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String corrected scalar field inflation compatible with the ACT data

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ABSTRACT

We consider the impact of the first string corrections of minimally coupled single scalar field theory on inflationary dynamics. Specifically we consider separately the string corrections $\sim \alpha' \xi(\phi) c_2 \left(\partial_\mu \phi \partial^\mu \phi\right)^2$ and $\sim \alpha' c \xi(\phi) \Box\phi \partial_\mu \phi \partial^\mu \phi$, where α' is the square of the string scale. Our aim is to develop a theory which is self consistent in the sense that the field equations reproduce themselves in the slow-roll approximation. Such a requirement for the theory with $\sim \alpha' \xi(\phi) c_2 \left(\partial_\mu \phi \partial^\mu \phi\right)^2$ resulted to a trivial non-minimal coupling function $\xi(\phi)$, however a self-consistent framework emerged from the theory with correction term $\sim \alpha' c \xi(\phi) \Box\phi \partial_\mu \phi \partial^\mu \phi$. The resulting theory can easily be worked out analytically and we obtained an inflationary theory that can easily be fitted with the Atacama Cosmology Telescope constraints on the scalar spectral index and the updated Planck constraints on the tensor-to-scalar ratio.

1. Introduction

The prominent candidate theory for early times, with observational relevance to present day's physics, is inflation [1–4]. This theory solves all the shortcomings of the hot Big Bang theory and will be thoroughly tested by ground based Cosmic Microwave Background (CMB) experiments like the Simons observatory [5], and from future gravitational wave experiments [6–13,Auclair]. The Simons observatory and the future LiteBird experiment [15] will directly probe the B-mode in the CMB, while the gravitational wave experiments will indirectly probe the inflationary era by determining a stochastic gravitational wave background compatible with some inflationary theory. So far the Pulsar Timing Array experiments confirmed the existence of a stochastic gravitational wave background in 2023 [16–19], but it is highly unlikely that inflation solely can explain this signal [20,21]. The recent Atacama Cosmology Telescope (ACT) data [22,23] combined with the DESI data [24], stirred things up in inflationary physics, since the scalar spectral index is in at least 2σ discordance with the corresponding Planck data [25]. To be specific, the scalar spectral index is constrained by the ACT data to be,

$$n_S = 0.9743 \pm 0.0034, \quad \frac{\mathrm{d} n_S}{\mathrm{d} \ln k} = 0.0062 \pm 0.0052\,. \tag{1}$$

In addition, the updated Planck constraint on the tensor-to-scalar ratio is [26],

$$r < 0.036\,, \tag{2}$$

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at 95% confidence. There is already a large stream of articles in the cosmology literature that aim to explain the ACT result [27–51], although caution is needed for the moment in trusting the ACT result [39]. In this work we aim to examine several scalar field theories with string origin, in view of the ACT result. In string theory, the higher order corrections to the low-energy effective action contain an infinite expansion with an expansion parameter $\alpha' = \lambda_s^2$ where λ_s is the fundamental string scale. Restricting the action to the lowest order gravitational action that ensures that the equations of motion are second order, the first scalar field related corrections to the scalar field action are $\sim \alpha'\xi(\phi)\Big(c\Box\phi\partial_\mu\phi\partial^\mu\phi + c_2\left(\partial_\mu\phi\partial^\mu\phi\right)^2\Big)$ [52,53]. However, we will not attempt any direct connection with a string theory framework, we will use the fact that certain string theories produce the terms just mentioned, that may appear in the low-energy scalar field Lagrangian. However, no contact with an actual string theoretic framework is attempted in this article. Specifically, in this work we aim to present a self-consistent framework for studying inflationary dynamics in such string corrected inflationary theories. We shall develop the formalism that enables us to study analytically such theories and the requirements for making such analytic manipulations of the theory. As we shall show, the resulting theory can be compatible with the ACT data for a wide range of the free parameters of the theory, for approximately 60 e-foldings of inflation.

For this study, we shall assume that the spacetime is a flat Friedmann-Robertson-Walker, with line element,

$$ds^2 = -dt^2 + a(t)^2 \sum_{i=1}^{3} (dx^i)^2 \,. \tag{3}$$

2. Self-consistent string corrected scalar field inflation formalism

In this section we shall develop the string corrected scalar field inflationary theory aiming to present a self-consistent theoretical framework. Before starting, let us note that the inflationary Lagrangian of any sort will depend on the curvature R, the scalar field and its derivatives denoted as $X = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$ and the two known fundamental constants of theoretical physics, the cosmological constant Λ and the reduced Planck mass M_p, that is,

$$\mathcal{S} = \int \mathrm{d}^4x \mathcal{L}(R, X, \phi, M_p, \Lambda)\,. \tag{4}$$

Having this in mind, let us consider the following general string-corrected scalar field action of the form,

$$\mathcal{S} = \int \mathrm{d}^4x\sqrt{-g}\Big(\frac{R}{2\kappa^2} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - c\xi(\phi)\Box\phi\partial_\mu\phi\partial^\mu\phi - c_2\xi(\phi)\big(\partial_\mu\phi\partial^\mu\phi\big)^2\Big)\,, \tag{5}$$

which contains solely higher derivatives of the scalar field and does not contain higher curvature corrections. In Eq. (5), $\kappa = \frac{1}{M_p}$ where M_p is the reduced Planck mass, and c and c_2 are dimensionful constants, which will depend on the fundamental constants of the theory, that is M_p and Λ and $\xi(\phi)$ is a dimensionless function of the scalar field. We shall consider separately the cases with string corrections $\sim c\xi(\phi)\Box\phi\partial_\mu\phi\partial^\mu\phi$ and $\sim c_2\xi(\phi)\big(\partial_\mu\phi\partial^\mu\phi\big)^2$ for simplicity.

2.1. Case I: String corrections of the form $\sim c\xi(\phi)\Box\phi\partial_\mu\phi\partial^\mu\phi$

Let us consider the string corrected scalar field theory with the following action,

$$\mathcal{S} = \int \mathrm{d}^4x\sqrt{-g}\Big(\frac{R}{2\kappa^2} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - c\xi(\phi)\Box\phi\partial_\mu\phi\partial^\mu\phi\Big)\,, \tag{6}$$

which will prove the most interesting case phenomenologically and also structurally since it provides a self-consistent theoretical framework. Upon varying the gravitational action (6) with respect to the metric tensor and the scalar field, we obtain the field equations,

$$\frac{3H^2}{\kappa^2} = \frac{\dot\phi^2}{2} + V + c\big(\dot\xi - 6H\xi\big)\dot\phi^3 \tag{7}$$

$$,-\frac{2\dot H}{\kappa^2} = \dot\phi^2 + c\big(\dot\xi - 6H\xi\big)\dot\phi^3 + c\dot\phi^2\big(2\xi\ddot\phi\xi + \dot\xi\dot\phi\big)\,, \tag{8}$$

$$\ddot\phi + 3H\dot\phi + V' + c\dot\phi\big(\ddot\xi\dot\phi + 3\dot\xi\dot\phi - 6\xi\big(\dot H\dot\phi + 3\dot\xi\dot\phi - 6\xi\big(\dot H\dot\phi + 2H\ddot\phi + 3H^2\dot\phi\big)\big)\big) = 0\,. \tag{9}$$

Now we will make a crucial assumption which can render the present inflationary theory a simple theory to tackle analytically and also a self-consistent theory. We assume that,

$$x = \frac{\dot\xi}{\xi H}\,, \tag{10}$$

where x is an arbitrary dimensionless number the value of which will be determined by demanding the self-consistency of the theory. The motivation for the condition (10) comes from observing the field equations which can be simplified if for example $\dot\xi = 6\xi H$. However we do not specify the value of x at this point, we leave it free to choose.

Due to the slow-roll conditions, it is natural to assume that the potential in the Friedmann Eq. (7) overwhelms the scalar field derivative terms, that is,

$$c\dot\xi\dot\phi^3 \ll V\,,\;\; 6H\xi\dot\phi^3 \ll V\,, \tag{11}$$

and also we assume that the slow-roll conditions hold true,

$$|\dot{H}| \ll H^2 \,. \tag{12}$$

In view of the slow-roll approximations and the conditions (10) and (11), the field equations become,

$$\frac{3H^2}{\kappa^2} \simeq V \,, \tag{13}$$

$$-\frac{2\dot{H}}{\kappa^2} \simeq c(2x-6)H\xi\dot{\phi}^3 \,, \tag{14}$$

$$\dot{\phi}^2 \simeq -\frac{V'}{c(x^2-18)H\xi} \,. \tag{15}$$

where we also assumed that the slow-roll conditions extend to the following quantities too,

$$\ddot{\phi} \ll \dot{\phi}^2, \;\; 3H\dot{\phi} \ll c\dot{\phi}^2\xi H^2 \,. \tag{16}$$

As it will prove, the present theoretical framework is self-consistent and the field equations produce one another for a specific value of x. The hierarchy between the terms in the field equations, is chosen as above, because the observation that if $\dot{\xi}$ and ξH scale in the same way as functions of time and produce a constant x, then the equations of motion are rendered a closed system of equations, as we now show. So basically, this is the motivation for assuming the dominance of the corrections over the friction terms, and of course the approximations must be checked in the end, if indeed they hold true at the first horizon crossing. As we will show, the approximations hold true for the values of the free parameters that guarantee a viable phenomenology. Let us see this explicitly, so starting from Eq. (13) and taking the derivative, we have,

$$\frac{6\dot{H}H}{\kappa^2} = V'\dot{\phi} \,, \tag{17}$$

and upon substituting $\dot{H}$ from Eq. (14) in Eq. (17), we obtain,

$$\frac{6(x-3)}{x^2-18} = 1 \,, \tag{18}$$

which has two solutions $x = 0$ and $x = 6$. The solution $x = 0$ results into a trivial theory, while $x = 6$ is quite intriguing because by looking Eq. (7) we can see that the field equation can be simplified without any constraint. So from now on we take $x = 6$. Let us continue showing that the present theoretical framework represented by the field Eqs. (13)–(15) is self-consistent. So by starting from Eq. (13) and taking the derivative, we get Eq. (17). Now from Eq. (14) due to Eq. (15) we have,

$$-\frac{\dot{H}}{\kappa^2} = -c(x-3)H\xi\dot{\phi}\frac{V'}{c(x^2-18)H^2\xi} \,, \tag{19}$$

so we have,

$$\frac{6\dot{H}}{\kappa^2} = \frac{\dot{\phi}}{H}V' \,, \tag{20}$$

which is identical to Eq. (17). So from Eq. (14) we ended up to Eq. (13) using Eq. (15). Now from Eq. (13) due to Eq. (15) we have,

$$-\frac{\dot{H}}{\kappa^2} = \frac{c(x^2-18)}{6}\xi H\dot{\phi}^3 \,, \tag{21}$$

which is identical with Eq. (14) for $x = 6$. Hence the system of field Eqs. (13)–(15) is closed to itself and self-consistent for $x = 6$.

Now from the constraint Eq. (10), we get,

$$\xi'\dot{\phi} = x\xi H \,, \tag{22}$$

so after some simple algebra, we get,

$$-\frac{(\xi')^2}{\xi^3} = \frac{c(x^2-18)x^2\kappa^4 V^2}{9V'} \,, \tag{23}$$

which apparently constraints the scalar field potential $V(\phi)$ and the non-minimal coupling function $\xi(\phi)$, which cannot be freely chosen, but must satisfy the differential equation (23). Now in order to study the inflationary phenomenology, one needs to find the slow-roll indices and the e-foldings numbers expressed in terms of the functions $\xi(\phi)$ and $V(\phi)$. The e-foldings number is defined to be,

$$N = \int_{\phi_i}^{\phi_f} \frac{H}{\dot{\phi}} \mathrm{d}\phi \,, \tag{24}$$

and by using Eq. (22) we have,

$$N = \frac{1}{x}\int_{\phi_i}^{\phi_f} \frac{\xi'}{\xi} \mathrm{d}\phi \,, \tag{25}$$

where ϕ_i is the value of the scalar field at the first horizon crossing when inflation commences and ϕ_f is the value of the scalar field at the end of the inflationary era. Now for the string corrected theory at hand, the slow-roll indices are [54],

$$\epsilon_1 = -\frac{\dot{H}}{H^2} \qquad \epsilon_2 = \frac{\ddot{\phi}}{H\dot{\phi}} \qquad \epsilon_3 = 0 \qquad \epsilon_4 = \frac{\dot{E}}{2HE} \qquad \epsilon_5 = \frac{Q_a}{2HQ_t} \qquad \epsilon_6 = \frac{\dot{Q}_t}{2HQ_t}\,, \tag{26}$$

with $Q_a = 2c\xi\dot{\phi}^3$, $Q_b = 0$, $E = \frac{1}{(\kappa\dot{\phi})^2}\left(\dot{\phi}^2 + \frac{3Q_a^2}{2Q_t} + Q_c\right)$, $Q_c = 4c\dot{\phi}^3(x-3)\xi H$, $Q_d = -4c\dot{\phi}^2(\dot{\xi}\dot{\phi} + \xi\ddot{\phi} - \xi\dot{\phi}H)$, $Q_e = -8c\xi\dot{\phi}^3$, $Q_f = 0$ and also $Q_t = \frac{1}{\kappa^2} + \frac{Q_b}{2}$. Regarding the observable quantities that determine the viability of the inflationary regime, namely the scalar spectral index of the primordial curvature perturbations and the tensor-to-scalar ratio, these are,

$$n_S = 1 + \frac{2(-2\epsilon_1 - \epsilon_2 - \epsilon_4)}{1-\epsilon_1}\,, \tag{27}$$

and also,

$$r = \left| \frac{16\left(c_A^3\left(\epsilon_1 - \frac{1}{4}\kappa^2\left(\frac{2Q_c+Q_d}{H^2} - \frac{Q_e}{H} + Q_f\right)\right)\right)}{c_T^3\left(\frac{\kappa^2 Q_b}{2} + 1\right)} \right|, \tag{28}$$

where c_A is the sound speed of the scalar perturbations, with its explicit form being,

$$c_A = \sqrt{\frac{\frac{Q_aQ_e}{\frac{2}{\kappa^2}+Q_b} + Q_f\left(\frac{Q_a}{\frac{2}{\kappa^2}+Q_b}\right)^2 + Q_d}{\dot{\phi}^2 + \frac{3Q_a^2}{\frac{2}{\kappa^2}+Q_b} + Q_c} + 1}\,, \tag{29}$$

and moreover c_T stands for the gravitational wave speed, defined as,

$$c_T^2 = 1 - \frac{Q_f}{2Q_t}\,, \tag{30}$$

hence for the theory at hand $c_T^2 = 1$, hence no issues with the GW170817 event emerge. This fact was expected, since the theory contains higher derivatives of the scalar field and not of the curvature. For this theory, the first slow-roll index acquires a very simple form, which is,

$$\epsilon_1 = -\frac{3(x-3)x}{x^2-18}\,\frac{\xi}{\xi'}\,\frac{V'}{V}\,. \tag{31}$$

In addition it is important to keep track of another quantity for the viability of the inflationary era for this theoretical framework, namely the amplitude of scalar perturbations $\mathcal{P}_\zeta(k_*)$ and the constraints on it imposed by the latest Planck data [25]. The definition of the amplitude of the scalar perturbations is,

$$\mathcal{P}_\zeta(k_*) = \frac{k_*^3}{2\pi^2}P_\zeta(k_*)\,, \tag{32}$$

which must be evaluated at the first horizon crossing, when the inflationary era commenced, and k_* stands for the CMB pivot scale. The Planck data constrain the amplitude of the scalar perturbations as follows $\mathcal{P}_\zeta(k_*) = 2.196^{+0.051}_{-0.06} \times 10^{-9}$ [25], evaluated at the CMB pivot scale. The amplitude of the scalar perturbations $\mathcal{P}_\zeta(k)$ can be expressed in terms of the two point function $\zeta(k)$ of the curvature perturbation, in the following way,

$$\langle\zeta(k)\zeta(k')\rangle = (2\pi)^3\delta^3(k-k')P_\zeta(k)\,. \tag{33}$$

For the string corrected scalar field theory at hand, the amplitude of the scalar perturbations $\mathcal{P}_\zeta(k)$ can be expressed in terms of the slow-roll parameters in the following way [54],

$$\mathcal{P}_\zeta(k) = \left(\frac{k\left((-2\epsilon_1 - \epsilon_2 - \epsilon_4)(0.57 + \log(|k\eta|) - 2 + \log(2)) - \epsilon_1 + 1\right)}{(2\pi)\left(zc_A^{\frac{4-n_S}{2}}\right)}\right)^2, \tag{34}$$

with $z = \frac{a\dot{\phi}\sqrt{\frac{E(\phi)}{\frac{1}{\kappa^2}}}}{H(\epsilon_5+1)}$ and all the above quantities have to be evaluated at the first horizon crossing, at which point, the conformal time η is equal to $\eta = -\frac{1}{aH}\frac{1}{-\epsilon_1+1}$ [54]. Now having all the above relations at hand, one can easily examine the inflationary phenomenology of the string corrected scalar field theory at hand. The condition $|\epsilon_1| \sim \mathcal{O}(1)$ will yield the value of the scalar field at the end of inflation, and from Eq. (25) one can evaluate the value of the scalar field at the beginning of inflation. After that, substituting ϕ_i in all the above quantities can give us a direct hint on whether the inflationary theory is viable or not, bearing also in mind the constraints on the amplitude of the scalar perturbations. We shall confront the theory at hand with the ACT data (1) and the updated Planck constraints (2). In the next section we shall give explicit examples of simple models that can generate a viable string corrected scalar field inflationary theory. Before doing that, in the next subsection we consider the theory with string corrections $\sim c_2\xi(\phi)\left(\partial_\mu\phi\partial^\mu\phi\right)^2$ and its consistency.

Before closing this section, let us discuss an interesting question. One may raise the concern that the dominance of the string-inspired correction terms over the canonical kinetic term is implying that the effective-field-theory expansion breaks, since the higher-order stringy corrections, would then be expected to contribute at the same order as the kinetic term. This argument however is not correct.

In general, the low-energy string effective action is basically an expansion in terms of the string scale α', and it is not an expansion in terms of the powers of the kinetic term $X \equiv \dot{\phi}^2/2$. The fact that a specific $\mathcal{O}(\alpha')$ operator may be assumed to dominate the background dynamics does not necessarily imply a loss of the effective field theory control, given that higher-order operators remain perturbatively suppressed by additional powers of the parameter α'. In particular, the effective field theory remains valid, as long as the following hierarchy

$$H^2 \ll M_s^2 \equiv \alpha'^{-1} \tag{35}$$

is satisfied, which is an assumption taken into account during the whole the inflationary evolution considered in this work.

We need to note that it is well known that regimes in which non-canonical kinetic terms may dominate over the canonical contribution are perfectly consistent within effective field theory frameworks. For examples of this sort, one may recall k-inflation, DBI inflation, and Horndeski theories [55–66], in which the higher-derivative operators control the background evolution, without however rendering the theory inconsistent. The present model we discussed, belongs to this class of models.

Also, the background dominance of a higher-derivative operator does not directly implies that higher-order $\mathcal{O}(\alpha'^2)$ corrections can become comparable. Such terms are in general suppressed by additional factors of the parameter $\alpha' H^2$ and thus remain subleading as long as $H^2/M_s^2 \ll 1$. The slow-roll conditions we imposed define a general hierarchy among background contributions, not a hierarchy in the effective field theory expansion parameter.

We also stress that the consistency of the effective field theory is mainly governed by the behavior of cosmological perturbations, and not solely by the background equations. In the present framework, the quadratic action for the scalar perturbations is well defined, and the scalar sound speed remains finite, and we have no strong-coupling inconsistencies. This confirms that the inflationary solutions are valid, despite the dominance of the leading string correction in the background dynamics.

Finally, let us also mention that the argument that corrections should not overwhelm canonical terms is not valid in other theories too, for example in R^2 theory, the R^2 term overwhelms the Einstein Hilbert term R, although the R^2 term is a correction. One may think these in the context of corrections of the standard canonical scalar field action, in which the R^2 term may dominate the leading order kinetic terms of the scalar field.

We need to note that the expressions for the spectral index, the tensor-to-scalar ratio and other observables, hold true if the cosmological perturbations freeze on superhorizon scales. This was proven that it indeed holds true in Ref. [54] and we quote here the proof, following closely [54], see [54] for more details. We perturb the flat FRW metric as follows,

$$ds^2 = -a^2(1+2\alpha)d\eta^2 - 2a^2\beta_{,\alpha}d\eta dx^\alpha + a^2\left(g^{(3)}_{\alpha\beta} + 2\varphi g^{(3)}_{\alpha\beta} + 2\gamma_{,\alpha|\beta} + 2C_{\alpha\beta}\right)dx^\alpha dx^\beta, \tag{36}$$

with $a(\eta)$ being the cosmic scale factor in terms of the conformal time η. Here α, β, γ and φ are scalar spacetime-dependent perturbations, and the tensor perturbation $C_{\alpha\beta}$ is a transverse and a trace-free tensor. The metric $g^{(3)}_{\alpha\beta}$ denotes the comoving three-space section of the FRW metric,

$$g^{(3)}_{\alpha\beta}dx^\alpha dx^\beta = \frac{1}{\left(1+\bar{r}^2\right)^2}\left(dx^2+dy^2+dz^2\right). \tag{37}$$

The kinematic quantities expressed in the normal frame are as follows,

$$\theta = 3H, \quad \sigma_{\alpha\beta} = \chi_{,\alpha|\beta} - \frac{1}{3}g^{(3)}_{\alpha\beta}\Delta\chi + a^2\dot{C}^{(t)}_{\alpha\beta}, \quad a_\alpha = \alpha_{,\alpha}, \quad R^{(h)} = \frac{1}{a^2}[6\,K - 4\Delta\varphi], \tag{38}$$

with

$$\chi \equiv a(\beta + a\dot{\gamma}), \tag{39}$$

and Δ is the Laplacian operator in $g^{(3)}_{\alpha\beta}$ and also θ stands for the expansion scalar, σ_{ab} stands for the shear tensor, and a_a denotes the acceleration vector. We use the following gauge transformation, $\hat{x}^a \equiv x^a + \tilde{\xi}^a(x^e)$ and we have the expressions for the perturbation variables,

$$\hat{\alpha} = \alpha - \dot{\xi}^t, \quad \hat{\beta} = \beta - \frac{1}{a}\xi^t + a\left(\frac{\xi}{a}\right)^{\cdot}, \quad \hat{\gamma} = \gamma - \frac{1}{a}\xi, \quad \hat{\varphi} = \varphi - H\xi^t, \quad \hat{\chi} = \chi - \xi^t, \quad \hat{\kappa} = \kappa + \left(3\dot{H} + \frac{\Delta}{a^2}\right)\xi^t,$$
$$\delta\hat{\mu} = \delta\mu - \dot{\mu}\xi^t, \quad \delta\hat{p} = \delta p - \dot{p}\xi^t, \quad \hat{v} = v - \frac{1}{a}\xi^t, \quad \hat{\Pi} = \Pi, \quad \delta\hat{\phi} = \delta\phi - \dot{\phi}\xi^t; \quad \hat{C}_{\alpha\beta} = C_{\alpha\beta}, \quad \hat{\Pi}^{(t)}_{\alpha\beta} = \Pi^{(t)}_{\alpha\beta}, \tag{40}$$

where $\xi^0 \equiv \frac{1}{a}\xi^t$ and in addition $\xi_\alpha \equiv \xi_{,\alpha}$. Furthermore, $\bar{\phi}$ and $\delta\phi$ denote the background and the perturbation part of the scalar field $\phi(\mathbf{x},t)$. We use the gauge-invariant variables,

$$\varphi_\chi \equiv \varphi - H\chi, \quad \varphi_v \equiv \varphi - aHv, \quad \delta_v \equiv \delta - a\frac{\dot{\mu}}{\mu}v, \quad \delta\phi_\varphi \equiv \delta\phi - \frac{\dot{\phi}}{H}\varphi \equiv -\frac{\dot{\phi}}{H}\varphi_{\delta\phi}, \quad v_\chi \equiv v - \frac{1}{a}\chi \equiv -\frac{1}{a}\chi_v, \tag{41}$$

where $\delta \equiv \delta\mu/\mu$. Also we use the following perturbation variable definitions,

$$\Phi \equiv \varphi_{\delta\phi}, \quad \Psi \equiv \varphi_{\chi} + \frac{\dot{F}+Q_a}{2F+Q_b}\frac{\delta F_{\chi}}{\dot{F}}. \tag{42}$$

and also,

$$c_A^2 = 1 + \frac{Q_d + \frac{\dot{F}+Q_a}{2F+Q_b}Q_e + \left(\frac{\dot{F}+Q_a}{2F+Q_b}\right)^2 Q_f}{\omega\dot{\phi}^2 + 3\frac{(\dot{F}+Q_a)^2}{2F+Q_b} + Q_c}. \tag{43}$$

The scalar-type perturbation equations considered in [54] for higher derivative theories, can be written as follows,

$$\dot{\Phi} = 2x_1\frac{\Delta}{a^2}\Psi, \tag{44}$$

$$\frac{1}{x_2}\left(x_2\Psi\right)^{\cdot} = \frac{1}{2}x_3\Phi, \tag{45}$$

where,

$$x_1 = \frac{\left(H + \frac{\dot{F}+Q_a}{2F+Q_b}\right)\left(F + \frac{1}{2}Q_b\right)}{\omega\dot{\phi}^2 + 3\frac{(\dot{F}+Q_a)^2}{2F+Q_b} + Q_c}, \tag{46}$$

$$x_2 = \frac{a(F + \frac{1}{2}Q_b)}{H + \frac{\dot{F}+Q_a}{2F+Q_b}}, \tag{47}$$

$$x_3 = \frac{1}{\left(H + \frac{\dot{F}+Q_a}{2F+Q_b}\right)\left(F + \frac{1}{2}Q_b\right)}x_4, \tag{48}$$

$$x_4 = \dot{\phi}^2 + 3\frac{(\dot{F}+Q_a)^2}{2F+Q_b} + Q_c + Q_d + \frac{\dot{F}+Q_a}{2F+Q_b}Q_e + \left(\frac{\dot{F}+Q_a}{2F+Q_b}\right)^2 Q_f, \tag{49}$$

and $F = \frac{1}{\kappa^2}$. After some normalization and redefinitions of variables, as follows,

$$z = \frac{a\dot{\phi}}{H}, \qquad \tilde{v} \equiv z\Phi, \quad u \equiv \frac{1}{\kappa^2}\frac{a}{H}\frac{1}{z}\Psi, \tag{50}$$

we end up to the perturbation equations,

$$\tilde{v}'' - \left(c_A^2\Delta + \frac{z''}{z}\right)\tilde{v} = a^2 z\left[\frac{1}{az^2}\left(az^2\dot{\Phi}\right)^{\cdot} - c_A^2\frac{\Delta}{a^2}\Phi\right] = 0, \tag{51}$$

$$u'' - \left[c_A^2\Delta + \frac{(1/\bar{z})''}{(1/\bar{z})}\right]u = \frac{a^2 x_2}{\bar{z}}\left\{\frac{\bar{z}^2}{ax_2}\left[\frac{a}{\bar{z}^2}\left(x_2\Psi\right)^{\cdot}\right]^{\cdot} - c_A^2\frac{\Delta}{a^2}\Psi\right\} = 0. \tag{52}$$

In these differential equations of the perturbations, c_A plays the role of the wave speed of the fluctuating fluid or field and the simultaneously excited metric.

For the tensor mode, using the following

$$z_t \equiv a\sqrt{Q_t}, \quad v_t \equiv z_t\Phi, \tag{53}$$

with $\Phi = C_{\alpha\beta}$ or $h_{\ell\mathbf{k}}$, we get

$$v_t'' - \left(c_T^2\Delta + \frac{z_t''}{z_t}\right)v_t = a^2 z_t\left[\frac{1}{az_t^2}\left(az_t^2\dot{\Phi}\right)^{\cdot} - c_T^2\frac{\Delta}{a^2}\Phi\right] = 0. \tag{54}$$

The differential equations (51,54) are generally valid for a wide variety of higher derivative gravity theories. In the large-scale limits (superhorizon scales), with $c_A^2 k^2 \ll z''/z$ and $(1/\bar{z})''/(1/\bar{z})$, we get

$$\Phi(k,\eta) = \frac{1}{z}\tilde{v} = C(k)\left\{1 + k^2\left[\int^{\eta}\bar{z}^2\left(\int^{\eta}\frac{d\eta}{z^2}\right)d\eta - \int^{\eta}\bar{z}^2\,d\eta\int^{\eta}\frac{d\eta}{z^2}\right]\right\} - 2\tilde{d}(k)k^2\int^{\eta}\frac{d\eta}{z^2}, \tag{55}$$

$$\Psi(k,\eta) = \frac{\bar{z}}{x_2}u = C(k)\frac{1}{2x_2}\int^{\eta}\bar{z}^2\,d\eta + \tilde{d}(k)\frac{1}{x_2}\left\{1 + k^2\left[\int^{\eta}\frac{1}{z^2}\left(\int^{\eta}\bar{z}^2\,d\eta\right)d\eta - \int^{\eta}\bar{z}^2\,d\eta\int^{\eta}\frac{d\eta}{z^2}\right]\right\}. \tag{56}$$

Notice that at leading order in the large-scale expansion, the C-mode of the perturbation Φ remains constant on superhorizon scales. Thus, by ignoring the transient mode, we get

$$\Phi(k,\eta) = C(k). \tag{57}$$

and it is remarkable to point out that it remains constant regardless the equation of state $p(\mu)$ (of the matter fluid content, with pressure p and energy density μ), the field potential $V(\phi)$, and the gravity theories $f(\phi,R,X)$, $\omega(\phi)$, $\xi(\phi)$.

2.2. Case II: String corrections of the form $\sim c_2\xi(\phi)\big(\partial_\mu\phi\partial^\mu\phi\big)^2$

Let us now consider another class of string corrected scalar field theory with the following action,

$$S=\int \mathrm{d}^4x\sqrt{-g}\Big(\frac{R}{2\kappa^2}-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi-V(\phi)-c_2\xi(\phi)\big(\partial_\mu\phi\partial^\mu\phi\big)^2\Big)\,, \tag{58}$$

which will prove not so interesting phenomenologically. Upon varying the gravitational action (6) with respect to the metric and the scalar field, we obtain the field equations,

$$\frac{3H^2}{\kappa^2}=\frac{\dot\phi^2}{2}+V-c_2\xi\dot\phi^4\,, \tag{59}$$

$$-\frac{2\dot H}{\kappa^2}=-2c_2\xi\dot\phi^4\,, \tag{60}$$

$$\ddot\phi+3H\dot\phi+V'+c_2\dot\phi^2\big(-3\dot x\dot\phi-12\xi\ddot\phi-12H\xi\dot\phi\big)=0\,. \tag{61}$$

Now we will make again the assumption (10) and also we assume that the slow-roll conditions hold true, so the field equations become,

$$\frac{3H^2}{\kappa^2}\simeq V\,, \tag{62}$$

$$\frac{\dot H}{\kappa^2}\simeq c_2\xi\dot\phi^4\,, \tag{63}$$

$$\dot\phi^3\simeq\frac{2V'}{3(x+4)\,c_2\xi H}\,. \tag{64}$$

Now let us see whether the above set of field equations is self-consistent and closed to itself. From Eq. (62) by taking the derivative we obtain,

$$\frac{6\dot H H}{\kappa^2}=V'\dot\phi\,, \tag{65}$$

and from Eq. (63), we obtain,

$$\frac{x+4}{4}=1\,, \tag{66}$$

which has the solutions $x=0$, however this is a trivial solution since it yields $\xi=\mathrm{constant}$. Thus we shall not further analyze this class of string corrected scalar field theories. We need to note that the choice $x=$constant is made after the field equations are derived and this is a restriction on the dynamics of inflation, but does not restrict the dynamics before the variation of the action. In some sense it is a restriction on some subspace of the total phase space of the system. In addition, we assumed that $x=$constant, in the scenario that x is not constant, the whole analysis collapses because the solution or the trajectories in the total phase space, actual propagate through the entire phase space. Furthermore, the system of equations would not be a closed system exactly reproducible. In addition, we tried the $x\neq\mathrm{constant}$ case, but it is truly impossible to derive any analytic results. There are too many terms that mix in a non-hierarchical or manageable way, hence it is too difficult to determine the dynamics analytically, even in the slow-roll case.

3. String corrected scalar field inflation models compatible with the ACT data

Let us now present some viable scenarios of string corrected scalar field theory, which are compatible with both the ACT data and the updated Planck constraints on the tensor-to-scalar ratio.

We start off with a power-law scenario for the scalar coupling function $\xi(\phi)$, in which case the function $\xi(\phi)$ is chosen to be,

$$\xi(\phi)=\frac{(b+\lambda(\kappa\phi))^n}{M}\,, \tag{67}$$

where $\lambda=\frac{\delta M_p^2}{\Lambda}$, b is a dimensionless parameter and $M=\frac{\Lambda^3\mu}{M_p^6}$ and μ is a dimensionless parameter. By solving the differential Eq. (23), given $\xi(\phi)$ in Eq. (67) we obtain the scalar potential $V(\phi)$,

$$V(\phi)=\frac{\kappa^2\lambda^2M(n+3)}{\frac{72c\kappa^3(b+\kappa\lambda\phi)^{n+3}}{\lambda n^2}+c_1}\,, \tag{68}$$

where c_1 is a dimension $[m]^{-4}$ integration constant of the form $c_1=\frac{\beta}{\Lambda^{-2}}$ and β is dimensionless. For the given scalar potential and coupling function $\xi(\phi)$, the first slow-roll index ϵ_1 from Eq. (31) reads in this case,

$$\epsilon_1=\frac{216c\kappa^3(n+3)(b+\kappa\lambda\phi)^{n+3}}{n\big(72c\kappa^3(b+\kappa\lambda\phi)^{n+3}+c_1\lambda n^2\big)}\,, \tag{69}$$

so upon solving $\epsilon_1(\phi_f)=1$ we obtain,

$$\phi_f=\frac{\Big(\frac{c_1\lambda n^3}{c\kappa^3(144n+648)}\Big)^{\frac{1}{n+3}}-b}{\kappa\lambda}\,. \tag{70}$$

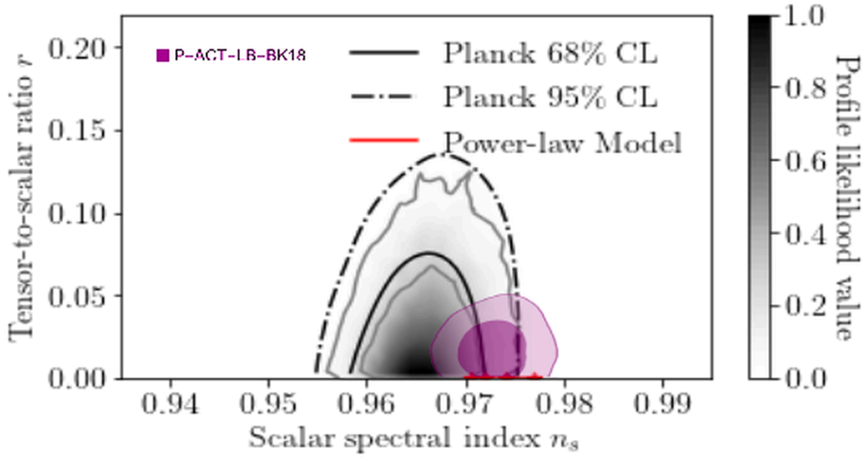

Fig. 1. Marginalized curves of the Planck 2018 data and power-law model (67) versus the ACT data, the Planck 2018 data and the updated Planck tensor-to-scalar ratio constraint.

Upon plugging this in the expression for the e-foldings number in Eq. (25), given $\xi(\phi)$, we get ϕ_i, which is for this model,

$$\phi_i = \frac{e^{-\frac{6N}{n}}\left(\frac{c_1\lambda n^3}{c\kappa^3(144n+648)}\right)^{\frac{1}{n+3}} - b}{\kappa\lambda}\,. \tag{71}$$

Now one can obtain a viable phenomenology compatible with the ACT data (1) and the updated Planck constraints (2), by choosing in this case for example $\delta = 9.08099\times 10^{-38}$ and $\beta = 4.55945\times 10^{71}$ for $N = 60$ and $b = 2$, in which case the scalar spectral index becomes $n_S = 0.97428$ and the tensor-to-scalar ratio becomes $r = 7.89\times 10^{-174}$, practically zero, and the amplitude of the scalar perturbations becomes $\mathcal{P}_\zeta(k_*) = 2.183\times 10^{-9}$. In Fig. 1 we confront the model at hand with the ACT and Planck 2018 likelihood curves and the updated Planck 2018 constraints on the tensor-to-scalar ratio for β chosen in the range $\beta = [6.839\times 10^{70}, 2.097\times 10^{72}]$.

As it can be seen in Fig. 1, the model (67) is well fitted inside the ACT data (1) and the Planck constraint (2).

Let us now consider an exponential model in which case we choose the coupling function $\xi(\phi)$ as follows,

$$\xi(\phi) = \lambda\exp(\gamma(\kappa\phi))\,, \tag{72}$$

where λ is a dimensionless constant and $\gamma = \frac{\delta M_p^2}{\Lambda}$. Again by solving the differential Eq. (23), given $\xi(\phi)$ in Eq. (72) we obtain the scalar potential $V(\phi)$,

$$V(\phi) = -\frac{\gamma^3}{\gamma^3 c_1 - 72c\kappa\lambda e^{\gamma\kappa\phi}}\,, \tag{73}$$

where c_1 is again a dimension $[m]^{-4}$ integration constant of the form $c_1 = \frac{\beta M_p^3}{\Lambda^{3/2}}$. For the given scalar potential and coupling function $\xi(\phi)$, the first slow-roll index ϵ_1 from Eq. (31) reads in this case,

$$\epsilon_1 = -\frac{216c\kappa\lambda e^{\gamma\kappa\phi}}{\gamma^3 c_1 - 72c\kappa\lambda e^{\gamma\kappa\phi}}\,, \tag{74}$$

so in this case, upon solving $\epsilon_1(\phi_f) = 1$ we obtain,

$$\phi_f = \frac{\log\left(-\frac{\gamma^3 c_1}{144c\kappa\lambda}\right)}{\gamma\kappa}\,. \tag{75}$$

Upon plugging this in the expression for the e-foldings number in Eq. (25), given $\xi(\phi)$, we get ϕ_i, which is for this model,

$$\phi_i = \frac{\log\left(-\frac{\gamma^3 c_1}{144c\kappa\lambda}\right) - 6N}{\gamma\kappa}\,. \tag{76}$$

Now one can obtain a viable phenomenology compatible with the ACT data (1) and the updated Planck constraints (2), by choosing in this case for example $\delta = 1.2557\times 10^{-42}$, $\beta = -7.585\times 10^{-20}$, for $N = 60$ and $\lambda = 10^{1.9}$ and $c = 10^{-3.2}$, in which case the scalar spectral index becomes $n_S = 0.974$ and the tensor-to-scalar ratio becomes $r = 4.0952\times 10^{-171}$ and in addition, the amplitude of the scalar perturbations becomes $\mathcal{P}_\zeta(k_*) = 2.198\times 10^{-9}$. In Fig. 2 we confront the model at hand with the ACT and Planck 2018 likelihood curves and the updated Planck 2018 constraints on the tensor-to-scalar ratio for δ in the range $\delta = [1.141\times 10^{-42}, 1.304\times 10^{-42}]$.

As it can be seen in Fig. 2, the model (72) is well fitted inside the ACT data (1) and the Planck constraint (2).

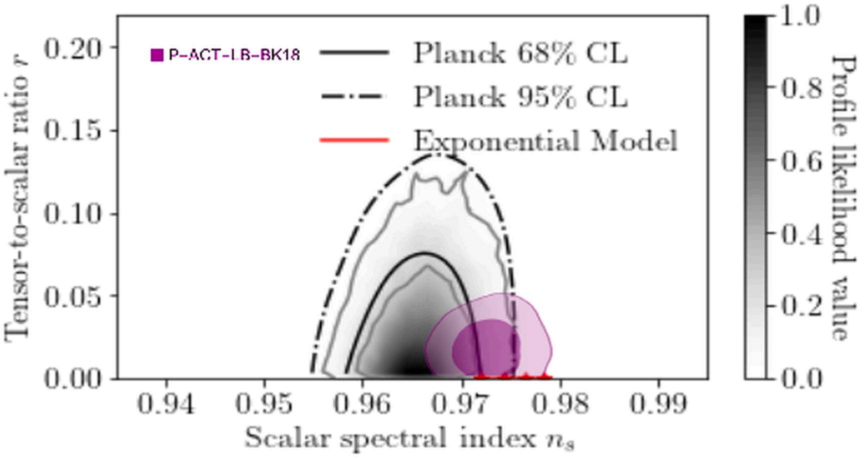

Fig. 2. Marginalized curves of the Planck 2018 data and power-law model (72) versus the ACT data, the Planck 2018 data and the updated Planck tensor-to-scalar ratio constraint.

Thus in this section we demonstrated that the string corrected scalar field theory framework can provide viable inflationary phenomenology compatible with both the ACT data and the updated Planck constraints. The characteristic of the two models we presented is the almost zero tensor-to-scalar ratio, but this has to be a model-dependent feature.

Before closing, let us briefly discuss the approximations we made in the field equations, and if these are justified in the end, for the values of the free parameters we chose for the exponential and the power-law model. In the first place, a justifiable objection is that the kinetic term $\frac{\dot{\phi}^2}{2}$ cannot be subleading compared to $c(2x-6)H\xi\dot{\phi}^3$ which originates from a string correction origin. This might be indeed logical, however using this assumption, we ended up to a closed set of field equations, which is self-consistent and can become ACT-compatible. Moreover, the approximation $\frac{\dot{\phi}^2}{2} \ll c(2x-6)H\xi\dot{\phi}^3$ is found numerically that it holds true, for example for the power-law model, using the set of values of the free parameters for which the ACT-compatibility is guaranteed we ended up to $\frac{\dot{\phi}^2}{2} = 10^{-26}M$ and $c(2x-6)H\xi\dot{\phi}^3 = 10^{-20}M$ in Planck units. Also for the exponential model we get $\frac{\dot{\phi}^2}{2} = 10^{-157}/c_1$ and $c(2x-6)H\xi\dot{\phi}^3 = 110^{-155}/c_1$, which validates the assumption.

However we need to note that the results indicate that a certain level of fine-tuning of the variables is required in order to obtain phenomenologically viable results. This feature counts on the downside of this theoretical framework.

In addition, since the theory studied in this work is a potential driven scalar field theory, one expects that the reheating era will be initiated by small oscillations of the scalar field. We will not go into details since the full treatment would require careful analysis, but the theory is a scalar theory and we expect that the field oscillations will drive the reheating stage.

4. Conclusions

In this work we considered the first low-energy string corrections to the single scalar field inflationary Lagrangian. Specifically, it is known that in string theory the higher order corrections to the low-energy effective action contain an infinite expansion with an expansion parameter $\alpha' = \lambda_s^2$ with λ_s being the fundamental string scale. If one restricts the action to the lowest order gravitational action, which ensures that the equations of motion are second order, the first corrections to the single scalar field action are $\sim \alpha'\xi(\phi)\left(c\Box\phi\partial_\mu\phi\partial^\mu\phi + c_2\left(\partial_\mu\phi\partial^\mu\phi\right)^2\right)$ [52,53]. We examined the effect of these two correction terms independently on the inflationary dynamics of single scalar field inflation. Using only the slow-roll assumption, we aimed to develop a formalism that is self-consistent, so the approximations used were not just leading order corrections to field equations, but the field equations reproduce one the other. The theory with correction term $\sim \alpha' c_2\left(\partial_\mu\phi\partial^\mu\phi\right)^2$ did not result to interesting results but yielded a trivial inflationary theory with constant non-minimal coupling function $\xi(\phi)$, which we did not further study. However, the theory with the correction term $\sim \alpha'\left(c\Box\phi\partial_\mu\phi\partial^\mu\phi\right)$ yielded quite interesting results, since the resulting theory was found self-consistent, with the field equations reproducing one another. We found analytically the slow-roll indices and the e-foldings number in closed forms and we examined two models, with the non-minimal coupling function $\xi(\phi)$ having a power-law form and an exponential form. Accordingly, the scalar potential was found via the differential equation that it is required to satisfy, which also contains the non-minimal coupling function $\xi(\phi)$. The models were confronted with both the ACT data and the updated Planck constraints on the tensor-to-scalar ratio and the results indicated that the models can easily be compatible with the ACT data, for 60 e-foldings. One non-trivial extension of this work is to try to develop a theory that simultaneously considers the above string corrections, or even including Einstein-Gauss-Bonnet terms. This task however exceeds the purposes of this letter. Finally, it is interesting to note that the class of models we studied in this work may generate interesting dark energy phenomena, such as phantom divide crossings [67]. Also it is interesting to note that small tensor-to-scalar ratios can also be achieved by scalar-tensor theories [45]. We further need to note that the ACT data assume overall base ΛCDM, however the DESI data indicated that the dark energy might be dynamical [68]. Such issues must be taken into account when the ACT inflationary data are considered, as for example in Yuennan et al. [69]. One thing is certain, it is too early to make a decisive conclusion about the tilt difference in the spectral index between Planck and ACT. There are strong hints though. This debatable difference also propagates on the running of the spectral index, at $1-\sigma$ significance. The ACT data point out a positive spectral index. This is sensational, so the future data are eagerly anticipated. In addition, it is important to stress that the value of the spectral index is inferred under the assumption that the ΛCDM model holds pre-recombination, and that any type of new physics introduced pre-recombination for example in order to solve the Hubble tension, (such as early dark energy), typically shifts the spectral index upwards towards the Harrison-Zeldovich scale invariant value usually to compensate for changes in the damping scale.

Data availability

No data was used for the research described in the article.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] A.D. Linde, Lect. Notes Phys. 738 (2008) 1. arXiv:0705.0164, [Hep-th].
[2] D.S. Gorbunov, V.A. Rubakov, Introduction to the theory of the early universe: Cosmological perturbations and inflationary theory, World Scientific, Hackensack, USA, Hackensack, USA, 2011 489.
[3] A. Linde, arxiv:1402.0526, [Hep-th].
[4] D.H. Lyth, A. Riotto, Phys. Rept. 314 (1999) 1, [hep-ph/9807278].
[5] M.H. Abitbol, Simons observatory], Bull. Am. Astron. Soc 51 (2019) 147. arXiv:1907.08284, [Astro-ph.IM].
[6] S. Hild, M. Abernathy, F. Acernese, P. Amaro-Seoane, N. Andersson, K. Arun, F. Barone, B. Barr, M. Barsuglia, M. Beker, Class. Quant. Grav. 28 (2011) 94013. arXiv:1012.0908. https://doi.org/10.1088/0264-9381/28/9/094013 [Gr-qc].
[7] J. Baker, J. Bellovary, P.L. Bender, E. Berti, R. Caldwell, J. Camp, J.W. Conklin, N. Cornish, C. Cutler, R. Derosa, arxiv:1907.06482, [Astro-ph.IM].
[8] T.L. Smith, R. Caldwell, Phys. Rev. D 100 (10) (2019), arXiv:1908.00546. https://doi.org/10.1103/PhysRevD.100.104055, [Astro-ph.CO].
[9] J. Crowder, N.J. Cornish, Phys. Rev. D 72 (2005) 83005. https://doi.org/10.1103/PhysRevD.72.083005,arXiv:gr-qc/0506015,[gr-qc].
[10] T.L. Smith, R. Caldwell, Phys. Rev. D 95, (4) (2017) 44036. arXiv:1609.05901. https://doi.org/10.1103/PhysRevD.95.044036, [Gr-qc].
[11] N. Seto, S. Kawamura, T. Nakamura, Phys. Rev. Lett. 87 (2001) 221103. arXiv:astro-ph/0108011. https://doi.org/10.1103/PhysRevLett.87.221103, [Astro-ph].
[12] S. Kawamura, M. Ando, N. Seto, S. Sato, M. Musha, I. Kawano, J. Yokoyama, T. Tanaka, K. Ioka, T. Akutsu, arxiv:2006.13545, [Gr-qc].
[13] A. Weltman, P. Bull, S. Camera, K. Kelley, H. Padmanabhan, J. Pritchard, A. Raccanelli, S.R.-S. rensen, L. Shao, S. Andrianomena, Astron. Soc. Austral. 37 (2020) 2. arXiv:1810.02680. https://doi.org/10.1017/pasa.2019.42, [Astro-ph.CO].
[Auclair] P. Auclair, LISA Cosmology Working Group arxiv:2204.05434 , [Astro-ph.CO].
[15] E. Allys, LiteBIRD, PTEP (2023). 4, 042F01, arXiv:2202.02773. https://doi.org/10.1093/ptep/ptac150, [astro-ph.IM]
[16] G. Agazie, NANOGrav] Astrophys. J. Lett. 951 (1) (2023). L8 arXiv:2306.16213. https://doi.org/10.3847/2041-8213/acdac6, [astro-ph.HE].
[17] J. Antoniadis, P. Arumugam, S. Arumugam, S. Babak, M. Bagchi, A.S.B. Nielsen, C.G. Bassa, A. Bathula, A. Berthereau, M. Bonetti, arxiv:2306.16214, [Astro-ph.HE].
[18] D.J. Reardon, A. Zic, R.M. Shannon, G.B. Hobbs, M. Bailes, V. Di, A. Marco, A.F. Kapur, E. Rogers, J. Thrane, Askew, arxiv:2306.16215, https://doi.org/10.3847/2041-8213/acdd02,[Astro-ph.HE].
[19] H. Xu, S. Chen, Y. Guo, J. Jiang, B. Wang, J. Xu, Z. Xue, R.N. Caballero, J. Yuan, Y. Xu, arxiv:2306.16216, https://doi.org/10.1088/1674-4527/acdfa5, [Astro-ph.HE].
[20] S. Vagnozzi, JHEAp 39 (2023) 81–98. arXiv:2306.16912. https://doi.org/10.1016/j.jheap.2023.07.001,[Astro-ph.CO].
[21] V.K. Oikonomou, Phys. Rev. D 108 (4) (2023) 43516. arXiv:2306.17351. https://doi.org/10.1103/PhysRevD.108.043516, [Astro-ph.CO].
[22] T. Louis, [ACT] arxiv:2503.14452, [Astro-ph.CO].
[23] E. Calabrese, [ACT] arxiv:2503.14454, [Astro-ph.CO].
[24] A.G. Adame, [[DESI],] JCAP 04 (2025). arXiv:2404.03000. https://doi.org/10.1088/1475-7516/2025/04/012, [Astro-ph.CO]
[25] Y. Akrami, Planck Astron. Astrophys. 641 (10) (2020). arXiv:1807.06211. https://doi.org/10.1051/0004-6361/201833887,[Astro-ph.CO].
[26] P.A.R. Ade, BICEP and Keck Phys. Rev. Lett. 127 (15) (2021). arXiv:2110.00483. https://doi.org/10.1103/PhysRevLett.127.151301, [Astro-ph.CO].
[27] R. Kallosh, A. Linde, D. Roest, arxiv:2503.21030, [Hep-th].
[28] Q. Gao, Y. Gong, Z. Yi, F. Zhang, arxiv:2504.15218 , [Astro-ph.CO].
[29] L. Liu, Z. Yi, Y. Gong, arxiv:2505.02407 , [Astro-ph.CO].
[30] A. Yogesh, Q. Mohammadi, T. Wu, Zhu, arxiv:2505.05363, [Astro-ph.CO].
[31] Z. Yi, X. Wang, Q. Gao, Y. Gong, arxiv:2505.10268, [Astro-ph.CO].
[32] Z.Z. Peng, Z.C. Chen, L. Liu, arxiv:2505.12816, [Astro-ph.CO].
[33] W. Yin, arxiv:2505.03004, [Hep-ph].
[34] C.T. Byrnes, M. Cortês, A.R. Liddle, arxiv:2505.09682, [Astro-ph.CO].
[35] W.J. Wolf, arxiv:2506.12436, [astro-ph.CO].
[36] S. Aoki, H. Otsuka, R. Yanagita, arxiv:2504.01622, [Hep-ph].
[37] Q. Gao, Y. Qian, Y. Gong, Z. Yi, arxiv:2506.18456, [Gr-qc].
[38] M. Zahoor, S. Khan, I.A. Bhat, arxiv:2507.18684, [Astro-ph.CO].
[39] E.G.M. Ferreira, E. Mcdonough, L. Balkenhol, R. Kallosh, L. Knox, A. Linde, arxiv:2507.12459, [Astro-ph.CO].
[40] A. Mohammadi, Yogesh, A. Wang, arxiv:2507.06544, [Astro-ph.CO].
[41] S. Choudhury, G. Bauyrzhan, S.K. Singh, K. Yerzhanov, arxiv:2506.15407, [Astro-ph.CO].
[42] S.D. Odintsov, V.K. Oikonomou, Phys. Lett. B 868 (2025) 139779. arXiv:2506.08193. https://doi.org/10.1016/j.physletb.2025.139779, [Gr-qc].
[43] D.A.Q. R, J. Chagoya, A.A. Roque, arxiv:2508.13273, [Gr-qc].
[44] Y. Zhu, Q. Gao, Y. Gong, Z. Yi, arxiv:2508.09707, [Astro-ph.CO].
[45] G. Kouniatalis, E.N. Saridakis, arxiv:2507.17721, [Astro-ph.CO].
[46] M. Hai, A.R. Kamal, N.F. Shamma, M.S.J. Shuvo, arxiv:2506.08083, [Hep-th].
[47] C. Dioguardi, A.J. Iovino, A. Racioppi, Phys. Lett. B 868 (2025) 139664. arXiv:2504.02809. https://doi.org/10.1016/j.physletb.2025.139664, [Gr-qc].
[48] J. Yuennan, P. Koad, F. Atamurotov, P. Channuie, arxiv:2508.17263, [Astro-ph.CO].

[49] H.J. Kuralkar, S. Panda, A. Vidyarthi, arxiv:2504.15061, [Gr-qc].
[50] H.J. Kuralkar, S. Panda, A. Vidyarthi, JCAP 05 (2025). 073 arXiv:2502.03573. https://doi.org/10.1088/1475-7516/2025/05/073,[Astro-ph.CO].
[51] S. Aoki, H. Otsuka, R. Yanagita, arxiv:2509.06739, [Hep-ph].
[52] R.R. Metsaev, A.A. Tseytlin, Nucl. Phys. B 293 (1987) 385–419. https://doi.org/10.1016/0550-3213(87)90077-0
[53] C. Cartier, J.C. Hwang, E.J. Copeland, Phys. Rev. D 64 (2001) 103504. arXiv:astro-ph/0106197. https://doi.org/10.1103/PhysRevD.64.103504, [Astro-ph].
[54] J.C. Hwang, H. Noh, Phys. Rev. D 71 (2005) 63536. https://doi.org/10.1103/PhysRevD.71.063536, [gr-qc/0412126].
[55] J. Ohashi, J. Soda, S. Tsujikawa, Phys. Rev. D 88 (2013) 103517. arXiv:1310.3053. https://doi.org/10.1103/PhysRevD.88.103517, [Hep-th].
[56] F. Arroja, T. Tanaka, JCAP 05 (2011). arXiv:1103.1102. https://doi.org/10.1088/1475-7516/2011/05/005,[Astro-ph.CO].
[57] V. Taveras, N. Yunes, Phys. Rev. D 78 (2008) 64070. arXiv:0807.2652. https://doi.org/10.1103/PhysRevD.78.064070, [Gr-qc].
[58] C. Armendariz-Picon, T. Damour, V.F. Mukhanov, Phys. Lett. B 458 (1999) 209–218. arXiv:hep-th/9904075. https://doi.org/10.1016/S0370-2693(99)00603-6,[Hep-th].
[59] J. Garriga, V.F. Mukhanov, Phys. Lett. B 458 (1999) 219–225. arXiv:hep-th/9904176. https://doi.org/10.1016/S0370-2693(99)00602-4,[Hep-th].
[60] S. Choudhury, S, Eur. Phys. J. C 75 (6) (2015) 241. arXiv:1210.4478. https://doi.org/10.1140/epjc/s10052-015-3452-3,[Hep-th].
[61] Y.F. Cai, J.B. Dent, D.A. Easson, Phys. Rev. D 83 (2011) 101301. arXiv:1011.4074. https://doi.org/10.1103/PhysRevD.83.101301,[Hep-th].
[62] D. Langlois, S. Renaux-Petel, D.A. Steer, T. Tanaka, Phys. Rev. D 78 (2008) 63523. arXiv:0806.0336. https://doi.org/10.1103/PhysRevD.78.063523, [Hep-th].
[63] X. Chen, Phys. Rev. D 72 (2005) 123518. arXiv:astro-ph/0507053. https://doi.org/10.1103/PhysRevD.72.123518,[Astro-ph].
[64] T. Kobayashi, Rept. Prog. Phys. 82 (8) (2019) 86901. arXiv:1901.07183. https://doi.org/10.1088/1361-6633/ab2429,[Gr-qc].
[65] H. Lu, Y. Pang, Phys. Lett. B 809 (2020) 135717. arXiv:2003.11552. https://doi.org/10.1016/j.physletb.2020.135717,[Gr-qc].
[66] D. Bettoni, S. Liberati, Phys. Rev. D 88 (2013) 84020. arXiv:1306.6724. https://doi.org/10.1103/PhysRevD.88.084020,[Gr-qc].
[67] S. Tsujikawa, arxiv:2508.17231, [astro-ph.CO].
[68] S. Capozziello, H. Chaudhary, T. Harko, G. Mustafa, Phys. Dark Univ. 51 (2026) 102196. arXiv:2512.10585. https://doi.org/10.1016/j.dark.2025.102196,[Astro-ph.CO].
[69] J. Yuennan, F. Atamurotov, S. Capozziello, P. Channuie, Eur. Phys. J. C 86 (3) (2026) 237. arXiv:2602.17380. https://doi.org/10.1140/epjc/s10052-026-15461-1, [Gr-qc].