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Letter

Multiloop spectra in general scalar EFTs and CFTs

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ABSTRACT

We consider the most general effective field theory (EFT) Lagrangian with scalar fields and derivatives, and renormalise it to substantially higher loop order than existing results in the literature. EFT Lagrangians have phenomenological applications, for example by encoding corrections to the Standard Model from unknown new physics. At the same time, scalar EFTs capture the spectrum of Wilson–Fisher conformal field theories (CFTs) in $4-\varepsilon$ dimensions. Our results are enabled by a more efficient version of the R^* method for renormalisation, in which the IR divergences are subtracted via a small-momentum asymptotic expansion. In particular, we renormalise the most general set of composite operators up to engineering dimension six and Lorentz rank two. We exhibit direct applications of our results to Ising (Z_2), $O(n)$, and hypercubic ($S_n \ltimes (Z_2)^n$) CFTs, relevant for a plethora of real-world critical phenomena. The computed scaling dimensions agree well with known non-perturbative results, and they lead to new predictions where such results do not yet exist. We thereby expand the understanding of generic EFTs and open new possibilities in diverse fields, such as the numerical conformal bootstrap.

1. Introduction

A remarkable feature of quantum field theory is its range of applications: from the search for new physics at energy scales above 1 TeV to emergent low-energy conformal field theories describing phase transitions in real-world fluids and crystals. Here we will show that these two disparate regimes not only fall under the same paradigm, but also benefit from the same concrete computational advances.

Consider a renormalisable Lagrangian in d spacetime dimensions perturbed by higher-dimensional operators,

$$\mathcal{L} = \mathcal{L}_{\text{renorm.}} + \sum_i c_i \chi_i \,. \tag{1}$$

This defines a low energy effective field theory (EFT). The couplings c_i scale naively with the inverse powers of the scale of new physics, $c_i \sim M^{d-[\chi_i]}$, where $[\chi_i]$ denotes the engineering dimension of χ_i. The naive scaling receives quantum corrections, leading to operator mixing and running determined by the anomalous dimensions. In e.g. the Standard Model (SM) EFT, such effects are under systematic study, leading to standardised operator bases [1,2] and multi-loop renormalisation results [3–10].

Conformal field theories (CFTs) [11–14] are believed to describe continuous phase transitions across classical and quantum critical phenomena [15]. In many cases they can be realised as IR fixed-points of quantum field theories, either by a long uncontrolled renormalisation group flow or in perturbative limits such as the ε-expansion [16,17]. The spectrum of the IR CFT can then be extracted by computing anomalous dimensions of primary operators. In EFT, a global symmetry is generally assumed, and only Lorentz scalar operators singlet under this symmetry are included in the sum in (1). However, to describe the full spectrum of an IR CFT, also non-singlets and non-scalar operators need to be considered (the c_i are viewed as probes). In particular, the modern axiomatic/bootstrap approach to CFT emphasises the set of local operators and their associated data as a defining property of the theory, where scaling dimensions (classical plus anomalous) are key pieces of data.

Here we shall renormalise the most general scalar field theory in $d = 4-\varepsilon$ dimensions in the (modified) minimal subtraction ($\overline{\text{MS}}$) scheme.

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The renormalizable part at zero spin,

$$\mathcal{L}_{\text{renorm.}} = \tfrac{1}{2}(\partial\phi)^2 - \Lambda - t_a\phi^a - \tfrac{1}{2}m^2_{ab}\phi^a\phi^b - \tfrac{1}{3!}h_{abc}\,\phi^a\phi^b\phi^c - \tfrac{1}{4!}\lambda_{abcd}\phi^a\phi^b\phi^c\phi^d, \tag{2}$$

was renormalised to six-loop order in Bednyakov and Pikelner [18], and to seven-loop order in Schnetz [19] under the $O(n)$ symmetric restriction with $\Lambda = t_a = h_{abc} = 0$. We make the conceptual and technical leap to the systematic treatment of higher-dimensional operators $c_i\chi_i$ at multi-loop order in this general theory, as well as spinning RG-relevant operators. Going beyond the typical EFT restriction to singlet scalar operators, we include all operators up to engineering dimension $\Delta = [\chi]$ six and Lorentz rank ℓ two,

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{1}{4!}\lambda_{abcd}\,\phi^a\phi^b\phi^c\phi^d + \sum_{\ell=0}^{2}\sum_{\Delta=2}^{6} c_i^{(\Delta,\ell)}\mathcal{O}_i^{(\Delta,\ell)}\,. \tag{3}$$

For example, four-field operators at spin-0 and spin-1 are

$$c^{(6,0)}\chi^{(6,0)} = c^{(6,0)}_{abcd}\,\phi^a\phi^b\partial_\mu\phi^c\partial^\mu\phi^d, \tag{4}$$

$$c^{(6,1)}\chi^{(6,1)} = u^\mu\, c^{(6,1)}_{abcd}\,\phi^a\phi^b\phi^c\partial_\mu\phi^d, \tag{5}$$

where u^μ is a reference vector. We provide the full Lagrangian in the supplemental material. For Z_2 and $O(n)$ symmetry, we extend the analysis to higher mass dimension; see Fig. 1 for a visual summary. Previous results in the general scalar EFT include two loops at mass dimension six (spin-zero) [20]. We restrict to single insertions of the higher-dimensional operator, sufficient to compute anomalous dimensions in the CFT.

We will work with a specific *primary operator basis* to systematically identify non-redundant degrees of freedom. This basis is well-suited for subsequent treatment in both EFT and CFT. We will compute anomalous dimensions to dramatically increased loop order compared to the literature. For CFT applications, the data is re-expanded as a series in $\varepsilon = 4 - d$ using the critical coupling. Typically we extend existing $O(\varepsilon)$ results to new estimates at $O(\varepsilon^5)$. The power of these estimates is demonstrated by comparing with non-perturbative results for three-dimensional CFTs ($\varepsilon = 1$), as shown in the figures and tables below. In these comparisons we use $\text{Padé}_{m,n}$ approximants, which are rational functions $(a_0 + a_1\varepsilon + \ldots + a_m\varepsilon^m)/(1 + b_1\varepsilon + \ldots + b_n\varepsilon^n)$ that reproduce the perturbative series up to order ε^{m+n}. We take $m \le n \le m+1$ unless otherwise stated. We leave a comparison of different resummation methods, including estimation of their respective errors, to future work.

2. Method

2.1. Primary operator basis

EFT Lagrangians are plagued by redundancies. For instance, integration by parts (at the action level) and field redefinitions alter the form of the Lagrangian without affecting the S-matrix. Since the set of independent operators that contribute to the EFT S-matrix coincides with the set of primary operators in CFT [21,22], we derive a minimal basis by imposing the primary condition,

$$[K_\mu, c_A^{(\Delta,\ell)}\mathcal{O}_A^{(\Delta,\ell)}(0)] = 0 \;\leftrightarrow\; c_A^{(\Delta,\ell)}\mathcal{O}_A^{(\Delta,\ell)}(x) \subset \mathcal{L}, \tag{6}$$

where K_μ is the generator of special conformal transformations and A stands for a collection of flavour indices. The primary condition imposes constraints on the coupling constant tensors. For instance, the coefficient of the operator in (4) must satisfy

$$c^{(6,0)}_{abcd} + c^{(6,0)}_{bcad} + c^{(6,0)}_{cabd} = 0, \tag{7}$$

as well as symmetry in the first and last pairs of indices. This removes all total derivatives from the basis. We call this basis of operators the *primary operator basis*.

Notably, in the primary basis we do not include operators proportional to $\partial^2\phi$. However, they are generated as counterterms and we take their effect into account. Computationally, at linear order in the couplings (except for λ_{abcd}), we subtract all operators proportional to the interacting equations of motion, $\partial^2\phi^a + \frac{1}{6}\lambda_{abcd}\phi^b\phi^c\phi^d = 0$. We follow the setup outlined in Cao et al. [23] for this procedure. In particular, by automatically subtracting subdivergences from Feynman integrals, we avoid the need for explicit counterterm graphs. In comparison to Cao et al. [23], we have considerably improved on the computation of UV divergences, as we will now describe.

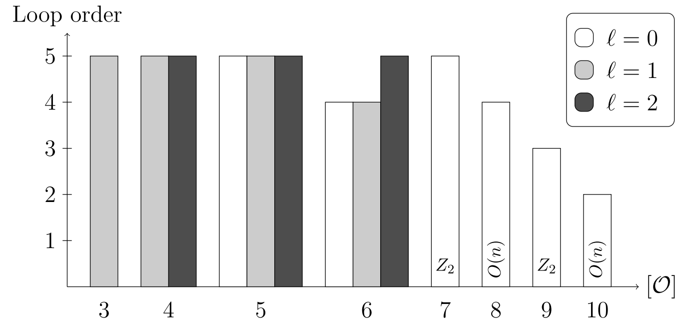

Fig. 1. Overview of our results for all operators in the general theory at spin ℓ and mass dimension $[\mathcal{O}]$. Beyond dimension six, we provide results for the single scalar EFT (Z_2) and the $O(n)$ model (singlets only). At dimension six, $\ell = 0, 1$ we have five-loop results for hypercubic and $O(n)$ symmetry; in the general theory we only present four-loop results since consistency checks beyond were more demanding.

2.2. Framework for computing UV counterterms

We extract the renormalisation constants from correlation functions of n fields ϕ^a with operator insertions using a new formulation of the R^* method [3,24–32], first presented in Chakraborty [33]. The key advantage of the R^* approach is that it allows to extract the renormalisation constants of L-loop n-point correlators from Feynman integrals of at most $L-1$ loops. This reduction in complexity is achieved by a suitable infrared rearrangement (IRR) that reroutes the external momenta in a diagram in such a way that the integration over loop momenta can be factorised [25].

The main difference between the original formulation of R^* and our implementation is in the way infrared divergences, which may arise due to IRR, are subtracted. Rather than using the local IR subtraction operation, we use a small-momentum asymptotic expansion as implemented via the expansion-by-subgraph [34–46]. We will refer to this variant of R^* as R^*_{ME}. In comparison to the previous approach used by one of the authors [3,31], R^*_{ME} leads to far less counterterms, especially in the context of higher-dimensional operators.

We start with Bogoliubov's recursive definition (BPHZ) [47–49] of the local UV counterterm $\mathcal{Z}(G)$ of a Feynman graph G in the MS (or $\overline{\text{MS}}$) scheme [50],

$$\mathcal{Z}(G) = -K_\varepsilon\bar{R}(G), \quad \bar{R}(G) = \sum_{\gamma\subsetneq G}\mathcal{Z}(\gamma) * G/\gamma, \tag{8}$$

where the sum goes over all UV-divergent bridgeless proper subgraphs, $\gamma = \sqcup\gamma_i$, of G. G/γ denotes the contracted graph obtained after contracting each connected component γ_i of γ into a vertex in G, into which the counterterm $\mathcal{Z}(\gamma) = \prod_i \mathcal{Z}(\gamma_i)$ is inserted, as indicated by the $*$-symbol. K_ε implements integration and projects out pole terms in ε.

Since $\mathcal{Z}(G)$ is polynomial (local) in the external momenta of degree $\omega(G)$, the superficial degree of divergence of G, the counterterm can be written as

$$\mathcal{Z}(G) = T^{(\omega)}_{\{p\}}\mathcal{Z}(G') = -K_\varepsilon\widetilde{T}^{(\omega)}_{\{p\}}\bar{R}(G')\,. \tag{9}$$

Here G' is the IRR version of G obtained by inserting an arbitrary momentum Q in and out of the diagram G. The original counterterm of G is recovered by projecting out the polynomial terms of *all* the original external momenta $\{p\}$ in the diagram via the degree-ω Taylor expansion operator $T^{(\omega)}_{\{p\}}$, which may be defined as

$$T^{(\omega)}_{p_1,\ldots,p_n} f(p_1,\ldots,p_n) = \frac{1}{\omega!}\frac{d^\omega}{d\lambda^\omega} f(\lambda p_1,\ldots,\lambda p_n)\Big|_{\lambda=0}\,. \tag{10}$$

Importantly, $T^{(\omega)}_{\{p\}}$ does not commute with integration. Instead, the corresponding asymptotic expansion operator $\widetilde{T}^{(\omega)}_{\{p\}}$ does and thus can act on the integrand before integration. More precisely, it is defined via the expansion-by-subgraph as follows:

$$\widetilde{T}^{(\omega)}_{\{p\}}(G) = \sum_{\gamma_A \subset G} T^{(\omega)}_{\{p\}}(\gamma_A) * G/\gamma_A, \tag{11}$$

where γ_A is an asymptotically irreducible (AI) subgraph; see ref. [45] for a precise definition. For massless graphs it is sufficient that the AI subgraph contains all hard external legs and that it becomes one-vertex-irreducible when these external lines are connected at an additional vertex. The vertices for Q going in and out of the diagram can be chosen such that the integration of the loop over G' can be factorized, leading to the aforementioned simplification of the IRR, i.e. the L-loop multi-scale integral effectively factorizes into a product of massless propagator-type integrals of at most $L-1$ loops. An example is presented in the supplementary material.

We have implemented $R^*_{\rm ME}$ in a program based on `Maple` [51] and `Form` [52,53]. The resulting massless propagator-type integrals, which were computed up to 4 loops in Baikov and Chetyrkin [54], Lee et al. [55], are evaluated using `Forcer` [56]. Our computational setup is therefore limited to 5 loops. Tensor reduction is performed with `Opiter` [57]. Feynman diagrams were generated with the algorithm of T. Kaneko [58,59] implemented in `Form5.0`.

3. Results and applications

Our complete set of data is shared at the `GitHub` repository https://github.com/jasperrn/EFT-RGE, where we also list our conventions. We now explain our results and apply them to specific theories.

The results are given with general field indices. For a chosen global symmetry, they first need to be contracted with tensor structures and the coupling constant needs to be substituted. For instance, the $O(n)$-symmetric coupling is $\lambda_{abcd} = \frac{\lambda}{3}(\delta_{ab}\delta_{cd} + \delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc})$. The second step, for CFT applications, is to evaluate the anomalous dimensions at the critical coupling $\lambda_\star$, which is a zero of the β function given as a series in $\varepsilon = 4 - d$. Depending on the symmetry, there may exist none, one, or multiple compatible fixed points. Here we specialize our results to Z_2 (Ising CFT), $O(n)$ and cubic symmetry; for which previous results were collected in Henriksson [60], Bednyakov et al. [61]. Other symmetries are also experimentally relevant [15], and can be extracted from our results.

3.1. Extraction using tensors

Let us consider the results for dimension-six scalars. They extend to four-loop order and take the form (the notation is `res[`Δ, ℓ, number of fields, operator label`]`)

$$\begin{aligned}
\texttt{res[6,0,4,1]} &= l\left(\frac{2}{3}D^{(6,4)}_{a_4a_3b_1b_2}\lambda_{a_1a_2b_1b_2} + \dots\right) + \dots \\
\texttt{res[6,0,6,1]} &= l\left(\frac{1}{3}D^{(6,4)}_{a_5a_6b_2b_3}\lambda_{a_1b_1b_2b_3}\lambda_{a_2a_3a_4b_1} \right.\\
&\quad \left. + D^{(6,6)}_{a_3a_4a_5a_6b_1b_2}\lambda_{a_1a_2b_1b_2} + \dots\right) + \dots,
\end{aligned} \tag{12}$$

where l is a loop-counting parameter. (We suppressed the classical term at l^0.) These are our largest files (40 MB). In the repository we also explain the conditions that the coupling constant tensors $D^{(6,i)}$ satisfy. For instance, to extract $O(n)$ singlets we use $D^{(6,4)}_{abcd} = d_4(\delta_{ab}\delta_{cd} - \frac{1}{2}\delta_{ac}\delta_{bd} - \frac{1}{2}\delta_{ad}\delta_{bc})$, and $D^{(6,6)}_{abcdef} = d_6(\delta_{ab}\delta_{cd}\delta_{ef} + \text{perms})$, compatible with (7) and total symmetry respectively. The coefficients of d_4 and d_6 in the RHS of (12) (modulo the tensor structures) define the matrix

$$\Gamma = \begin{pmatrix} -2\varepsilon + \frac{2n+4}{3}\lambda - \frac{17n+34}{18}\lambda^2 + \dots & \frac{5(n+2)(n+4)}{81}\lambda^3 \\ \frac{10(1-n)}{9}\lambda^3 + \dots & -3\varepsilon + (n+14)\lambda + \frac{49n+378}{6}\lambda^2 \dots \end{pmatrix}, \tag{13}$$

where the top-right corner represents four-loop order, while the bottom left starts at two loops. The full dimension is $\Delta = 6 + \text{eigs}(\Gamma)$, and in this particular case we can check that the results agree with Jenkins et al. [5], Cao et al. [23], Derkachov and Manashov [62], Roosmale Nepveu [63].

For a non-singlet $O(n)$ representation, consider the $[2,2]$ Young tableau (called B_4 in Henriksson [60]). Only the $D^{(6,4)}$ term produces a non-zero operator, yielding a one-dimensional entry (i.e. no mixing matrix):

$$\gamma = -2\varepsilon + 2\lambda - \frac{7n+44}{18}\lambda^2 + \left(\frac{1516+346n-11n^2}{216} + \frac{4n+20}{9}\zeta_3\right)\lambda^3 + \dots. \tag{14}$$

Evaluating (13) and (14) at $n = 4$ in fact gives results for the three singlet operators of the Higgs sector in the SMEFT. That is because the scalar sector of the SM is invariant under $O(4)$ custodial symmetry, and the custodial-violating operator in the B_4 representation is invariant under $SU(2) \times U(1)$. We thereby extend the results in Jenkins et al. [5] to five loops [64]. Evaluating them instead at the critical coupling, $\lambda_\star = \frac{3\varepsilon}{n+8} + \frac{9(3n+14)\varepsilon^2}{(n+8)^3} + \dots$, gives results for operators in the $O(n)$ Wilson–Fisher CFT, where the full scaling dimension is $6 + \gamma$.

3.2. Spectrum of the 3d Ising CFT

Table 1
Our results at $\Delta \le 6$ in the Ising CFT, compared with numerical bootstrap results in 3d with statistical and rigorous error intervals respectively.

Operator	ϕ^5	$\phi^2\partial_\mu\partial_\nu\phi$	$\phi^3\partial_\mu\partial_\nu\phi$
Previous loop order	ε^3 [65]	ε^2 [66]	ε^1 [67]
New loop order	ε^5	ε^5	ε^5
Padé approximant	5.257395	4.162978	5.465027
Statistical error [68]	5.2906(11)	4.180305(18)	5.50915(44)
Rigorous error [69]	5.262(89)	—	5.499(17)

The Ising CFT, the canonical Wilson–Fisher fixed-point, is of central experimental and theoretical importance and has been studied with a variety of methods. In Table 1, we compare our new results up to $\Delta = 6$ with the most precise non-perturbative results from the conformal bootstrap.

In this case, our results also include Lorentz scalars with engineering dimension $[\mathcal{O}] \le 10$, which read

$$\begin{aligned}
\Delta_{\phi^5} &= 5 + \tfrac{5}{6}\varepsilon - \tfrac{685}{324}\varepsilon^2 + 6.64393\varepsilon^3 \\
&\quad - 26.1423\varepsilon^4 + 121.213\varepsilon^5 + O(\varepsilon^6), \\
\Delta_{\phi^6} &= 6 + 2\varepsilon - \tfrac{257}{54}\varepsilon^2 + 17.8246\varepsilon^3 \\
&\quad - 82.9519\varepsilon^4 + 447.314\varepsilon^5 + O(\varepsilon^6), \\
\Delta_{\phi^7} &= 7 + \tfrac{7}{2}\varepsilon - \tfrac{959}{108}\varepsilon^2 + 38.6637\varepsilon^3 \\
&\quad - 208.437\varepsilon^4 + 1291.46\varepsilon^5 + O(\varepsilon^6), \\
\Delta_{(\partial\phi)^4} &= 8 - \tfrac{8}{9}\varepsilon + \tfrac{22}{81}\varepsilon^2 - 0.31768\varepsilon^3 \\
&\quad + 0.749246\varepsilon^4 + O(\varepsilon^5), \\
\Delta_{\phi^8} &= 8 + \tfrac{16}{3}\varepsilon - \tfrac{1198}{81}\varepsilon^2 + 73.4366\varepsilon^3 - 450.966\varepsilon^4 + O(\varepsilon^5), \\
\Delta_{\phi(\partial\phi)^4} &= 9 - \tfrac{5}{18}\varepsilon - \tfrac{62}{243}\varepsilon^2 + 0.812882\varepsilon^3 + O(\varepsilon^4), \\
\Delta_{\phi^9} &= 9 + \tfrac{15}{2}\varepsilon - \tfrac{821}{36}\varepsilon^2 + 127.189\varepsilon^3 + O(\varepsilon^4), \\
\Delta_{\partial^2(\partial\phi)^4} &= 10 - \tfrac{5}{3}\varepsilon + \tfrac{1}{648}\varepsilon^2 + O(\varepsilon^3), \\
\Delta_{\phi^2(\partial\phi)^4} &= 10 + \tfrac{2}{3}\varepsilon - \tfrac{269}{162}\varepsilon^2 + O(\varepsilon^3), \\
\Delta_{\phi^{10}} &= 10 + 10\varepsilon - \tfrac{1795}{54}\varepsilon^2 + O(\varepsilon^3),
\end{aligned} \tag{15}$$

where we numerically evaluated the Riemann zeta values ζ_3, ζ_4, …. The five-loop result for ϕ^6 agrees with Cao et al. [23], and in all other cases

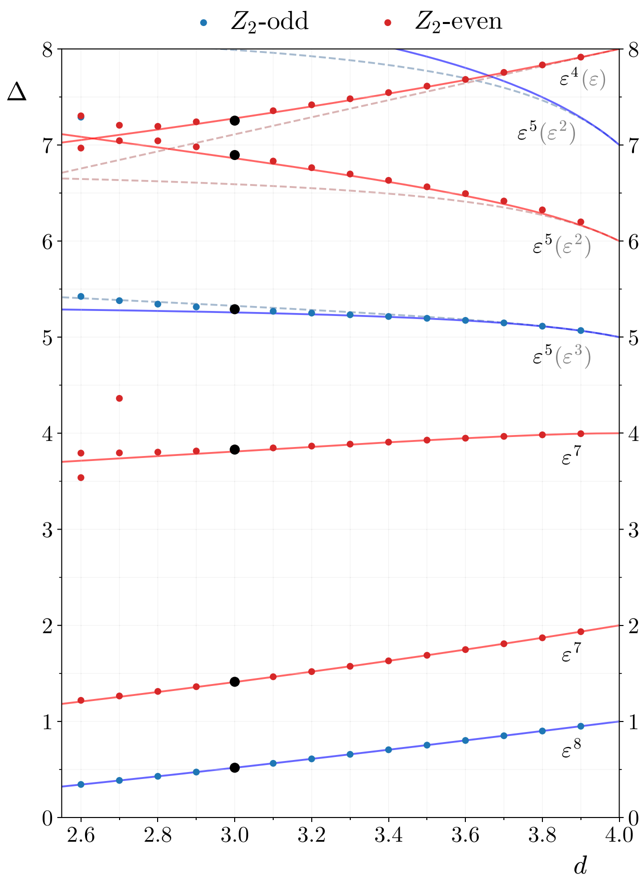

Fig. 2. Scalar spectrum in the Ising CFT with $\Delta < 8$ for spacetime dimension $2.6 \leq d < 4$, using results up to $O(\varepsilon^n)$. Dashed lines show previous leading determinations at $O(\varepsilon^m)$. Numerical bootstrap results (the dots) are from Simmons-Duffin [68] ($d = 3$) and Henriksson et al. [70]. These works did not detect the operator ϕ^7.

we improve on previous lower-order results from Derkachov and Manashov [62], Roosmale Nepveu [63], Zhang and Zia [65], Kehrein and Wegner [67].

We present in Fig. 2 a comprehensive picture of the scalar spectrum up to $\Delta = 8$ and compare it to bootstrap results across spacetime dimensions [68,70]. We find excellent agreement up to $\varepsilon = 1$ and good agreement beyond. The worse agreement at $\varepsilon > 1$ may be assigned to larger coupling constant, or non-perturbative effects.

3.3. *O*(3) *CFT and quantum-critical corrections to scaling*

For the $O(n)$ CFT, we discuss the case $n = 3$. In Table 2, we present the leading spectrum of the 3d $O(3)$ CFT, complete up to $\Delta < 4$, and compare with different non-perturbative determinations directly in $d = 3$.

A particularly interesting operator in the $O(3)$ CFT is $Q = \phi^{[a}\partial_{[\mu}\phi^b\partial_{\nu]}\phi^{c]}$, which in vector notation has a component $\vec{\phi}\cdot(\partial_x\vec{\phi}\times\partial_t\vec{\phi})$. It is rank-three antisymmetric under $O(3)$ (and more generally $O(n)$) and rank-two antisymmetric under the Lorentz group, and has been proposed to contribute a substantial correction to scaling in quantum-critical behaviour described by the 3d $O(3)$ CFT [76]. We now report a five-loop result for this operator (displaying the result at $n = 3$):

$$\Delta_Q = 5 - \frac{3\varepsilon}{2} - \frac{15\varepsilon^2}{484} - \frac{5585\varepsilon^3}{234256} + \left(\frac{2725\zeta_3}{161051} - \frac{3507905}{340139712}\right)\varepsilon^4 - \left(\frac{104650\zeta_5}{1771561} - \frac{8175\zeta_4}{644204} - \frac{14015335\zeta_3}{935384208} + \frac{1051598975}{493882861824}\right)\varepsilon^5 + O(\varepsilon^6), \tag{16}$$

Table 2
All operators in the 3d $O(3)$ CFT with $\Delta < 4$. Bootstrap values are either from Chester et al. [75] or private correspondence with the authors of that paper. The operator referenced with Han et al. [74] was missing in the original paper, but the authors of that paper confirm the existence of the state and gave the value 2.67 without quoting error bars. Notation: old loop order → new loop order (previous order in parenthesis).

R	ℓ	χ	order	Padé	Bootstrap	Monte Carlo
V	0	ϕ	(ε^8)	.5188246	.518942(51)	.518920(25) [71]
T	0	ϕ^2	(ε^6)	1.210809	1.20954(32)	1.2094(3) [72]
S	0	ϕ^2	(ε^7)	1.571279	1.59489(59)	1.5948(2) [71]
A	1	$\partial\phi^2$	exact	2		
T_3	0	ϕ^3	(ε^6)	2.042931	2.03867(23)	2.0385(3) [73]
H_3	1	$\partial\phi^3$	$\varepsilon \to \varepsilon^5$	2.766426	2.77025(22)	2.67 [74]
T_4	0	ϕ^4	(ε^6)	2.991664	< 2.99056	2.9857(9) [73]
S	2	$\partial^2\phi^2$	exact	3		
T	2	$\partial^2\phi^2$	$\varepsilon^4 \to \varepsilon^5$	3.015701	3.013(18)	
V	1	$\partial\phi^3$	$\varepsilon^2 \to \varepsilon^5$	3.015815	3.03120(32)	
A_3	1^-	$\partial^2\phi^3$	$\varepsilon \to \varepsilon^5$	3.446361	NA	
T	0	ϕ^4	(ε^6)	3.550026	3.561(13)	
V	2	$\partial^2\phi^3$	$\varepsilon^2 \to \varepsilon^5$	3.630425	3.633(4)	
H_4	1	$\partial\phi^4$	$\varepsilon \to \varepsilon^5$	3.676439	3.713(9)	
S	0	ϕ^4	(ε^7)	3.793620	3.7667(10)	3.759(2) [71]
T_3	2	$\partial^2\phi^3$	$\varepsilon^2 \to \varepsilon^5$	3.837674	3.841(19)	

where previously only the $O(\varepsilon)$ term was known [60,67]. A Padé$_{2,3}$ approximant gives $\Delta_Q^{3d} = 3.44636$, of use for future studies [77–81].

3.4. *Cubic CFT and conformal bootstrap*

The conformal bootstrap [82–84] produces rigorous error bars for critical exponents and other conformal data, including the most precise determinations for the Ising [85] and $O(n)$ [75,86] CFTs. However, results for *any* other scalar CFT have been considerably less precise, one reason being the increasingly complicated spectrum of less symmetric CFTs. Numerical bootstrap studies require input in the form of spectral gaps, which can be guided by approximate knowledge of the spectrum of the candidate theory.

Our results provide precisely this insight, which we exemplify in $C_3 = S_3 \ltimes (Z_2)^3$ symmetric scalar field theories, referred to as cubic. In Table 3 we present the entire spectrum up to engineering dimension six in the B (two-index antisymmetric) and S (singlet) representations of the cubic group. These proved of crucial importance in Kousvos and Stergiou [87], where a gap on the first B scalar operator ($\Delta_B \geq 4.0$) was used to exclude the $O(3)$ theory from parameter space, and the leading S spin-2 operator after the stress tensor ($\Delta_{T'_{\mu\nu}} \geq 4.0$) was used to obtain a bootstrap island. Without these spectral gap assumptions, the results of Kousvos and Stergiou [87] could not have been obtained. Both gaps required justification by our present work, since information on these operators was not available in the literature, with the exception of the 1-loop results of Bednyakov et al. [61], which do not suffice for resummations.

We thus exemplified how our results enable the implementation of spectral gaps specifically in cubic theories, but we emphasise that the applicability of our theory-independent results is much broader.

4. Discussion and outlook

In this paper, we have derived new multi-loop anomalous dimensions that provide a comprehensive picture of the spectrum of composite operators in scalar EFTs and CFTs. A central point is that, while many recent computational methods were derived with EFT applications in mind, they can also be used to derive high-quality estimates for CFT operator spectra. This then renders the study of new theories via non-perturbative methods, such as the conformal bootstrap, approachable. For example, bootstrapping the cubic theory of the ε expansion had long remained an open problem in the bootstrap community, before perturbative intuition and our state-of-the-art data finally enabled its study [87]. We

Table 3
Operators in the B and S representations of the cubic group. The used Padé resummations are: Padé$_{3,2}$ for B $\partial\phi^2$ and the first two S $\partial^2\phi^4$; Padé$_{2,2}$ for the first B $\partial\phi^4$, $\partial^{[1,1]}\phi^4$, $\partial^2\phi^4$ and the last S $\partial^2\phi^4$; Padé$_{3,3}$ for S ϕ^2 and ϕ^4 (using Bednyakov et al. [61]); and Padé$_{2,3}$ for all other operators. The last column states the irrep of each operator under the $O(3)$ group.

Antisymmetric $B_{[ab]}$				Singlets S			
	Δ_{C_3}	$\Delta_{O(3)}$	$O(n)$		Δ_{C_3}	$\Delta_{O(3)}$	$O(n)$
ϕ^6	5.30113	5.27248	T_6	ϕ^2	1.56416	1.56246	S
ϕ^6	5.91670	5.93449	T_4	ϕ^4	3.01080	2.99166	T_4
$\partial\phi^2$	2.01615	2	A	ϕ^4	3.78431	3.78198	S
$\partial\phi^4$	3.70481	3.68387	H_4	ϕ^6	5.28692	5.27248	T_6
$\partial\phi^4$	4.09212	4.09582	A	$\Box\phi^4$	5.02294	5.02573	S
$\partial^{[1,1]}\phi^4$	4.45226	4.44619	A	ϕ^6	5.89107	5.93449	T_4
$\partial^2\phi^4$	4.51586	4.50492	H_4	ϕ^6	6.56760	6.55755	S
$\partial^2\phi^4$	4.78803	4.78899	T_4	$T^{\mu\nu}$	3	3	S
				$\partial^2\phi^4$	4.73644	4.78778	T_4
				$\partial^2\phi^4$	4.77219	4.71329	S
				$\partial^2\phi^4$	5.51728	5.51633	S

expect that this will be the case for many other theories. Full details of our results will be presented in a subsequent publication [64].

We used an improved R^* method to access particularly high loop orders in a manageable way. R^* is applicable to local QFTs with particles and operators of arbitrary spin and mass dimension, and for any spacetime dimension $d = n - \varepsilon$. Furthermore, it reduces the complexity of integrals and it circumvents the need for explicit counterterms for subdivergences. The improved R^*_{ME} retains all these advantages, but is more efficient, especially when applied to theories with higher-dimensional operators. The momentum expansion can also be used in a more global manner, recently to renormalize twist-2 operators in QCD [88–91] up to spin 20.

Future applications of R^*_{ME} within scalar theories can include targeted studies towards specific operators of interest, for instance parity-odd scalars [85,92,93] and singlet vector operators [94,95], which are typically found at even larger engineering dimensions than considered here. Another direction would be to derive a general two-loop dilation operator, in the spirit of Kehrein and Wegner [67] for $O(n)$ CFTs and [96,97] for $\mathcal{N} = 4$ SYM, ideally already within the primary operator basis. The general results could also be used to test ideas in geometrised field spaces, e.g. [98,99].

Having demonstrated the applicability and impact of our present results, another important extension would be to go beyond scalar theories. Relevant to this is the three-loop renormalisation of general 4d QFTs [100–106], the inclusion of field potentials [107–110], and recent one-loop renormalisation of higher-dimensional operators in such theories [111–113]. Importantly, there are also CFT applications within this class, for instance 4d conformal gauge theories, including Banks–Zaks fixed-points [114–116] and the ϵ expansion for 3d fermionic and gauge theories [117–119]. While working with general (flavour) index structure is the first step towards CFT applications since it allows for arbitrary global-symmetry representations, the inclusion of non-scalar operators is the next step needed to be worked out systematically (beyond twist 2).

Data availability

All results are available in a public GitHub repository.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Supplementary material

Supplementary material associated with this article can be found in the online version at 10.1016/j.physletb.2026.140235.

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