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Erratum: Quark and gluon helicity evolution at small x: revised and updated

Florian Cougoulic,[a] **Yuri V. Kovchegov**,[b] **Andrey Tarasov**[b,c] **and Yossathorn Tawabutr**[b]

[a]*Department of Physics, University of Jyväskylä, P.O. Box 35, Survontie 9 C, Jyväskylä 40014, Finland*

[b]*Department of Physics, The Ohio State University, 191 W. Woodruff Ave, Columbus, OH 43210, U.S.A.*

[c]*Joint BNL-SBU Center for Frontiers in Nuclear Science (CFNS) at Stony Brook University, 100 Nicolls Road, Stony Brook, NY 11794, U.S.A.*

E-mail: florian.f.cougoulic@jyu.fi, kovchegov.1@osu.edu, tarasov.3@osu.edu, tawabutr.1@osu.edu

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Abstract: This erratum compiles the corrections for [1], which mainly result from several sign changes in the intermediate steps of the calculation. All the final results remain unchanged.

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The following corrections regarding the original paper should be taken into consideration:

- Eq. (3.16) should be modified by replacing pol $\to$ G in the superscripts.
- Eq. (3.29) should read:

$$\bar{v}_{\sigma_1}(k_1) \left(\hat{V}^\dagger_{\underline{w}}\right)^{ji} v_{\sigma_2}(k_2) \to 2\sqrt{k_1^- k_2^-} \int d^2 z \, \left(V^\dagger_{\underline{z},\underline{w};-\sigma_2,-\sigma_1}\right)^{ji}. \tag{1}$$

Specifically, the signs of σ_1 and σ_2 in the subscript of $V^\dagger$ have been reversed.

- Immediately before eq. (3.30), insert the following footnote:

"*The overall sign difference between our eq. (2) and eq. (15) in [3] is due to the need to correct the on-shell anti-quark factor in the latter such that $-\not{k}_2\, 2\pi\delta(k_2^2) \to \not{k}_2\, 2\pi\delta(k_2^2)$.*"

- Eq. (3.30) should read:

$$\begin{aligned} &g_{1L}^q(x,k_T^2) \\ &= \frac{2P^+}{(2\pi)^3} \int d^2\zeta\, d^2w\, d^2z\, \frac{d^2k_1\, dk_1^-}{(2\pi)^3}\, e^{i\underline{k}_1\cdot(\underline{w}-\underline{\zeta})+i\underline{k}\cdot(\underline{z}-\underline{\zeta})}\, \theta(k_1^-) \sum_{\sigma_1,\sigma_2} \bar{v}_{\sigma_2}(k_2) \tfrac{1}{2}\gamma^+\gamma^5 v_{\sigma_1}(k_1)\, 2\sqrt{k_1^- k_2^-} \\ &\times \left\langle \mathrm{T\,tr}\left[V_{\underline{\zeta}} V^\dagger_{\underline{z},\underline{w};-\sigma_2,-\sigma_1}\right]\right\rangle \frac{1}{\left[2k_1^- xP^+ + \underline{k}_1^2 - i\epsilon k_1^-\right]\left[2k_1^- xP^+ + \underline{k}^2 + i\epsilon k_1^-\right]}\Bigg|_{k_2^-=k_1^-,\, k_1^2=0,\, k_2^2=0,\, \underline{k}_2=-\underline{k}} + \text{c.c.} \end{aligned} \tag{2}$$

Besides the sign change in σ_1 and σ_2 in the subscript of $V^\dagger$, eq. (3.30) also receives an overall sign change, per the footnote mentioned above.

- Next, eq. (3.32) becomes

$$\begin{aligned} g_{1L}^q(x,k_T^2) = \frac{4P^+}{(2\pi)^3} &\int d^2\zeta\, d^2w\, d^2z\, \frac{d^2k_1\, dk_1^-}{(2\pi)^3}\, e^{i\underline{k}_1\cdot(\underline{w}-\underline{\zeta})+i\underline{k}\cdot(\underline{z}-\underline{\zeta})}\, \theta(k_1^-)\, \frac{1}{\underline{k}_1^2\, \underline{k}^2} \\ &\times \left[\underline{k}\cdot\underline{k}_1\, \delta^2(\underline{z}-\underline{w}) \left\langle \mathrm{T\,tr}\left[V_{\underline{\zeta}} V^{\mathrm{pol}[1]\,\dagger}_{\underline{w}}\right]\right\rangle + i\, \underline{k}\times\underline{k}_1 \left\langle \mathrm{T\,tr}\left[V_{\underline{\zeta}} V^{\mathrm{pol}[2]\,\dagger}_{\underline{z},\underline{w}}\right]\right\rangle\right] + \text{c.c.} \end{aligned} \tag{3}$$

The change is a direct consequence of the change outlined above to eq. (3.30).

- Similarly, eq. (3.33) also receives a sign correction and should now read

$$\begin{aligned} &g_{1L}^q(x,k_T^2) \\ &= -\frac{4iP^+}{(2\pi)^5} \int d^2\zeta\, d^2w \int_0^{p_2^-} dk_1^- \Bigg\{ e^{i\underline{k}\cdot(\underline{w}-\underline{\zeta})}\, \frac{\underline{k}}{\underline{k}^2}\cdot\frac{\underline{\zeta}-\underline{w}}{|\underline{\zeta}-\underline{w}|^2} \left\langle \mathrm{T\,tr}\left[V_{\underline{\zeta}} V^{\mathrm{pol}[1]\,\dagger}_{\underline{w}}\right] + \bar{\mathrm{T}}\,\mathrm{tr}\left[V^{\mathrm{pol}[1]}_{\underline{\zeta}} V^\dagger_{\underline{w}}\right]\right\rangle \\ &\quad + i\, \frac{\underline{k}}{\underline{k}^2}\times\frac{\underline{\zeta}-\underline{w}}{|\underline{\zeta}-\underline{w}|^2} \int d^2z \left\langle e^{i\underline{k}\cdot(\underline{z}-\underline{\zeta})}\, \mathrm{T\,tr}\left[V_{\underline{\zeta}} V^{\mathrm{pol}[2]\,\dagger}_{\underline{z},\underline{w}}\right] + e^{-i\underline{k}\cdot(\underline{z}-\underline{\zeta})}\, \bar{\mathrm{T}}\,\mathrm{tr}\left[V^{\mathrm{pol}[2]}_{\underline{z},\underline{w}} V^\dagger_{\underline{\zeta}}\right]\right\rangle\Bigg\}. \end{aligned} \tag{4}$$

- Subsequently, this results in an overall sign change in eqs. (3.37) and (3.38). However, due to another sign correction between eqs. (3.38) and (3.39), the latter equation and the ones that follow are correct as originally stated in [1].

JHEP09(2024)052

- Similar sign correction from eq. (1) of this document requires eq. (3.58) to be revised to:

$$\begin{aligned}\sigma^{\gamma^* p}(\lambda,\Sigma) = -\int \frac{d^2x_1\, d^2x_{1'}\, d^2x_0}{4\pi} \int\limits_0^1 \frac{dz}{z\,(1-z)} \\ \times \sum_{\sigma,\sigma',f} 2\,\mathrm{Re}\Big\{ \Psi^{\gamma^*\to q\bar q}_{\sigma,\sigma';\lambda}(\underline{x}_{10},z) \left[\Psi^{\gamma^*\to q\bar q}_{\sigma,\sigma';\lambda}(\underline{x}_{1'0},z)\right]^* \left\langle \mathrm{T\,tr}\left[V^{\mathrm{pol}}_{\underline{1}',\underline{1};\sigma,\sigma}\, V_{\underline{0}}^\dagger\right]\right\rangle(z) \\ - \Psi^{\gamma^*\to q\bar q}_{\sigma',\sigma;\lambda}(\underline{x}_{01},1-z)\left[\Psi^{\gamma^*\to q\bar q}_{\sigma',\sigma;\lambda}(\underline{x}_{01'},1-z)\right]^*\left\langle \mathrm{T\,tr}\left[V_{\underline{0}}\, V^{\mathrm{pol}\,\dagger}_{\underline{1}',\underline{1};-\sigma,-\sigma}\right]\right\rangle(z)\Big\}.\end{aligned} \tag{5}$$

- As a result, eq. (3.60) now reads:

$$\begin{aligned}\sigma^{\gamma^* p}(+,+) - \sigma^{\gamma^* p}(-,+) = -\sum_f \frac{2\,\alpha_{EM}\, Z_f^2}{\pi^2} \int d^2x_1\, d^2x_{1'}\, d^2x_0 \int\limits_0^1 dz \\ \times\ \mathrm{Re}\Big\{ -i\,[z^2+(1-z)^2]\, a_f^2\, \frac{\underline{x}_{10}\times\underline{x}_{1'0}}{x_{10}\, x_{1'0}}\, K_1(x_{10}\, a_f)\, K_1(x_{1'0}\, a_f) \left\langle \mathrm{T\,tr}\left[V^{\mathrm{G}[2]}_{\underline{1}',\underline{1}}\, V_{\underline{0}}^\dagger\right] - \mathrm{T\,tr}\left[V_{\underline{0}}\, V^{\mathrm{G}[2]\,\dagger}_{\underline{1}',\underline{1}}\right]\right\rangle(z) \\ + \delta^2(\underline{x}_{11'}) \left[(2z-1)\, a_f^2\, [K_1(x_{10}\, a_f)]^2 + m_f^2\, [K_0(x_{10}\, a_f)]^2\right] \left\langle \mathrm{T\,tr}\left[V^{\mathrm{pol}[1]}_{\underline{1}}\, V^\dagger_{\underline{0}}\right] + \mathrm{T\,tr}\left[V_{\underline{0}}\, V^{\mathrm{pol}[1]\,\dagger}_{\underline{1}}\right]\right\rangle(z)\Big\}.\end{aligned} \tag{6}$$

- Due to another sign correction, eq. (3.61) and the ones that follow are correct as originally stated in [1].

- Similar sign correction from eq. (1) of this document dictates that eq. (4.11) must be modified to:

$$\begin{aligned}\int\limits_{-\infty}^{0} dx^-_{2'} \int\limits_{0}^{\infty} dx_2^-\, \overline{\bar\psi^i_\alpha(x_2^-,\underline{x}_1)\, \psi^j_\beta}(x^-_{2'},\underline{x}_1) \\ = \sum_{\sigma,\sigma'} \int d^2x_2\, d^2x_{2'} \left[\int\limits_{-\infty}^{0} dx^-_{2'} \int \frac{d^4k_{2'}}{(2\pi)^4}\, e^{ik^+_{2'}x^-_{2'}}\, e^{i\underline{k}_{2'}\cdot\underline{x}_{2'1}}\, \frac{i}{k^2_{2'}+i\epsilon}\, (v_{\sigma'}(k_{2'}))_\beta\right] \\ \times \left[\left(-V^\dagger_{\underline{2},\underline{2}';-\sigma,-\sigma'}\right)^{ji} (2k_2^-)\,(2\pi)\,\delta(k_2^- - k^-_{2'})\right] \\ \times \left[\int\limits_0^\infty dx_2^- \int \frac{d^4k_2}{(2\pi)^4}\, e^{-ik_2^+x_2^-}\, e^{-i\underline{k}_2\cdot\underline{x}_{21}}\, \frac{i}{k_2^2+i\epsilon}\, (\bar v_\sigma(k_2))_\alpha\right].\end{aligned} \tag{7}$$

- As a result, eq. (4.12) should now read:

$$\begin{aligned}\int\limits_{-\infty}^{0} dx^-_{2'} \int\limits_{0}^{\infty} dx_2^-\, \overline{\bar\psi^i_\alpha(x_2^-,\underline{x}_1)\, \psi^j_\beta}(x^-_{2'},\underline{x}_1) \\ = -\frac{1}{\pi}\sum_{\sigma} \int dk^-\, k^- \int d^2x_2\, d^2x_{2'} \left[\int \frac{d^2k_{2'}}{(2\pi)^2}\, e^{i\underline{k}_{2'}\cdot\underline{x}_{2'1}}\, \frac{1}{\underline{k}^2_{2'}}\, (v_\sigma(k_{2'}))_\beta\right] \\ \times \left(\sigma\, V^{\mathrm{pol}[1]\,\dagger}_{\underline{2}}\, \delta^2(\underline{x}_{22'}) - V^{\mathrm{pol}[2]\,\dagger}_{\underline{2},\underline{2}'}\right)^{ji} \left[\int \frac{d^2k_2}{(2\pi)^2}\, e^{-i\underline{k}_2\cdot\underline{x}_{21}}\, \frac{1}{\underline{k}_2^2}\, (\bar v_\sigma(k_2))_\alpha\right].\end{aligned} \tag{8}$$

- Due to another sign correction, there is no change to eq. (4.13) and beyond.

JHEP09(2024)052

- Finally, a separate minor correction is warranted for eq. (4.38), which should now read:

$$G_{10}(zs) = G_{10}^{(0)}(zs) + \frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int d^2 x_2 \tag{9}$$
$$\times \Bigg\{ 2 \left[\frac{1}{x_{21}^2} - \frac{\underline{x}_{21}}{x_{21}^2} \cdot \frac{\underline{x}_{20}}{x_{20}^2} \right] \left[S_{20}(z's)\, G_{21}(z's) + S_{21}(z's)\, \Gamma_{20,21}^{\text{gen}}(z's) \right]$$
$$+ \left[2 \frac{\epsilon^{ij}\, x_{21}^j}{x_{21}^4} - \frac{\epsilon^{ij}\,(x_{20}^j + x_{21}^j)}{x_{20}^2\, x_{21}^2} - \frac{2\, \underline{x}_{20} \times \underline{x}_{21}}{x_{20}^2\, x_{21}^2} \left(\frac{x_{21}^i}{x_{21}^2} - \frac{x_{20}^i}{x_{20}^2} \right) \right]$$
$$\times \left[S_{20}(z's)\, G_{21}^i(z's) + S_{21}(z's)\, \Gamma_{20,21}^{i\,\text{gen}}(z's) \right]$$
$$+ \frac{x_{10}^2}{x_{21}^2\, x_{20}^2} \left[S_{20}(z's)\, G_{12}(z's) - \Gamma_{10,21}^{\text{gen}}(z's) \right] \Bigg\}.$$

This correction does not propagate.

References

[1] F. Cougoulic, Y.V. Kovchegov, A. Tarasov and Y. Tawabutr, *Quark and gluon helicity evolution at small x: revised and updated*, *JHEP* **07** (2022) 095 [arXiv:2204.11898] [INSPIRE].

JHEP09(2024)052