Published for SISSA by Springer

Received: November 18, 2025
Accepted: December 31, 2025
Published: February 13, 2026

Maxwell Chern-Simons gravity in 3D: thermodynamics of cosmological solutions and black holes with torsion

Luis Avilés[a,b] **Patrick Concha**[c,d] **Javier Matulich**[e,d] **Evelyn Rodríguez**[c,d] **and David Tempo**[a,b]

[a] *Instituto de Ciencias Exactas y Naturales (ICEN), Universidad Arturo Prat, Playa Brava 3256, 1111346 Iquique, Chile*
[b] *Facultad de Ciencias, Universidad Arturo Prat, Avenida Arturo Prat Chacón 2120, 1110939 Iquique, Chile*
[c] *Departamento de Matemática y Física Aplicadas, Universidad Católica de la Santísima Concepción, Alonso de Ribera 2850, Concepción, Chile*
[d] *Grupo de Investigación en Física Teórica, GIFT, Universidad Católica de la Santísima Concepción, Alonso de Ribera 2850, Concepción, Chile*
[e] *Instituto de Física Teórica UAM/CSIC, Universidad Autónoma de Madrid, C/ Nicolás Cabrera 13-15, Cantoblanco, Madrid 28049, Spain*

E-mail: luaviles@unap.cl, patrick.concha@ucsc.cl, javier.matulich@csic.es, erodriguez@ucsc.cl, jtempol@unap.cl

Abstract: We construct generalized sets of asymptotic conditions for both three-dimensional Maxwell Chern-Simons gravity and a novel extension that incorporates torsion through a deformation of the Maxwell algebra. These boundary conditions include the most general temporal components of the gauge fields that consistently preserve the corresponding asymptotic Maxwell algebras with identical classical central charges, while allowing for the inclusion of chemical potentials conjugate to the conserved charges. We show that both sets of asymptotic configurations admit nontrivial solutions carrying not only mass and angular momentum but also an additional global spin-2 charge. In the torsionless case, the theory admits locally flat cosmological spacetimes, whereas in the presence of torsion, it generalizes to BTZ-like black hole geometries. For each case, the thermodynamic properties are consistently derived in terms of the gauge fields and the topology of the Euclidean manifold, shown to correspond to a solid torus. Furthermore, we obtain a general expression for the entropy, depending on both the horizon area and its spin-2 analogues, which can be written as a reparametrization-invariant integral of the induced spin-2 fields on the spacelike section of the horizon.

Keywords: Classical Theories of Gravity, Space-Time Symmetries, Chern-Simons Theories, Gauge Symmetry

ArXiv ePrint: 2510.25559

https://doi.org/10.1007/JHEP02(2026)143

JHEP02(2026)143

Contents

JHEP02(2026)143

1 Introduction

Three-dimensional gravity provides a fertile setting for exploring conceptual and structural aspects of gravitational theories. Its topological nature and the absence of local degrees of freedom simplify the dynamics, allowing for the explicit construction of exact solutions and a detailed analysis of their physical properties. Notably, when formulated as a Chern-Simons (CS) gauge theory, gravity in three dimensions provides a natural frame in which symmetries, topology, and geometry are intrinsically linked [1]. The three-dimensional CS gravity framework constitutes a powerful theoretical laboratory for exploring various aspect of higher-dimensional gravity theories, including black hole solutions and their associated thermodynamics properties [2–5].

The inclusion of additional gauge symmetries has proven to be a fruitful avenue for extending General Relativity (GR). In particular, the formulation of CS (super)gravity theories through extensions, expansions, and deformations of the Poincaré and AdS symmetries has led to significant results [6–8]. Within this framework, the Maxwell algebra [9–11], which corresponds to an extension and deformation of the Poincaré algebra [12], has attracted considerable attention and has been studied in various contexts [13–33]. Its incorporation into the CS formulation of gravity give rise to Maxwell CS gravity [8], which is characterized by a new gauge field denoted as the gravitational Maxwell field.[1] The corresponding field equations derived from the CS action comprise the Poincaré ones, describing asymptotically flat and torsionless geometries, together with an additional equation involving the gravitational Maxwell field.

[1]The CS theory based on Maxwell algebra in $2+1$ was initially considered in [13, 34] as a notable model leading to the two-dimensional linear gravities referred in [35–39] by a dimensional reduction.

The Maxwell CS gravity theory can alternatively be obtained as a vanishing cosmological constant limit from two different gravity theories: the so-called AdS-Lorentz gravity [40–42], whose underlying symmetry corresponds to a semi-simple extension of the Poincaré algebra, and the Maxwell algebra with torsion [30], which is invariant under the deformed Maxwell algebra introduced in [27]. Although both symmetries allow the inclusion of a negative cosmological constant in the CS actions, the resulting gravity theories are physically distinct. Specifically, Maxwell CS with torsion can be regarded as a Maxwellian extension of Teleparallel gravity [43–46] when the deformation parameter is fixed to $\varepsilon = -2/\ell$, with ℓ related to the cosmological constant by $\Lambda = -\frac{1}{\ell^2}$. Notably, under this identification, the cosmological constant effectively acts as a source of torsion.

In the study of asymptotic symmetries, Maxwell CS gravity has been shown to possess a modified asymptotic structure, leading to an enlargement of the $\mathfrak{bms}_3$ algebra [47–49], commonly denoted as $\mathfrak{max}$-$\mathfrak{bms}_3$ [24]. This extended $\mathfrak{bms}_3$ algebra can also be recovered as a flat limit of three copies of the Virasoro algebra [21, 42],[2] which appears as the asymptotic symmetry algebra of the AdS-Lorentz CS gravity [42]. Moreover, when torsion is switched on within the Maxwell CS gravity framework, the asymptotic symmetry algebra undergoes an infinite-dimensional enhancement, taking the form of $\mathfrak{bms}_3 \oplus \mathfrak{vir}$, which can be interpreted as an extended version of the deformed Maxwell algebra [27].

Despite their algebraic richness and theoretical appeal, the thermodynamic properties of Maxwell solutions have remained largely unexplored. In this work, we address this gap by analyzing the entropy and thermodynamic consistency of asymptotically flat cosmological solutions in three-dimensional Maxwell CS theory. We show that the gravitational Maxwell field modifies the structure of canonical generators such that solutions are endowed not only with mass and angular momentum but also with an additional global spin-2 charge, which contributes to the first law of thermodynamics, thus providing new insights into the structure of the theory. In particular, the entropy we obtain extends the standard Bekenstein-Hawking formula of general relativity reported in [50, 51]. We then consider an extension of the theory by including non-vanishing torsion, achieved via a deformation of the Maxwell algebra that leads to a new CS theory of gravity with non-Riemannian geometry. As we have said before, this torsional theory possesses a modified asymptotic symmetry algebra and admits a novel black hole solution, which we construct explicitly. We compute its conserved charges and entropy, demonstrating that the solution is a natural generalization of the BTZ-like black hole with torsion [52–56]. Importantly, we show that the previously discussed cosmological solution is recovered as the limit $\varepsilon \to 0$ of this black hole configuration. This limit can be interpreted as a vanishing cosmological constant limit, i.e. $\ell \to \infty$, when we set $\varepsilon = -2/\ell$. To the best of our knowledge, this is the first explicit construction of a black hole solution with torsion in Maxwell,[3] along with a complete thermodynamic analysis in the torsional and torsionless sectors.

[2] Also described as a semi-simple enlargement of the $\mathfrak{bms}_3$ algebra.

[3] A complete study of the stationary black hole solution, along with its thermodynamic properties, of a CS action based on the semi-simple extension of the Poincaré gauge group (or AdS-Lorentz) was considered in [57].

This paper is organized as follows. In section 2, we review Maxwell CS gravity and study the thermodynamics of its asymptotically flat cosmological solutions. Section 3 introduces the torsional extension and analyzes its asymptotic symmetries and black hole thermodynamics. Section 4 concludes our work with a discussion of our results and future directions.

2 Three-dimensional Maxwell Chern-Simons gravity

In this section, we briefly review the so-called Maxwell CS gravity theory formulated using the Chern-Simons formalism. We review the asymptotic structure of the theory, whose algebra is given by an extension and deformation of the $\mathfrak{bms}_3$ algebra [24], and is denoted here by $\mathfrak{max}$-$\mathfrak{bms}_3$. We also address the explicit computation of the entropy for an asymptotically flat cosmological solution. The Maxwell algebra generated by $\{J_a, P_a, Z_a\}$ is given by the following non-vanishing commutators:

$$
\begin{aligned}
[J_a, J_b] &= \epsilon_{abc} J^c\,, & [J_a, P_b] &= \epsilon_{abc} P^c\,, \\
[J_a, Z_b] &= \epsilon_{abc} Z^c\,, & [P_a, P_b] &= \epsilon_{abc} Z^c\,.
\end{aligned} \tag{2.1}
$$

where $a, b, \cdots = 0, 1, 2$ are Lorentz indices raised and lowered with the Minkowski metric η_{ab} and ϵ_{abc} is the Levi-Civita tensor. In order to write down a CS action for this algebra, we define the one-form gauge connection

$$
A = \omega^a J_a + e^a P_a + \sigma^a Z_a\,, \tag{2.2}
$$

where e^a, ω^a and σ^b correspond to the vielbein, spin connection, and gravitational Maxwell field, respectively.

The non-vanishing components of the invariant tensor are given by:

$$
\begin{aligned}
\langle J_a J_b \rangle &= \alpha_0 \eta_{ab}\,, & \langle P_a P_b \rangle &= \alpha_2 \eta_{ab}\,, \\
\langle J_a P_b \rangle &= \alpha_1 \eta_{ab}\,, & \langle J_a Z_b \rangle &= \alpha_2 \eta_{ab}\,,
\end{aligned} \tag{2.3}
$$

where α_0, α_1 and α_2 are real dimensionless constants. The bilinear form is non-degenerate provided that $\alpha_2 \neq 0$. Then, considering the previous invariant tensor and the one-form gauge connection (2.2) in the CS action

$$
I[A] = \frac{k}{4\pi} \int_{\mathcal{M}} \left\langle A dA + \frac{2}{3} A^3 \right\rangle\,, \tag{2.4}
$$

defined on a three-dimensional manifold $\mathcal{M}$, and where $k = \frac{1}{4G}$ is the level of the theory related to the gravitational constant G, we obtain

$$
I_{\mathfrak{max}} = \frac{k}{4\pi} \int \alpha_0 \left(\omega^a d\omega_a + \frac{1}{3} \epsilon_{abc} \omega^a \omega^b \omega^c \right) + 2\alpha_1 R_a e^a + \alpha_2 \left(e^a T_a + 2\sigma^a R_a \right)\,. \tag{2.5}
$$

Here $R^a = d\omega^a + \frac{1}{2}\epsilon^{abc}\omega_b\omega_c$ and $T^a = de^a + \epsilon^{abc}\omega_b e_c$, are the usual Lorentz curvature and torsion two-forms. The first term along α_0 in the action (2.5) is the gravitational CS term. Next term with coupling constant α_1 is the Einstein-Hilbert term. The last term, with the strength of interaction α_2, gives the dynamics to the gravitational Maxwell field and

JHEP02(2026)143

also contributes to the dynamics of other fields. Indeed, when $\alpha_2 \neq 0$, the field equations are given by

$$
\begin{aligned}
R^a &= 0\,,\\
T^a &= 0\,,\\
D\sigma^a + \frac{1}{2}\epsilon^{abc}e_b e_c &= 0\,,
\end{aligned}
\tag{2.6}
$$

where $D\Phi^a = d\Phi^a + \epsilon^a{}_{bc}\omega^b\Phi^c$ denotes the Lorentz covariant derivative. These field equations describe geometries which are Riemannian and locally flat, as it can be seen from the above first two expressions. The third equation shows the dynamics of the gravitational Maxwell field σ^a, referred here as a "spin-2" field [29].[4] As it was shown in [24] the coupling of this one-form field to the geometry leads to nontrivial effects compared to GR that can be regarded as a deformation [29] of the "exotic" Einstein gravity [58]. In what follows the asymptotic structure of the three-dimensional Maxwell CS gravity is explored including the thermodynamics features of locally flat cosmological spacetimes.

2.1 Asymptotic conditions with chemical potentials

Here we review and generalize the set of boundary conditions for Maxwell CS gravity in [24] to incorporate chemical potentials conjugated to the conserved charges. The study of the behavior of dynamic fields in the asymptotic region become crucial for computing the thermodynamic properties configurations with a sensible thermodynamics.

As explained in [59] the radial dependence of the gauge field can be entirely gauged away asymptotically by virtue of a suitable choice of a gauge group element $g = g\,(r)$, so that $A = g^{-1}ag + g^{-1}dg$, where a is an auxiliary connection given by

$$
a = a_\phi\,(t,\phi)\,d\phi + a_t\,(t,\phi)\,dt\ , \tag{2.7}
$$

The asymptotic behavior for a_ϕ can be written as follows [24, 60][5]

$$
a_\phi = J_1 + \frac{1}{2}\,\mathcal{M}\,J_0 + \frac{1}{2}\mathcal{N}\,P_0 + \frac{1}{2}\mathcal{F}\,Z_0\,, \tag{2.8}
$$

where $\mathcal{N}, \mathcal{M}$, and $\mathcal{F}$ stand for arbitrary functions of the boundary coordinates (t, ϕ). The asymptotic form of a_ϕ is preserved under a restricted set of gauge transformations, $\delta a = d\lambda + [a, \lambda]$, with the Lie-algebra-valued parameter $\lambda_{(0)} = \lambda_{(0)}[y, f, h]$ generated by

$$
\begin{aligned}
\lambda_{(0)}[y,f,h] = yJ_1 - y'J_2 + fP_1 - f'P_2 + hZ_1 - h'Z_2 + \left(\frac{\mathcal{M}}{2}f + \frac{\mathcal{N}}{2}y - f''\right)P_0 \\
+ \left(\frac{\mathcal{M}}{2}y - y''\right)J_0 + \left(\frac{1}{2}\mathcal{M}h + \frac{1}{2}\mathcal{F}y + \frac{1}{2}\mathcal{N}f - h''\right)Z_0\,,
\end{aligned}
\tag{2.9}
$$

[4] Here, the statement that the gravitational Maxwell field σ^a describes a "spin-2" field is based on the fact that in the metric formulation only the symmetric part remains so that its field equation reduces to the massless Fierz-Pauli equation.

[5] Hereafter, our conventions are such that the nonvanishing components of the Minkowski metric in tangent space η_{ab} read $\eta_{01} = \eta_{10} = \eta_{22} = 1$, and the Levi-Civita symbol fulfills $\epsilon_{012} = 1$.

provided that the functions $\mathcal{M}, \mathcal{N}$ and $\mathcal{F}$ transform according to:

$$\begin{aligned}
\delta_{(0)}\mathcal{M} &= \mathcal{M}'y + 2\mathcal{M}y' - 2y''' \,, \\
\delta_{(0)}\mathcal{N} &= \mathcal{M}'f + 2\mathcal{M}f' - 2f''' + \mathcal{N}'y + 2\mathcal{N}y' \,, \\
\delta_{(0)}\mathcal{F} &= \mathcal{M}'h + 2\mathcal{M}h' - 2h''' + \mathcal{N}'f + 2\mathcal{N}f' + \mathcal{F}'y + 2\mathcal{F}y' \,,
\end{aligned} \tag{2.10}$$

where $y = y(\phi, t)$, $f = f(\phi, t)$, and $h = h(\phi, t)$ are arbitrary functions and prime denotes derivative with respect to ϕ. Following refs. [61, 62], the asymptotic symmetries are preserved under time evolution by choosing the asymptotic form of the Lagrange multiplier a_t according to

$$a_t = \lambda_{(0)}[\mu, \xi, \nu] \,, \tag{2.11}$$

with λ defined through (2.9) and where μ, ξ, and ν stand for arbitrary functions of the boundary coordinates (t, ϕ) which are assumed to be fixed at the boundary. In the thermodynamic analysis these functions are identified with the "chemical potentials" conjugated to the charges [61]. The asymptotic form of the temporal components of the connection in (2.11) is maintained by the asymptotic symmetries provided both that the field equations are fulfilled in the asymptotic region, i.e.,

$$\begin{aligned}
\dot{\mathcal{M}} &= 2\mathcal{M}\mu' + \mathcal{M}'\mu - 2\mu''' \,, \\
\dot{\mathcal{N}} &= 2\mathcal{M}\xi' + \mathcal{M}'\xi + 2\mathcal{N}\mu' + \mathcal{N}'\mu - 2\xi''' \,, \\
\dot{\mathcal{F}} &= 2\mathcal{M}\nu' + \mathcal{M}'\nu + 2\mathcal{N}\xi' + \mathcal{N}'\xi + 2\mathcal{F}\mu' + \mathcal{F}'\mu - 2\nu''' \,,
\end{aligned} \tag{2.12}$$

and that parameters μ, ξ, and ν, which describe the asymptotic symmetries, satisfy suitable differential equations of first order in time given by

$$\begin{aligned}
&\dot{y} = y\mu' - \mu y' \; ; \qquad \dot{\xi} = f\mu' - \mu f' + y\xi' - \xi y' \,, \\
&\dot{\nu} = h\mu' - h'\mu + f\xi' - f'\xi + y\nu' - y'\nu \,.
\end{aligned} \tag{2.13}$$

The asymptotic symmetry generators can be computed in the Regge-Teitelboim approach [63, 64] such that their variations reduce to

$$\delta Q[\lambda] = -\frac{k}{2\pi} \int\limits_{\partial\Sigma} \langle \lambda \delta a \rangle \, d\phi \,, \tag{2.14}$$

where $\partial\Sigma$ stands for the boundary of the spacelike section Σ. The explicit form of the generators associated to the asymptotic symmetries can be integrated and are found to be given by

$$Q[y, f, h] = -\int d\phi \, (y\mathcal{J} + f\mathcal{P} + h\mathcal{Z}) \,, \tag{2.15}$$

where the dynamical fields $\mathcal{J}$, $\mathcal{P}$ and $\mathcal{Z}$ are determined in terms of the functions $\mathcal{N}, \mathcal{M}$, and $\mathcal{F}$, according to the relations:

$$\mathcal{J} = \frac{k}{4\pi} \left(\alpha_2\mathcal{F} + \alpha_1\mathcal{N} + \alpha_0\mathcal{M}\right) \; ; \qquad \mathcal{P} = \frac{k}{4\pi} \left(\alpha_2\mathcal{N} + \alpha_1\mathcal{M}\right) \; ; \qquad \mathcal{Z} = \frac{k}{4\pi} \alpha_2\mathcal{M} \,. \tag{2.16}$$

The transformations law of the dynamical variables $\{\mathcal{J}, \mathcal{P}, \mathcal{Z}\}$ are found using (2.16) and (2.10) and they explicitly read

$$\begin{aligned}
\delta\mathcal{J} &= 2\mathcal{J}y' + \mathcal{J}'y + 2\mathcal{P}f' + \mathcal{P}'f + 2\mathcal{Z}h' + \mathcal{Z}'h - \frac{\alpha_0 k}{2\pi}y''' - \frac{\alpha_1 k}{2\pi}f''' - \frac{\alpha_2 k}{2\pi}h''' \,, \\
\delta\mathcal{P} &= 2\mathcal{P}y' + \mathcal{P}'y + 2\mathcal{Z}f' + \mathcal{Z}'f - \frac{\alpha_1 k}{2\pi}y''' - \frac{\alpha_2 k}{2\pi}f''' \,, \\
\delta\mathcal{Z} &= 2\mathcal{Z}y' + \mathcal{Z}'y - \frac{\alpha_2 k}{2\pi}y''' \,.
\end{aligned} \tag{2.17}$$

The algebra of the conserved charges (2.15) can be readily obtained from the transformation law of the dynamical fields in (2.17) by virtue of $\delta_{\lambda_2}Q[\lambda_1] = \{Q[\lambda_1], Q[\lambda_2]\}$. Thus, expanding in Fourier modes according to $X = \frac{1}{2\pi}\sum X_m e^{im\phi}$, and providing that the zero modes X_0 being shifted as $X_0 \to X_0 + \frac{c_i}{24}$, the nontrivial Poisson brackets are given by [24, 60]

$$\begin{aligned}
i\{\mathcal{J}_m, \mathcal{J}_n\} &= (m-n)\,\mathcal{J}_{m+n} + c_{\mathcal{J}}\, m(m^2-1)\delta_{m+n,0} \,, \\
i\{\mathcal{J}_m, \mathcal{P}_n\} &= (m-n)\,\mathcal{P}_{m+n} + c_{\mathcal{P}}\, m(m^2-1)\delta_{m+n,0} \,, \\
i\{\mathcal{J}_m, \mathcal{Z}_n\} &= (m-n)\,\mathcal{Z}_{m+n} + c_{\mathcal{Z}}\, m(m^2-1)\delta_{m+n,0} \,, \\
i\{\mathcal{P}_m, \mathcal{P}_n\} &= (m-n)\,\mathcal{Z}_{m+n} + c_{\mathcal{Z}}\, m(m^2-1)\delta_{m+n,0} \,,
\end{aligned} \tag{2.18}$$

where the central extensions $c_{\mathcal{J}}$, $c_{\mathcal{P}}$ and $c_{\mathcal{Z}}$ are fully determined in terms of the constants of the action (2.5) according to

$$c_{\mathcal{J}} = k\alpha_0 \qquad c_{\mathcal{P}} = k\alpha_1 \,, \qquad c_{\mathcal{Z}} = k\alpha_2 \,. \tag{2.19}$$

It is worth noting that this asymptotic symmetry algebra corresponds to an extension and deformation of $\mathfrak{bms}_3$ algebra [47–49] at which both the abelian generators $\mathcal{Z}_n$ and the generators $\mathcal{P}_n$ have conformal weight 2. This deformation introduces an additional central extension $c_{\mathcal{Z}}$ which is proportional to the coupling constant of the Maxwell gravitational action. It also is reassuring that the asymptotic symmetry algebra (2.18) contains the wedge algebra in (2.1) for generators X_m, with $m = -1, 0, 1$, providing $i\{,\} \to [,]$.

It should be noted that gauge field configurations with $\mathcal{M}$, $\mathcal{N}$, $\mathcal{F}$, as well as the chemical potentials ξ, μ, ν fixed to constants, solve the field equations in (2.12). Indeed, this class of configurations explicitly reads

$$\begin{aligned}
a = &\left(J_1 + \frac{1}{2}\mathcal{M}\,J_0 + \frac{1}{2}\mathcal{N}\,P_0 + \frac{1}{2}\mathcal{F}\,Z_0\right)d\phi + \Big[\mu J_1 + \frac{\mu}{2}\mathcal{M}J_0 \\
&+ \frac{1}{2}\left(\xi\mathcal{M} + \mu\mathcal{N}\right)P_0 + \xi P_1 + \frac{1}{2}\left(\nu\mathcal{M} + \mu\mathcal{F} + \xi\mathcal{N}\right)Z_0 + \nu Z_1\Big]\,dt \,,
\end{aligned} \tag{2.20}$$

In appendix A by restoring the radial dependence of the gauge fields, it shown that this stationary solution describes a locally flat cosmological spacetime endowed with a spin-2 gauge field, named here as the Maxwell CS field. It also follows from (2.15) that this solution not only possesses mass and angular momentum, being parametrized by the spin-2 charges $\mathcal{P}$, $\mathcal{J}$, respectively, but also has an extra global charge of spin 2, determined by $\mathcal{Z}$. In the next section thermodynamic properties of this solution is analyzed.

2.2 Thermodynamics of asymptotically flat cosmological solutions

This section is devoted to the study of the thermodynamics of cosmological solutions in the Maxwell CS gravity theory. In this context, following the canonical approach the entropy can be suitably computed along the lines of [62, 65], by the general formula

$$\begin{aligned} S &= \frac{k}{2\pi}\left[\int_{r_+} d\tau d\phi \langle A_\tau A_\phi\rangle\right]_{\text{on-shell}} \\ &= k\left[\langle a_\tau a_\phi\rangle\right]_{\text{on-shell}} \ , \end{aligned} \tag{2.21}$$

where A_τ stands for the Euclidean continuation of A_t by the replacement $\tau = it$, and where its final form is obtained by imposing the regularity conditions that require the holonomies around the thermal circle to be trivial.

For the field configuration in (2.20) the entropy reads

$$S = k\left[\alpha_0\mu\mathcal{M} + \alpha_1\left(\mu\mathcal{N} + \xi\mathcal{M}\right) + \alpha_2\left(\nu\mathcal{M} + \xi\mathcal{N} + \mu\mathcal{F}\right)\right] \ , \tag{2.22}$$

where it is assumed that the chemical potentials (ξ, μ, ν) are constrained to fulfill regularity conditions. Since all the chemical potentials are explicitly incorporated along the temporal components of the gauge fields, the analysis can be carried out for a fixed range of the angular coordinates of the torus, i.e., we assume that $0 < \tau \leq 1$, and $0 < \phi \leq 2\pi$. Thus, these regularity conditions, can be computed following the prescription in [65], which in our case consists in finding a permissible gauge transformation in (2.20), that allows to gauge away the components along the generators P_a and Z_a. The latter implies that a_t reduces such that it only takes values in the Lorentz subalgebra of the gauge group spanned by the Maxwell algebra (2.1), so that the remaining regularity condition results in requiring that the holonomy along the thermal circle to be trivial.

In the present case, the required group element reads

$$g = e^{\lambda P_2 + \rho Z_2} \ , \tag{2.23}$$

so that the angular and temporal components of the gauge field now read

$$\begin{aligned} a_\phi &= J_1 + \frac{1}{2}\mathcal{M}J_0 + \lambda P_1 + \left(\frac{\lambda^2}{2} + \rho\right)Z_1 + \frac{1}{2}\left(\mathcal{N} - \lambda\mathcal{M}\right)P_0 + \frac{1}{2}\left[\mathcal{F} + \left(\frac{\lambda^2}{2} - \rho\right)\mathcal{M} - \lambda\mathcal{N}\right]Z_0 \, , \\ a_t &= \mu\left(J_1 + \frac{1}{2}\mathcal{M}J_0\right) + (\xi + \mu\lambda)P_1 + \left[\nu + \xi\lambda + \frac{1}{2}\mu(\lambda^2 + 4\rho)\right]Z_1 + \frac{1}{2}\left(\xi\mathcal{M} + \mu\mathcal{N} - \mu\lambda\mathcal{M}\right)P_0 \\ &\quad + \frac{1}{4}\left[2\mu\mathcal{F} + \left(2\nu - \xi\lambda + \mu\lambda^2 - 4\mu\rho\right)\mathcal{M} + 2\left(\xi - \mu\lambda\right)\mathcal{N}\right]Z_0 \, , \end{aligned} \tag{2.24}$$

from which it follows that the contributions along the generators P_1 and Z_1 in (2.24) can be removed provided that λ and ρ are chosen as follow

$$\lambda = -\frac{\xi}{\mu} \, , \qquad \rho = \frac{1}{2}\frac{\xi^2}{\mu^2} - \frac{\nu}{\mu} \, . \tag{2.25}$$

This subsequently allows for fixing the Lagrange multiplier parameters μ and ν (chemical potentials) according to,

$$\mu = -2\frac{\mathcal{M}}{\mathcal{N}}\xi \, , \qquad \nu = \left(\frac{\mathcal{F}}{\mathcal{N}} - \frac{3}{4}\frac{\mathcal{N}}{\mathcal{M}}\right)\xi \, , \tag{2.26}$$

the terms along P_0 and Z_0 in the temporal component of the gauge field in (2.24) vanish so that

$$a_t = -2\xi\frac{\mathcal{M}}{\mathcal{N}}\left(J_1 + \frac{1}{2}\mathcal{M}J_0\right) . \tag{2.27}$$

Hence, imposing that the holonomy of the spin connection around the thermal circle is trivial,

$$H = e^{a_\tau}\Big|_{\text{on-shell}} = (-1)^{n_\mathfrak{m}}\mathbb{1}_{2\times 2} , \tag{2.28}$$

in the fundamental representation of $sl(2,\mathbb{R})$, this requirement leads to

$$\xi = -\frac{n_\mathfrak{m}\pi\mathcal{N}}{\mathcal{M}^{3/2}} , \tag{2.29}$$

where $n_\mathfrak{m}$ is an integer. It is important to note that, since we are dealing with a cosmological horizon, the orientation of the solid torus is reversed relative to that of the black hole. As a result, the chemical potential ξ is such that $\xi = -1/T_C$. This choice is consistent with the requirement that the Hawking temperature be positive, fixing $n_\mathfrak{m}$ to be positive or negative, depending on the sign of the function $\mathcal{N}$.

In summary, the regularity conditions of the Euclidean gauge fields are successfully implemented, leading to the chemical potentials being fixed as specific functions of the global charges, according to (2.16), as follows

$$\xi = -\frac{n_\mathfrak{m}\pi\mathcal{N}}{\mathcal{M}^{3/2}} , \qquad \mu = \frac{2\pi n_\mathfrak{m}}{\mathcal{M}^{1/2}} , \qquad \nu = -n_\mathfrak{m}\pi\left[\frac{\mathcal{F}}{\mathcal{M}^{3/2}} - \frac{3\mathcal{N}^2}{4\mathcal{M}^{5/2}}\right] . \tag{2.30}$$

It is reassuring that for $n_\mathfrak{m} = \mathrm{sgn}(\mathcal{N})$ the above result for the regularity conditions is in complete agreement with those found in the metric formalism (see appendix A), which stem from requiring the absence of conical singularities of the Euclidean fields at the cosmological horizon, as it is shown in (A.19). In fact, it is found that the existence of the cosmological horizon implies that $\mathcal{M} > 0$, and that the Euclidean fields possess around the horizon the topology of a solid torus leads to the condition:

$$\frac{\mathcal{N}}{\mathcal{M}}\left(\mathcal{F} - \frac{\mathcal{N}^2}{4\mathcal{M}}\right) > 0 . \tag{2.31}$$

The entropy can then be readily obtained by plugging the chemical potentials (2.30) in eq. (2.22), so in terms of the variables $\mathcal{M}$, $\mathcal{N}$ and $\mathcal{F}$, it reads

$$S = 2\pi n_\mathfrak{m} k\left[\alpha_0\sqrt{\mathcal{M}} + \alpha_1\frac{\mathcal{N}}{2\sqrt{\mathcal{M}}} + \frac{\alpha_2}{2}\left(\frac{\mathcal{F}}{\sqrt{\mathcal{M}}} - \frac{\mathcal{N}^2}{4\mathcal{M}^{3/2}}\right)\right] . \tag{2.32}$$

This result for the entropy not only extends the one for General Relativity (along α_1) [50, 51, 65] in the presence of the CS term ($\alpha_0 \neq 0$) [66, 67], but also points out an explicit contribution (along α_2) of the gravitational Maxwell field to the entropy of the cosmological spacetimes in this gravity theory.

In terms of the (extensive) global charges $\mathcal{J}, \mathcal{P}$ and $\mathcal{Z}$ defined through eqs. (2.16), the entropy in (2.32) can be rewritten as follows

$$S_C = 2\pi n_\mathfrak{m}\sqrt{\pi k}\sqrt{\alpha_2}\left[\frac{\mathcal{J}}{\sqrt{\mathcal{Z}}} + \frac{\alpha_0}{\alpha_2}\sqrt{\mathcal{Z}} - \frac{1}{4}\left(\frac{\mathcal{P}}{\mathcal{Z}} - \frac{\alpha_1}{\alpha_2}\right)^2\sqrt{\mathcal{Z}}\right] . \tag{2.33}$$

In order to have a sensible thermodynamics, the entropy must be a real function, therefore $\mathcal{Z} > 0$ as well as the coupling constants $\alpha_i \geq 0$ with $i = 0, 1, 2$. In addition, positivity of the entropy implies that the angular momentum has to be such that for $n_{\mathfrak{m}} > 0$ then $\mathcal{J} > \mathcal{J}_c$, and for $n_{\mathfrak{m}} < 0$ then $\mathcal{J} < \mathcal{J}_c$, where[6]

$$\mathcal{J}_c = \frac{(\alpha_2 \mathcal{P} - \alpha_1 \mathcal{Z})^2 - 4\alpha_0 \alpha_2 \mathcal{Z}^2}{4\alpha_2^2 \mathcal{Z}} \,. \tag{2.34}$$

Thus, in order to determine the temperature and the chemical potentials in the micro-canonical ensemble, we use the thermodynamic relations

$$\beta_C = \left(\frac{\partial S_C}{\partial \mathbb{M}}\right)\Big|_{\mathbb{J},\mathbb{W}} = -T_C^{-1} = -n_{\mathfrak{m}} \frac{1}{2}\sqrt{\frac{\pi \alpha_2 k}{\mathcal{Z}}}\left(\frac{\mathcal{P}}{\mathcal{Z}} - \frac{\alpha_1}{\alpha_2}\right) \,, \tag{2.35}$$

$$\Omega_C = -\,\beta_C^{-1}\left(\frac{\partial S_C}{\partial \mathbb{J}}\right)\Big|_{\mathbb{M},\mathbb{W}} = -2\left(\frac{\mathcal{P}}{\mathcal{Z}} - \frac{\alpha_1}{\alpha_2}\right)^{-1} \,, \tag{2.36}$$

$$\Phi_C = -\,\beta_C^{-1}\left(\frac{\partial S_C}{\partial \mathbb{Z}}\right)\Big|_{\mathbb{M},\mathbb{J}} = \frac{1}{4}\left(3\frac{\mathcal{P}}{\mathcal{Z}} + \frac{\alpha_1}{\alpha_2}\right) - \frac{\alpha_2 \mathcal{J} - \alpha_0 \mathcal{Z}}{\alpha_2 \mathcal{P} - \alpha_1 \mathcal{Z}} \,, \tag{2.37}$$

where $\mathbb{M} = 2\pi\mathcal{P}$, $\mathbb{J} = -2\pi\mathcal{J}$, and $\mathbb{W} = 2\pi\mathcal{Z}$, such that the first law of thermodynamics is found to fulfilled according to

$$\delta S_C = \beta_C(\delta\mathbb{M} - \Omega_C \delta\mathbb{J} - \Phi_C \delta\mathbb{W}) \,, \tag{2.38}$$

such that the chemical potentials $\beta_C = \xi$, $\Omega_C = \mu/\xi$, and $\Phi_C = -\nu/\xi$ correspond to the conjugated to the mass ($\mathbb{M}$), angular momentum ($\mathbb{J}$) and the additional spin-2 charge ($\mathbb{W}$), respectively. It is important to emphasize that the mass of the cosmological solution is found to be bounded from below for $\mathcal{N} > 0$, which turns out to be compatible with the condition in (2.31).

As a final remark of this section, note that the entropy depends not only on the horizon area but also on the values of the spin connection and the gravitational Maxwell fields at the horizon (see appendix A). Actually, one verifies that the values of the purely angular components of the spin-2 fields at the cosmological horizon are given by

$$\left(\frac{\mathcal{A}_{\text{metric}}}{2\pi}\right)^2 = e_{a\phi} e^a_{\ \phi}|_{r_c} = \frac{\mathcal{N}^2}{4\mathcal{M}} \,, \tag{2.39}$$

$$\left(\frac{\mathcal{A}_{\text{Max}}}{2\pi}\right)^2 = \sigma_{a\phi} \sigma^a_{\ \phi}|_{r_c} = \left(\frac{\mathcal{F}}{\sqrt{\mathcal{M}}} - \frac{\mathcal{N}^2}{4\mathcal{M}^{3/2}}\right)^2 \,, \tag{2.40}$$

$$\left(\frac{\mathcal{A}_{\text{CS}}}{2\pi}\right)^2 = \omega_{a\phi} \omega^a_{\ \phi}|_{r_c} = \mathcal{M} \,. \tag{2.41}$$

From this one concludes that the entropy (2.32) can be fully expressed as the sum of the horizon area element and its spin-2 analogues, thereby extending the standard Bekenstein-Hawking formula for the entropy in our case to the form

$$S = k\left(\alpha_0 \mathcal{A}_{\text{CS}} + \alpha_1 \mathcal{A}_{\text{metric}} + \alpha_2 \mathcal{A}_{\text{Max}}\right) \,, \tag{2.42}$$

[6] In the metric formulation $n_{\mathfrak{m}}$ appears determined in terms the global charges according to $n_{\mathfrak{m}} = \text{sgn}(\alpha_2 \mathcal{P} - \alpha_1 \mathcal{Z})$.

As depicted in appendix A, these particular contributions in (2.40) appear naturally defined in terms of the pullback of the metric and the spin-2 fields at the spacelike section of the horizon [68, 69].

In the following section, we will deal with the Maxwell generalization of three-dimensional gravity with torsion. We first present the CS gravity theory invariant under a deformed Maxwell algebra. Subsequently, we will compute the asymptotic symmetry algebra considering consistent boundary conditions that incorporate chemical potentials, and finally we analyze the black hole solution of the theory including its corresponding thermodynamics features. We will show that the results obtained in this section can be recovered in the torsionless limit in a particular case.

3 Three-dimensional Maxwell Chern-Simons gravity with torsion

In this section, we present a generalization of Maxwell CS gravity by including torsion. For this purpose, we will consider a particular deformation of the Maxwell algebra, which was first presented in [27], and subsequently approached in [30]. Interestingly, despite the deformed Maxwell symmetry is isomorphic to the direct sum of $\mathfrak{iso}(2,1)$ with $\mathfrak{so}(2,1)$, the resulting CS action leads to a non-vanishing torsion as equation of motion [30].

3.1 Maxwell Chern-Simons gravity with torsion

The deformed Maxwell algebra is spanned by the set of generators $\{J_a, P_a, Z_a\}$, which satisfy the following non-vanishing commutation relations:

$$\begin{aligned} [J_a, J_b] &= \epsilon_{abc} J^c\,, & [J_a, P_b] &= \epsilon_{abc} P^c\,, \\ [J_a, Z_b] &= \epsilon_{abc} Z^c\,, & [P_a, P_b] &= \epsilon_{abc}\left(Z^c + \varepsilon P^c\right)\,. \end{aligned} \tag{3.1}$$

Note that when the deformation parameter ε is fixed to zero, the Maxwell algebra in (2.1) is recovered. In fact, the algebra (3.1) can be seen as the Maxwell extension of the teleparallel algebra [46]. As was pointed out in [27], the previous algebra can be written as the direct sum of the $\mathfrak{iso}(2,1)$ and $\mathfrak{so}(2,1)$ algebras. Indeed, considering the following redefinition of the generators,

$$L_a \equiv J_a - \varepsilon^{-1} P_a - \varepsilon^{-2} Z_a\,; \qquad N_a \equiv \varepsilon^{-1} Z_a\,; \qquad S_a \equiv \varepsilon^{-1} P_a + \varepsilon^{-2} Z_a\,, \tag{3.2}$$

the $\mathfrak{iso}(2,1) \otimes \mathfrak{so}(2,1)$ algebra is revealed,

$$\begin{aligned} [L_a, L_b] &= \epsilon_{abc} L^c\,, \\ [L_a, N_b] &= \epsilon_{abc} N^c\,, \\ [S_a, S_b] &= \epsilon_{abc} S^c\,. \end{aligned} \tag{3.3}$$

Three-dimensional gravity action based on the deformed Maxwell algebra (3.1) can be formulated as a CS form by considering (2.4), and where the gauge connection one-form A reads

$$A = e^a P_a + \omega^a J_a + \sigma^a Z_a\,, \tag{3.4}$$

where e^a, ω^a and σ^a are the dreibein, the spin connection, and the gravitational Maxwell field, respectively. The corresponding curvature two-form $F = dA + \frac{1}{2}[A, A]$ is given by

$$F = \hat{T}^a P_a + R^a J_a + F^a Z_a , \tag{3.5}$$

with

$$\begin{aligned} R^a &= d\omega^a + \frac{1}{2}\epsilon^{abc}\omega_b\omega_c \,, \\ \hat{T}^a &= T^a + \frac{\varepsilon}{2}\epsilon^{abc}e_b e_c \,, \\ F^a &= D\sigma^a + \frac{1}{2}\epsilon^{abc}e_b e_c \,. \end{aligned} \tag{3.6}$$

Here, $T^a = De^a$ is the torsion two-form, R^a is the curvature two-form, and D is the Lorentz covariant derivative defined above. Note that the limit $\varepsilon \to 0$ reproduces, as expected, the Maxwell curvature two-form in (2.6).

On the other hand, the non-vanishing components of an invariant bilinear form of the deformed Maxwell algebra read

$$\begin{aligned} \langle J_a J_b \rangle &= \alpha_0 \eta_{ab} \,, \qquad & \langle P_a P_b \rangle &= (\varepsilon\alpha_1 + \alpha_2)\eta_{ab} \,, \\ \langle J_a P_b \rangle &= \alpha_1 \eta_{ab} \,, \qquad & \langle J_a Z_b \rangle &= \alpha_2 \eta_{ab} \,, \end{aligned} \tag{3.7}$$

where α_0, α_1 and α_2 are arbitrary constants. In this case, non-degeneracy of the bilinear form requires $\alpha_2 \neq 0$ and $\alpha_2 \neq -\varepsilon\alpha_1$. Naturally, the limit $\varepsilon \to 0$ applied to (3.7) leads to the non-vanishing components of the invariant tensor for the Maxwell algebra (2.3).

From the gauge potential (3.4) and the non-vanishing components of the invariant tensor (3.7), the Chern-Simons action (2.4) reduces to the torsional Maxwell CS action

$$\begin{aligned} I_{\mathfrak{tor-max}} = \frac{1}{16\pi G}\int_{\mathcal{M}} & \alpha_0 \left(\omega^a d\omega_a + \frac{1}{3}\epsilon^{abc}\omega_a\omega_b\omega_c\right) + \alpha_1 \left(2R_a e^a + \frac{\varepsilon^2}{3}\epsilon^{abc}e_a e_b e_c + \varepsilon T^a e_a\right) \\ & + \alpha_2 \left(T^a e_a + 2R^a\sigma_a + \frac{\varepsilon}{3}\epsilon^{abc}e_a e_b e_c\right) , \end{aligned} \tag{3.8}$$

up to a surface term. One can see that the CS action is proportional to three independent sectors, each one with its respective coupling constant. In particular, the first two terms along α_0 and α_1 correspond to those of the Riemann-Cartan gravity, namely gravity with non-vanishing torsion. The piece along the α_2 contains a torsional term, a cosmological term and a term involving the gravitational Maxwell field σ_a.

Varying the action (3.8) with respect to the fundamental fields yields the field equations of Maxwell CS with torsion,

$$\begin{aligned} \delta e^a :& \qquad 0 = \alpha_1\left(R_a + \varepsilon\hat{T}_a\right) + \alpha_2\hat{T}_a \,, \\ \delta\omega^a :& \qquad 0 = \alpha_0 R_a + \alpha_1\hat{T}_a + \alpha_2 F_a \,, \\ \delta\sigma^a :& \qquad 0 = \alpha_2 R_a \,. \end{aligned} \tag{3.9}$$

Imposing the conditions $\alpha_2 \neq 0$ and $\alpha_2 \neq -\varepsilon\alpha_1$ to avoid degeneracy, the previous equations reduce to the vanishing of the curvature two-forms (3.6),

$$\begin{aligned} R^a &= 0\,, \\ T^a + \frac{\varepsilon}{2}\epsilon^{abc}e_b e_c &= 0\,, \\ D\sigma^a + \frac{1}{2}\epsilon^{abc}e_b e_c &= 0\,. \end{aligned} \tag{3.10}$$

As we can see, the field equations are those of Riemann-Cartan gravity (with zero curvature and constant torsion) plus the equation of motion involving the gravitational Maxwell field. Then, the geometries described by the equations (3.10) are non-Riemannian (with non-vanishing torsion) and locally flat. As expected, when $\varepsilon \to 0$, the above equations reduce to the field equations for the Maxwell CS gravity theory (2.6). Furthermore, note that the spin connection ω^a can be decomposed as

$$\omega^a = \tilde{\omega}^a + k^a\,, \tag{3.11}$$

where $\tilde{\omega}^a$ corresponds to the torsion-free Levi-Civita connection, and k^a is the contorsion. By the second equation in (3.10), the contorsion is fixed as follows

$$k^a = -\frac{\varepsilon}{2}e^a\,. \tag{3.12}$$

Then, the equation of motion of the gravitational Maxwell field and the Riemann-Cartan curvature R^a expressed in terms of its Riemannian part $\tilde{R}^a$, can be written as

$$\tilde{R}^a = \frac{\Lambda_{\text{eff}}}{2}\epsilon^{abc}e_b e_c\,; \qquad \tilde{D}\sigma^a + \frac{1}{2}\epsilon^{abc}\left(e_b - 2\sqrt{-\Lambda_{\text{eff}}}\,\sigma_b\right)e_c\,, \tag{3.13}$$

where the effective cosmological constant is

$$\Lambda_{\text{eff}} = -\frac{\varepsilon^2}{4}\,. \tag{3.14}$$

When the effective cosmological constant is negative, $\Lambda_{\text{eff}} = -\frac{1}{\ell^2}$, the metric is given by the anti-de Sitter solution since $\Lambda_{\text{eff}} < 0$. Indeed, in this case, the solution of the first two equations in (3.10) is given by the so-called Riemman-Cartan black hole [52]. In particular, for $\varepsilon = -2/\ell$, this solution leads to the teleparallel black hole [44, 53].

Note that each term of the action (3.8) is invariant under the gauge transformation laws of the algebra (3.1). Indeed, considering the following gauge parameter

$$\Lambda = \varepsilon^a P_a + \rho^a J_a + \chi^a Z_a, \tag{3.15}$$

we have that the gauge transformations $\delta A = d\Lambda + [A, \Lambda]$ of the theory are given by

$$\begin{aligned} \delta_\Lambda e^a &= D\varepsilon^a - \epsilon^{abc}\rho_b e_c + \varepsilon\epsilon^{abc}e_b\varepsilon_c, \\ \delta_\Lambda \omega^a &= D\rho^a, \\ \delta_\Lambda \sigma^a &= D\chi^a + \epsilon^{abc}e_b\varepsilon_c - \epsilon^{abc}\rho_b\sigma_c. \end{aligned} \tag{3.16}$$

JHEP02(2026)143

In the next section, we shall demonstrate that three-dimensional Maxwell gravity with torsion admits black hole solutions. We show that the aforementioned solution constitutes a generalization of the teleparallel black hole presented in [43, 44, 52, 70], whose thermodynamic properties are directly influenced by the presence of the gravitational Maxwell field σ^a. Prior to this analysis, we will present a detailed study of the asymptotic structure of the previously discussed theory of gravity with torsion.

3.2 Asymptotic symmetries

The asymptotic conditions for Maxwell CS gravity theory with non-vanishing torsion were explored in [30]. Here we generalized this analysis to incorporate chemical potentials conjugated to the conserved charges and for a generic deformation parameter ε. Thus, along the line of [59] the radial dependence of the gauge field is eliminated asymptotically for an appropriate gauge choice, so that

$$A = g^{-1}dg + g^{-1}a_{(\varepsilon)}g \,, \tag{3.17}$$

with $a_{(\varepsilon)} = a_t(t,\phi)dt + a_\phi(t,\phi)d\phi$. In this case, the form of a_ϕ is chosen to be given by (2.8) [30]. It is found then that the asymptotic form of angular component a_ϕ is left invariant for the Lie-algebra-valued parameter $\lambda_{(\varepsilon)} = \lambda_{(\varepsilon)}[y,f,h]$ given by

$$\lambda_{(\varepsilon)} = \lambda_{(0)} + \frac{\varepsilon}{2}\mathcal{N} f P_0 \,, \tag{3.18}$$

and providing that the functions $\mathcal{M},\mathcal{N}$ and $\mathcal{F}$ now transform according to:

$$\begin{aligned} \delta_{(\varepsilon)}\mathcal{M} &= \delta_{(0)}\mathcal{M} \,, \\ \delta_{(\varepsilon)}\mathcal{N} &= \delta_{(0)}\mathcal{N} + \varepsilon\left(\mathcal{N}'f + 2\mathcal{N}f'\right) \,, \\ \delta_{(\varepsilon)}\mathcal{F} &= \delta_{(0)}\mathcal{F} \,, \end{aligned} \tag{3.19}$$

where $\lambda_{(0)}$ and $\delta_{(0)}(\cdot)$ are specified by (2.9) and (2.10), respectively. Thus, to preserve the asymptotic symmetries under evolution in time, the asymptotic form of the Lagrange multiplier a_t must to be of the form

$$a_t = \lambda_{(\varepsilon)}[\mu,\xi,\nu] \,, \tag{3.20}$$

where the arbitrary functions $\mu(t,\phi)$, $\xi(t,\phi)$ and $\nu(t,\phi)$ are identified with the chemical potentials conjugated to the charges and which are assumed to be fixed at the boundary [61, 62]. The asymptotic behavior of a_t is then preserved under the asymptotic symmetries considering that the field equations:

$$\begin{aligned} \dot{\mathcal{M}} &= 2\mathcal{M}\mu' + \mathcal{M}'\mu - 2\mu''' \,, \\ \dot{\mathcal{N}} &= 2\mathcal{M}\xi' + \mathcal{M}'\xi + 2\mathcal{N}\mu' + \mathcal{N}'\mu - 2\xi''' + \varepsilon\left(\mathcal{N}'\xi + 2\mathcal{N}\xi'\right) \,, \\ \dot{\mathcal{F}} &= 2\mathcal{M}\nu' + \mathcal{M}'\nu + 2\mathcal{N}\xi' + \mathcal{N}'\xi + 2\mathcal{F}\mu' + \mathcal{F}'\mu - 2\nu''' \,, \end{aligned} \tag{3.21}$$

are satisfied in the asymptotic region and the parameters y, f, and h fulfill the following conditions:

$$\begin{aligned} &\dot{y} = y\mu' - \mu y' \,; \qquad \dot{\xi} = f\mu' - \mu f' + y\xi' - \xi y' + \varepsilon\left(f\xi' - \xi f'\right) \,, \\ &\dot{\nu} = h\mu' - h'\mu + f\xi' - f'\xi + y\nu' - y'\nu \,. \end{aligned} \tag{3.22}$$

The asymptotic symmetry generators are then obtained through the Regge-Teitelboim method [63] through the expression in (2.14) considering the invariant tensor in (3.7), the angular component of gauge field a_ϕ in (2.8), and the algebra-valued parameter defined in (3.18). Thus, one finds that

$$\delta Q[y,f,h] = -\int d\phi \left(y\delta\mathcal{J} + f\delta\mathcal{P} + h\delta\mathcal{Z} \right) , \tag{3.23}$$

where the canonical generators of the asymptotic symmetry are found to be determined by

$$\mathcal{J} = \frac{k}{4\pi}\left(\alpha_2\mathcal{F} + \alpha_1\mathcal{N} + \alpha_0\mathcal{M}\right) \; ; \quad \mathcal{P} = \frac{k}{4\pi}\left[(\varepsilon\alpha_1 + \alpha_2)\mathcal{N} + \alpha_1\mathcal{M}\right] \; ; \quad \mathcal{Z} = \frac{k}{4\pi}\alpha_2\mathcal{M} . \tag{3.24}$$

The corresponding transformations laws of these latter canonical generators can be obtained using (3.19), leading to

$$\begin{aligned}
\delta\mathcal{J} &= \mathcal{J}'y + 2\mathcal{J}y' + \mathcal{P}'f + 2\mathcal{P}f' + \mathcal{Z}'h + 2\mathcal{Z}h' - \frac{\alpha_0 k}{2\pi}y''' - \frac{\alpha_1 k}{2\pi}f''' - \frac{\alpha_2 k}{2\pi}h''' , \\
\delta\mathcal{P} &= \mathcal{P}'y + 2\mathcal{P}y' + (\mathcal{Z}' + \varepsilon\mathcal{P}')f + 2(\mathcal{Z} + \varepsilon\mathcal{P})f' - \frac{\alpha_1 k}{2\pi}y''' - \frac{(\alpha_2 + \varepsilon\alpha_1)k}{2\pi}f''' , \\
\delta\mathcal{Z} &= \mathcal{Z}'y + 2\mathcal{Z}y' - \frac{\alpha_2 k}{2\pi}y''' .
\end{aligned} \tag{3.25}$$

Under the assumption that the functions y, f, and h exhibit no field dependence, the integration of the variation (3.23) becomes trivial, resulting in

$$Q[y,f,h] = -\int d\phi \left(y\mathcal{J} + f\mathcal{P} + h\mathcal{Z} \right) . \tag{3.26}$$

As in the case without torsion, expanding in Fourier modes according to $X = \frac{1}{2\pi}\sum_m X_m e^{im\theta}$, one finds that $\mathcal{J}$, $\mathcal{P}$ and $\mathcal{Z}$ obey, in terms of the Poisson bracket, the following asymptotic symmetry algebra

$$\begin{aligned}
i\left\{\mathcal{J}_m, \mathcal{J}_n\right\} &= (m-n)\,\mathcal{J}_{m+n} + c_{\mathcal{J}}\, m(m^2-1)\delta_{m+n,0} , \\
i\left\{\mathcal{J}_m, \mathcal{P}_n\right\} &= (m-n)\,\mathcal{P}_{m+n} + c_{\mathcal{P}}\, m(m^2-1)\delta_{m+n,0} , \\
i\left\{\mathcal{P}_m, \mathcal{P}_n\right\} &= (m-n)\left(\mathcal{Z}_{m+n} + \varepsilon\mathcal{P}_{m+n}\right) + \left(c_{\mathcal{Z}} + \varepsilon c_{\mathcal{P}}\right) m(m^2-1)\delta_{m+n,0} , \\
i\left\{\mathcal{J}_m, \mathcal{Z}_n\right\} &= (m-n)\,\mathcal{Z}_{m+n} + c_{\mathcal{Z}}\, m(m^2-1)\delta_{m+n,0} ,
\end{aligned} \tag{3.27}$$

where classical central charges $c_{\mathcal{J}}, c_{\mathcal{P}}$ and $c_{\mathcal{Z}}$ are defined in (2.19). The infinite-dimensional algebra (3.27) can be seen as an infinite-dimensional enhancement of the deformed Maxwell algebra (3.1) in which both the abelian generators $\mathcal{Z}_n$ and the generators $\mathcal{P}_n$ have conformal weight 2. In particular, in the limit $\varepsilon \to 0$ we recover the asymptotic symmetry algebra in (2.18) of the three-dimensional Maxwell CS gravity theory reviewed in section 2.1. Furthermore, as it was shown in [30] the algebra (3.27) is isomorphic to the $\mathfrak{bms}_3 \oplus \mathfrak{vir}$ algebra.

In this context, it is important to highlight those gauge field configurations with $\mathcal{M}$, $\mathcal{N}$, $\mathcal{F}$, and also the chemical potentials ξ, μ, ν fixed to constants, since these fulfill the field equations in (3.21). The gauge field for this class of configurations $a_{(\varepsilon)} = a_t dt + a_\phi d\phi$, explicitly reads

$$\begin{aligned}
a_{(\varepsilon)} = &\left(J_1 + \frac{1}{2}\mathcal{M}\,J_0 + \frac{1}{2}\mathcal{N}\,P_0 + \frac{1}{2}\mathcal{F}\,Z_0 \right) d\phi + \Big[\mu J_1 + \frac{\mu}{2}\mathcal{M}J_0 \\
&+ \frac{1}{2}\left(\xi\mathcal{M} + \mu\mathcal{N} + \frac{\varepsilon}{2}\mathcal{N}\xi \right) P_0 + \xi P_1 + \frac{1}{2}\left(\nu\mathcal{M} + \mu\mathcal{F} + \xi\mathcal{N}\right) Z_0 + \nu Z_1 \Big] dt .
\end{aligned} \tag{3.28}$$

In appendix B by restoring the radial dependence of the gauge field (3.28) is shown that for $\varepsilon = -2/\ell$, where ℓ is the AdS$_3$ radius, this stationary solution corresponds to the BTZ black hole [3] together to a spin-2 gauge field, named here as the gravitational Maxwell field. Generically, as follows from (3.26) this solution not only possesses mass and angular momentum, parametrized by the spin-2 charges $\mathcal{P}$, $\mathcal{J}$, respectively, but also possesses an extra global charge of spin 2, determined by $\mathcal{Z}$. In what follows, we will carry out the calculation and analysis of the entropy associated with the black hole solution (3.28) for Maxwell CS gravity with torsion, as well as examine the corresponding first law of thermodynamics.

3.3 Thermodynamics

Let us move now to the study of the thermodynamics of the solution (3.28) of the Maxwell CS theory with torsion. As in the Maxwell case, the entropy can be determined from the expression (2.21). Hence, plugging (3.28) into the expression for the entropy (2.21), and using the non-vanishing components of the invariant tensor (3.7), one finds

$$S = k\left[\alpha_0 \mu \mathcal{M} + \alpha_1 \left(\mu \mathcal{N} + \xi \mathcal{M} + \varepsilon \xi \mathcal{N}\right) + \alpha_2 \left(\nu \mathcal{M} + \xi \mathcal{N} + \mu \mathcal{F}\right)\right]_{\text{on-shell}} , \tag{3.29}$$

where it is assumed that the regularity conditions are imposed.

As in the Maxwell case, and along the lines of [65], we have to find an permissible gauge transformation allowing to gauge away some temporal components of the gauge field so that it takes values only in the Lorentz subalgebras of the gauge group (3.1). In this case, for simplicity and exploiting the fact that the deformed Maxwell algebra can be expressed as the direct sum of the Poincaré and Lorentz algebras (see (3.2)), we consider the following group element

$$g = e^{\rho T_2} , \tag{3.30}$$

so that the angular and temporal components of (3.28), take the form,

$$\begin{aligned} a_\phi &= L_1 + S_1 + \rho T_1 + \frac{1}{2}\mathcal{M} L_0 + \frac{1}{2}\left(\varepsilon \mathcal{F} - \rho \mathcal{M} - \mathcal{N}\right) T_0 + \frac{1}{2}\left(\mathcal{M} + \varepsilon \mathcal{N}\right) S_0 , \\ a_t &= \mu L_1 + \frac{1}{2}\mu \mathcal{M} L_0 + (\mu + \varepsilon \xi) S_1 + \frac{1}{2}(\mathcal{M} + \varepsilon \mathcal{N})(\mu + \varepsilon \xi) S_0 + (\rho \mu - \xi + \varepsilon \nu) T_1 \\ &\quad + \left(\varepsilon \mu \mathcal{F} - \mu \mathcal{N} - \mathcal{M}(\rho \mu + \xi - \varepsilon \nu)\right) T_0 . \end{aligned} \tag{3.31}$$

It is thus found that the term along T_1 in a_t vanishes when ρ is given by

$$\rho = \frac{\xi - \varepsilon \nu}{\mu} , \tag{3.32}$$

and that the term along T_0 is eliminated by fixing the Lagrange multiplier ν according to

$$\nu = \frac{\xi}{\varepsilon} - \frac{(\varepsilon \mathcal{F} - \mathcal{N})}{2\varepsilon \mathcal{M}} \mu , \tag{3.33}$$

so that the temporal component a_t reduces to

$$a_t = \mu \left(L_1 + \frac{1}{2}\mathcal{M} L_0\right) + (\mu + \varepsilon \xi)\left[S_1 + \frac{1}{2}\left(\mathcal{M} + \varepsilon \mathcal{N}\right) S_0\right] . \tag{3.34}$$

JHEP02(2026)143

Thus, imposing that the holonomy of the a_τ around the thermal circle is trivial, which in this case is written as

$$H = e^{a_\tau}\Big|_{\text{on-shell}} = \begin{pmatrix} (-1)^n \mathbb{1}_{2\times 2} & 0 \\ 0 & (-1)^m \mathbb{1}_{2\times 2} \end{pmatrix} , \tag{3.35}$$

the chemical potentials ξ, μ, ν become fixed as specific functions of $\mathcal{M}, \mathcal{N}$ and $\mathcal{F}$, obtaining the following results:

$$\begin{aligned} \mu &= -\frac{2\pi n}{\sqrt{\mathcal{M}}} , \\ \xi &= \frac{2\pi}{\varepsilon}\left(\frac{n}{\sqrt{\mathcal{M}}} - \frac{m}{\sqrt{\mathcal{M}+\varepsilon\mathcal{N}}}\right) , \\ \nu &= \frac{2\pi n}{\sqrt{\mathcal{M}}}\left[\frac{1}{\varepsilon^2} + \frac{1}{2\mathcal{M}}\left(\mathcal{F} - \frac{\mathcal{N}}{\varepsilon}\right)\right] - \frac{2\pi m}{\varepsilon^2\sqrt{\mathcal{M}+\varepsilon\mathcal{N}}} , \end{aligned} \tag{3.36}$$

where n, m are integers. Note that in the present case, as we are dealing with a black hole solution, the orientation of the solid torus is reversed relative to that of the cosmological solution analyzed in the section 2.2. Indeed, it is shown in appendix B that for $\mathcal{M} > 0$ and $\mathcal{M} + \varepsilon\mathcal{N} > 0$ the spacetime develops an inner and an event horizon. Furthermore, for $n = -m = 1$, the regularity conditions in (3.36) are in complete agreement with those found in the metric formalism in (B.21) that stem from requiring the absence of conical singularities of the Euclidean fields at the event horizon. This means that in the present case $\xi = 1/T_{\rm BH}$, with $T_{\rm BH}$ the Hawking temperature.

Thus, substituting (3.36) into the entropy (3.29), one obtains

$$S_{\rm BH} = 2\pi k\sqrt{\mathcal{M}}\left\{\left(\frac{\alpha_1\varepsilon + \alpha_2}{\varepsilon^2}\right)\left(n + m\sqrt{1 + \frac{\varepsilon\mathcal{N}}{\mathcal{M}}}\right) - n\left[\alpha_0 + \frac{\alpha_2}{2\varepsilon}\left(\frac{\varepsilon\mathcal{F} - \mathcal{N}}{\mathcal{M}}\right)\right]\right\} , \tag{3.37}$$

It can be readily checked that, upon imposing the condition $m = n = -n_{\mathfrak{m}}$, the expression reduces to (2.32) in the limit $\varepsilon \to 0$. In terms of the global charges $\mathcal{P}, \mathcal{J}$ y $\mathcal{Z}$, the entropy finally reads

$$S_{\rm BH} = \frac{4\pi\sqrt{\pi k}\sqrt{\alpha_2}}{\varepsilon}\left[\frac{n}{2}\left(\frac{\mathcal{P} - \varepsilon\mathcal{J}}{\sqrt{\mathcal{Z}}}\right) + n\left(\frac{1}{\varepsilon} + \frac{\alpha_1 - \varepsilon\alpha_0}{2\alpha_2}\right)\sqrt{\mathcal{Z}} - \frac{m}{\varepsilon}\sqrt{\frac{\alpha_2 + \varepsilon\alpha_1}{\alpha_2}}\sqrt{\mathcal{Z} + \varepsilon\mathcal{P}}\right] . \tag{3.38}$$

The entropy $S_{\rm BH}$ is real for conditions $\alpha_2 > 0$, $\alpha_2 + \varepsilon\alpha_1 > 0$, $\mathcal{Z} > 0$ and $\mathcal{Z} + \varepsilon\mathcal{P} > 0$. Moreover, for $\varepsilon < 0$, we can see that this quantity is positive for the additional conditions:

- When $m = n$, for $n > 0$, $\mathcal{J} > \mathcal{J}_c$ and for $n < 0$, $\mathcal{J} < \mathcal{J}_c$, with

$$\mathcal{J}_c = \frac{\mathcal{P}}{\varepsilon} - 2\sqrt{\frac{\mathcal{Z}(\alpha_1\varepsilon + \alpha_2)(\varepsilon\mathcal{P} + \mathcal{Z})}{\alpha_2\varepsilon^4}} + \frac{\mathcal{Z}(\varepsilon(\alpha_1 - \alpha_0\varepsilon) + 2\alpha_2)}{\alpha_2\varepsilon^2} . \tag{3.39}$$

- When $m = -n$, for $n > 0$, $\mathcal{J} > \mathcal{J}_c$ and for $n < 0$, $\mathcal{J} < \mathcal{J}_c$, with

$$\mathcal{J}_c = \frac{\mathcal{P}}{\varepsilon} + 2\sqrt{\frac{\mathcal{Z}(\alpha_1\varepsilon + \alpha_2)(\varepsilon\mathcal{P} + \mathcal{Z})}{\alpha_2\varepsilon^4}} + \frac{\mathcal{Z}(\varepsilon(\alpha_1 - \alpha_0\varepsilon) + 2\alpha_2)}{\alpha_2\varepsilon^2} . \tag{3.40}$$

As in the previous analysis, the temperature and chemical potentials within the microcanonical ensemble are determined using the thermodynamic relations,

$$\beta_{\mathrm{BH}} = \left(\frac{\partial S_{\mathrm{BH}}}{\partial \mathbb{M}}\right)\Big|_{\mathbb{J},\mathbb{W}} = T_{\mathrm{BH}}^{-1} = \frac{\sqrt{\pi k}}{\varepsilon}\left(\frac{n\sqrt{\alpha_2}}{\sqrt{\mathcal{Z}}} - \frac{m\sqrt{\alpha_2+\varepsilon\alpha_1}}{\sqrt{\varepsilon\mathcal{P}+\mathcal{Z}}}\right), \tag{3.41}$$

$$\Omega_{\mathrm{BH}} = -\,\beta_{\mathrm{BH}}^{-1}\left(\frac{\partial S_{\mathrm{BH}}}{\partial \mathbb{J}}\right)\Big|_{\mathbb{M},\mathbb{W}} = \frac{n\varepsilon\sqrt{\alpha_2}\sqrt{\varepsilon\mathcal{P}+\mathcal{Z}}}{m\sqrt{\mathcal{Z}}\sqrt{\alpha_1\varepsilon+\alpha_2} - n\sqrt{\alpha_2}\sqrt{\varepsilon\mathcal{P}+\mathcal{Z}}}, \tag{3.42}$$

$$\Phi_{\mathrm{BH}} = -\,\beta_{\mathrm{BH}}^{-1}\left(\frac{\partial S_{\mathrm{BH}}}{\partial \mathbb{W}}\right)\Big|_{\mathbb{M},\mathbb{J}} = \frac{n\left[\alpha_2\left(\varepsilon\mathcal{J}-\mathcal{P}\right)+\mathcal{Z}\left(\alpha_1-\varepsilon\alpha_0\right)\right]\sqrt{\varepsilon\mathcal{P}+\mathcal{Z}}}{2\sqrt{\alpha_2}\mathcal{Z}\left(m\sqrt{\mathcal{Z}}\sqrt{\alpha_1\varepsilon+\alpha_2} - n\sqrt{\alpha_2}\sqrt{\varepsilon\mathcal{P}+\mathcal{Z}}\right)} - \frac{1}{\varepsilon}, \tag{3.43}$$

where $\mathbb{M} = 2\pi\mathcal{P}$, $\mathbb{J} = -2\pi\mathcal{J}$, and $\mathbb{W} = 2\pi\mathcal{Z}$. These thermodynamical quantities satisfy the first law of thermodynamics.

$$\delta S_{\mathrm{BH}} = \beta_{\mathrm{BH}}(\delta\mathbb{M} - \Omega_{\mathrm{BH}}\delta\mathbb{J} - \Phi_{\mathrm{BH}}\delta\mathbb{W}), \tag{3.44}$$

such that the chemical potentials $\beta_{\mathrm{BH}} = \xi$, $\Omega_{\mathrm{BH}} = \mu/\xi$, and $\Phi_{\mathrm{BH}} = -\nu/\xi$ correspond to the conjugated to the mass ($\mathbb{M}$), angular momentum ($\mathbb{J}$) and the additional spin-2 charge ($\mathbb{W}$), respectively. It is worth pointing out that the mass of the black hole turns out to be bounded from below when

$$\alpha_1\left(r_+ - r_-\right)^2 + 2\alpha_2 \mathrm{sgn}(\mathcal{N}) r_+ r_- > 0\,, \tag{3.45}$$

where r_+ and r_- correspond to the outer and inner horizons, respectively, given in (B.9).

To recover the results of the Maxwell theory without torsion in the limit $\varepsilon \to 0$, it is necessary to impose the condition $m = n$. As a remark, in the present case we have $\xi = 1/T_{\mathrm{BH}}$, whereas in the cosmological solution of Maxwell CS, the corresponding relation is $\xi = -1/T_C$. As we have mentioned, with the choice $m = n = -n_{\mathfrak{m}}$, it is straightforward to show that the limit $\varepsilon \to 0$ applied to the black hole entropy (3.38) leads to the entropy of the cosmological solution of the Maxwell CS theory (2.33).

It must be emphasized that we can express the black hole entropy (3.37) in terms of the outer and inner horizons of the black hole given in (B.9) as follows

- $m = n$

$$S_{\mathrm{inn}} = 2n\pi k\left(\frac{\alpha_0}{\ell}\left(r_+ - r_-\right) - \alpha_1 r_- - \frac{\alpha_2(r_-^2 - \mathcal{F})\ell}{2(r_+ - r_-)}\right), \tag{3.46}$$

- $m = -n$

$$S_{\mathrm{out}} = 2n\pi k\left(\frac{\alpha_0}{\ell}\left(r_+ - r_-\right) + \alpha_1 r_+ - \frac{\alpha_2(r_+^2 - \mathcal{F})\ell}{2(r_+ - r_-)}\right), \tag{3.47}$$

where we have fixed $\varepsilon = -2/\ell$. We conclude that S_{out} corresponds to a Maxwell extension (along α_2) of the entropy of the teleparallel black hole extensively analyzed in [52–55], and recently studied in [71]. The first law in this case takes the form

$$\delta\mathbb{M} = T_{\mathrm{BH}}\delta S_{\mathrm{out}} + \Omega_H\delta\mathbb{J} + \Phi\delta\mathbb{W} \tag{3.48}$$

where the extensive variables can be expressed as follows

$$T_{\mathrm{BH}}=\frac{r_+^2-r_-^2}{2\pi n\ell^2 r_+}\,,\qquad \Omega_H=\frac{1}{\ell}+\frac{r_-}{\ell r_+}\,,\qquad \Phi=\frac{\left(r_+^2+(1+r_-/r_+)\mathcal{F}-3r_+r_-\right)\ell}{2(r_+-r_-)^2}\,. \tag{3.49}$$

As we have already mentioned, it is not possible to take the flat limit $\ell\to\infty$ in S_{out} since $r_+(M,J,\ell)$ is pushed out to infinity and consequently does not have a well-defined limit. We can consider instead the thermodynamics of the inner horizon of the Maxwell BTZ-like black hole with torsion as follows,

$$T_{\mathrm{inn}}=\frac{r_+^2-r_-^2}{2\pi n\ell^2 r_-}\,,\qquad \Omega_{\mathrm{inn}}=\frac{1}{\ell}+\frac{r_+}{\ell r_-}\,,\qquad \Phi_{\mathrm{inn}}=\frac{\left(r_-^2+(1+r_+/r_-)\mathcal{F}-3r_+r_-\right)\ell}{2(r_+-r_-)^2}\,, \tag{3.50}$$

with a first law of the form

$$\delta\mathbb{M}=-T_{\mathrm{inn}}\delta S_{\mathrm{inn}}+\Omega_{\mathrm{inn}}\delta\mathbb{J}+\Phi_{\mathrm{inn}}\delta\mathbb{W}\,. \tag{3.51}$$

The thermodynamics of the inner (Cauchy) horizon of black holes has been studied in [72, 73]. As happens in the standard BTZ black hole, the horizon of the cosmological solution in the Maxwell case, corresponds to the limit of the inner horizon of the Maxwell BTZ-like black holes with torsion. Furthermore, as we have mentioned before all thermodynamic quantities of the Maxwell cosmological solutions are recovered in the flat limit $\ell\to\infty$ of the quantities of the inner horizon of the Maxwell torsional BTZ-like black hole (when $m=n$).

It is also important to point out that in the metric formalism ($n=-m=1$) the entropy in (3.37) and (3.46)–(3.47) can be also fully expressed as the sum of the horizon area element and its spin-2 analogues [68, 69] leading to an extended version of the standard Bekenstein-Hawking entropy formula given by

$$S_{\mathrm{BH}}=k\left(\alpha_0\mathcal{A}_{\mathrm{CS}}+\alpha_1\mathcal{A}_{\mathrm{metric}}+\alpha_2\mathcal{A}_{\mathrm{Max}}\right), \tag{3.52}$$

where $\mathcal{A}_{\mathrm{CS}}$, $\mathcal{A}_{\mathrm{Max}}$ and $\mathcal{A}_{\mathrm{metric}}$ are related to the values of the purely angular components of the spin-2 fields at horizons:

$$\begin{aligned}\left(\frac{\mathcal{A}_{\mathrm{metric}}}{2\pi}\right)^2&=e_{a\phi}e^a{}_\phi|_{r_\pm}=\frac{1}{\varepsilon^2}\left(\sqrt{\mathcal{M}}\pm\sqrt{\mathcal{M}+\varepsilon\mathcal{N}}\right)^2=r_\pm^2\,,\\ \left(\frac{\mathcal{A}_{\mathrm{Max}}}{2\pi}\right)^2&=\sigma_{a\phi}\sigma^a{}_\phi|_{r_\pm}=\frac{\varepsilon^2}{4\mathcal{M}}\left[\mathcal{F}-\frac{1}{\varepsilon^2}\left(\sqrt{\mathcal{M}}\pm\sqrt{\mathcal{M}+\varepsilon\mathcal{N}}\right)^2\right]^2=\frac{(\mathcal{F}-r_\pm^2)^2}{\varepsilon^2(r_+-r_-)^2}\,,\\ \left(\frac{\mathcal{A}_{\mathrm{CS}}}{2\pi}\right)^2&=\omega_{a\phi}\omega^a{}_\phi|_{r_\pm}=\mathcal{M}=\frac{1}{4}\varepsilon^2\left(r_++\mathrm{sgn}\,(\mathcal{N})r_-\right)^2\,,\end{aligned} \tag{3.53}$$

where these particular contributions in (3.53) are defined in terms of the pullback of the metric and the spin-2 fields at the spacelike section of the horizon defined in (A.21) (see refs. [68, 69]).

4 Discussion

In this work, we performed a comprehensive analysis of the thermodynamics of three-dimensional gravity solutions based on Maxwell symmetry, considering both cases with and without torsion. As mentioned in the Introduction, despite the algebraic richness and theoretical relevance of Maxwell CS gravity, the thermodynamic behavior of its solutions had not been thoroughly investigated. In this article, we contributed to closing this gap by examining the entropy and establishing the thermodynamic consistency of asymptotically flat cosmological solutions and black holes in three-dimensional Maxwell Chern-Simons.

We first examined the behavior of the dynamical fields in the asymptotic region, which was essential for computing the thermodynamic properties of the solutions. In both scenarios, we considered asymptotic conditions incorporating chemical potentials conjugate to the conserved charges. In the case without torsion, we found that the entropy of the cosmological solution extends the one for General Relativity in the presence of the CS term, and has a new contribution proportional to the α_2 constant, showing that the gravitational Maxwell field modifies not only the conserved charges but also the entropy of the solution.

We then extended the analysis to the thermodynamics of the Maxwell BTZ-like black hole with torsion, and we found that the entropy generalizes the one of the torsional BTZ black hole, exhaustively studied in [52–55]. By evaluating the entropy through the first law, we verified that the resulting expressions consistently satisfy black hole thermodynamics in both torsional and torsionless theories. In the appropriate limit, the charges and entropy of the Maxwell BTZ-like black hole with torsion reduce smoothly to the results of Maxwell CS without torsion, which confirms the robustness of our formulation. Taken together, these results establish a coherent thermodynamic framework for Maxwell CS gravity theories, clarifying how torsion affects the structure of conserved charges and the interpretation of physical parameters in stationary black hole solutions. It would be interesting to extend our study and the asymptotic symmetry analysis to the non-relativistic and ultra-relativistic regimes of the Maxwell CS gravity theory [74]. Of particular interest is the emergence of an infinite-dimensional extension of the generalized Maxwell algebra as the asymptotic symmetry algebra of the AdS-Carroll CS gravity [75]. One could investigate the boundary dynamics of Maxwellian generalizations of the Galilean/Carroll (super)gravity introduced in [74, 76–82], together with the space of solutions satisfying the set of asymptotic conditions. Furthermore, it would be natural to consider alternative boundary conditions in order to explore their impact on both the asymptotic symmetry algebra and the solution space.

On the other hand, the presence of non-vanishing torsion in a non-relativistic regime has also been shown to be particularly interesting. In fact, non-vanishing torsion in such a setting was first encountered in the context of Lifshitz holography [83] and the Quantum Hall Effect [84]. Subsequently, diverse strategies have been implemented to construct non-relativistic gravity models with non-vanishing torsion [85–89]. Thus, it would be worthwhile to explore the boundary dynamics of torsional non-relativistic gravity models.

It should also be emphasized that the study presented in this article provides the missing elements necessary to carry out a complete analysis of the energy bounds and asymptotic Killing spinors in the supersymmetric extensions of both Maxwell/Hietarinta and AdS-Lorentz gravity theories [25, 60, 90].

Acknowledgments

We thank Rodrigo Aros and Ricardo Troncoso for interesting remarks and discussions. This work was funded by the National Agency for Research and Development ANID —

FONDECYT grants 1250642, 1231133, 1221624 and 1250487. This work was supported by the Research project Code DIREG 04/2025 (P.C.) and FGII05/2024 (P.C., J.M. and E.R.) of the Universidad Católica de la Santisima Concepción (UCSC), Chile. P.C., J.M. and E.R. would also like to thank the Dirección de Investigación and Vice-rectoría de Investigación of the Universidad Católica de la Santísima Concepción, Chile, for their constant support. J.M. has been supported by the MCI, AEI, FEDER (UE) grants PID2021-125700NB-C21 ("Gravity, Supergravity and Superstrings" (GRASS)) and IFT Centro de Excelencia Severo Ochoa CEX2020-001007-S. L.A. has been supported by SIA-ANID grant 85220027.

A Chemical potentials in the metric formalism (torsionless case)

In order to recover the spacetime metric $g_{\mu\nu}$ as well as the gravitational Maxwell field $\sigma_{\mu\nu}$ the radial dependence in the gauge field (2.20) must be introduced through a permissible gauge transformation. This procedure can be achieved by choosing the following group element

$$g_{\mathcal{C}} = e^{F(r)P_2 + G(r)Z_2} \,, \tag{A.1}$$

where the functions $F(r)$ and $G(r)$ are defined according to

$$F(r) = \frac{\mathcal{N}}{2\mathcal{M}}\left(1+\sqrt{1-\frac{4\mathcal{M}}{\mathcal{N}^2}r^2}\right) \;;\quad G(r) = F(r) + \frac{1}{2\mathcal{M}}\left(\mathcal{F} - \mathcal{N} - \frac{\mathcal{N}^2}{2\mathcal{M}}\right) \,, \tag{A.2}$$

Thus, the full gauge field

$$A = \omega^a J_a + e^a P_a + \sigma^a Z_a = g_{\mathcal{C}}^{-1} a g_{\mathcal{C}} + g_{\mathcal{C}}^{-1} d g_{\mathcal{C}} \,. \tag{A.3}$$

is so that their spacetime components can be suitably written as follows

$$A_\phi = a_\phi + c_\phi(r) \;; \qquad A_t = a_t + c_t(r) \;; \qquad A_r = c_r(r) \,, \tag{A.4}$$

where a_ϕ and a_t given in (2.20) while $c_\phi(r)$, $c_\phi(r)$ and $c_r(r)$ given by

$$\begin{aligned}
c_\phi &= F(r)\left(P_1 - \frac{1}{2}\mathcal{M}P_0 - \frac{1}{2}\mathcal{N}Z_0\right) + \left(G(r) + \frac{1}{2}F(r)^2\right)\left(Z_1 - \frac{1}{2}\mathcal{M}Z_0\right) + \frac{1}{2}F(r)^2\mathcal{M}Z_0 \,, \\
c_t &= F(r)\left[\left(P_1 - \frac{1}{2}\mathcal{M}P_0 - \frac{1}{2}\mathcal{N}Z_0\right)\mu + \left(\xi + \frac{G(r)}{F(r)}\mu\right)\left(Z_1 - \frac{1}{2}\mathcal{M}Z_0\right)\right] \\
&\quad + \frac{1}{2}\mu F(r)^2\left(Z_1 + \frac{1}{2}\mathcal{M}Z_0\right) \,, \\
c_r &= F(r)' P_2 + G(r)' Z_2 \,.
\end{aligned} \tag{A.5}$$

The spacetime metric defined by

$$ds^2 = \eta_{ab} e^a{}_\mu e^b{}_\nu dx^\mu dx^\nu \,, \tag{A.6}$$

is then directly obtained in Schwarzschild-like coordinates,

$$ds^2 = -\left(\frac{\mathcal{N}^2}{4r^2} - \mathcal{M}\right)\xi^2 dt^2 + \left(\frac{\mathcal{N}^2}{4r^2} - \mathcal{M}\right)^{-1} dr^2 + r^2\left[d\phi + \left(\mu + \frac{\mathcal{N}\xi}{2r}\right)dt\right]^2 \,, \tag{A.7}$$

where for $\mathcal{M} > 0$, it possesses a cosmological horizon located at

$$r = r_c = \frac{|\mathcal{N}|}{2\sqrt{\mathcal{M}}} \, . \tag{A.8}$$

In this frame the nontrivial components of spin connection are given by

$$\omega^0 = \frac{1}{2}\mathcal{M}\left(d\phi + \mu dt\right) \ , \qquad \omega^1 = d\phi + \mu dt \, . \tag{A.9}$$

The Maxwell CS field defined here as $\sigma_{\mu\nu} = \eta_{ab} e^a{}_\mu \sigma^b{}_\nu$, can be suitably expressed as follows

$$d\sigma^2 = \eta_{ab} e^a{}_\mu \sigma^b{}_\nu dx^\mu dx^\nu = \left\{ \varrho_{\mu\nu} + \frac{3}{4}\xi \left[r^2 + \frac{1}{3}\left(\mathcal{F} - \frac{\nu}{\xi}\mathcal{N}\right)\right] \epsilon_{r\mu\nu} \right\} dx^\mu dx^\nu \ , \tag{A.10}$$

where its symmetry part $\sigma_{(\mu\nu)}$ reads

$$\begin{aligned} \varrho_{\mu\nu} dx^\mu dx^\nu = {}& \xi\mathcal{M}\left(\nu + \frac{1}{2}\frac{\mathcal{N}}{\mathcal{M}}\xi\right) dt^2 + \left[\left(1 + \frac{1}{4}\frac{\mathcal{N}}{\mathcal{M}}\right) r^2 + \frac{1}{4}\frac{\mathcal{N}}{\mathcal{M}}\left(\mathcal{F} - \mathcal{N} - \frac{\mathcal{N}^2}{2\mathcal{M}}\right)\right] \left(d\phi + \mu dt\right)^2 \\ & + \frac{1}{2}\left(\xi r^2 + \xi\mathcal{F} + \nu\mathcal{N}\right)\left(d\phi + \mu dt\right) dt + \left(\frac{\mathcal{N}^2}{4r^2} - \mathcal{M}\right)^{-1} dr^2 \, . \end{aligned} \tag{A.11}$$

Note that the chemical potentials ξ, μ and ν have been explicitly incorporated in the metric $g_{\mu\nu}$ and the gravitational CS field $\sigma_{\mu\nu}$, so regularity analysis of the fields around the cosmological horizon at $r = r_c$ can be carried out for a fixed range of the angular coordinates of the solid torus, where it is assumed to be $0 < \tau \leq 1$, and $0 < \phi \leq 2\pi$. It follows that around the cosmological horizon the rotating frame in (A.7) can be fixing by choosing the Lagrange multiplier μ according to

$$\mu = -\frac{\mathcal{N}\xi}{2r_c} = -\mathrm{sgn}\left(\mathcal{N}\right)\sqrt{\mathcal{M}}\xi \ , \tag{A.12}$$

so that expanding around the cosmological horizon the metric reduces to Rindler space,[7] that is

$$ds^2 \approx -\frac{4\mathcal{M}^3\xi^2}{\mathcal{N}^2}\rho^2 dt^2 + d\rho^2 + \frac{1}{4}\frac{\mathcal{N}^2}{\mathcal{M}} d\phi^2 \ , \tag{A.13}$$

from which the temperature can be directly read off and it is given by

$$\xi^2 = \frac{\mathcal{N}^2\pi^2}{\mathcal{M}^3} \, . \tag{A.14}$$

The gravitational Maxwell field (A.10) around the horizon behaves as follows

$$d\sigma^2 \approx \left[\varrho_{\mu\nu} + \frac{1}{4}\left(\nu - \left(\mathcal{F} - \frac{3}{4}\frac{\mathcal{N}^2}{\mathcal{M}}\right)\frac{\xi}{\mathcal{N}}\right)\mathcal{N}\epsilon_{r\mu\nu}\right] dx^\mu dx^\nu \ , \tag{A.15}$$

where

$$\begin{aligned} \varrho_{\mu\nu} dx^\mu dx^\nu \approx {}& -\frac{4\mathcal{M}^3\xi^2}{\mathcal{N}^2}\rho^2 dt^2 + d\rho^2 + \frac{1}{2}\mathcal{N}\left[\nu - \left(\mathcal{F} - \frac{3\mathcal{N}^2}{4\mathcal{M}}\right)\mathcal{N}^{-1}\xi\right] dt d\phi \\ & + \frac{1}{4}\frac{\mathcal{N}}{\mathcal{M}}\left(\mathcal{F} - \frac{\mathcal{N}^2}{4\mathcal{M}}\right) d\phi^2 \ , \end{aligned} \tag{A.16}$$

[7]This step can be carried out by redefining the radial coordinate according to $r = r_c \,\mathrm{sech}\left(\frac{2\mathcal{M}}{\mathcal{N}}\rho\right)$.

Thus, fixing the rotating frame in the Maxwell CS field (A.15) at the horizon leads to that the Lagrange multiplier ν becoming determined by

$$\nu = \pi \left(\mathcal{F} - \frac{3}{4}\frac{\mathcal{N}^2}{\mathcal{M}} \right) \frac{\xi}{\mathcal{N}} \,. \tag{A.17}$$

It is worth pointing out that both the metric and the gravitational Maxwell field are finite and well-behaved at the horizon. Indeed, around the horizon both the metric (A.13) and the gravitational Maxwell field (A.15) by virtue of the regularity conditions (A.12) and (A.17) reduce to Rindler spacetimes with the same Hawking temperature which becomes fixed to $\xi = -1/T_C$, where the minus sign amounts to the fact that orientation of the solid torus is reversed compared to that of the black hole; so that the chemical potential ξ corresponds to the minus branch of (A.14). This latter feature provides the additional condition $\varrho_{\phi\phi}\left(r_c\right) > 0$, which would mean that both fields really share the same topology around the horizon, that is,

$$\varrho_{\phi\phi}\left(r_c\right) = \frac{\mathcal{N}}{\mathcal{M}} \left(\mathcal{F} - \frac{\mathcal{N}^2}{4\mathcal{M}} \right) > 0 \,, \tag{A.18}$$

which follows from (A.16).

Thus, in the metric formalism the regularity conditions on the fields are found to be given by

$$\xi = -\frac{\pi\left|\mathcal{N}\right|}{\mathcal{M}^{3/2}} \,, \qquad \mu = \frac{2\pi}{\mathcal{M}^{1/2}} \mathrm{sgn}\left(\mathcal{N}\right) \,, \qquad \nu = -\pi \left[\frac{\mathcal{F}}{\mathcal{M}^{3/2}} - \frac{3\mathcal{N}^2}{4\mathcal{M}^{5/2}} \right] \mathrm{sgn}\left(\mathcal{N}\right) \,. \tag{A.19}$$

As a close remark one can verifies from (A.13), (A.16) and (A.9) that the purely angular components of following spin-2 fields at the cosmological horizon,

$$\begin{aligned} g_{\phi\phi}\left(r_c\right) &= e_{a\phi} e^{a}{}_{\phi}|_{r_c} = \left(\frac{\mathcal{A}_{\text{metric}}}{2\pi}\right)^2 = \frac{\mathcal{N}^2}{4\mathcal{M}} \,, \\ h_{\phi\phi}\left(r_c\right) &= \sigma_{a\phi} \sigma^{a}{}_{\phi}|_{r_c} = \left(\frac{\mathcal{A}_{\text{Max}}}{2\pi}\right)^2 = \left(\frac{\mathcal{F}}{\sqrt{\mathcal{M}}} - \frac{\mathcal{N}^2}{4\mathcal{M}^{3/2}}\right)^2 \,, \\ w_{\phi\phi}\left(r_c\right) &= \omega_{a\phi} \omega^{a}{}_{\phi}|_{r_c} = \left(\frac{\mathcal{A}_{\text{CS}}}{2\pi}\right)^2 = \mathcal{M} \,, \end{aligned} \tag{A.20}$$

appear to be finite and along the lines of [68, 69] can be precisely defined in terms of the pullback of the metric and the additional spin-2 fields at the spacelike section of the horizon according to

$$\mathcal{A} = \int_{\partial\Sigma_+} \left(X_{\mu\nu} \frac{dx^\mu}{d\sigma} \frac{dx^\nu}{d\sigma} \right)^{1/2} d\sigma \,, \tag{A.21}$$

which turns out to be reparametrization invariant. Indeed, it is reassuring to find in the same manner as in [91] (see e.g. [92]) that $\mathcal{A}_{\text{CS}}$ turns out to be Lorentz invariant in spite the fact that the Lorentz connection enters explicitly.

B Chemical potentials in the metric formalism with torsion

For the Maxwell CS theory with torsion (3.8) the corresponding spacetime metric $g_{\mu\nu}$ as well as the gravitational Maxwell field $\sigma_{\mu\nu}$ are constructed through the restoring the radial dependence in the full gauge field. For the solution in (3.28) the latter can be carried out by the following group element

$$g_{\mathcal{B}} = e^{F(r)P_2 + G(r)Z_2} \,, \tag{B.1}$$

with $F(r)$ and $G(r)$ given by

$$F(r) = \varepsilon^{-1}\log\left[1 - \frac{\varepsilon r}{\mathcal{M}}\left(\frac{\sigma r^2 - \mathcal{N}}{2r} - f(r)\right)\right] \;;\quad G(r) = F(r) + \frac{\varepsilon\mathcal{F} - \mathcal{N}}{2\varepsilon\mathcal{M}} \,, \tag{B.2}$$

and where the function $f(r)$ reads

$$f(r)^2 = \frac{\varepsilon^2}{4}r^2 - \left(\mathcal{M} + \frac{1}{2}\varepsilon\mathcal{N}\right) + \frac{\mathcal{N}^2}{4r^2} = \left(\frac{\varepsilon r^2 - \mathcal{N}}{2r}\right)^2 - \mathcal{M} \,. \tag{B.3}$$

Thus, the components of the full gauge field,

$$A = \omega^a J_a + e^a P_a + \sigma^a Z_a = g_{\mathcal{B}}^{-1} a g_{\mathcal{B}} + g_{\mathcal{B}}^{-1} d g_{\mathcal{B}} \,, \tag{B.4}$$

are suitably written as follows

$$A_\phi = a_\phi + c_\phi(r) \;; \qquad A_t = a_t + c_t(r) \;; \qquad A_r = c_r(r) \,, \tag{B.5}$$

where a_ϕ and a_t given in (3.28) while $c_\phi(r)$, $c_\phi(r)$ and $c_r(r)$ read

$$\begin{aligned} c_\phi &= 2\sigma^{-2}e^{\varepsilon F(r)/2}\sinh\left(\frac{\varepsilon}{2}F(r)\right)\left[Z_1 + \varepsilon P_1 - \frac{1}{2}\left(\mathcal{M} + \varepsilon\mathcal{N}\right)e^{-\varepsilon F(r)}\left(Z_0 + \sigma P_0\right)\right] \\ &\quad - \varepsilon^{-1}\left(F(r) - \varepsilon G(r)\right)\left[Z_1 - \frac{1}{2}\mathcal{M}Z_0\right] \,, \\ c_t &= \left(\mu + \varepsilon\xi\right)c_\phi - \xi\left(F(r) - \varepsilon G(r)\right)\left[Z_1 - \frac{1}{2}\mathcal{M}Z_0\right] \,, \\ c_r &= F(r)' P_2 + G(r)' Z_2 \,. \end{aligned} \tag{B.6}$$

The spacetime metric $ds^2 = \eta_{ab}e^a{}_\mu e^b{}_\nu dx^\mu dx^\nu$, is then found to be given by

$$ds^2 = -f(r)^2\xi^2 dt^2 + \frac{dr^2}{f(r)^2} + r^2\left(d\phi + N^\phi(r)dt\right)^2 \,, \tag{B.7}$$

where $f(r)$ is given by (B.3) while $N^\phi(r)$ reads

$$N^\phi(r) = \mu + \frac{1}{2}\varepsilon\xi + \frac{\mathcal{N}}{2r^2}\xi \,. \tag{B.8}$$

It is reassuring that this solution corresponds to the three-dimensional black hole solution [2], which goes hand in hand with the field equations in (3.13). It also follows from (B.3) that for

$\mathcal{M} > 0$ together with $\mathcal{M} + \varepsilon\mathcal{N} > 0$, this spacetime develops an inner and an event horizon which are located at $r = r_\pm$, being determined according to

$$r_\pm^2 = \frac{1}{\varepsilon^2}\left(\sqrt{\mathcal{M}} \pm \sqrt{\mathcal{M}+\varepsilon\mathcal{N}}\right)^2 \ . \tag{B.9}$$

In this frame the nontrivial components of spin connection are found to be given by

$$\omega^0 = \frac{1}{2}\mathcal{M}\left(d\phi + \mu dt\right) \ , \qquad \omega^1 = d\phi + \mu dt \ . \tag{B.10}$$

The corresponding Maxwell CS field defined here as $\sigma_{\mu\nu} = \varepsilon\eta_{ab}e^a{}_\mu\sigma^b{}_\nu$, is appropriately expressed as follows

$$d\sigma^2 = \varepsilon\eta_{ab}e^a{}_\mu\sigma^b{}_\nu dx^\mu dx^\nu = \left(\varrho_{\mu\nu} + \Phi(r)\epsilon_{r\mu\nu}\right)dx^\mu dx^\nu \ , \tag{B.11}$$

with

$$\Phi(r) = \varepsilon\frac{1}{4}\xi\left[\left(\frac{\varepsilon\mathcal{F}-\mathcal{N}}{2\mathcal{M}} + \frac{1}{\varepsilon} - \frac{\nu}{\xi}\right)\left(r^2\varepsilon - \mathcal{N}\right) + \frac{1}{\varepsilon}\left(\varepsilon\mathcal{F}-\mathcal{N}\right)\right] \ , \tag{B.12}$$

and where its symmetric part $\sigma_{(\mu\nu)}$ is given by

$$\begin{aligned}\varrho_{\mu\nu}dx^\mu dx^\nu = {} & \frac{\varepsilon}{2}\left[\left(1-\varepsilon\frac{\nu}{\xi}\right)\left(r^2\varepsilon-\mathcal{N}\right) + 2\left(\mathcal{N}+\frac{\nu}{\xi}\mathcal{M}\right)\right]\xi^2dt^2 + \frac{dr^2}{f(r)^2} \\ & + \left[r^2 - \left(\frac{\varepsilon\mathcal{F}-\mathcal{N}}{4\mathcal{M}}\right)\left(\varepsilon r^2 - \mathcal{N}\right)\right]\left(d\phi+\mu dt\right)^2 \\ & + \frac{\varepsilon}{2}\left[3r^2 + \mathcal{F} - \left(\frac{\varepsilon\mathcal{F}-\mathcal{N}}{2\mathcal{M}} + \frac{\nu}{\xi}\right)\left(r^2\varepsilon-\mathcal{N}\right)\right]\xi\left(d\phi+\mu dt\right)dt \ .\end{aligned} \tag{B.13}$$

The regularity of the fields around the event horizon $r = r_+$ can be carried out for a fixed range of the angular coordinates of the solid torus, i.e., $0 < \tau \leq 1$, and $0 < \phi \leq 2\pi$. The latter can be implemented by fixing the rotating frame around r_+ in the metric (B.7), which leads to choosing the Lagrange multiplier μ, according to

$$\mu = -\frac{1}{2}\varepsilon\xi - \frac{\mathcal{N}}{2r_+^2}\xi = -\frac{\varepsilon}{2}\left(1 - \frac{r_-}{r_+}\right)\xi \ , \tag{B.14}$$

so that around the event horizon r_+ the metric behaves as follows[8]

$$ds^2 \approx -\frac{\varepsilon^4\left(r_+^2 - r_-^2\right)^2}{4^2r_+^2}\xi^2\rho^2dt^2 + d\rho^2 + r_+^2d\phi^2 \ , \tag{B.15}$$

from which one can read that the Hawking temperature $\xi = 1/T_{\mathrm{BH}}$ is found to be

$$\xi = \frac{2^3\pi r_+}{\varepsilon^2\left(r_+^2 - r_-^2\right)} = \frac{2\pi}{|\varepsilon|}\left(\frac{1}{\sqrt{\mathcal{M}}} + \frac{1}{\sqrt{\mathcal{M}+\varepsilon\mathcal{N}}}\right) \ . \tag{B.16}$$

Along the same line, expanding the Maxwell field (B.11) around r_+ with the previous fixing (B.14), it acquires the behavior

$$d\sigma^2 \approx \left[\varrho_{\mu\nu} - \frac{\varepsilon}{4}r_+\left(r_+ + r_-\right)\left(1 - \varepsilon\frac{\nu}{\xi} + \frac{\left(r_+ - r_-\right)\left(\mathcal{F} + r_+r_-\right)}{r_+\left(r_+ + r_-\right)^2}\right)\xi\epsilon_{r\mu\nu}\right]dx^\mu dx^\nu \ , \tag{B.17}$$

[8]This is carried out by defining a new radial coordinate according to $r = \sqrt{\frac{1}{2}\left[r_+^2 + r_-^2 + \left(r_+^2 - r_-^2\right)\cosh\left(\sigma\rho\right)\right]}$.

JHEP02(2026)143

where

$$\varrho_{\mu\nu}dx^{\mu}dx^{\nu} \approx -\frac{\varepsilon^4\left(r_+^2-r_-^2\right)^2}{4^2r_+^2}\xi^2\rho^2dt^2+d\rho^2+r_+\left(\frac{r_+^2-\mathcal{F}}{r_++r_-}\right)d\phi^2$$
$$-\frac{\varepsilon}{4}r_+\left(r_++r_-\right)\left(1-\varepsilon\frac{\nu}{\xi}+\frac{\left(r_+-r_-\right)\left(\mathcal{F}+r_+r_-\right)}{r_+\left(r_++r_-\right)^2}\right)\xi dtd\phi\,. \tag{B.18}$$

Thus, by fixing the rotating frame in the gravitational Maxwell field around r_+, introduces the following condition on the Lagrange multiplier ν, given by

$$\nu=\frac{1}{\varepsilon}\left[\frac{\left(r_+-r_-\right)\left(\mathcal{F}+r_+r_-\right)}{r_+\left(r_++r_-\right)^2}+1\right]\xi\,. \tag{B.19}$$

It should be emphasized that both the metric and the gravitational Maxwell field are finite and well-behaved at the horizon r_+. Furthermore, around the horizon both the metric (B.15) and the gravitational Maxwell field (B.17) by virtue of the regularity conditions (B.14) and (B.19) reduce to Rindler spacetimes with the same Hawking temperature determined in (B.16). It is important to note that this latter feature provides also the condition that $\varrho_{\phi\phi}\left(r_c\right)>0$, which would mean that both fields really share the same topology around the horizon, that is,

$$\varrho_{\phi\phi}\left(r_+\right)=r_+\left(\frac{r_+^2-\mathcal{F}}{r_++r_-}\right)>0\,, \tag{B.20}$$

which follows from (B.18). It is also verified that expanding the fields around the inner horizon r_- one obtains also regularity conditions being those in (B.16), (B.14) and (B.19) by in changing $r_\pm\to r_\mp$.

In sum, in the metric formalism the regularity conditions on the fields around r_+ are found to be given by

$$\xi=\frac{2\pi}{|\varepsilon|}\left(\frac{1}{\sqrt{\mathcal{M}}}+\frac{1}{\sqrt{\mathcal{M}+\varepsilon\mathcal{N}}}\right)\,,\qquad \mu=-\frac{2\pi}{\mathcal{M}^{1/2}}\,,\qquad \nu=\frac{\pi\left(\varepsilon\mathcal{F}-\mathcal{N}\right)}{\varepsilon\mathcal{M}^{3/2}}+\frac{\xi}{\varepsilon}\,. \tag{B.21}$$

Data Availability Statement. This article has no associated data or the data will not be deposited.

Code Availability Statement. This article has no associated code or the code will not be deposited.

References

[1] J. Zanelli, *Lecture notes on Chern-Simons (super-)gravities. Second edition (February 2008)*, in the proceedings of the *7th Mexican Workshop on Particles and Fields*, Merida, Mexico (2005) [hep-th/0502193] [INSPIRE].

[2] M. Banados, C. Teitelboim and J. Zanelli, *The Black hole in three-dimensional space-time*, *Phys. Rev. Lett.* **69** (1992) 1849 [hep-th/9204099] [INSPIRE].

[3] M. Banados, M. Henneaux, C. Teitelboim and J. Zanelli, *Geometry of the* $(2+1)$ *black hole*, *Phys. Rev. D* **48** (1993) 1506 [*Erratum ibid.* **88** (2013) 069902] [gr-qc/9302012] [INSPIRE].

[4] M. Banados, C. Teitelboim and J. Zanelli, *Dimensionally continued black holes*, *Phys. Rev. D* **49** (1994) 975 [gr-qc/9307033] [INSPIRE].

[5] J. Crisostomo, R. Troncoso and J. Zanelli, *Black hole scan*, *Phys. Rev. D* **62** (2000) 084013 [hep-th/0003271] [INSPIRE].

[6] J.D. Edelstein, M. Hassaine, R. Troncoso and J. Zanelli, *Lie-algebra expansions, Chern-Simons theories and the Einstein-Hilbert Lagrangian*, *Phys. Lett. B* **640** (2006) 278 [hep-th/0605174] [INSPIRE].

[7] F. Izaurieta et al., *Standard General Relativity from Chern-Simons Gravity*, *Phys. Lett. B* **678** (2009) 213 [arXiv:0905.2187] [INSPIRE].

[8] P. Salgado, R.J. Szabo and O. Valdivia, *Topological gravity and transgression holography*, *Phys. Rev. D* **89** (2014) 084077 [arXiv:1401.3653] [INSPIRE].

[9] H. Bacry, P. Combe and J.L. Richard, *Group-theoretical analysis of elementary particles in an external electromagnetic field. 1. the relativistic particle in a constant and uniform field*, *Nuovo Cim. A* **67** (1970) 267 [INSPIRE].

[10] R. Schrader, *The Maxwell group and the quantum theory of particles in classical homogeneous electromagnetic fields*, *Fortsch. Phys.* **20** (1972) 701 [INSPIRE].

[11] J. Gomis and A. Kleinschmidt, *On free Lie algebras and particles in electro-magnetic fields*, *JHEP* **07** (2017) 085 [arXiv:1705.05854] [INSPIRE].

[12] H.R. Safari, *Deformation of Asymptotic Symmetry Algebras and Their Physical Realizations*, Ph.D. thesis, Institute for Research in Fundamental Sciences, Tehran, Iran (2020) [arXiv:2011.02318] [INSPIRE].

[13] C. Duval, Z. Horvath and P.A. Horvathy, *Chern-Simons gravity, based on a non-semisimple group*, arXiv:0807.0977 [INSPIRE].

[14] J.A. de Azcarraga, K. Kamimura and J. Lukierski, *Generalized cosmological term from Maxwell symmetries*, *Phys. Rev. D* **83** (2011) 124036 [arXiv:1012.4402] [INSPIRE].

[15] R. Durka, J. Kowalski-Glikman and M. Szczachor, *Gauged AdS-Maxwell algebra and gravity*, *Mod. Phys. Lett. A* **26** (2011) 2689 [arXiv:1107.4728] [INSPIRE].

[16] J.A. de Azcarraga, K. Kamimura and J. Lukierski, *Maxwell symmetries and some applications*, *Int. J. Mod. Phys. Conf. Ser.* **23** (2013) 01160 [arXiv:1201.2850] [INSPIRE].

[17] P.K. Concha, D.M. Peñafiel, E.K. Rodríguez and P. Salgado, *Even-dimensional General Relativity from Born-Infeld gravity*, *Phys. Lett. B* **725** (2013) 419 [arXiv:1309.0062] [INSPIRE].

[18] P.K. Concha, D.M. Penafiel, E.K. Rodriguez and P. Salgado, *Chern-Simons and Born-Infeld gravity theories and Maxwell algebras type*, *Eur. Phys. J. C* **74** (2014) 2741 [arXiv:1402.0023] [INSPIRE].

[19] S. Hoseinzadeh and A. Rezaei-Aghdam, *(2+1)-dimensional gravity from Maxwell and semisimple extension of the Poincaré gauge symmetric models*, *Phys. Rev. D* **90** (2014) 084008 [arXiv:1402.0320] [INSPIRE].

[20] P.K. Concha and E.K. Rodríguez, $N=1$ *Supergravity and Maxwell superalgebras*, *JHEP* **09** (2014) 090 [arXiv:1407.4635] [INSPIRE].

[21] R. Caroca, P. Concha, E. Rodríguez and P. Salgado-Rebolledo, *Generalizing the $\mathfrak{bms}_3$ and 2D-conformal algebras by expanding the Virasoro algebra*, Eur. Phys. J. C **78** (2018) 262 [arXiv:1707.07209] [INSPIRE].

[22] R. Caroca et al., *Generalized Chern-Simons higher-spin gravity theories in three dimensions*, Nucl. Phys. B **934** (2018) 240 [arXiv:1712.09975] [INSPIRE].

[23] L. Ravera, *Hidden role of Maxwell superalgebras in the free differential algebras of $D = 4$ and $D = 11$ supergravity*, Eur. Phys. J. C **78** (2018) 211 [arXiv:1801.08860] [INSPIRE].

[24] P. Concha et al., *Asymptotic symmetries of three-dimensional Chern-Simons gravity for the Maxwell algebra*, JHEP **10** (2018) 079 [arXiv:1805.08834] [INSPIRE].

[25] P. Concha, D.M. Peñafiel and E. Rodríguez, *On the Maxwell supergravity and flat limit in $2+1$ dimensions*, Phys. Lett. B **785** (2018) 247 [arXiv:1807.00194] [INSPIRE].

[26] P. Concha, L. Ravera and E. Rodríguez, *On the supersymmetry invariance of flat supergravity with boundary*, JHEP **01** (2019) 192 [arXiv:1809.07871] [INSPIRE].

[27] P. Concha and H.R. Safari, *On Stabilization of Maxwell-BMS Algebra*, JHEP **04** (2020) 073 [arXiv:1909.12827] [INSPIRE].

[28] P. Salgado-Rebolledo, *The Maxwell group in $2+1$ dimensions and its infinite-dimensional enhancements*, JHEP **10** (2019) 039 [arXiv:1905.09421] [INSPIRE].

[29] D. Chernyavsky, N.S. Deger and D. Sorokin, *Spontaneously broken 3d Hietarinta/Maxwell Chern-Simons theory and minimal massive gravity*, Eur. Phys. J. C **80** (2020) 556 [arXiv:2002.07592] [INSPIRE].

[30] H. Adami, P. Concha, E. Rodriguez and H.R. Safari, *Asymptotic symmetries of Maxwell Chern-Simons gravity with torsion*, Eur. Phys. J. C **80** (2020) 967 [arXiv:2005.07690] [INSPIRE].

[31] R. Caroca et al., *Hypersymmetric extensions of Maxwell-Chern-Simons gravity in $2+1$ dimensions*, Phys. Rev. D **104** (2021) 064011 [arXiv:2105.12243] [INSPIRE].

[32] P. Concha, J. Matulich, D. Pino and E. Rodríguez, *Asymptotic structure of three-dimensional Maxwell Chern-Simons gravity coupled to spin-3 fields*, JHEP **02** (2025) 148 [arXiv:2412.04992] [INSPIRE].

[33] F. Höfenstock and P. Salgado-Rebolledo, *Boundary dynamics of Maxwell-invariant three-dimensional Chern-Simons gravity*, JHEP **01** (2026) 022 [arXiv:2506.07651] [INSPIRE].

[34] D. Cangemi, *One formulation for both lineal gravities through a dimensional reduction*, Phys. Lett. B **297** (1992) 261 [gr-qc/9207004] [INSPIRE].

[35] T. Fukuyama and K. Kamimura, *Gauge Theory of Two-dimensional Gravity*, Phys. Lett. B **160** (1985) 259 [INSPIRE].

[36] K. Isler and C.A. Trugenberger, *A Gauge Theory of Two-dimensional Quantum Gravity*, Phys. Rev. Lett. **63** (1989) 834 [INSPIRE].

[37] A.H. Chamseddine and D. Wyler, *Gauge Theory of Topological Gravity in (1+1)-Dimensions*, Phys. Lett. B **228** (1989) 75 [INSPIRE].

[38] D. Cangemi and R. Jackiw, *Gauge invariant formulations of lineal gravity*, Phys. Rev. Lett. **69** (1992) 233 [hep-th/9203056] [INSPIRE].

[39] D. Cangemi and R. Jackiw, *Poincare gauge theory for gravitational forces in $(1+1)$-dimensions*, Annals Phys. **225** (1993) 229 [hep-th/9302026] [INSPIRE].

JHEP02(2026)143

[40] D.V. Soroka and V.A. Soroka, *Semi-simple extension of the (super)Poincare algebra*, *Adv. High Energy Phys.* **2009** (2009) 234147 [hep-th/0605251] [INSPIRE].

[41] P. Salgado and S. Salgado, $\mathfrak{so}(D-1,1)\otimes\mathfrak{so}(D-1,2)$ *algebras and gravity*, *Phys. Lett. B* **728** (2014) 5 [INSPIRE].

[42] P. Concha et al., *Semi-simple enlargement of the* $\mathfrak{bms}_3$ *algebra from a* $\mathfrak{so}(2,2)\oplus\mathfrak{so}(2,1)$ *Chern-Simons theory*, *JHEP* **02** (2019) 002 [arXiv:1810.12256] [INSPIRE].

[43] M. Blagojevic and M. Vasilic, *3-D gravity with torsion as a Chern-Simons gauge theory*, *Phys. Rev. D* **68** (2003) 104023 [gr-qc/0307078] [INSPIRE].

[44] M. Blagojevic and M. Vasilic, *Asymptotic symmetries in 3-d gravity with torsion*, *Phys. Rev. D* **67** (2003) 084032 [gr-qc/0301051] [INSPIRE].

[45] A. Giacomini, R. Troncoso and S. Willison, *Three-dimensional supergravity reloaded*, *Class. Quant. Grav.* **24** (2007) 2845 [hep-th/0610077] [INSPIRE].

[46] R. Caroca, P. Concha, D. Peñafiel and E. Rodríguez, *Three-dimensional teleparallel Chern-Simons supergravity theory*, *Eur. Phys. J. C* **81** (2021) 762 [arXiv:2103.06717] [INSPIRE].

[47] A. Ashtekar, J. Bicak and B.G. Schmidt, *Asymptotic structure of symmetry reduced general relativity*, *Phys. Rev. D* **55** (1997) 669 [gr-qc/9608042] [INSPIRE].

[48] G. Barnich and G. Compere, *Classical central extension for asymptotic symmetries at null infinity in three spacetime dimensions*, *Class. Quant. Grav.* **24** (2007) F15 [gr-qc/0610130] [INSPIRE].

[49] G. Barnich and C. Troessaert, *Aspects of the BMS/CFT correspondence*, *JHEP* **05** (2010) 062 [arXiv:1001.1541] [INSPIRE].

[50] G. Barnich, *Entropy of three-dimensional asymptotically flat cosmological solutions*, *JHEP* **10** (2012) 095 [arXiv:1208.4371] [INSPIRE].

[51] A. Bagchi, S. Detournay, R. Fareghbal and J. Simón, *Holography of 3D Flat Cosmological Horizons*, *Phys. Rev. Lett.* **110** (2013) 141302 [arXiv:1208.4372] [INSPIRE].

[52] A.A. Garcia, F.W. Hehl, C. Heinicke and A. Macias, *Exact vacuum solution of a (1+2)-dimensional Poincare gauge theory: BTZ solution with torsion*, *Phys. Rev. D* **67** (2003) 124016 [gr-qc/0302097] [INSPIRE].

[53] M. Blagojevic and B. Cvetkovic, *Black hole entropy in 3-D gravity with torsion*, *Class. Quant. Grav.* **23** (2006) 4781 [gr-qc/0601006] [INSPIRE].

[54] M. Blagojevic and B. Cvetkovic, *Covariant description of the black hole entropy in 3D gravity*, *Class. Quant. Grav.* **24** (2007) 129 [gr-qc/0607026] [INSPIRE].

[55] M. Blagojevic and B. Cvetkovic, *Black hole entropy from the boundary conformal structure in 3D gravity with torsion*, *JHEP* **10** (2006) 005 [gr-qc/0606086] [INSPIRE].

[56] M. Blagojević, B. Cvetković and M. Vasilić, *"Exotic" black holes with torsion*, *Phys. Rev. D* **88** (2013) 101501 [arXiv:1310.1412] [INSPIRE].

[57] L. Avilés, D. Hidalgo and O. Valdivia, *Thermodynamics of the three-dimensional black hole with torsion*, *JHEP* **09** (2023) 185 [arXiv:2308.09114] [INSPIRE].

[58] E. Witten, $(2+1)$-*Dimensional Gravity as an Exactly Soluble System*, *Nucl. Phys. B* **311** (1988) 46 [INSPIRE].

JHEP02(2026)143

[59] O. Coussaert, M. Henneaux and P. van Driel, *The Asymptotic dynamics of three-dimensional Einstein gravity with a negative cosmological constant*, Class. Quant. Grav. **12** (1995) 2961 [gr-qc/9506019] [INSPIRE].

[60] J. Matulich and E. Rodríguez, *Enlarged super-bms3 algebra and its flat limit*, Phys. Rev. D **110** (2024) 064064 [arXiv:2310.16614] [INSPIRE].

[61] M. Henneaux, A. Perez, D. Tempo and R. Troncoso, *Chemical potentials in three-dimensional higher spin anti-de Sitter gravity*, JHEP **12** (2013) 048 [arXiv:1309.4362] [INSPIRE].

[62] C. Bunster et al., *Generalized Black Holes in Three-dimensional Spacetime*, JHEP **05** (2014) 031 [arXiv:1404.3305] [INSPIRE].

[63] T. Regge and C. Teitelboim, *Role of Surface Integrals in the Hamiltonian Formulation of General Relativity*, Annals Phys. **88** (1974) 286 [INSPIRE].

[64] M. Banados, *Global charges in Chern-Simons field theory and the* $(2+1)$ *black hole*, Phys. Rev. D **52** (1996) 5816 [hep-th/9405171] [INSPIRE].

[65] J. Matulich, A. Perez, D. Tempo and R. Troncoso, *Higher spin extension of cosmological spacetimes in 3D: asymptotically flat behaviour with chemical potentials and thermodynamics*, JHEP **05** (2015) 025 [arXiv:1412.1464] [INSPIRE].

[66] A. Bagchi and R. Basu, *3D Flat Holography: Entropy and Logarithmic Corrections*, JHEP **03** (2014) 020 [arXiv:1312.5748] [INSPIRE].

[67] M. Riegler, *Flat space limit of higher-spin Cardy formula*, Phys. Rev. D **91** (2015) 024044 [arXiv:1408.6931] [INSPIRE].

[68] A. Perez, D. Tempo and R. Troncoso, *Higher spin black hole entropy in three dimensions*, JHEP **04** (2013) 143 [arXiv:1301.0847] [INSPIRE].

[69] D. Grumiller et al., *Higher Spin Black Holes with Soft Hair*, JHEP **10** (2016) 119 [arXiv:1607.05360] [INSPIRE].

[70] E.W. Mielke and A.A. Rincon Maggiolo, *Rotating black hole solution in a generalized topological 3-D gravity with torsion*, Phys. Rev. D **68** (2003) 104026 [INSPIRE].

[71] M. Geiller, C. Goeller and N. Merino, *Most general theory of 3d gravity: Covariant phase space, dual diffeomorphisms, and more*, JHEP **02** (2021) 120 [arXiv:2011.09873] [INSPIRE].

[72] A. Castro and M.J. Rodriguez, *Universal properties and the first law of black hole inner mechanics*, Phys. Rev. D **86** (2012) 024008 [arXiv:1204.1284] [INSPIRE].

[73] S. Detournay, *Inner Mechanics of 3d Black Holes*, Phys. Rev. Lett. **109** (2012) 031101 [arXiv:1204.6088] [INSPIRE].

[74] L. Avilés et al., *Non-Relativistic Maxwell Chern-Simons Gravity*, JHEP **05** (2018) 047 [arXiv:1802.08453] [INSPIRE].

[75] L. Avilés, O. Fuentealba, D. Hidalgo and P. Rodríguez, *AdS_3 Carroll gravity: asymptotic symmetries and C-thermal configurations*, JHEP **05** (2025) 174 [arXiv:2503.18818] [INSPIRE].

[76] D.M. Peñafiel and P. Salgado-Rebolledo, *Non-relativistic symmetries in three space-time dimensions and the Nappi-Witten algebra*, Phys. Lett. B **798** (2019) 135005 [arXiv:1906.02161] [INSPIRE].

[77] P. Concha, L. Ravera and E. Rodríguez, *Three-dimensional Maxwellian extended Bargmann supergravity*, JHEP **04** (2020) 051 [arXiv:1912.09477] [INSPIRE].

JHEP02(2026)143

[78] P. Concha, L. Ravera, E. Rodríguez and G. Rubio, *Three-dimensional Maxwellian Extended Newtonian gravity and flat limit*, JHEP **10** (2020) 181 [arXiv:2006.13128] [INSPIRE].

[79] P. Concha, M. Ipinza, L. Ravera and E. Rodríguez, *Non-relativistic three-dimensional supergravity theories and semigroup expansion method*, JHEP **02** (2021) 094 [arXiv:2010.01216] [INSPIRE].

[80] P. Concha, D. Peñafiel, L. Ravera and E. Rodríguez, *Three-dimensional Maxwellian Carroll gravity theory and the cosmological constant*, Phys. Lett. B **823** (2021) 136735 [arXiv:2107.05716] [INSPIRE].

[81] R. Caroca, D.M. Peñafiel and P. Salgado-Rebolledo, *Nonrelativistic spin-3 symmetries in* $2+1$ *dimensions from expanded and extended Nappi-Witten algebras*, Phys. Rev. D **107** (2023) 064034 [arXiv:2208.00602] [INSPIRE].

[82] P. Concha, E. Rodríguez and S. Salgado, *3D Carrollian gravity from 2D Euclidean symmetry*, Eur. Phys. J. C **85** (2025) 517 [arXiv:2501.00205] [INSPIRE].

[83] M.H. Christensen, J. Hartong, N.A. Obers and B. Rollier, *Torsional Newton-Cartan Geometry and Lifshitz Holography*, Phys. Rev. D **89** (2014) 061901 [arXiv:1311.4794] [INSPIRE].

[84] M. Geracie, K. Prabhu and M.M. Roberts, *Curved non-relativistic spacetimes, Newtonian gravitation and massive matter*, J. Math. Phys. **56** (2015) 103505 [arXiv:1503.02682] [INSPIRE].

[85] E. Bergshoeff, J. Rosseel and T. Zojer, *Newton-Cartan supergravity with torsion and Schrödinger supergravity*, JHEP **11** (2015) 180 [arXiv:1509.04527] [INSPIRE].

[86] E. Bergshoeff, A. Chatzistavrakidis, L. Romano and J. Rosseel, *Newton-Cartan Gravity and Torsion*, JHEP **10** (2017) 194 [arXiv:1708.05414] [INSPIRE].

[87] D. Van den Bleeken, *Torsional Newton-Cartan gravity from the large c expansion of general relativity*, Class. Quant. Grav. **34** (2017) 185004 [arXiv:1703.03459] [INSPIRE].

[88] P. Concha, E. Rodríguez, G. Rubio and P. Yañez, *Three-dimensional Newtonian gravity with cosmological constant and torsion*, Eur. Phys. J. C **83** (2023) 47 [arXiv:2204.11763] [INSPIRE].

[89] P. Concha, N. Merino and E. Rodríguez, *Non-relativistic limit of the Mielke-Baekler gravity theory*, Eur. Phys. J. C **84** (2024) 407 [arXiv:2309.00500] [INSPIRE].

[90] P. Concha, O. Fierro and E. Rodríguez, *Hietarinta Chern-Simons supergravity and its asymptotic structure*, Eur. Phys. J. C **84** (2024) 102 [arXiv:2312.14686] [INSPIRE].

[91] S.N. Solodukhin, *Holography with gravitational Chern-Simons*, Phys. Rev. D **74** (2006) 024015 [hep-th/0509148] [INSPIRE].

[92] L. Bonora et al., *Gravitational Chern-Simons Lagrangians and black hole entropy*, JHEP **07** (2011) 085 [arXiv:1104.2523] [INSPIRE].