad \mathcal{F}^{++5}(\zeta), \quad \mathcal{W}_{(\alpha\beta)}(x, \theta, u), \quad \bar{\mathcal{W}}_{(\dot{\alpha}\dot{\beta})}(x, \bar{\theta}, u), \quad (8.1)$$

which contain, respectively, the linearized gravity curvatures:

$$\mathcal{G}_{\alpha\beta\dot{\alpha}\dot{\beta}}, \quad R, \quad \mathcal{R}_{(\alpha\beta\gamma\delta)}, \quad \bar{\mathcal{R}}_{(\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta})}. \quad (8.2)$$

From these supercurvatures we constructed all linearized quadratic invariants and presented some examples of the linearized invariants beyond the quadratic order. All the constructed objects are expressed in terms of unconstrained analytical prepotentials, which is the main feature of our approach in contrast to the previous superspace studies [30–32,35,36]. This allows us to systematically derive the component form of the invariants by solving the harmonic zero-curvature equations in WZ gauge. It is interesting to note that all the linearized higher-derivative supergravity invariants constructed have an obvious tensor structure and are manifestly invariant with respect to the gauge transformations. It is the basic difference of these invariants from the action of  $\mathcal{N} = 2$  Einstein supergravity (4.18), which is gauge invariant up to total derivatives and so represents a Chern-Simons-like action.

These results raise a number of interesting and natural issues which we hope to address in the near future.

(1) Nonlinear invariants in harmonic superspace:

The most direct task is to construct nonlinear generalizations of the supercurvatures and invariants introduced here. Since there exists no analytical integration measure in this version of  $\mathcal{N} = 2$  Einstein supergravity [16], one can expect that the corresponding nonlinear quadratic invariants can be written in the full harmonic superspace as

$$\begin{aligned} & \int d^4x d^8\theta du E H^{--\alpha\dot{\alpha}}(\mathfrak{D}^{+})^4 H_{\alpha\dot{\alpha}}^{--}, \\ & \int d^4x d^8\theta du E H^{--5}(\mathfrak{D}^{+})^4 H^{--5}. \end{aligned} \quad (8.3)$$

Checking the invariance of such expressions implies making use of the properties of the integration measure  $E$  and integrations by parts. We hope to achieve some progress toward this goal elsewhere.

It is interesting to establish the connection of such nonlinear invariants with those found in other approaches [30–32]. Despite the fact that the invariants constructed supersymmetrize the same gravitational invariants, it is problematic to compare the results obtained at the superfield level, since in our

approach the invariants are constructed in terms of analytic prepotentials, while the standard approach uses supercurvatures and supertorsions. This issue is discussed in more detail in the next item.

- (2) Differential geometry of curved  $\mathcal{N} = 2$  harmonic superspace:

The natural way to construct higher-derivative  $\mathcal{N} = 2$  supergravity invariants is to use the supertorsions and supercurvatures appearing in the algebra of covariant derivatives [52]. Such a method was adopted in [30,31]. The covariant supergeometry of  $\mathcal{N} = 2$  supergravity in harmonic superspace was considered in [18,53]. In the harmonic superspace approach all the basic supergravity constraints are automatically solved in terms of analytic prepotentials. In a separate publication, we shall present such nonlinear invariants constructed out of the covariant torsions and curvatures, starting from the full harmonic superspace algebra of the covariant derivatives.

In this regard, one can also ask which of the linearized supercurvatures (8.1) have nonlinear extension and how these objects are related to the prepotential superfields solving the  $\mathcal{N} = 2$  supergravity constraints. It is well known how the well-defined nonlinear generalizations of invariants  $\mathcal{F}^{++5}$  and  $\mathcal{W}_{(\alpha\beta)}$  appear in the covariant superfield formulations of  $\mathcal{N} = 2$  supergravity [32–34,52]. As one would expect, a nonlinear extension of the supercurvature  $\mathcal{F}^{++a\dot{a}}$  defined in (1.2) can be obtained as a descendant of the curvature tensor  $T^i_\alpha$  of the superspace formulation of  $\mathcal{N} = 2$  Poincaré supergravity given in [54,55]. This conjecture is based on the fact that this superspace formulation of  $\mathcal{N} = 2$  Einstein supergravity is directly related to the Fradkin-Vasiliev–de Wit–van Holten [46–48] off-shell  $\mathcal{N} = 2$  supergravity multiplet, which is described just by the unconstrained analytical prepotentials (3.10) used through the paper. Checking this amounts to an additional analysis at the full nonlinear level and is beyond the scope of the present work.

- (3)  $\mathcal{N} = 2$  Gauss-Bonnet theorem in harmonic superspace:

In the pure gravity, there are two topological invariants quadratic in the Riemann tensor: Pontryagin and Gauss-Bonnet (GB) invariants. Gauss-Bonnet invariant is given as the integral of Euler density:

$$I_{\text{GB}} = \int d^4x \sqrt{g} (R^{mnkr} R_{mnkr} - 4R^{mn} R_{mn} + R^2). \quad (8.4)$$

Such invariants are topological in the sense that  $\delta I_{\text{GB}} / \delta g_{mn} = 0$ .

$\mathcal{N} = 1$  Gauss-Bonnet invariant was constructed in [51] and its  $\mathcal{N} = 2$  cousin in [30] in the framework of conventional  $\mathcal{N} = 2$  superspace. It

would be interesting to explicitly construct  $\mathcal{N} = 2$  generalization of these invariants in harmonic superspace and to explicitly check that such invariants are topological; i.e., they satisfy the relation

$$\delta I = \int d^4x d^8\theta du E \frac{\delta I}{\delta h^{++M}} \delta h^{++M} = 0. \quad (8.5)$$

In [30], the topological nature of  $\mathcal{N} = 2$  GB term has been established only at the component level.

- (4)  $\mathcal{N} = 2$  conformal supergravity invariants:

$\mathcal{N} = 2$  conformal supergravity formulated in harmonic superspace [19] involves an additional analytic prepotential  $h^{(4)}$ , so the structure of the linearized Weyl-squared invariant will be modified as compared to  $\mathcal{N} = 2$  Einstein supergravity. It is quite simple to extend the construction of Sec. V to the conformal case. We shall present this result and its higher-spin generalizations elsewhere.

The problem of constructing nonlinear  $\mathcal{N} = 2$  Weyl-squared action in harmonic superspace remains a challenge.

- (5) Other versions of  $\mathcal{N} = 2$  supergravity:

It is well known that various types of  $\mathcal{N} = 2$  Einstein supergravity can be constructed from the conformal  $\mathcal{N} = 2$  Weyl multiplet using superconformal compensators [1,16,17]. The  $\mathcal{N} = 2$  Maxwell multiplet must necessarily be present as one of the compensators. The second compensator may be the hypermultiplet, the nonlinear multiplet, the tensor or central-charged multiplets. These compensators lead to different versions of supergravity. The “principal” version (when one of the compensators is the hypermultiplet) contains an infinite number of off-shell components, other versions contain  $40_{\text{B}} + 40_{\text{F}}$  off-shell degrees of freedom. In this article, we dealt with the version corresponding to the nonlinear compensating multiplet. It is natural to consider the construction of invariants also in other versions of supergravity.

In this regard, we note the work [27] in which various linearized versions of  $\mathcal{N} = 2$  Einstein supergravity were considered. It is instructive to reproduce such versions within the harmonic superspace framework. Such formulations can lead to the alternative versions of  $\mathcal{N} = 2$  higher spins [25] and can clarify their relationships with the off-shell superconformal multiplets of  $\mathcal{N} = 2$  higher spins recently considered in Ref. [56].

It is also of interest to construct the linearized versions of  $\mathcal{N} = 2$  anti-de Sitter supergravities, which were investigated in Ref. [57]. The harmonic formulation of such theories is so far unknown.

- (6)  $\mathcal{N} = 2$  higher-spin generalizations:

Generalizing the invariants constructed to  $\mathcal{N} = 2$  higher-spin off-shell supermultiplets goes straightforwardly. For example, in terms of the

covariant superfields  $G^{\pm\pm\alpha(s-1)\dot{\alpha}(s-1)}$  and  $G^{\pm\pm\alpha(s-2)\dot{\alpha}(s-2)}$  introduced in [25], we can immediately derive the following generalizations of the invariants  $I_1$  and  $I_2$ :

$$I_1^{(s)} = \int d\zeta^{(-4)} \mathcal{F}^{++\alpha(s-1)\dot{\alpha}(s-1)} \mathcal{F}_{\alpha(s-1)\dot{\alpha}(s-1)}^{++},$$

$$\mathcal{F}^{++\alpha(s-1)\dot{\alpha}(s-1)} := (\mathcal{D}^+)^4 G^{--\alpha(s-1)\dot{\alpha}(s-1)}, \quad (8.6)$$

$$I_2^{(s)} = \int d\zeta^{(-4)} \mathcal{F}^{++\alpha(s-2)\dot{\alpha}(s-2)} \mathcal{F}_{\alpha(s-2)\dot{\alpha}(s-2)}^{++},$$

$$\mathcal{F}^{++\alpha(s-2)\dot{\alpha}(s-2)} := (\mathcal{D}^+)^4 G^{--\alpha(s-2)\dot{\alpha}(s-2)}. \quad (8.7)$$

These invariants are the higher-spin generalizations of the  $R^2$  and  $R^{mn}R_{mn}$  invariants of  $\mathcal{N} = 2$  supergravity. In a similar way, one can generalize other invariants to  $\mathcal{N} = 2$  higher spins. In the bosonic case, the higher-spin invariants of this kind were discussed in [58].

(7) Induced actions:

It is known that the higher-derivative gravity and higher-spin invariants can be recovered as induced actions by starting from couplings of the higher-spin gauge fields to quantum matter fields and then integrating out the matter fields from the generating functional (see, e.g., [59]). The hypermultiplet  $q^+$  is the most general  $\mathcal{N} = 2$  matter multiplet. Using cubic interacting vertices of hypermultiplet with gauge higher-spin superfields [44], one can consider an effective action of the form:

$$\int Dq^+ e^{-\frac{1}{2} \int d\zeta^{(-4)} q^{+a} (\mathcal{D}^{++} + \kappa \hat{\mathcal{H}}^{++}) q_a^+} \quad (8.8)$$

(see [44] for details of the notation). Doing this functional integral must yield some higher-derivative invariants of the higher-spin  $\mathcal{N} = 2$  gauge multiplets. In particular, it would be instructive to reproduce in such a way the invariants introduced at the linearized level in the present paper.

(8)  $6D, \mathcal{N} = (1, 0)$  supergravity invariants:

Various higher-derivative invariants of  $6D, \mathcal{N} = (1, 0)$  supergravity have been actively studied in the superfield approach (see, e.g., [60–63]).  $6D$  supergravity in harmonic superspace was constructed by Sokatchev in [64]. However, the higher-derivative invariants of  $\mathcal{N} = (1, 0)$  off-shell supergravity were never tackled in detail by the harmonic superspace methods.<sup>11</sup> It is worth noting that the construction of such invariants in the formulations of  $\mathcal{N} = 2$  supergravity without analytic prepotentials requires utilizing new action principles based on the use of

closed covariant superforms [60]. It is of interest to learn whether such invariants admit a reformulation in harmonic superspace. Recall once more that the latter provides the universal methods of setting up superfield invariants, based upon the analytic gauge prepotentials and the harmonic zero-curvature equations.

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## APPENDIX A: LINEARIZED GRAVITY INVARIANTS IN SPINOR NOTATIONS

We use the standard definition of the linearized gravity field  $g_{mn} = \eta_{mn} + h_{mn}$ ,  $h := \eta^{mn} h_{mn}$ . The linearizations of Riemann curvature tensor, Ricci curvature and scalar curvature are given by

$$R_{krmn} = 2\partial_{[r}\partial_{[m}h_{n]k]}$$

$$= \frac{1}{2}(\partial_m\partial_r h_{nk} - \partial_m\partial_k h_{nr} - \partial_n\partial_r h_{mk} + \partial_n\partial_k h_{mr}), \quad (A1a)$$

$$R_{mn} = \frac{1}{2}(\partial^r\partial_m h_{nr} + \partial^r\partial_n h_{mr} - \square h_{mn} - \partial_m\partial_n h), \quad (A1b)$$

$$R = \partial^m\partial^n h_{mn} - \square h. \quad (A1c)$$

All curvatures are invariant under the linearized gauge transformations  $\delta h_{mn} = \partial_m a_n + \partial_n a_m$ .

The spin 2 field  $\Phi_{\beta\dot{\beta}}^{\alpha\dot{\alpha}}$  from WZ gauge (3.10) is related to the fierbein as  $\delta_{\beta\dot{\beta}}^{\alpha\dot{\alpha}} + \Phi_{\beta\dot{\beta}}^{\alpha\dot{\alpha}} := \frac{1}{2}\sigma_m^{\alpha\dot{\alpha}}\sigma_{\beta\dot{\beta}}^m e_a^m$ . The fierbein expansion near the flat Minkowski background is given by  $e_a^m = \delta_a^m + \varepsilon_a^m$  so that  $h_{mn} = \varepsilon_{mn} + \varepsilon_{nm}$ . After fixing the local Lorentz freedom, in the symmetric gauge  $\varepsilon_{mn} = \varepsilon_{(mn)}$  we have

$$h_{mn} = 2\varepsilon_{mn} = \sigma_m^{\alpha\dot{\alpha}}\sigma_n^{\beta\dot{\beta}}(\Phi_{(\alpha\beta)(\dot{\alpha}\dot{\beta})} + \varepsilon_{\alpha\beta}\varepsilon_{\dot{\alpha}\dot{\beta}}\Phi)$$

$$= \sigma_m^{\alpha\dot{\alpha}}\sigma_n^{\beta\dot{\beta}}\Phi_{(\alpha\beta)(\dot{\alpha}\dot{\beta})} + 2\eta_{mn}\Phi. \quad (A2)$$

From this relation it follows that  $h = 8\Phi$ .

One can rewrite<sup>12</sup> the curvatures (A1) in terms of the irreducible spin 2 fields  $\Phi_{(\alpha\beta)(\dot{\alpha}\dot{\beta})}$  and  $\Phi$ :

<sup>12</sup>Here we use standard notations, see, e.g., [16]:

$$\sigma_{[mn]}^{(\alpha\beta)} = \frac{i}{2}((\sigma_m)_a^{\alpha}\sigma_n^{\dot{\alpha}\beta} - (\sigma_n)_a^{\alpha}\sigma_m^{\dot{\alpha}\beta}),$$

$$\tilde{\sigma}_{[mn]}^{(\dot{\alpha}\dot{\beta})} = \frac{i}{2}(\sigma_m^{\alpha\dot{\alpha}}(\sigma_n)_{\dot{\alpha}}^{\dot{\beta}} - \sigma_n^{\alpha\dot{\alpha}}(\sigma_m)_{\dot{\alpha}}^{\dot{\beta}}).$$

<sup>11</sup>In Ref. [65] the harmonic approach was applied for analysis of UV divergences of  $6D$  supergravities.

$$R_{krmn} = \sigma_{[kr]}^{(\alpha\beta)} \sigma_{[mn]}^{(\gamma\delta)} \left( -\frac{1}{2} \mathcal{R}_{(\alpha\beta\gamma\delta)} - \frac{1}{48} \epsilon_{\alpha\gamma} \epsilon_{\beta\delta} R \right) - \frac{1}{4} \sigma_{[kr]}^{(\alpha\beta)} \tilde{\sigma}_{[mn]}^{(\dot{\alpha}\dot{\beta})} \mathcal{R}_{(\alpha\beta)(\dot{\alpha}\dot{\beta})} + \text{c.c.}, \quad (\text{A3})$$

$$R_{mn} = \sigma_m^{\alpha\dot{\alpha}} \sigma_n^{\beta\dot{\beta}} \left( \mathcal{R}_{(\alpha\beta)(\dot{\alpha}\dot{\beta})} + \frac{1}{8} \epsilon_{\alpha\beta} \epsilon_{\dot{\alpha}\dot{\beta}} R \right), \quad (\text{A4})$$

$$R = 4\partial^{\alpha\dot{\alpha}} \partial^{\beta\dot{\beta}} \Phi_{(\alpha\beta)(\dot{\alpha}\dot{\beta})} - 6\Box\Phi. \quad (\text{A5})$$

Here we used the following notations for the irreducible parts of the linearized curvatures:

$$\mathcal{R}_{(\alpha\beta\gamma\delta)} := \partial_{(\alpha}^{\dot{\alpha}} \partial_{\beta}^{\dot{\beta}} \Phi_{\gamma\delta)(\dot{\alpha}\dot{\beta})}, \quad \bar{\mathcal{R}}_{(\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta})} := \partial_{(\dot{\alpha}}^{\alpha} \partial_{\dot{\beta}}^{\beta} \Phi_{\dot{\gamma}\dot{\delta})(\alpha\beta)}, \quad (\text{A6})$$

$$\mathcal{R}_{(\alpha\beta)(\dot{\alpha}\dot{\beta})} := 2\partial_{(\alpha(\dot{\alpha}} \partial^{\rho\dot{\rho}} \Phi_{(\beta)\rho)(\dot{\beta})\dot{\rho})} - \frac{1}{2} \Box \Phi_{(\alpha\beta)(\dot{\alpha}\dot{\beta})} - 2\partial_{(\alpha(\dot{\alpha}} \partial_{\beta)\dot{\beta})} \Phi. \quad (\text{A7})$$

Note that the linearized invariants (A6) are constructed from the conformal graviton and so can be identified with the self-dual and anti-self-dual parts of the linearized Weyl tensor. For completeness, we give explicit relations for the irreducible decomposition of the Weyl tensor:

$$W_{krmn} := R_{krmn} - (g_{k[m} R_{n]r} - g_{r[m} R_{n]k}) + \frac{1}{3} R g_{k[m} g_{n]r}, \quad (\text{A8})$$

$$W_{krmn} = -\frac{1}{2} \sigma_{[kr]}^{(\alpha\beta)} \sigma_{[mn]}^{(\gamma\delta)} \mathcal{R}_{(\alpha\beta\gamma\delta)} - \frac{1}{2} \tilde{\sigma}_{[kr]}^{(\dot{\alpha}\dot{\beta})} \tilde{\sigma}_{[mn]}^{(\dot{\gamma}\dot{\delta})} \bar{\mathcal{R}}_{(\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta})}. \quad (\text{A9})$$

The linearized Bianchi identities read

$$\partial^{\beta\dot{\beta}} \mathcal{R}_{(\alpha\beta)(\dot{\alpha}\dot{\beta})} = \frac{1}{8} \partial_{\alpha\dot{\alpha}} R, \quad (\text{A10})$$

so the linearized Einstein tensor has the form

$$\mathcal{G}_{\alpha\beta\dot{\alpha}\dot{\beta}} = \mathcal{R}_{(\alpha\beta)(\dot{\alpha}\dot{\beta})} - \frac{1}{8} \epsilon_{\alpha\beta} \epsilon_{\dot{\alpha}\dot{\beta}} R, \quad \partial^{\beta\dot{\beta}} \mathcal{G}_{\alpha\beta\dot{\alpha}\dot{\beta}} = 0. \quad (\text{A11})$$

Using the linearized Einstein tensor one can represent the linearized Einstein-Gilbert action as

$$S_2 \sim \int d^4x \Phi^{\alpha\beta\dot{\alpha}\dot{\beta}} \mathcal{G}_{\alpha\beta\dot{\alpha}\dot{\beta}}. \quad (\text{A12})$$

We make use of the above notations for curvatures when writing solutions of the harmonic zero-curvature equations in the gravity sector and also in the component reduction of  $\mathcal{N} = 2$  invariants. In such a formulation, the component invariants are explicitly invariant with respect to the linearized gauge transformations.

## APPENDIX B: SPIN $\frac{3}{2}$ LINEARIZED CURVATURES

In this appendix, we present the expressions for the linearized curvatures of the Rarita-Schwinger spin  $\frac{3}{2}$  gauge field. Such curvatures appear in solutions of the zero curvature equations in the fermionic spin  $\frac{3}{2}$  sector and in the component representation of  $\mathcal{N} = 2$  supergravity invariants.

In accord with Eq. (3.25), the irreducible parts of the spin  $\frac{3}{2}$  Rarita-Schwinger field undergo the following linearized gauge transformations:

$$\psi^{\beta\alpha\dot{\alpha}i} = \psi^{(\beta\alpha)\dot{\alpha}i} + \epsilon^{\beta\alpha} \psi^{\dot{\alpha}i}, \quad \delta_\lambda \psi^{(\beta\alpha)\dot{\alpha}i} = -\partial^{(\beta\alpha} \epsilon^{\dot{\alpha})i}, \quad (\text{B1a})$$

$$\delta_\lambda \psi^{\dot{\alpha}i} = -\frac{1}{2} \partial_{\dot{\alpha}}^{\alpha} \epsilon^{\alpha i}$$

and, for the complex-conjugated field,

$$\bar{\psi}^{\dot{\beta}\dot{\alpha}ai} = \bar{\psi}^{(\dot{\beta}\dot{\alpha})ai} + \epsilon^{\dot{\beta}\dot{\alpha}} \bar{\psi}^{ai}, \quad \delta_\lambda \bar{\psi}^{(\dot{\beta}\dot{\alpha})ai} = -\partial^{(\dot{\beta}\dot{\alpha}} \bar{\epsilon}^{a)i}, \quad (\text{B1b})$$

$$\delta_\lambda \bar{\psi}^{ai} = -\frac{1}{2} \partial_a^{\dot{\alpha}} \bar{\epsilon}^{\dot{\alpha}i}.$$

From these irreducible parts one can construct one-derivative irreducible linearized gauge invariants:

$$\mathcal{R}_{(\dot{\alpha}\dot{\beta})\alpha}^i := \partial_{(\dot{\alpha}}^{\beta} \psi_{(\dot{\beta})\beta}^i + \partial_{(\dot{\alpha}\dot{\beta}} \psi_{\alpha)}^i, \quad \mathcal{R}_\alpha^i := \partial^{\beta\dot{\beta}} \psi_{(\alpha\beta)\dot{\beta}}^i - 3\partial_{\alpha}^{\dot{\beta}} \psi_{\dot{\beta}}^i, \quad (\text{B2a})$$

$$\bar{\mathcal{R}}_{(\alpha\beta)\dot{\alpha}}^i := \partial_{(\alpha}^{\dot{\beta}} \bar{\psi}_{(\beta)\dot{\beta}}^i + \partial_{(\alpha\dot{\beta}} \bar{\psi}_{\dot{\beta})}^i, \quad \bar{\mathcal{R}}_{\dot{\alpha}}^i := \partial^{\beta\dot{\beta}} \bar{\psi}_{(\beta\dot{\beta})\dot{\alpha}}^i - 3\partial_{\dot{\alpha}}^{\beta} \bar{\psi}_{\beta}^i. \quad (\text{B2b})$$

The Einstein-like spin  $\frac{3}{2}$  tensors have the form given below and satisfy the linearized Bianchi identities:

$$\mathcal{G}_{\dot{\alpha}\dot{\beta}\alpha}^i := \mathcal{R}_{(\dot{\alpha}\dot{\beta})\alpha}^i + \frac{1}{2} \epsilon_{\dot{\alpha}\dot{\beta}} \mathcal{R}_\alpha^i, \quad \partial^{\alpha\dot{\alpha}} \mathcal{G}_{\dot{\alpha}\dot{\beta}\alpha}^i = 0, \quad (\text{B3a})$$

$$\bar{\mathcal{G}}_{\alpha\beta\dot{\alpha}}^i := \bar{\mathcal{R}}_{(\alpha\beta)\dot{\alpha}}^i + \frac{1}{2} \epsilon_{\alpha\beta} \bar{\mathcal{R}}_{\dot{\alpha}}^i, \quad \partial^{\alpha\dot{\alpha}} \bar{\mathcal{G}}_{\alpha\beta\dot{\alpha}}^i = 0. \quad (\text{B3b})$$

Using these linearized tensors, one can construct the gauge-invariant spin  $\frac{3}{2}$  action as

$$S_{\frac{3}{2}} \sim \int d^4x i \bar{\psi}^{\dot{\alpha}\dot{\beta}ai} \bar{\mathcal{G}}_{\dot{\alpha}\dot{\beta}ai}. \quad (\text{B4})$$

## APPENDIX C: SOLUTIONS OF HARMONIC ZERO-CURVATURE EQUATIONS

To perform the component reduction of the supergravity invariants, one needs solutions of four linear harmonic equations (4.15):

$$\begin{aligned}
 \mathcal{D}^{++}G^{--\alpha\dot{\alpha}} &= \mathcal{D}^{--}G^{++\alpha\dot{\alpha}}, \\
 \mathcal{D}^{++}G^{--5} &= \mathcal{D}^{--}G^{++5}, \\
 \mathcal{D}^{++}G^{--\hat{\alpha}+} &= \mathcal{D}^{--}G^{++\hat{\alpha}+} \\
 \mathcal{D}^{++}G^{--\hat{\alpha}+} + G^{--\hat{\alpha}+} &= 0.
 \end{aligned} \tag{C1}$$

Because of linearity, one can solve these equations for each field in WZ gauge (3.10) separately.

Note the important property

$$\delta[(\mathcal{D}^+)^4 G^{--\alpha\dot{\alpha}}] = 0, \quad \delta[(\mathcal{D}^+)^4 G^{--5}] = 0. \tag{C2}$$

So all  $(\theta^-)^4$  components of  $G^{--\alpha\dot{\alpha}}$  and  $G^{--5}$  are invariant under gauge transformations (4.17). This useful observation makes it possible to verify the correctness of the solutions obtained. In order to explicitly identify gauge invariant terms in the solutions of zero-curvature equations, we shall use the notation of Appendixes A and B.

In this appendix, we explicitly quote solutions of the zero-curvature equations necessary for the component reduction of the  $\mathcal{N} = 2$  supergravity curvatures and invariants defined earlier. We are working with the redefined auxiliary fields  $P_{\alpha\dot{\alpha}}$ ,  $\rho_{\dot{\beta}}^i$  and  $\chi_{\dot{\beta}}^i$ , see Eq. (3.26).

Gravity sector:

$$\begin{aligned}
 G_{(\Phi)}^{++\alpha\dot{\alpha}} &= -4i\theta^{+\beta}\bar{\theta}^{+\dot{\beta}}\Phi_{\beta\dot{\beta}}^{\alpha\dot{\alpha}} + 4(\theta^+)^2\bar{\theta}^{+\dot{\beta}}\bar{\theta}^{-\dot{\alpha}}B_{\dot{\beta}}^{\alpha} \\
 &\quad - 4(\bar{\theta}^+)^2\theta^{+\beta}\theta^{-\alpha}B_{\beta}^{\dot{\alpha}},
 \end{aligned} \tag{C3a}$$

$$\begin{aligned}
 G_{(\Phi)}^{--\alpha\dot{\alpha}} &= -4i\theta^{-\beta}\bar{\theta}^{-\dot{\beta}}\Phi_{\beta\dot{\beta}}^{\alpha\dot{\alpha}} \\
 &\quad + 8(\theta^-)^2\bar{\theta}^{-\dot{\rho}}\bar{\theta}^{+\dot{\beta}}\left(\partial_{(\dot{\rho}}^{\beta}\Phi_{\beta\dot{\beta}}^{\alpha\dot{\alpha}} - \frac{1}{2}\delta_{\dot{\rho}}^{\dot{\alpha}}B_{\dot{\beta}}^{\alpha}\right) \\
 &\quad - 8(\bar{\theta}^-)^2\theta^{-\rho}\theta^{+\beta}\left(\partial_{(\rho}^{\dot{\beta}}\Phi_{\beta\dot{\beta}}^{\alpha\dot{\alpha}} - \frac{1}{2}\delta_{\rho}^{\alpha}B_{\dot{\beta}}^{\dot{\alpha}}\right) \\
 &\quad - 8i(\theta^-)^4\theta_{\rho}^{+}\bar{\theta}_{\dot{\rho}}^{+}\left(\mathcal{R}^{(\alpha\rho)(\dot{\alpha}\dot{\rho})} - \frac{1}{8}\epsilon^{\alpha\rho}\epsilon^{\dot{\alpha}\dot{\rho}}R\right),
 \end{aligned} \tag{C3b}$$

$$G_{(\Phi)}^{++5} = 2(\theta^+)^2\bar{\theta}^{+\dot{\beta}}\theta^{-\beta}B_{\beta\dot{\beta}} + 2(\bar{\theta}^+)^2\theta^{+\beta}\bar{\theta}^{-\dot{\beta}}B_{\beta\dot{\beta}}, \tag{C3c}$$

$$\begin{aligned}
 G_{(\Phi)}^{--5} &= -2(\theta^-)^2\bar{\theta}^{-\dot{\beta}}\theta^{+\beta}B_{\beta\dot{\beta}} - \frac{i}{2}(\theta^-)^4(\theta^+)^2R \\
 &\quad - 2(\bar{\theta}^-)^2\theta^{-\beta}\bar{\theta}^{+\dot{\beta}}B_{\beta\dot{\beta}} + \frac{i}{2}(\theta^-)^4(\bar{\theta}^+)^2R,
 \end{aligned} \tag{C3d}$$

$$G_{(\Phi)}^{++\alpha+} = -i(\theta^+)^2\bar{\theta}_{\dot{\beta}}^{+}B^{\alpha\dot{\beta}}, \tag{C3e}$$

$$\begin{aligned}
 G_{(\Phi)}^{--\alpha+} &= -\frac{i}{2}(\theta^-)^2\bar{\theta}_{\dot{\beta}}^{+}B^{\alpha\dot{\beta}} - i(\theta^+\theta^-)\bar{\theta}_{\dot{\beta}}^{-}B^{\alpha\dot{\beta}} \\
 &\quad + (\theta^+)^2(\bar{\theta}^-)^2\theta^{-\rho}\partial_{\rho\dot{\beta}}B^{\alpha\dot{\beta}} - (\theta^-)^2(\bar{\theta}^+\bar{\theta}^-)\theta^{+\rho}\partial_{\rho\dot{\beta}}B^{\alpha\dot{\beta}} \\
 &\quad + 2(\theta^-)^2\theta^{+\rho}\bar{\theta}^{+(\dot{\rho}}\theta^{-\dot{\beta}})\partial_{\rho\dot{\beta}}B_{\beta}^{\alpha} - \frac{i}{2}(\theta^-)^4(\theta^+)^2\bar{\theta}^{+\dot{\beta}}\square B_{\dot{\beta}}^{\alpha},
 \end{aligned} \tag{C3f}$$

$$G_{(\Phi)}^{--\alpha-} = -\frac{i}{2}(\theta^-)^2\bar{\theta}_{\dot{\beta}}^{-}B^{\alpha\dot{\beta}} + (\theta^-)^4\theta^{+\beta}\partial_{\beta\dot{\beta}}B^{\alpha\dot{\beta}}. \tag{C3g}$$

Maxwell sector:

$$G_{(C)}^{++5} = -4i\theta^{+\beta}\bar{\theta}^{+\dot{\beta}}C_{\beta\dot{\beta}}, \tag{C4a}$$

$$\begin{aligned}
 G_{(C)}^{--5} &= -4i\theta^{-\beta}\bar{\theta}^{-\dot{\beta}}C_{\beta\dot{\beta}} + 8(\theta^-)^2\bar{\theta}^{-(\dot{\rho}}\bar{\theta}^{+\dot{\beta}})\bar{\mathcal{F}}_{(\dot{\rho}\dot{\beta})} \\
 &\quad - 8(\bar{\theta}^-)^2\theta^{-(\rho}\theta^{+\beta})\mathcal{F}_{(\rho\beta)} \\
 &\quad - 8i(\theta^-)^4\theta^{+\rho}\bar{\theta}^{+\dot{\rho}}(\partial_{\rho}^{\dot{\beta}}\bar{\mathcal{F}}_{(\dot{\rho}\dot{\beta})} + \partial_{\dot{\rho}}^{\beta}\mathcal{F}_{(\rho\beta)}).
 \end{aligned} \tag{C4b}$$

Auxiliary field  $T^{(\alpha\beta)}$ :

$$G_{T^{(\alpha\beta)}}^{++\alpha\dot{\alpha}} = 4i(\bar{\theta}^+)^2\theta_{\beta}^{+}\bar{\theta}^{-\dot{\alpha}}T^{(\alpha\beta)} - 4i(\theta^+)^2\bar{\theta}_{\dot{\beta}}^{+}\theta^{-\alpha}\bar{T}^{(\dot{\alpha}\dot{\beta})}, \tag{C5a}$$

$$\begin{aligned}
 G_{T^{(\alpha\beta)}}^{--\alpha\dot{\alpha}} &= -4i(\bar{\theta}^-)^2\theta_{\beta}^{-}\bar{\theta}^{+\dot{\alpha}}T^{(\alpha\beta)} + 4(\theta^-)^4(\bar{\theta}^+)^2\partial_{\rho}^{\dot{\alpha}}T^{(\alpha\rho)} \\
 &\quad + 4i(\theta^-)^2\bar{\theta}_{\dot{\beta}}^{-}\theta^{+\alpha}\bar{T}^{(\dot{\alpha}\dot{\beta})} + 4(\theta^-)^4(\theta^+)^2\partial_{\rho}^{\alpha}\bar{T}^{(\dot{\alpha}\dot{\rho})},
 \end{aligned} \tag{C5b}$$

$$G_{T^{(\alpha\beta)}}^{++5} = -2i(\bar{\theta}^+)^2\theta_{\alpha}^{+}\theta_{\beta}^{-}T^{(\alpha\beta)} - 2i(\theta^+)^2\bar{\theta}_{\dot{\alpha}}^{+}\bar{\theta}_{\dot{\beta}}^{-}\bar{T}^{(\dot{\alpha}\dot{\beta})}, \tag{C5c}$$

$$\begin{aligned}
 G_{T^{(\alpha\beta)}}^{--5} &= -2i(\bar{\theta}^-)^2\theta_{\alpha}^{+}\theta_{\beta}^{-}T^{(\alpha\beta)} - 4(\theta^-)^4\theta_{\alpha}^{+}\bar{\theta}^{+\dot{\sigma}}\partial_{\dot{\sigma}\dot{\beta}}T^{(\alpha\beta)} \\
 &\quad - 2i(\theta^-)^2\bar{\theta}_{\dot{\alpha}}^{+}\bar{\theta}_{\dot{\beta}}^{-}\bar{T}^{(\dot{\alpha}\dot{\beta})} - 4(\theta^-)^4\bar{\theta}_{\dot{\alpha}}^{+}\theta^{+\sigma}\partial_{\sigma\dot{\beta}}\bar{T}^{(\dot{\alpha}\dot{\rho})},
 \end{aligned} \tag{C5d}$$

$$G_{T^{(\alpha\beta)}}^{++\alpha+} = -(\bar{\theta}^+)^2\theta_{\beta}^{+}T^{(\alpha\beta)}, \tag{C5e}$$

$$\begin{aligned}
 G_{T^{(\alpha\beta)}}^{--\alpha+} &= -\frac{1}{2}(\bar{\theta}^-)^2\theta_{\beta}^{+}T^{(\alpha\beta)} - (\bar{\theta}^+\bar{\theta}^-)\theta_{\beta}^{-}T^{(\alpha\beta)} \\
 &\quad + i(\bar{\theta}^-)^2(\theta^+\theta^-)\bar{\theta}^{+\dot{\rho}}\partial_{\rho\dot{\beta}}T^{(\alpha\beta)} \\
 &\quad - 2i(\bar{\theta}^-)^2\theta^{+(\beta}\theta^{-\rho)}\bar{\theta}^{+\dot{\rho}}\partial_{\rho\dot{\beta}}T_{\beta}^{(\alpha)} \\
 &\quad - i(\bar{\theta}^+)^2(\theta^-)^2\bar{\theta}^{-\dot{\rho}}\partial_{\rho\dot{\beta}}T^{(\alpha\beta)} \\
 &\quad + \frac{1}{2}(\theta^-)^4(\bar{\theta}^+)^2\theta_{\beta}^{+}\square T^{(\alpha\beta)},
 \end{aligned} \tag{C5f}$$

$$G_{T^{(\alpha\beta)}}^{--\alpha-} = \frac{1}{2}(\bar{\theta}^-)^2\theta_{\beta}^{-}T^{(\alpha\beta)} - i(\theta^-)^4\bar{\theta}^{+\dot{\rho}}\partial_{\rho\dot{\beta}}T^{(\alpha\beta)}. \tag{C5g}$$

Auxiliary field  $T$ :

$$G_{(T)}^{++\alpha\dot{\alpha}} = 4i(\bar{\theta}^+)^2\theta^{+\alpha}\bar{\theta}^{-\dot{\alpha}}T - 4i(\theta^+)^2\bar{\theta}^{+\dot{\alpha}}\theta^{-\alpha}\bar{T}, \tag{C6a}$$

$$\begin{aligned}
 G_{(T)}^{--\alpha\dot{\alpha}} &= -4i(\bar{\theta}^-)^2\theta^{-\alpha}\bar{\theta}^{+\dot{\alpha}}T + 4(\theta^-)^4(\bar{\theta}^+)^2\partial^{\alpha\dot{\alpha}}T \\
 &\quad + 4i(\theta^-)^2\bar{\theta}^{-\dot{\alpha}}\theta^{+\alpha}\bar{T} + 4(\theta^-)^4(\theta^+)^2\partial^{\alpha\dot{\alpha}}\bar{T},
 \end{aligned} \tag{C6b}$$

$$G_{(T)}^{++5} = -2i(\bar{\theta}^+)^2(\theta^+\theta^-)T + 2i(\theta^+)^2(\bar{\theta}^+\bar{\theta}^-)\bar{T}, \tag{C6c}$$

$$G_{(T)}^{-5} = -2i(\theta^-)^2(\bar{\theta}^+\bar{\theta}^-)T + 2i(\bar{\theta}^-)^2(\theta^+\theta^-)\bar{T}, \quad (\text{C6d})$$

$$G_{(T)}^{++\alpha+} = -(\bar{\theta}^+)^2\theta^{\alpha+}T, \quad (\text{C6e})$$

$$\begin{aligned} G_{(T)}^{-\alpha+} = & -\frac{1}{2}(\bar{\theta}^-)^2\theta^{\alpha+}T - (\bar{\theta}^+\bar{\theta}^-)\theta^{-\alpha}T \\ & - i(\bar{\theta}^+)^2(\theta^-)^2\bar{\theta}^{-\dot{\rho}}\partial_{\rho}^{\alpha}T + i(\bar{\theta}^-)^2(\theta^+\theta^-)\bar{\theta}^{+\dot{\rho}}\partial_{\rho}^{\alpha}T \\ & - 2i(\bar{\theta}^-)^2\theta^{+(\alpha}\theta^{-\rho)}\bar{\theta}^{+\dot{\rho}}\partial_{\rho\dot{\rho}}T + \frac{1}{4}(\theta^-)^4(\bar{\theta}^+)^2\theta^{-\alpha}\square T, \end{aligned} \quad (\text{C6f})$$

$$G_{(T)}^{-\alpha-} = \frac{1}{2}(\bar{\theta}^-)^2\theta^{-\alpha}T - \frac{i}{2}(\theta^-)^4\bar{\theta}^{+\dot{\rho}}\partial_{\rho}^{\alpha}T. \quad (\text{C6g})$$

Auxiliary field  $P^{\alpha\dot{\alpha}}$ :

$$G_{(P)}^{++\alpha\dot{\alpha}} = 4i(\theta^+)^2\bar{\theta}^+\bar{\theta}^{-\dot{\alpha}}P^{\alpha\dot{\beta}} + 4i(\bar{\theta}^+)^2\theta_{\beta}^+\theta^{-\alpha}\bar{P}^{\beta\dot{\alpha}}, \quad (\text{C7a})$$

$$\begin{aligned} G_{(P)}^{-\alpha\dot{\alpha}} = & 2i(\bar{\theta}^-)^2(\theta^+\theta^-)P^{\alpha\dot{\alpha}} + 4i(\theta^-)^2\bar{\theta}^+\bar{\theta}^{-\dot{\alpha}}P^{\alpha\dot{\beta}} \\ & - 8(\theta^-)^4\theta^{+\rho}\bar{\theta}^+\bar{\theta}^{\dot{\rho}}\partial_{\rho}^{\alpha}P^{\alpha\dot{\beta}} - 2i(\theta^-)^2(\bar{\theta}^+\bar{\theta}^-)\bar{P}^{\beta\dot{\alpha}} \\ & + 4i(\bar{\theta}^-)^2\theta_{(\beta}^+\theta^{-\alpha)}\bar{P}^{\beta\dot{\alpha}} - 8(\theta^-)^4\bar{\theta}^{+\dot{\rho}}\theta_{(\beta}^+\partial_{\rho}^{\alpha)}\bar{P}^{\beta\dot{\alpha}}, \end{aligned} \quad (\text{C7b})$$

$$G_{(P)}^{++5} = 2i(\theta^+)^2\theta_{\beta}^-\bar{\theta}_{\dot{\beta}}^+P^{\beta\dot{\beta}} + 2i(\bar{\theta}^+)^2\theta_{\beta}^+\bar{\theta}_{\dot{\beta}}^-\bar{P}^{\beta\dot{\beta}}, \quad (\text{C7c})$$

$$\begin{aligned} G_{(P)}^{-5} = & -2i(\theta^-)^2\theta_{\beta}^+\bar{\theta}_{\dot{\beta}}^-P^{\beta\dot{\beta}} + 2(\theta^-)^4(\theta^+)^2\partial_{\beta\dot{\beta}}P^{\beta\dot{\beta}} \\ & - 2i(\bar{\theta}^-)^2\theta_{\beta}^-\bar{\theta}_{\dot{\beta}}^+P^{\beta\dot{\beta}} + 2(\theta^-)^4(\bar{\theta}^+)^2\partial_{\beta\dot{\beta}}\bar{P}^{\beta\dot{\beta}}, \end{aligned} \quad (\text{C7d})$$

$$G_{(P)}^{++\alpha+} = -(\theta^+)^2\bar{\theta}_{\beta}^+P^{\alpha\dot{\beta}}, \quad (\text{C7e})$$

$$\begin{aligned} G_{(P)}^{-\alpha+} = & -\frac{1}{2}(\theta^-)^2\bar{\theta}_{\beta}^+P^{\alpha\dot{\beta}} - (\theta^+\theta^-)\bar{\theta}_{\beta}^-P^{\alpha\dot{\beta}} \\ & - i(\theta^+)^2(\bar{\theta}^-)^2\theta^{-\rho}\partial_{\rho\dot{\beta}}P^{\alpha\dot{\beta}} \\ & + i(\theta^-)^2(\bar{\theta}^+\bar{\theta}^-)\theta^{+\rho}\partial_{\rho\dot{\beta}}P^{\alpha\dot{\beta}} \\ & - 2i(\theta^-)^2\theta^{+\rho}\bar{\theta}^{+\dot{\rho}}\partial_{\rho\dot{\rho}}P_{\dot{\beta}}^{\alpha} \\ & - \frac{1}{2}(\theta^-)^4(\theta^+)^2\bar{\theta}^{+\dot{\beta}}\square P_{\dot{\beta}}^{\alpha}, \end{aligned} \quad (\text{C7f})$$

$$G_{(P)}^{-\alpha-} = -\frac{1}{2}(\theta^-)^2\bar{\theta}_{\beta}^-P^{\alpha\dot{\beta}} - i(\theta^-)^4\theta^{+\beta}\partial_{\beta\dot{\beta}}P^{\alpha\dot{\beta}}. \quad (\text{C7g})$$

Auxiliary field  $V^{\alpha\dot{\alpha}(ij)}$ :

$$G_{(V)}^{++\alpha\dot{\alpha}} = (\theta^+)^4V^{\alpha\dot{\alpha}(ij)}u_i^-u_j^-, \quad (\text{C8a})$$

$$\begin{aligned} G_{(V)}^{-\alpha\dot{\alpha}} = & \frac{1}{3}(\theta^-)^2(\bar{\theta}^+)^2V^{\alpha\dot{\alpha}(ij)}u_i^-u_j^- + \frac{1}{3}(\bar{\theta}^-)^2(\theta^+)^2V^{\alpha\dot{\alpha}(ij)}u_i^-u_j^- + \frac{4}{3}(\theta^+\theta^-)(\bar{\theta}^+\bar{\theta}^-)V^{\alpha\dot{\alpha}(ij)}u_i^-u_j^- - \frac{2}{3}(\theta^-)^2(\bar{\theta}^+\bar{\theta}^-)V^{\alpha\dot{\alpha}(ij)}u_i^+u_j^- \\ & - \frac{2}{3}(\bar{\theta}^-)^2(\theta^+\theta^-)V^{\alpha\dot{\alpha}(ij)}u_i^+u_j^- + \frac{4}{3}i(\theta^+)^2(\bar{\theta}^-)^2\theta^{-\rho}\bar{\theta}^{+\dot{\rho}}\partial_{\rho\dot{\rho}}V^{\alpha\dot{\alpha}(ij)}u_i^-u_j^- + \frac{4}{3}i(\bar{\theta}^+)^2(\theta^-)^2\theta^{+\rho}\bar{\theta}^{-\dot{\rho}}\partial_{\rho\dot{\rho}}V^{\alpha\dot{\alpha}(ij)}u_i^-u_j^- \\ & + \frac{1}{3}(\theta^-)^4V^{\alpha\dot{\alpha}(ij)}u_i^+u_j^+ + \frac{4}{3}i(\theta^-)^4\theta^{+\rho}\bar{\theta}^{+\dot{\rho}}\partial_{\rho\dot{\rho}}V^{\alpha\dot{\alpha}(ij)}u_i^+u_j^- - \frac{2}{3}(\theta^-)^4(\theta^+)^4\square V^{\alpha\dot{\alpha}(ij)}u_i^-u_j^-. \end{aligned} \quad (\text{C8b})$$

Auxiliary field  $S^{(ij)}$ :

$$G_{(S)}^{++5} = (\theta^+)^4S^{(ij)}u_i^-u_j^-, \quad (\text{C9a})$$

$$\begin{aligned} G_{(S)}^{-5} = & \frac{1}{3}(\theta^-)^2(\bar{\theta}^+)^2S^{(ij)}u_i^-u_j^- + \frac{1}{3}(\bar{\theta}^-)^2(\theta^+)^2S^{(ij)}u_i^-u_j^- + \frac{4}{3}(\theta^+\theta^-)(\bar{\theta}^+\bar{\theta}^-)S^{(ij)}u_i^-u_j^- - \frac{2}{3}(\theta^-)^2(\bar{\theta}^+\bar{\theta}^-)S^{(ij)}u_i^+u_j^- \\ & - \frac{2}{3}(\bar{\theta}^-)^2(\theta^+\theta^-)S^{(ij)}u_i^+u_j^- + \frac{4}{3}i(\theta^+)^2(\bar{\theta}^-)^2\theta^{-\rho}\bar{\theta}^{+\dot{\rho}}\partial_{\rho\dot{\rho}}S^{(ij)}u_i^-u_j^- + \frac{4}{3}i(\bar{\theta}^+)^2(\theta^-)^2\theta^{+\rho}\bar{\theta}^{-\dot{\rho}}\partial_{\rho\dot{\rho}}S^{(ij)}u_i^-u_j^- \\ & + \frac{1}{3}(\theta^-)^4S^{(ij)}u_i^+u_j^+ + \frac{4}{3}i(\theta^-)^4\theta^{+\rho}\bar{\theta}^{+\dot{\rho}}\partial_{\rho\dot{\rho}}S^{(ij)}u_i^+u_j^- - \frac{2}{3}(\theta^-)^4(\theta^+)^4\square S^{(ij)}u_i^-u_j^-. \end{aligned} \quad (\text{C9b})$$

Rarita-Schwinger field  $\psi^{\alpha\beta\dot{\alpha}i} = \psi^{(\alpha\beta)\dot{\alpha}i} + \epsilon^{\alpha\beta}\psi^{\dot{\alpha}i}$ :

$$G_{(\psi)}^{++\alpha\dot{\alpha}} = 16(\bar{\theta}^+)^2\theta^{\alpha\beta}\psi_{\beta}^{\dot{\alpha}i}u_i^- - 16(\theta^+)^2\bar{\theta}^{+\dot{\beta}}\bar{\psi}_{\dot{\beta}}^{\alpha\dot{\alpha}i}u_i^- - 16i(\theta^+)^4\bar{\theta}^{-\dot{\beta}}\partial_{\beta}^{\dot{\beta}}\psi_{\beta}^{\alpha\dot{\alpha}i}u_i^- - 16i(\theta^+)^4\theta^{-\beta}\partial_{\beta}^{\dot{\beta}}\bar{\psi}_{\dot{\beta}}^{\alpha\dot{\alpha}i}u_i^-, \quad (\text{C10a})$$

$$\begin{aligned}
 G_{(\psi)}^{-\alpha\dot{\alpha}} = & 16(\bar{\theta}^+\bar{\theta}^-)\theta^{-\beta}\psi_{\beta}^{\alpha\dot{\alpha}i}u_i^- + 8(\bar{\theta}^-)^2\theta^{+\beta}\psi_{\beta}^{\alpha\dot{\alpha}i}u_i^- - 8(\bar{\theta}^-)^2\theta^{-\beta}\psi_{\beta}^{\alpha\dot{\alpha}i}u_i^+ - 16i(\bar{\theta}^-)^2(\theta^+\theta^-)\bar{\theta}^{+\dot{\sigma}}(\partial_{\sigma}^{\beta}\psi_{\beta}^{\alpha\dot{\alpha}i} - \delta_{\sigma}^{\dot{\alpha}}\partial_{\beta}^{\beta}\psi_{\beta}^{\alpha\dot{\beta}i})u_i^- \\
 & + 8i(\bar{\theta}^+)^2(\theta^-)^2\bar{\theta}^{-\dot{\sigma}}(2\partial_{\sigma}^{\beta}\psi_{\beta}^{\alpha\dot{\alpha}i} - \delta_{\sigma}^{\dot{\alpha}}\partial_{\beta}^{\beta}\psi_{\beta}^{\alpha\dot{\beta}i})u_i^- - 32i(\bar{\theta}^-)^2\theta^{-(\beta}\theta^{+\sigma)}\bar{\theta}^{+\dot{\sigma}}\partial_{\sigma\dot{\sigma}}\psi_{\beta}^{\alpha\dot{\alpha}i}u_i^- - 16i(\theta^-)^4\bar{\theta}^{+\dot{\sigma}}\mathcal{G}^{\dot{\sigma}\dot{\alpha}\alpha i}u_i^+ \\
 & + 32(\theta^-)^4(\bar{\theta}^+)^2\theta^{+\rho}\partial_{\rho\dot{\rho}}\mathcal{G}^{\dot{\rho}\dot{\alpha}\alpha i}u_i^- - 16(\theta^+\theta^-)\bar{\theta}^{-\dot{\beta}}\bar{\psi}_{\dot{\beta}}^{\alpha\dot{\alpha}i}u_i^- - 8(\theta^-)^2\bar{\theta}^{+\dot{\beta}}\bar{\psi}_{\dot{\beta}}^{\alpha\dot{\alpha}i}u_i^- + 8(\theta^-)^2\bar{\theta}^{-\dot{\beta}}\bar{\psi}_{\dot{\beta}}^{\alpha\dot{\alpha}i}u_i^+ \\
 & - 16i(\theta^-)^2(\bar{\theta}^+\bar{\theta}^-)\theta^{+\sigma}(\partial_{\sigma}^{\dot{\beta}}\bar{\psi}_{\dot{\beta}}^{\alpha\dot{\alpha}i} - \delta_{\sigma}^{\alpha}\partial_{\dot{\beta}}^{\dot{\beta}}\bar{\psi}_{\dot{\beta}}^{\beta\dot{\alpha}i})u_i^- + 8i(\theta^+)^2(\bar{\theta}^-)^2\theta^{-\sigma}(2\partial_{\sigma}^{\dot{\beta}}\bar{\psi}_{\dot{\beta}}^{\alpha\dot{\alpha}i} - \delta_{\sigma}^{\alpha}\partial_{\dot{\beta}}^{\dot{\beta}}\bar{\psi}_{\dot{\beta}}^{\beta\dot{\alpha}i})u_i^- \\
 & + 32i(\theta^-)^2\bar{\theta}^{-(\dot{\beta}}\bar{\theta}^{+\dot{\sigma}})\theta^{+\sigma}\partial_{\sigma\dot{\sigma}}\bar{\psi}_{\dot{\beta}}^{\alpha\dot{\alpha}i}u_i^- - 16i(\theta^-)^4\theta_{\sigma}^{+}\bar{\mathcal{G}}^{\sigma\alpha\dot{\alpha}i}u_i^+ - 32(\theta^-)^4(\theta^+)^2\bar{\theta}^{+\dot{\rho}}\partial_{\rho\dot{\rho}}\bar{\mathcal{G}}^{\rho\alpha\dot{\alpha}i}u_i^-, \quad (C10b)
 \end{aligned}$$

$$G_{(\psi)}^{++5} = -16(\theta^+)^2\bar{\theta}^{+\dot{\beta}}\psi_{\dot{\beta}}^i u_i^- + 8i(\theta^+)^4\theta_{\alpha}^{-}\partial_{\beta}^{\beta}\psi_{\beta}^{\alpha\dot{\beta}i}u_i^- + 16(\bar{\theta}^+)^2\theta^{+\beta}\bar{\psi}_{\beta}^i u_i^- - 8i(\theta^+)^4\bar{\theta}_{\alpha}^{-}\partial_{\beta}^{\dot{\beta}}\bar{\psi}_{\dot{\beta}}^{\beta\dot{\alpha}i}u_i^-, \quad (C10c)$$

$$\begin{aligned}
 G_{(\psi)}^{-5} = & -16(\theta^+\theta^-)\bar{\theta}^{-\dot{\beta}}\psi_{\dot{\beta}}^i u_i^- - 8(\theta^-)^2\bar{\theta}^{+\dot{\beta}}\psi_{\dot{\beta}}^i u_i^- + 8(\theta^-)^2\bar{\theta}^{-\dot{\beta}}\psi_{\dot{\beta}}^i u_i^+ - 8i(\theta^-)^2(\bar{\theta}^+\bar{\theta}^-)\theta_{\alpha}^{+}\mathcal{R}^{ai}u_i^- \\
 & - 4i(\theta^+)^2(\bar{\theta}^-)^2\theta_{\alpha}^{-}(\mathcal{R}^{ai} + 2\partial^{\dot{\alpha}\dot{\beta}}\psi_{\dot{\beta}}^{-})u_i^- + 4i(\theta^-)^4\theta_{\alpha}^{+}\mathcal{R}^{ai}u_i^+ + 8(\theta^-)^4(\theta^+)^2\bar{\theta}^{+\dot{\rho}}\partial_{\rho\dot{\rho}}\mathcal{R}^{ai}u_i^- \\
 & + 16(\bar{\theta}^+\bar{\theta}^-)\theta^{-\beta}\bar{\psi}_{\beta}^i u_i^- + 8(\bar{\theta}^-)^2\theta^{+\beta}\bar{\psi}_{\beta}^i u_i^- - 8(\bar{\theta}^-)^2\theta^{-\beta}\bar{\psi}_{\beta}^i u_i^+ - 8i(\bar{\theta}^-)^2(\theta^+\theta^-)\bar{\theta}_{\alpha}^{+}\bar{\mathcal{R}}^{ai}u_i^- \\
 & - 4i(\bar{\theta}^+)^2(\theta^-)^2\bar{\theta}_{\alpha}^{-}(\bar{\mathcal{R}}^{ai} + 2\partial^{\dot{\alpha}\dot{\beta}}\bar{\psi}_{\dot{\beta}}^{-})u_i^- - 4i(\theta^-)^4\bar{\theta}_{\alpha}^{+}\bar{\mathcal{R}}^{ai}u_i^+ + 8(\theta^-)^4(\bar{\theta}^+)^2\theta^{+\rho}\partial_{\rho\dot{\rho}}\bar{\mathcal{R}}^{ai}u_i^-. \quad (C10d)
 \end{aligned}$$

Auxiliary fermionic field  $\rho_{\beta}^i$ :

$$G_{(\rho)}^{++5} = 8(\bar{\theta}^+)^2\theta^{+\beta}\rho_{\beta}^i u_i^- - 8(\theta^+)^2\bar{\theta}^{+\dot{\beta}}\bar{\rho}_{\dot{\beta}}^i u_i^-, \quad (C11a)$$

$$\begin{aligned}
 G_{(\rho)}^{-5} = & 8(\bar{\theta}^+\bar{\theta}^-)\theta^{-\beta}\rho_{\beta}^i u_i^- + 4(\bar{\theta}^-)^2\theta^{+\beta}\rho_{\beta}^i u_i^- - 4(\bar{\theta}^-)^2\theta^{-\beta}\rho_{\beta}^i u_i^+ - 8i(\bar{\theta}^-)^2(\theta^+\theta^-)\bar{\theta}^{+\dot{\sigma}}\partial_{\sigma}^{\beta}\rho_{\beta}^i u_i^- + 8i(\bar{\theta}^+)^2(\theta^-)^2\bar{\theta}^{-\dot{\sigma}}\partial_{\sigma}^{\beta}\rho_{\beta}^i u_i^- \\
 & - 16i(\bar{\theta}^-)^2\theta^{-(\beta}\theta^{+\sigma)}\bar{\theta}^{+\dot{\sigma}}\partial_{\sigma\dot{\sigma}}\rho_{\beta}^i u_i^- + 8i(\theta^-)^4\bar{\theta}^{+\dot{\sigma}}\partial_{\sigma}^{\dot{\beta}}\rho_{\beta}^i u_i^+ - 4(\theta^-)^4(\bar{\theta}^+)^2\theta^{+\beta}\square\rho_{\beta}^i u_i^- - 8(\theta^+\theta^-)\bar{\theta}^{-\dot{\beta}}\bar{\rho}_{\dot{\beta}}^i u_i^- \\
 & - 4(\theta^-)^2\bar{\theta}^{+\dot{\beta}}\bar{\rho}_{\dot{\beta}}^i u_i^- + 4(\theta^-)^2\bar{\theta}^{-\dot{\beta}}\bar{\rho}_{\dot{\beta}}^i u_i^+ - 8i(\theta^-)^2(\bar{\theta}^+\bar{\theta}^-)\theta^{+\sigma}\partial_{\sigma}^{\dot{\beta}}\bar{\rho}_{\dot{\beta}}^i u_i^- + 8i(\theta^+)^2(\bar{\theta}^-)^2\theta^{-\sigma}\partial_{\sigma}^{\dot{\beta}}\bar{\rho}_{\dot{\beta}}^i u_i^- \\
 & + 16i(\theta^-)^2\bar{\theta}^{-(\dot{\beta}}\bar{\theta}^{+\dot{\sigma}})\theta^{+\sigma}\partial_{\sigma\dot{\sigma}}\bar{\rho}_{\dot{\beta}}^i u_i^- + 8i(\theta^-)^4\theta^{+\sigma}\partial_{\sigma}^{\dot{\beta}}\bar{\rho}_{\dot{\beta}}^i u_i^+ + 4(\theta^-)^4(\theta^+)^2\bar{\theta}^{+\dot{\rho}}\square\bar{\rho}_{\dot{\beta}}^i u_i^-. \quad (C11b)
 \end{aligned}$$

Auxiliary fermionic field  $\chi_{\beta}^i$ :

$$G_{(\chi)}^{++\alpha\dot{\alpha}} = -4i(\theta^+)^4\bar{\theta}^{-\dot{\alpha}}\chi^{\alpha i}u_i^- + 4i(\theta^+)^4\theta^{-\alpha}\bar{\chi}^{\dot{\alpha}i}u_i^-, \quad (C12a)$$

$$\begin{aligned}
 G_{(\chi)}^{-\alpha\dot{\alpha}} = & 4i(\bar{\theta}^-)^2(\theta^+\theta^-)\bar{\theta}^{+\dot{\alpha}}\chi^{\alpha i}u_i^- - 2i(\theta^-)^2(\bar{\theta}^+)^2\bar{\theta}^{-\dot{\alpha}}\chi^{\alpha i}u_i^- - 2i(\theta^-)^4\bar{\theta}^{+\dot{\alpha}}\chi^{\alpha i}u_i^+ - 4(\theta^-)^4(\bar{\theta}^+)^2\theta^{+\rho}\partial_{\rho}^{\dot{\alpha}}\chi^{\alpha i}u_i^- \\
 & - 4i(\theta^-)^2(\bar{\theta}^+\bar{\theta}^-)\theta^{+\alpha}\bar{\chi}^{\dot{\alpha}i}u_i^- + 2i(\bar{\theta}^-)^2(\theta^+)^2\theta^{-\alpha}\bar{\chi}^{\dot{\alpha}i}u_i^- + 2i(\theta^-)^4\theta^{+\alpha}\bar{\chi}^{\dot{\alpha}i}u_i^+ - 4(\theta^-)^4(\theta^+)^2\bar{\theta}^{+\dot{\rho}}\partial_{\rho}^{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}i}u_i^-, \quad (C12b)
 \end{aligned}$$

$$G_{(\chi)}^{++5} = -2i(\theta^+)^4\theta^{-\alpha}\chi_{\alpha}^i u_i^- + 2i(\theta^+)^4\bar{\theta}^{-\dot{\alpha}}\bar{\chi}_{\dot{\alpha}}^i u_i^-, \quad (C12c)$$

$$\begin{aligned}
 G_{(\chi)}^{-5} = & 2i(\theta^-)^2(\bar{\theta}^+\bar{\theta}^-)\theta^{+\alpha}\chi_{\alpha}^i u_i^- - i(\theta^+)^2(\bar{\theta}^-)^2\theta^{-\alpha}\chi_{\alpha}^i u_i^- - i(\theta^-)^4\theta^{+\alpha}\chi_{\alpha}^i u_i^+ - 2(\theta^-)^4(\theta^+)^2\bar{\theta}^{+\dot{\rho}}\partial_{\rho}^{\dot{\alpha}}\chi_{\alpha}^i u_i^- \\
 & + -2i(\bar{\theta}^-)^2(\theta^+\theta^-)\bar{\theta}^{+\dot{\alpha}}\bar{\chi}_{\dot{\alpha}}^i u_i^- + i(\bar{\theta}^+)^2(\theta^-)^2\theta^{-\dot{\alpha}}\bar{\chi}_{\dot{\alpha}}^i u_i^- + i(\theta^-)^4\bar{\theta}^{+\dot{\alpha}}\bar{\chi}_{\dot{\alpha}}^i u_i^+ - 2(\theta^-)^4(\bar{\theta}^+)^2\theta^{+\rho}\partial_{\rho}^{\dot{\alpha}}\bar{\chi}_{\dot{\alpha}}^i u_i^-. \quad (C12d)
 \end{aligned}$$

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