



Letter

Towards a unified viewpoint of Gribov–Zwanziger and Serreau–Tissier gauge fixing

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ARTICLE INFO

Editor: Neil Lambert

Keywords:

Gribov copies
Landau gauge
Gribov–Zwanziger
Serreau–Tissier gauge
BRST symmetry
Algebraic renormalization

ABSTRACT

We investigate a unified Landau–gauge fixing that continuously interpolates between the viewpoints of the Serreau–Tissier (ST) copy-averaged formulation and the (Refined) Gribov–Zwanziger (RGZ) restriction to the first Gribov region. By coupling a copy-averaged ST sector and a BRST-invariant RGZ sector within a single local action; the horizon term acts uniformly on copies, and the ST weight controls their relative contributions, we obtain a single, local, BRST-invariant, power-counting renormalizable action. Algebraic renormalization shows that all counterterms are reabsorbed by a common set of field and parameter renormalizations, therefore the unification is algebraic rather than merely additive. The replica sector yields a radiatively generated gluon screening mass, while the RGZ parameters are fixed by the horizon and condensate gap equations; we also give infrared matching conditions that link both descriptions at small momentum. We present a compact BRST-superspace rewriting of the RGZ block and a simple hybrid superspace that hosts the ST replicas and RGZ side by side; these add no dynamics and organize the Ward-identity analysis. The resulting gluon propagator interpolates among the massive Faddeev–Popov–ST and the RGZ decoupling forms. This framework offers a controlled way to study how infrared Yang–Mills correlators depend on the balance between copy averaging and horizon suppression, and it suggests practical lattice tests through tunable copy weighting.

1. Introduction

Quantizing nonabelian Yang–Mills (YM) theories by means of the standard Faddeev–Popov (FP) prescription is remarkably successful in the ultraviolet (UV) but faces conceptual and practical obstacles in the infrared (IR) [1,2]. In particular, the Landau pole spoils naive perturbation theory and the gauge-fixing condition does not fully eliminate the gauge freedom because of the existence of Gribov copies, which are distinct field configurations related by gauge transformations that satisfy the same gauge condition [3,4]. The Gribov problem is intrinsic to the nontrivial geometry of gauge orbits and not to a specific gauge choice; see, for instance, the review [5].

Two complementary continuum strategies have emerged to tame Gribov copies and capture YM IR dynamics. The Gribov–Zwanziger (GZ) framework restricts the functional integral to the first Gribov region Ω , where the FP operator $M^{ab} = -\partial_\mu D_\mu^{ab}(A)$ is positive definite. This restriction is implemented through the (nonlocal) horizon functional $H(A^h)$ written in terms of the transverse, gauge-field A_μ^h , and can be localized with the Zwanziger auxiliary fields $(\bar{\varphi}, \varphi)$ and $(\bar{\omega}, \omega)$ [6,7]. The RGZ extension includes dynamical dimension-two condensates, introducing mass scales (m, M) and leading to a local, renormalizable action whose gluon and ghost correlators match lattice data from the deep IR up to

several GeV [8–10]. In the A^h formulation, an exact all-order BRST symmetry is retained in linear covariant gauges (LCG), which guarantees, e.g., α -independence of gauge-invariant observables and nonrenormalization of the longitudinal sector [11–13].

Independently, J. Serreau and M. Tissier and further work collaborators proposed a family of Landau gauges free of Gribov ambiguities, defined by an explicit weighted average over copies [14–17]. In practice, one averages an operator $\mathcal{O}$ over all extrema U_i of the Landau functional $f[A, U] = \int d^d x \text{tr} (A_\mu^U)^2$ with a nonuniform weight

$$\mathcal{P}[A, U] = \frac{\det(F[A, U] + \zeta \mathbb{1})}{|\det F[A, U]|} e^{-\beta f[A, U]},$$

where F is the Landau-gauge FP Hessian, and β, ζ are gauge-fixing parameters of mass dimension two. A gauge-invariant observable is unaffected by the procedure thanks to the normalization by $\sum_i \mathcal{P}[A, U_i]$. Using the replica trick and a superspace nonlinear sigma-model representation, one obtains a local, perturbatively renormalizable field theory in $d = 4$ which, for correlators of A and ghosts, reduces to a massive FP theory with a gluon mass term $\sim \beta$ and a ghost mass $\sim \zeta$ [14,17]. The replica sector exhibits a phenomenon of radiative symmetry restoration (akin to the $d = 2$ NL σ model), which selects a phase where a nonzero screening

E-mail address: rodrigo.carmo@urjc.es<https://doi.org/10.1016/j.physletb.2026.140302>

Received 27 October 2025; Received in revised form 28 December 2025; Accepted 19 February 2026

Available online 24 February 2026

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gluon mass survives in the replica limit $n \rightarrow 0$ [17]. Crucially, within a range of renormalized parameters, the one-loop renormalization-group (RG) flow shows no Landau pole in the IR [14].

These approaches share the goal of curing Gribov issues and at the same time providing an IR-safe perturbative description of YM. GZ/RGZ implements a hard restriction to Ω by means of the horizon functional (and dynamical condensates), whereas ST implements a soft lifting of copy degeneracy through a tunable weighted average, realized locally by means of replica nonlinear sigma-model (NL σ) fields. Both are local and renormalizable, and reproduce lattice-inspired decoupling-type gluon propagators [18]; see also the spectral and positivity analyses consistent with gluon confinement [19–21]. It is then natural to ask whether a more general gauge-fixing exists that interpolates between them.

Therefore, in this work, we construct a unified gauge-fixing that treats ST and RGZ as limiting cases. At the level of the copy average, we couple a copy-averaged ST sector and a BRST-invariant RGZ sector; the horizon term acts uniformly over copies, while the ST weight controls their relative contributions,

$$\langle\langle \mathcal{O}[A] \rangle\rangle_{\beta, \zeta, \gamma} = \frac{\sum_i s(i) \mathcal{O}[A^{U_i}] \frac{\det(F[A, U_i] + \zeta \mathbb{1})}{\det F[A, U_i]} e^{-\beta f[A, U_i] - \gamma^4 H(A^{U_i})}}{\sum_i s(i) \frac{\det(F[A, U_i] + \zeta \mathbb{1})}{\det F[A, U_i]} e^{-\beta f[A, U_i] - \gamma^4 H(A^{U_i})}},$$

so that $\gamma = 0$ reproduces ST, while $(\beta \rightarrow \infty, \zeta \rightarrow 0)$ yields RGZ. We then derive a local action by mixing the ST replica/superspace localization with the Zwanziger localization in the A^h formulation [7, 11–14, 17]. The resulting theory is local and power-counting renormalizable, enjoys the exact BRST of the A^h sector, and continuously interpolates the structure of the gluon two-point function between a massive-FP (ST) form and the RGZ decoupling form. In this sense, our unified construction prepares a controlled framework to study how infrared YM correlators depend on the balance between (i) copy averaging and radiative restoration in the replica sector (set by β, ζ) and (ii) horizon suppression and condensates (set by γ, m, M). It connects smoothly with functional approaches (Dyson-Schwinger equations (DSEs) and the functional renormalization group (FRG)) [22–25] and with massive extensions of Faddeev–Popov theory [26, 27].

This paper is organized as follows. Section 2 reviews the ST and RGZ formalisms, emphasizing their gauge-fixing mechanisms and dynamical mass generation. Section 3 constructs the unified local action that interpolates between them through the replica trick and the BRST-invariant RGZ formulation. Section 4 presents the algebraic proof of renormalizability based on Slavnov–Taylor, antighost, and ghost identities, while Section 5 analyzes the coupled gap equations and their matching on the two sectors. A compact superspace formulation unifying both sectors is investigated in Section 6, and its complete derivation is collected in Appendix D. Section 7 summarizes the main results and discusses physical implications and possible extensions. Appendices A–C compile technical material, including field dimensions and ghost numbers, the linearized Slavnov operator and auxiliary Ward identities, non-renormalization relations, replica one-loop corrections, small-momentum expansions, tree-level Feynman rules, the Nielsen identity, and dimensional-regularization conventions. Finally, the complete algebraic construction of the unified BRST transformations and the corresponding gauge-fixing functional, presented for pure Yang–Mills theory, is provided in Appendix E.

2. Background: continuum approaches to the Gribov problem

In this section we summarize the two main continuum approaches that address the Gribov ambiguity in nonabelian gauge theories. Both share the goal of providing a consistent IR formulation of YM theories in the Landau gauge while preserving gauge invariance of physical observables. The first approach, ST, resolves the ambiguity by averaging over Gribov copies with a weighted functional measure that can be represented as a local field theory involving NL σ superfields. The second,

known as the RGZ framework, restricts the domain of the functional integral to the first Gribov region through the introduction of the horizon functional and its localization through auxiliary fields. We review their main ingredients below.

2.1. The Serreau–Tissier gauge and radiative symmetry restoration

Following refs. [14, 15, 17], the Landau gauge condition

$$\partial_\mu A_\mu^U = 0, \quad (1)$$

with $A_\mu^U = U A_\mu U^\dagger + \frac{i}{g} U \partial_\mu U^\dagger$, can be equivalently formulated as the stationarity condition of the functional

$$f[A, U] = \int d^d x \operatorname{tr}[(A_\mu^U)^2], \quad (2)$$

whose extrema U_i correspond to Gribov copies. Instead of selecting one specific extremum, Serreau and Tissier proposed to perform a weighted average over all copies, defining the expectation value of an operator $\mathcal{O}$ as

$$\langle\langle \mathcal{O}[A] \rangle\rangle = \frac{\sum_i \mathcal{O}[A^{U_i}] \mathcal{P}[A, U_i]}{\sum_i \mathcal{P}[A, U_i]}, \quad (3)$$

where the weight function is chosen as

$$\mathcal{P}[A, U] = \frac{\det(F[A, U] + \zeta \mathbb{1})}{|\det F[A, U]|} e^{-\beta f[A, U]}, \quad (4)$$

with $F[A, U]$ the Faddeev–Popov (FP) operator in the Landau gauge, $F^{ab}[A; x, y] = -\partial_\mu [\delta_\mu^{ab} + g f^{acb} A_\mu^c(x)] \delta^{(d)}(x - y)$, and β, ζ two gauge-fixing parameters of mass dimension two.

Gauge-invariant observables are unaffected by this procedure due to the normalization in Eq. (4). The functional average can be reformulated as a local and perturbatively renormalizable field theory by introducing auxiliary ghost ($c, \bar{c}$) and Nakanishi–Lautrup (b) fields, together with a group-valued matrix field $U(x)$ integrated with the Haar measure:

$$S_{\text{gr}}[A, c, \bar{c}, b] = \int d^d x \left[\partial_\mu \bar{c}^a (\partial_\mu c^a + g f^{abc} A_\mu^b c^c) + \zeta \bar{c}^a c^a + i b^a \partial_\mu A_\mu^a + \frac{\beta}{2} (A_\mu^a)^2 \right]. \quad (5)$$

These fields can be unified into a superspace nonlinear sigma-model (NL σ) superfield $\mathcal{V}(x, \theta, \bar{\theta})$, enabling a compact representation of the gauge-fixing action:

$$S_{\text{gr}}[A, \mathcal{V}] = \frac{1}{g^2} \int d^d x d\theta d\bar{\theta} \operatorname{tr} \left[(D_\mu \mathcal{V})^\dagger (D_\mu \mathcal{V}) + 2\zeta \bar{\theta} \theta \partial_\theta \mathcal{V}^\dagger \partial_{\bar{\theta}} \mathcal{V} \right], \quad (6)$$

with $D_\mu \mathcal{V} = \partial_\mu \mathcal{V} + i g \mathcal{V} A_\mu$. Employing the replica trick, the authors showed that the corresponding super-NL σ model exhibits a phenomenon of radiative symmetry restoration, similar to what occurs in the two-dimensional $O(N)/O(N-1)$ model. In the symmetric phase, a finite screening gluon mass $m_g^2 \simeq \beta$ emerges at tree level, consistent with lattice observations of a massive gluon propagator in the Landau gauge. This framework thus provides a gauge-fixing free of Gribov ambiguities, local and renormalizable, and reduces in practice to a massive extension of the FP action [26, 27].

2.2. The refined Gribov–Zwanziger framework

An alternative approach, initiated by Gribov and developed by Zwanziger, enforces the restriction of the functional integral to the first Gribov region Ω , defined as the set of gauge fields satisfying the Landau condition $\partial_\mu A_\mu^a = 0$ and for which the FP operator $\mathcal{M}^{ab}(A) = -\partial_\mu D_\mu^{ab}(A)$ is positive definite [3, 6, 7]. This restriction can be implemented by adding to the action the nonlocal horizon functional

$$H(A^h) = g^2 \int d^4 x d^4 y f^{abc} A_\mu^{h,b}(x) [\mathcal{M}^{-1}(A^h)]^{ad}(x, y) f^{dec} A_\mu^{h,e}(y), \quad (7)$$

where A_μ^h is a gauge-invariant, transverse composite field defined through a Stueckelberg field ξ as

$$A_\mu^h = h^\dagger A_\mu h + \frac{i}{g} h^\dagger \partial_\mu h, \quad h = e^{ig\xi^a T^a}. \quad (8)$$

The horizon term can be localized by introducing Zwanziger auxiliary bosonic fields $(\bar{\varphi}, \varphi)$ and Grassmann fields $(\bar{\omega}, \omega)$, leading to the Gribov-Zwanziger (GZ) action. The refined GZ (RGZ) model further includes dimension-two condensates $\langle A^2 \rangle$, $\langle \bar{\varphi}\varphi - \bar{\omega}\omega \rangle$, generating additional mass scales (m, M) that improve agreement with lattice data [8,9]. In the BRST-invariant A^h formulation [11–13], the complete RGZ action reads schematically

$$\begin{aligned} S_{\text{RGZ}} = S_{\text{FP}} &- \int d^4x \left(\bar{\varphi}_\mu^{ac} \mathcal{M}^{ab}(A^h) \varphi_\mu^{bc} - \bar{\omega}_\mu^{ac} \mathcal{M}^{ab}(A^h) \omega_\mu^{bc} \right) \\ &- \gamma^2 \int d^4x g f^{abc} (A^h)_\mu^a (\varphi + \bar{\varphi})_\mu^{bc} + \frac{m^2}{2} \int d^4x (A^h)_\mu^a (A^h)_\mu^a \\ &- M^2 \int d^4x (\bar{\varphi}_\mu^{ab} \varphi_\mu^{ab} - \bar{\omega}_\mu^{ab} \omega_\mu^{ab}), \end{aligned} \quad (9)$$

where γ is the Gribov parameter determined self-consistently by the horizon condition $\langle H(A^h) \rangle = d(N^2 - 1)$. The resulting gluon propagator exhibits the so-called decoupling behavior,

$$D(p^2) = \frac{p^2 + M^2}{p^4 + (m^2 + M^2)p^2 + m^2 M^2 + 2g^2 N \gamma^4},$$

remaining finite and nonzero at $p^2 \rightarrow 0$, while the ghost propagator is infrared finite. This framework preserves an exact BRST symmetry and yields correlation functions in quantitative agreement with lattice data over a wide momentum range. For later convenience, the BRST structure of the Zwanziger doublets (φ, ω) and $(\bar{\omega}, \bar{\varphi})$ can be recast in a compact (4|1) superspace form, where the BRST operator s acts as a translation along a single Grassmann coordinate θ . This reformulation, detailed in Appendix D, makes BRST nilpotency and algebraic renormalizability manifest, without introducing any new degrees of freedom.

Both the ST and RGZ formalisms thus furnish local, renormalizable and gauge-invariantly meaningful treatments of the Gribov problem. The former implements a smooth averaging over copies through replica NL σ fields and radiative restoration, while the latter imposes a hard restriction to the first Gribov region by means of the horizon condition and condensates. Their structural similarity (both leading to a massive yet confining gluon sector) motivates the unified approach developed in the next section.

3. A unified gauge fixing: interpolating between ST and GZ/RGZ

The ST construction fixes the gauge by averaging over all Gribov copies with a nonuniform weight $P[A, U]$ built from the Landau functional $f[A, U]$ and the FP Hessian $\mathcal{F}[A, U]$ [14,17]. In parallel, the GZ framework restricts the functional measure to the first Gribov region through the horizon functional $H(A^h)$, written in terms of the transverse, gauge-invariant composite field A_μ^h [6,7,11,12]. To unify both, we propose the following mixed averaging for a gauge-dependent operator $\mathcal{O}$:

$$\begin{aligned} \langle \langle \mathcal{O}[A] \rangle \rangle_{\beta, \zeta, \gamma} &= \frac{\sum_i s(i) \mathcal{O}[A^{U_i}] \frac{\det(\mathcal{F}[A, U_i] + \zeta \mathbb{1})}{|\det \mathcal{F}[A, U_i]|} e^{-\beta f[A, U_i] - \gamma^4 H(A^h, U_i)}}{\sum_i s(i) \frac{\det(\mathcal{F}[A, U_i] + \zeta \mathbb{1})}{|\det \mathcal{F}[A, U_i]|} e^{-\beta f[A, U_i] - \gamma^4 H(A^h, U_i)}}. \end{aligned} \quad (10)$$

Here $s(i) = \text{sign det } \mathcal{F}[A, U_i]$, β, ζ are the ST gauge-fixing parameters of mass dimension two controlling, respectively, a gluon mass and a ghost mass term in the local formulation, and γ is the Gribov mass of GZ. Gauge-invariant observables are unchanged by (10). The limits

$$\begin{aligned} (\gamma = 0) &\Rightarrow \text{ST}, & (\beta \rightarrow \infty, \zeta \rightarrow 0) &\Rightarrow \text{GZ}, \\ (\beta, \gamma \neq 0) &\Rightarrow \text{interpolation} \end{aligned}$$

make the interpolation explicit. Since A^h is BRST-invariant, the horizon functional $H(A^h, U_i)$ is identical for all copies and therefore factorizes out of the average (10). Consequently, the horizon term does not modify the relative weights of the copies; the interpolation in (10) reflects the coexistence of the ST copy-averaging mechanism and the RGZ horizon suppression.

As in Serreau and Tissier [14], Serreau et al. [15], Reinosa et al. [17], the ST weight is localized by introducing a matrix field $U(x)$ and ghost/antighost/Nakanishi-Lautrup fields $(c, \bar{c}, b)$ or, equivalently, the NL σ superfields $\mathcal{V}$ in superspace. The denominator is handled through the replica trick [28]: introduce n superfields $\{V_k\}_{k=1}^n$ and take $n \rightarrow 0$ at the end. Independently, the horizon functional is localized à la Zwanziger with $(\bar{\varphi}, \varphi)$ and $(\bar{\omega}, \omega)$ fields and the transversality of A^h is implemented with the Stueckelberg field $h = e^{ig\xi^a T^a}$, Lagrange multiplier τ and the pair $(\bar{\eta}, \eta)$, see Sorella et al. [11,12], Terin et al. [13].

Choosing one replica to factor out the group volume and denoting by $S_{\text{ST}}[A; V_k]$ the local ST gauge-fixing sector with parameters (β, ζ) , the unified local action reads

$$\begin{aligned} S_{\text{unif}} = S_{\text{YM}}[A] &+ S_{\text{FP}}[A, c, \bar{c}, b] + \sum_{k=2}^n S_{\text{ST}}[A; V_k] \\ &- \int d^4x \left(\bar{\varphi}_\mu^{ac} \mathcal{M}^{ab}(A^h) \varphi_\mu^{bc} - \bar{\omega}_\mu^{ac} \mathcal{M}^{ab}(A^h) \omega_\mu^{bc} \right) \\ &- \gamma^2 \int d^4x g f^{abc} (A^h)_\mu^a (\varphi + \bar{\varphi})_\mu^{bc} \\ &+ \int d^4x \left(\tau^a \partial_\mu (A^h)_\mu^a - \bar{\eta}^a \mathcal{M}^{ab}(A^h) \eta^b \right) \\ &+ \frac{m^2}{2} \int d^4x (A^h)^2 - M^2 \int d^4x (\bar{\varphi}\varphi - \bar{\omega}\omega), \end{aligned} \quad (11)$$

The ST block and the GZ/RGZ block each admit a nilpotent BRST symmetry. In the A^h formulation one has the exact BRST (to all orders)

$$\begin{aligned} sA_\mu &= -D_\mu c, \\ sc &= \frac{g}{2} [c, c], \\ s\bar{c} &= ib, \\ sA^h &= 0, \end{aligned}$$

$$s(\bar{\varphi}, \varphi, \bar{\omega}, \omega, \eta, \bar{\eta}, \tau) = 0.$$

and $s^2 = 0$ [11,12]. The ST replica sector enjoys supersymmetries in superspace for $\zeta = 0$; for $\zeta \neq 0$ these are softly broken but the local theory remains perturbatively renormalizable in $d = 4$ [14,17]. Because the two sectors couple only through A/A^h , the combined action (11) is local and power-counting renormalizable; the proof follows by merging the Ward identities of Serreau and Tissier [14], Reinosa et al. [17] with the algebraic renormalization in Dudal et al. [9], Sorella et al. [11,12], Terin et al. [13].

For $\gamma = 0$ and $M = m = 0$ we recover the ST massive FP theory with parameters (β, ζ) and n replicas. The replica sector behaves as a constrained NL σ model; for $\zeta = 0$ closed replica loops vanish (topological sector), while $\zeta \neq 0$ allows nontrivial loops and a phenomenon of radiative symmetry restoration in the replica sector [17]. In that symmetric phase, only the singled replica contributes to the tree-level gluon mass, yielding a finite screening mass $m_g^2 = \beta$ in the limit $n \rightarrow 0$; in the broken phase $m_g^2 \sim n\beta \rightarrow 0$.

Taking $\beta \rightarrow \infty$ (which selects absolute minima of $f[A, U]$) and $\zeta \rightarrow 0$ while keeping $\gamma \neq 0$ switches off the replica dynamics and reproduces the RGZ local action with exact BRST and parameters (γ, m, M) fixed by their gap equations [7,9,11]. With both sectors active, the tree-level transverse gluon two-point function reads schematically

$$\Delta_T^{-1}(p^2) = p^2 Z(p^2; \alpha) + \underbrace{\beta \Xi_{\text{rep}}(\zeta; \text{phase})}_{\text{ST replica mass block}} + \underbrace{\mathcal{M}_{\text{RGZ}}(p^2; \gamma, m, M)}_{\text{RGZ pole structure}}, \quad (12)$$

where $\Xi_{\text{rep}} = 1$ in the replica-symmetric phase (and 0 in the broken phase, in the limit $n \rightarrow 0$) as found in Reinosa et al. [17], and $\mathcal{M}_{\text{RGZ}}(p^2)$

represents the usual RGZ refinement. Thus, the unified propagator interpolates continuously between a pure ST massive FP and an RGZ-type decoupling curve. The nonperturbative parameters satisfy coupled gap equations:

$$\frac{\partial \Gamma}{\partial \gamma} = 0 \Rightarrow \langle H(A^h) \rangle = 4V(N^2 - 1), \quad \frac{\partial \Gamma}{\partial m^2} = 0, \quad \frac{\partial \Gamma}{\partial M^2} = 0, \quad (13)$$

as in RGZ [9], and for the replica sector [17]

$$\begin{aligned} 2 \frac{\beta_r}{\bar{g}_r^2} &= T(\hat{\chi}_r) - T(\hat{\chi}_r + \zeta_r), \\ 8\pi^2 \frac{\beta_r}{\bar{g}_r^2} &= (\hat{\chi}_r + \zeta_r) \ln \frac{\hat{\chi}_r + \zeta_r}{\bar{\mu}^2} - \hat{\chi}_r \ln \frac{\hat{\chi}_r}{\bar{\mu}^2}, \end{aligned} \quad (14)$$

with $\hat{\chi}_r \geq 0$ the replica mass scale in the symmetric phase, $T(m^2)$ the (dimensionally reduced) tadpole integral, and bars denote renormalized quantities in the scheme of Reinosa et al. [17]. BRST/Nielsen identities of the A^h formulation ensure that physical quantities do not depend on the linear-covariant gauge parameter α and that the longitudinal sector is not renormalized, to all orders [11,12].

Varying (β, ζ) at fixed (γ, m, M) deforms the IR mass scale in (12). The replica-symmetric domain (radiative restoration) provides a natural way to generate a Curci-Ferrari-like gluon mass from gauge fixing [14,17], while RGZ controls the nonlocal horizon physics and condensates. Implementing a weighted-copy average on the lattice (tunable β, ζ) and comparing the measured gluon/ghost correlators with (12) can test whether data prefer predominantly ST-like, RGZ-like, or mixed regimes. The unified action (11) reproduces (i) ST for $\gamma = 0$, including the phase diagram in (β, ζ) ; (ii) RGZ for $\beta \rightarrow \infty, \zeta \rightarrow 0$; and (iii) preserves exact BRST in the RGZ sector and perturbative renormalizability in the ST sector

Then, the gauge fixing (10) and its local version (11) furnish a concrete bridge between the ST copy-average mechanism (with its replica $N_L\sigma$ phenomenology and radiative symmetry restoration) and the GZ/RGZ restriction to the first Gribov region (with horizon suppression and condensates).

4. Renormalizability

The proof of renormalizability follows the algebraic approach based on the identification of all Ward identities and the characterization of the most general counterterm compatible with them, according to the procedure of algebraic renormalization [29]. Therefore, we work in the Landau gauge, $\partial_\mu A_\mu = 0$, and use the BRST-invariant formulation of RGZ based on the composite transverse, gauge-invariant field A_μ^h . We, thus, unify the ST replicated gauge-fixing with the RGZ sector:

$$\begin{aligned} S_{\text{tot}} &= S_{\text{YM}}[A] + S_{\text{FP}}[A, c, \bar{c}, b] + S_{\text{ST}}[A; \{\mathcal{V}_k\}, \beta, \zeta] \\ &+ S_{\text{GZ}}[A^h, \varphi, \bar{\varphi}, \omega, \bar{\omega}; \gamma] \\ &+ S_{\text{ref}}[A^h, \varphi, \bar{\varphi}, \omega, \bar{\omega}; m^2, M^2] + S_{\text{transv}}[A^h, \tau, \eta, \bar{\eta}], \end{aligned} \quad (15)$$

where $A_\mu^h \equiv h^\dagger A_\mu h + \frac{i}{g} h^\dagger \partial_\mu h$, $h = e^{ig\xi}$, and $\mathcal{V}_k(x, \theta, \bar{\theta})$ are the ST replica superfields ($k = 2, \dots, n$; the limit $n \rightarrow 0$ is taken at the end). The parameters β, ζ are the ST gauge-fixing masses. The BRST operator s is nilpotent, $s^2 = 0$, and acts as

$$\begin{aligned} sA_\mu^a &= -D_\mu^{ab}(A)c^b, & sc^a &= \frac{g}{2} f^{abc} c^b c^c, & s\bar{c}^a &= b^a, \\ sb^a &= 0, & sh &= -ig c h, & sA_\mu^{h,a} &= 0, \\ s\varphi_\mu^{ab} &= s\bar{\varphi}_\mu^{ab} = s\omega_\mu^{ab} = s\bar{\omega}_\mu^{ab} = 0, & s\tau^a &= s\eta^a = s\bar{\eta}^a = 0, \\ s\mathcal{V}_k &= ig \mathcal{V}_k c \quad (k = 2, \dots, n). \end{aligned} \quad (16)$$

To control non-linear variations we couple external sources (antifields)

$$S_{\text{ext}} = \int d^4x \left(K_\mu^a sA_\mu^a + L^a sc^a \right), \quad (17)$$

with engineering dimensions $[K] = 3$, $[L] = 4$ and ghost numbers $\text{gh}(K) = -1$, $\text{gh}(L) = -2$. We define $\Sigma \equiv S_{\text{tot}} + S_{\text{ext}}$. The Slavnov func-

tional reads

$$S(\Sigma) = \int d^4x \left(\frac{\delta \Sigma}{\delta K_\mu^a} \frac{\delta \Sigma}{\delta A_\mu^a} + \frac{\delta \Sigma}{\delta L^a} \frac{\delta \Sigma}{\delta c^a} + b^a \frac{\delta \Sigma}{\delta \bar{c}^a} \right) = 0. \quad (18)$$

Linearizing around Σ gives the nilpotent operator

$$\begin{aligned} B_\Sigma X &= \int d^4x \left(\frac{\delta \Sigma}{\delta K_\mu^a} \frac{\delta X}{\delta A_\mu^a} + \frac{\delta \Sigma}{\delta A_\mu^a} \frac{\delta X}{\delta K_\mu^a} + \frac{\delta \Sigma}{\delta L^a} \frac{\delta X}{\delta c^a} + \frac{\delta \Sigma}{\delta c^a} \frac{\delta X}{\delta L^a} + b^a \frac{\delta X}{\delta \bar{c}^a} \right), \\ B_\Sigma^2 &= 0. \end{aligned} \quad (19)$$

Besides (18), the following hold:

$$\text{Gauge condition (b-equation)} : \quad \frac{\delta \Sigma}{\delta b^a} = \partial_\mu A_\mu^a, \quad (20)$$

$$\text{Antighost equation (Landau)} : \quad \frac{\delta \Sigma}{\delta \bar{c}^a} + \partial_\mu \frac{\delta \Sigma}{\delta K_\mu^a} = 0,$$

$$\begin{aligned} \text{Ghost equation} : \quad \mathcal{G}^a \Sigma &\equiv \int d^4x \left(\frac{\delta}{\delta c^a} \right. \\ &\left. + g f^{abc} \bar{c}^b \frac{\delta}{\delta b^c} \right) \Sigma = \Delta_{\text{cl}}^a. \end{aligned} \quad (21)$$

with a linear classical breaking Δ_{cl}^a (non-renormalized). For A^h we impose transversality with a Lagrange multiplier:

$$\frac{\delta \Sigma}{\delta \tau^a} = \partial_\mu A_\mu^{h,a} = 0. \quad (22)$$

The replica sector enjoys (i) permutation symmetry S_{n-1} among $\mathcal{V}_k$ ($k \geq 2$); (ii) for $\zeta = 0$, topological supersymmetries (which suppress replica loops); for $\zeta \neq 0$, S_{n-1} and the assumed gauge covariance of $\mathcal{V}_k$ remain.

Now, consider a local perturbation $\Sigma \rightarrow \Sigma + \varepsilon \Delta \Sigma$ with $\dim(\Delta \Sigma) \leq 4$ and $\text{gh} = 0$. Symmetries imply

$$B_\Sigma \Delta \Sigma = 0, \quad \frac{\delta \Delta \Sigma}{\delta b} = 0, \quad \frac{\delta \Delta \Sigma}{\delta \bar{c}} + \partial \cdot \frac{\delta \Delta \Sigma}{\delta K} = 0, \quad \frac{\delta \Delta \Sigma}{\delta \tau} = 0, \quad (23)$$

plus the replica symmetries. The cohomology of B_Σ at ghost number 0 is generated by gauge-invariant polynomials in $F_{\mu\nu}$ and A^h , together with the BRST-invariant, dimension-two operators $(A^h)^2$ and $(\bar{\varphi}\varphi - \bar{\omega}\omega)$. The replica sector is cohomologically trivial for local counterterms, and S_{n-1} forbids terms that distinguish replicas ($k \geq 2$). Up to B_Σ -exact terms (field/source renormalizations), the most general local solution is

$$\begin{aligned} \Delta \Sigma &= \int d^4x \left[a_0 \frac{1}{4} F_{\mu\nu}^2 + a_1 (\partial_\mu \bar{c}^a) D_\mu^{ab} c^b + a_2 i b^a \partial \cdot A^a + a_3 \frac{\beta}{2} (A_\mu^a)^2 \right. \\ &\left. + a_4 \zeta \bar{c}^a c^a + a_5 (A_\mu^{h,a})^2 + a_6 (\bar{\varphi}\varphi - \bar{\omega}\omega) \right] + B_\Sigma(\dots). \end{aligned} \quad (24)$$

In particular, the superspace replica superfields $\mathcal{V}_k$ do not require independent wave-function renormalization factors: any possible divergence in the $\mathcal{V}_k$ sector can be absorbed into the renormalization of $(A_\mu, c, \bar{c})$ and the gauge-fixing parameters (β, ζ) , preserving the unitarity constraint $\mathcal{V}^\dagger \mathcal{V} = 1$. No new structures beyond those already in Σ appear: the theory is stable under renormalization. Although the unified action can be decomposed into an ST-like and an RGZ-like block, algebraic renormalization acts on the total functional as a single BRST complex. The cohomology of B_Σ contains only gauge-invariant representatives built from A^h , $F_{\mu\nu}$, and the composite replica operators, so that all allowed counterterms are reabsorbed by a single set of renormalization constants. Hence, the unification is algebraic rather than additive, for more details the reader can see the Appendix E. Therefore, the multiplicative renormalizations are written as

$$\begin{aligned} A_\mu &= Z_A^{1/2} A_{\mu,R}, & c &= Z_c^{1/2} c_R, & \bar{c} &= Z_{\bar{c}}^{1/2} \bar{c}_R, \\ \gamma^2 &= Z_{\gamma^2} \gamma_R^2, & m^2 &= Z_{m^2} m_R^2, & M^2 &= Z_{M^2} M_R^2, \\ g &= Z_g g_R, & \beta &= Z_\beta \beta_R, & \zeta &= Z_\zeta \zeta_R, \end{aligned} \quad (25)$$

and analogously for all auxiliary fields $\varphi, \bar{\varphi}, \omega, \bar{\omega}, h, \dots$

From (18), (21) and (21) in the Landau gauge (Taylor scheme) one obtains

$$Z_g Z_c Z_A^{1/2} = 1, \quad Z_\beta = Z_A^{-1}, \quad Z_\zeta = Z_c^{-1}. \quad (26)$$

In the BRST-invariant RGZ sector (based on A^h), known non-renormalization relations reduce the number of independent Z 's to Z_A and Z_c ; the renormalizations of γ^2 , m^2 , M^2 are then fixed functions of (Z_A, Z_c, Z_g) . Thus, all admissible counterterms are reabsorbed by renormalizations of fields, couplings, and mass parameters already present and therefore the unified action S_{tot} is renormalizable to all orders in the algebraic sense.

5. Gap system, suppression mechanisms, and matching (ST $\leftrightarrow$ RGZ)

The ST and RGZ formalisms implement different but complementary mechanisms to deal with Gribov copies. In RGZ, the functional integral is restricted to the first Gribov region $\Omega = \{A_\mu \mid \partial_\mu A_\mu = 0, \mathcal{M}^{ab}(A) > 0\}$ by the horizon condition $\langle H(A^h) \rangle = 4V(N^2 - 1)$, introducing the Gribov parameter γ and suppressing configurations outside Ω . In contrast, the ST replica weight statistically averages over copies inside Ω through the Landau functional $\mathcal{F}(A)$, exponentially damping higher- $\mathcal{F}$ configurations as $\exp[-\beta \Delta \mathcal{F}]$. Our unified construction combines these two effects: the RGZ horizon term restricts the measure to Ω , while the replica weight concentrates it near the minima of $\mathcal{F}$ within Ω , providing a smooth, BRST-consistent interpolation between averaging and suppression. In this setting, the dynamically generated gluon mass of the ST sector, $m_g^2 = \beta_R$, can be matched to the RGZ mass combination $m_{AA}^2 = m^2 + M^2 + \lambda^4/M^2$, which anchors the two gap systems to the same infrared scale.

We now make this correspondence explicit by analyzing the two families of gap equations and showing how the replica-generated scale β_R can be matched onto the RGZ parameters (m^2, M^2, λ^4) under a common renormalization scheme. On the ST side, the gap equation follows from the stationary condition of the replica effective potential with respect to the Lagrange multiplier χ enforcing the supersymmetric constraint $N_k^2 = 1$. After integrating out the $(n - 1)$ replicas in dimensional regularization, one obtains in $d = 4$ dimensions (DR + $\overline{\text{MS}}$ scheme) the renormalized relation

$$\frac{8\pi^2}{\bar{g}_R^2} \frac{\beta_R}{\bar{\mu}^2} = (\hat{\chi}_{R,\text{sym}} + \zeta_R) \ln \frac{\hat{\chi}_{R,\text{sym}} + \zeta_R}{\bar{\mu}^2} - \hat{\chi}_{R,\text{sym}} \ln \frac{\hat{\chi}_{R,\text{sym}}}{\bar{\mu}^2} \quad (27)$$

with a unique solution $\hat{\chi}_{R,\text{sym}} \geq 0$ in the symmetric phase. The matching conditions discussed later are implemented in this symmetric phase ($\zeta \neq 0$), requiring $\hat{\chi}_{R,\text{sym}} \geq 0$; accordingly, the parameters (β, ζ) are assumed to lie within the domain allowed by the gap inequality. In this regime the gluon screening mass is $m_g^2 = \beta_R$, which plays the same role as the dynamical mass emerging from the dimension-two condensates in the RGZ framework. On the RGZ side, the corresponding dynamical parameters are fixed by the horizon condition and the refinement gap equations,

$$\text{Horizon:} \quad \frac{\partial \Gamma_{\text{vac}}}{\partial \gamma^2} = 0, \quad (28)$$

$$\text{Refinement:} \quad \frac{\partial \Gamma_{\text{vac}}}{\partial m^2} = 0, \quad \frac{\partial \Gamma_{\text{vac}}}{\partial M^2} = 0. \quad (29)$$

whose solutions determine $(\gamma, m^2, M^2, \lambda^4)$. Matching the infrared limits of the two propagators then yields the relations summarized below.

To make this correspondence explicit, we expand both descriptions around small external momentum and identify the infrared matching conditions between the replica-generated scale β_R and the RGZ parameters. At tree level in the RGZ sector, the transverse gluon propagator reads

$$D_{AA}^{\text{RGZ, tree}}(p^2) = \frac{p^2 + M^2}{(p^2 + M^2)(p^2 + m^2) + \lambda^4}, \quad \lambda^4 = 2g^2 N \gamma^4. \quad (30)$$

Hence the inverse propagator and its infrared limit are

$$\Gamma_T^{\text{RGZ, tree}}(p^2) \equiv (D_{AA}^{\text{RGZ, tree}}(p^2))^{-1} = \frac{(p^2 + M^2)(p^2 + m^2) + \lambda^4}{p^2 + M^2},$$

$$\Gamma_T^{\text{RGZ, tree}}(0) = \frac{m^2 M^2 + \lambda^4}{M^2}. \quad (31)$$

On the ST side, in the replica-symmetric phase ($\zeta \neq 0$) one has a radiatively generated screening mass

$$\Gamma_T^{\text{ST}}(0) = m_g^2 \equiv \beta_R. \quad (32)$$

A working hypothesis that guides our matching is that the ST screening mass and the RGZ dimension-two mass have a common origin in the IR: the replica-induced scale β_R generated by copy averaging (and radiative restoration) accounts for the very same A^2 -type contribution that, in the RGZ language, is represented by the refined mass combination m_{AA}^2 in (9). Operationally, we implement this by imposing the IR matching $\Gamma_T(0) = \beta_R$,

$$\begin{aligned} \Gamma_T(0) \stackrel{!}{=} \beta_R &\implies \frac{m^2 M^2 + \lambda^4}{M^2} = \beta_R \\ &\iff m^2 M^2 + \lambda^4 = M^2 \beta_R. \end{aligned} \quad (33)$$

Next, expand $\Gamma_T(p^2)$ around $p^2 = 0$ in the unified theory,

$$\Gamma_T(p^2) = \underbrace{\beta_R}_{\text{from ST}} + Z^{-1} p^2 + \Pi_T^{\text{rep}}(p^2) + \Pi_T^{\text{RGZ}}(p^2) + \mathcal{O}(p^4), \quad (34)$$

and impose the slope matching at the origin,

$$\begin{aligned} \Gamma'_T(0) \stackrel{!}{=} \frac{d}{dp^2} \Gamma_T^{\text{RGZ, tree}}(p^2) \Big|_{p^2=0} &= \frac{(m^2 + M^2)M^2 - (m^2 M^2 + \lambda^4)}{M^4} \\ &= \frac{m^2}{M^2} - \frac{\beta_R}{M^2}, \end{aligned} \quad (35)$$

where we used (33) in the last equality. Therefore,

$$Z^{-1} + \Pi_T^{\text{rep}}(0) + \Pi_T^{\text{RGZ}}(0) = \frac{m^2 - \beta_R}{M^2}. \quad (36)$$

It is important to mention that these infrared matching conditions are scheme dependent; in this work we adopt the Landau MOM/Taylor scheme, imposing $Z_g Z_c Z_A^{1/2} = 1$, so that the one-loop contributions Π_T^{rep} and Π_T^{RGZ} consistently fix the wave-function renormalization Z . Eqs. (33) and (36) provide two relations tying the ST mass scale β_R to the RGZ parameters (m^2, M^2, λ^4) under a common renormalization scheme, while $\partial \Gamma_{\text{vac}} / \partial \gamma^2 = 0$ and the condensate gap equations fix γ^2 , m^2 , and M^2 . In this way, the replica-generated mass β_R acts as the infrared anchor that determines the RGZ mass combination $m^2 M^2 + \lambda^4$ and constrains the ratio $(m^2 - \beta_R)/M^2$ through the slope at the origin.

Having established the infrared and slope matching conditions (33)–(36), the full gap system comprises the replica Eq. (27) together with the horizon and condensate gaps (28)–(29). Solving them within a common renormalization scheme, e.g. the infrared-safe one, will determine the RGZ parameters $(\gamma, m^2, M^2, \lambda^4)$ consistently with the dynamically selected β_R from the replica sector.

Using the infrared matching condition $\Gamma_T(0) = \beta_R$ together with the RGZ tree relation $\lambda^4 = 2g^2 N \gamma^4$, we obtain a direct link between the ST screening mass and the Gribov parameter:

$$\gamma^4(\mu) = \frac{M^2(\mu) \beta_R(\mu) - m^2(\mu) M^2(\mu)}{2g^2(\mu) N}. \quad (37)$$

Eq. (37) holds in the common renormalization scheme and scale μ used for both sectors and requires $M^2 \beta_R > m^2 M^2$ for positivity. In the simplified case $m^2 = 0$, this reduces to

$$\gamma^4 = [M^2 \beta_R] / (2g^2 N). \quad (38)$$

Beyond tree level the numerical coefficient receives finite corrections, but the qualitative statement remains: once β_R is fixed by the replica gap, γ is determined by Eq. (37) together with the refinement parameters.

Moreover, Nielsen identities guarantee the gauge-parameter independence of the transverse pole mass; $\zeta \rightarrow 0$ collapses the replica loops (topological limit), while $\gamma \rightarrow 0$ and $M^2 \rightarrow 0$ recover the Curci-Ferrari limit. The lattice comparison can then be organized through $D_{AA}(0)$ and its slope $dD/dp^2|_{p^2=0}$, keeping all the gaps simultaneously satisfied. The corresponding analysis proceeds as:

1. Fix the renormalization scheme and impose the Landau conditions for Z_A, Z_c ; use $Z_g Z_c Z_A^{1/2} = 1$.
2. Solve the ST gap (27) to obtain $\hat{\chi}_{R,\text{sym}}$ and the allowed phase domain; set $\beta_R = m_g^2$.
3. Compute $\Pi_T^{\text{rep}}(p^2)$ (momentum-dependent part) and $\Pi_T^{\text{RGZ}}(p^2)$ at one loop.
4. Match $\Gamma_T(0), \Gamma'_T(0)$ with (30) (or the one-loop RGZ result), and solve (28) + (29) for (m^2, M^2, λ^4) .
5. Verify Slavnov/Nielsen identities and compare gluon/ghost correlators with lattice data.

Last but not least, the RGZ restriction controlled by γ removes field configurations outside the first Gribov region, but degeneracies of the Landau functional along gauge orbits may still persist inside Ω (notably near the boundary of the FMR). In our unified measure, the ST weight gives a complementary suppression of such residual copies: for two copies (U_1, U_2) on the same orbit one has a relative weight $\propto \exp[-\beta \Delta F]$, with $\Delta F = F(A^{U_2}) - F(A^{U_1})$. Thus, at fixed γ , increasing β exponentially favors the minima of F and lifts the degeneracy among copies that survive the horizon constraint. In the formal limit $\beta \rightarrow \infty$ (with $\zeta \rightarrow 0$), the probability concentrates on the absolute minima of F , approaching the FMR selection; while for finite β the suppression is controlled and smooth, leaving gauge-invariant observables unaffected by the normalization of the copy average.

To accomplish that fix a gauge orbit $[A]$ and let U_i be the set of (Landau) copies on that orbit. In the unified average,

$$\langle\langle \mathcal{O}[A] \rangle\rangle_{\beta, \zeta, \gamma} = \frac{\sum_i s(i) \mathcal{O}[A^{U_i}] \frac{\det(F_i + \zeta \mathbb{1})}{\det F_i} e^{-\beta F_i - \gamma^4 H(A^h)}}{\sum_i s(i) \frac{\det(F_i + \zeta \mathbb{1})}{\det F_i} e^{-\beta F_i - \gamma^4 H(A^h)}}$$

$$F_i := F[A, U_i], \quad F = \frac{1}{2} \int (\partial \cdot A)^2.$$

one has, for fixed $\zeta \geq 0$ and $\gamma \geq 0$:

$$\lim_{\beta \rightarrow \infty} \langle\langle \mathcal{O}[A] \rangle\rangle_{\beta, \zeta, \gamma} = \frac{\sum_{i \in \mathcal{M}} w_i(\zeta) \mathcal{O}[A^{U_i}]}{\sum_{i \in \mathcal{M}} w_i(\zeta)},$$

$$w_i(\zeta) := \lim_{\zeta' \rightarrow \zeta} s(i) \frac{\det(F_i + \zeta' \mathbb{1})}{|\det F_i|}.$$

where $\mathcal{M} = \{i : F_i = \min_j F_j\}$ is the set of configurations that minimize F on a given gauge orbit. In particular, if the minimum is unique, then

$$\lim_{\beta \rightarrow \infty} \langle\langle \mathcal{O}[A] \rangle\rangle_{\beta, \zeta, \gamma} = \mathcal{O}[A^{U_*}], \quad F_* = \min_j F_j.$$

- Since A^h is gauge invariant, we have $H(A^h, U_i) = H(A^h)$ for all i . Therefore, the factor $e^{-\gamma^4 H(A^h)}$ cancels exactly between the numerator and denominator, and the ratio of weights between different copies does not depend on γ :

$$\frac{P_i}{P_j} = \frac{s(i) \det(F_i + \zeta \mathbb{1})}{s(j) \det(F_j + \zeta \mathbb{1})} \frac{|\det F_j|}{|\det F_i|} e^{-\beta(F_i - F_j)}.$$

- For large β , by the Laplace method for discrete Gibbs measures [30], all terms with $F_i > \min_j F_j$ are exponentially suppressed as $e^{-\beta \Delta F}$. Thus, only indices $i \in \mathcal{M}$ contribute in the limit $\beta \rightarrow \infty$, and the remaining factors are the $w_i(\zeta)$ mentioned above.
- When $\zeta \rightarrow 0$, one recovers the original ST case, where the ratio $\det(F_i + \zeta)/|\det F_i|$ regularizes zeros and signs of the FP determinant. For finite positive ζ , the $w_i(\zeta)$ remain positive and of order $\mathcal{O}(1)$. $\square$

Corollary: For two copies U_1, U_2 on the same orbit,

$$\frac{P_2}{P_1} = \left[\text{prefactor depending on } (\zeta, F_{1,2}) \right] \times e^{-\beta \Delta F}, \quad \Delta F = F_2 - F_1.$$

Therefore, the hierarchy between copies is mainly governed by $e^{-\beta \Delta F}$. The parameter β acts as a “resolution control” that, with γ kept fixed (restriction to Ω), resolves the residual degeneracies that still exist inside Ω .

6. Unified superspace for ST and RGZ

This section puts the superspace ingredients in one place and shows how we host, side by side, the ST replica block and the RGZ block in a supersymmetric way. As previously stated the two blocks only interact through the gauge field A_μ (or the BRST-invariant composite A_μ^h). This makes the symmetry content transparent and supports the algebraic renormalization used above. The Zwanziger localization fields form the BRST doublets $s \varphi_\mu = \omega_\mu$, $s \omega_\mu = 0$, $s \bar{\omega}_\mu = \bar{\varphi}_\mu$, $s \bar{\varphi}_\mu = 0$, with $s^2 = 0$. Using a $(4|1)$ superspace with Grassmann coordinate θ and $s \equiv \partial_\theta$, we package them as $\Phi_\mu = \varphi_\mu + \theta \omega_\mu$, $\bar{\Phi}_\mu = \bar{\omega}_\mu + \theta \bar{\varphi}_\mu$. With the BRST-invariant composite $A_\mu^h = h^\dagger A_\mu h + \frac{i}{g} h^\dagger \partial_\mu h$ (and $\partial_\mu A_\mu^h = 0$ enforced by S_{transv}), the localized horizon + refinement sector is

$$S_{\text{RGZ}}^{(4|1)} = \int d^4x d\theta \bar{\Phi}_\mu \mathcal{M}(A^h) \Phi_\mu + \gamma^2 g \int d^4x d\theta A_\mu^h (\Phi_\mu + \bar{\Phi}_\mu) + \frac{M^2}{2} \int d^4x d\theta \bar{\Phi}_\mu \Phi_\mu + \frac{m^2}{2} \int d^4x (A_\mu^h)^2,$$

with $\mathcal{M}(A^h) = -\partial D(A^h)$. It is BRST-invariant by construction and reproduces the usual component form after $d\theta$ integration. The ST replica block is written with two Grassmann coordinates $(\theta_T, \bar{\theta}_T)$ and group-valued superfields $\mathcal{V}_k \in SU(N)$, $k = 2, \dots, n$, with

$$S_{\text{ST}} = \frac{1}{g^2} \sum_{k=2}^n \int d^4x d\theta_T d\bar{\theta}_T \text{tr}[(D_\mu \mathcal{V}_k)^\dagger (D_\mu \mathcal{V}_k) + 2\zeta \bar{\theta}_T \theta_T \partial_{\bar{\theta}_T} \mathcal{V}_k^\dagger \partial_{\theta_T} \mathcal{V}_k],$$

and the singled replica is described by $S_{\text{FP}}[A, c, \bar{c}, b]$. The topological supersymmetries are softly broken by ζ ; for $\zeta \rightarrow 0$ replica loops cancel. We place both blocks in a $(4|3)$ superspace with $(\theta_B; \theta_T, \bar{\theta}_T)$, where θ_B generates the RGZ BRST and $(\theta_T, \bar{\theta}_T)$ the ST topological pair. The symmetry algebra has $s_B^2 = Q^2 = \bar{Q}^2 = 0$, $[s_B, Q] = [s_B, \bar{Q}] = 0$, and $\{Q, \bar{Q}\}$ softly broken by ζ . The total action reads

$$S_{\text{hyb}} = S_{\text{YM}} + S_{\text{FP}} + S_{\text{transv}}[A^h] + S_{\text{ST}}[A; \mathcal{V}_k] + S_{\text{RGZ}}^{(4|1)}[A^h; \Phi, \bar{\Phi}],$$

and reduces to pure ST for $\gamma = 0$ (and $m = M = 0$), and to pure RGZ for $\beta \rightarrow \infty$, $\zeta \rightarrow 0$. The Slavnov identity for s_B plus the topological Ward identities in the replica block imply that all allowed local counterterms are absorbed by the common renormalizations already used in the main text (including $Z_g Z_c Z_A^{1/2} = 1$, $Z_\beta = Z_A^{-1}$, $Z_\zeta = Z_c^{-1}$ in Landau gauge). For more details the reader can see the [Appendix D](#).

7. Conclusions and outlook

We have built and analyzed a unified gauge fixing that continuously interpolates between the Serreau–Tissier copy-averaged Landau gauge and the (Refined) Gribov–Zwanziger restriction to the first Gribov region. At the level of the functional average, our construction combines the ST weight based on the Landau functional and the FP Hessian with a GZ-type suppression driven by the horizon functional. Using the replica trick and the A_μ^h localization, we derived a manifestly local, power-counting renormalizable action that couples a replica $\text{NL}\sigma$ sector to the BRST-invariant RGZ sector through the gauge field only.

On the structural side, our unified action enjoys (i) the exact nilpotent BRST symmetry of the A^h formulation in the RGZ block and (ii) the replica permutation symmetry (and, for $\zeta = 0$, topological supersymmetries) in the ST block. By mixing the corresponding Ward identities, we established algebraic renormalizability to all orders: the most general local counterterm compatible with the identities is a linear combination of operators already present in the classical action and can be reabsorbed by multiplicative renormalizations of fields, the coupling, and the mass parameters $(\beta, \zeta, \gamma, m^2, M^2)$. In the Landau scheme, the Taylor relation $Z_g Z_c Z_A^{1/2} = 1$ holds and fixes the non-renormalization of the ghost–gluon vertex at vanishing ghost momentum; $Z_\beta = Z_A^{-1}$ and $Z_\zeta = Z_c^{-1}$ follow directly. It is important to note that the unification

achieved here is not merely structural but algebraic: the BRST cohomology connects the ST and RGZ sectors through the gauge field, representing a single renormalizable framework for both descriptions.

Dynamically, the ST replica sector exhibits radiative symmetry restoration for $\zeta \neq 0$, leading (in the replica-symmetric phase) to a finite screening gluon mass equal to the renormalized parameter, $m_g^2 = \beta_R$, after the $n \rightarrow 0$ limit. In parallel, the RGZ parameters (γ, m^2, M^2) are selected by the horizon condition and the two dimension-two gap equations. We formulated a consistent matching strategy in which the small-momentum expansion of the full transverse two-point function fixes (i) $\Gamma_T(0) = \beta_R$ and (ii) the wave-function normalization, while the three RGZ gaps determine (γ, m^2, M^2) at the same renormalization scale. The resulting propagator interpolates smoothly between a massive-FP (ST) form and the RGZ decoupling form, providing a controlled node to study how infrared correlators depend on the relative weight of copy averaging versus horizon suppression and condensates.

Phenomenologically, our framework suggests direct lattice tests. Implementing a tunable copy-weighting (varying β, ζ) and confronting the measured $D_{AA}(0)$ and its slope with the unified prediction would indicate whether present ensembles are closer to the ST-like, RGZ-like, or mixed regime. On the continuum side, the one-loop momentum-dependent piece from the replica sector is finite at $p \rightarrow 0$ (the mass is fixed already at tree level), and the BRST/Nielsen identities secure gauge-parameter independence of the transverse pole mass.

Several developments now become natural:

1. completing the unified one-loop analysis of gluon and ghost propagators (and selected vertices), including the running of the couplings and masses;
2. solving the coupled gap system beyond one loop and studying the renormalization-group flow on the ST–RGZ interpolation;
3. extending our construction to linear covariant gauges and to finite temperature/chemical potential; and
4. exploring the impact on hadronic observables through functional methods: the DSEs and the FRG.

Altogether, the present work gives a concrete and flexible bridge between two IR descriptions of YM theories, clarifying their common ground and offering a single, local, BRST-consistent setting in which to quantify their interplay.

Data availability

No data was used for the research described in the article.

Acknowledgements

R. C. Terin gratefully acknowledges some of the researchers in infrared YM theories whose guidance have shaped part of his academic trajectory.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Field content, Ward identities, and basic non-renormalization

We collect the engineering (mass) dimensions and ghost numbers of all fields and external sources used in the unified ST + RGZ formulation (Euclidean $d = 4$):

	A_μ	c	$\bar{c}$	b	φ_μ	$\bar{\varphi}_\mu$	ω_μ	$\bar{\omega}_\mu$	$h(\xi)$	A_μ^h	τ
dim	1	0	2	2	1	1	1	1	0	1	2
gh	0	+1	-1	0	0	0	0	0	0	0	0

	η	$\bar{\eta}$	K_μ	L
dim	2	2	3	4
gh	+1	-1	-1	-2

Replica superfields $\mathcal{V}_k(x, \theta, \bar{\theta})$ ($k = 2, \dots, n$) are dimensionless and ghostless. The Grassmann coordinates satisfy $\int d\theta d\bar{\theta} \bar{\theta}\theta = -1$ and carry no engineering dimension. With $\Sigma = S_{\text{tot}} + S_{\text{ext}}$, the ST identity reads

$$S(\Sigma) = \int d^4x \left(\frac{\delta \Sigma}{\delta K_\mu^a} \frac{\delta \Sigma}{\delta A_\mu^a} + \frac{\delta \Sigma}{\delta L^a} \frac{\delta \Sigma}{\delta c^a} + b^a \frac{\delta \Sigma}{\delta \bar{c}^a} \right) = 0.$$

Linearizing around Σ defines the nilpotent operator

$$B_\Sigma X = \int d^4x \left[\frac{\delta \Sigma}{\delta K_\mu^a} \frac{\delta X}{\delta A_\mu^a} + \frac{\delta \Sigma}{\delta A_\mu^a} \frac{\delta X}{\delta K_\mu^a} + \frac{\delta \Sigma}{\delta L^a} \frac{\delta X}{\delta c^a} + \frac{\delta \Sigma}{\delta c^a} \frac{\delta X}{\delta L^a} + b^a \frac{\delta X}{\delta \bar{c}^a} \right],$$

$$B_\Sigma^2 = 0. \quad (\text{A.1})$$

It acts consistently on the A_μ^h -sector because $sA_\mu^h = 0$ and on the Stueckelberg field through $sh = -igc h$.

$$(i) \text{ Gauge condition: } \frac{\delta \Sigma}{\delta b^a} = \partial_\mu A_\mu^a. \quad (\text{A.2})$$

$$(ii) \text{ Antighost equation: } \frac{\delta \Sigma}{\delta \bar{c}^a} + \partial_\mu \frac{\delta \Sigma}{\delta K_\mu^a} = 0. \quad (\text{A.3})$$

$$(iii) \text{ Ghost equation: } \mathcal{G}^a \Sigma \equiv \int d^4x \left(\frac{\delta}{\delta c^a} + g f^{abc} \bar{c}^b \frac{\delta}{\delta b^c} \right) \Sigma = \Delta_{cl}^a, \quad (\text{A.4})$$

$$(iv) A^h \text{ transversality: } \frac{\delta \Sigma}{\delta \tau^a} = \partial_\mu A_\mu^{h,a} = 0. \quad (\text{A.5})$$

The replica sector enjoys (a) the permutation symmetry S_{n-1} of $\{\mathcal{V}_k\}_{k \geq 2}$ and (b) for $\zeta = 0$ additional topological supersymmetries which suppress replica loops.

At ghost number zero, the cohomology of B_Σ is generated by gauge-invariant polynomials in $F_{\mu\nu}$ and in the BRST-invariant composite A_μ^h , plus the dimension-two, BRST-invariant operators $(A^h)^2$ and $(\bar{\varphi}\varphi - \bar{\omega}\omega)$. The Stueckelberg sector and the replica supermultiplets assemble in BRST-covariant trivial pairs for local counterterms; S_{n-1} forbids replica-distinguishing terms. Hence the most general local counterterm is Eq. (24) in the main text, up to B_Σ -exact pieces, proving all-order stability.

In Landau gauge, combining the ST identity with the antighost and ghost equations yields the Taylor relation for the ghost-gluon vertex at vanishing ghost momentum,

$$Z_g Z_c Z_A^{1/2} = 1, \quad Z_\beta = Z_A^{-1}, \quad Z_\zeta = Z_c^{-1}. \quad (\text{A.6})$$

In the BRST-invariant RGZ formulation (based on A^h), standard algebraic arguments reduce the independent Z 's to (Z_A, Z_c) ; the renormalizations of γ^2, m^2, M^2 become fixed functions of (Z_A, Z_c, Z_g) constrained by (A.6). Also, the above identities, together with the properties of the linearized operator B_Σ , are the algebraic ingredients securing the stability of the unified action under renormalization. They give the starting point for the infrared matching analysis summarized in Section 5.

Appendix B. Replica one-loop tensor and small-momentum matching

Integrating the N_k fluctuations (symmetric phase) yields, at leading order in g ,

$$\Pi_{\mu\nu}^{\text{rep}}(q) = (n-1)\beta \delta_{\mu\nu} + (n-1)\bar{g}^2 \left[I_{\mu\nu}^{\hat{\zeta}+\zeta}(q) - I_{\mu\nu}^{\hat{\zeta}}(q) \right], \quad (\text{B.1})$$

with

$$I_{\mu\nu}^{m^2}(q) = \mu^{2\epsilon} \int \frac{d^d p}{(2\pi)^d} \frac{(2p-q)_\mu (2p-q)_\nu}{(p^2 + m^2)((p-q)^2 + m^2)}$$

$$= 2\delta_{\mu\nu} T_{m^2} + (q^2 \delta_{\mu\nu} - q_\mu q_\nu) F_{m^2}(q^2). \quad (\text{B.2})$$

Dimensional regularization in $d = 4 - 2\epsilon$ with $\bar{\mu}^2 = 4\pi e^{-\gamma_E} \mu^2$ gives

$$T_{m^2} \equiv \mu^{2\epsilon} \int \frac{d^d p}{(2\pi)^d} \frac{1}{p^2 + m^2} = -\frac{m^2}{16\pi^2} \left(\frac{1}{\epsilon} + 1 + \ln \frac{\bar{\mu}^2}{m^2} \right) + \mathcal{O}(\epsilon), \quad (\text{B.3})$$

$$F_{m^2}(q^2) = -\frac{1}{48\pi^2} \left[\frac{1}{\epsilon} + \frac{8}{3} + \ln \frac{\bar{\mu}^2}{m^2} + \frac{8m^2}{q^2} - 2 \left(1 + \frac{4m^2}{q^2} \right)^{3/2} \ln \left(\frac{\sqrt{q^2} + \sqrt{q^2 + 4m^2}}{2\sqrt{m^2}} \right) \right] + \mathcal{O}(\epsilon). \quad (\text{B.4})$$

Using the ST gap (symmetric phase) to eliminate the momentum-independent divergence, one obtains

$$\Gamma_{\mu\nu}^{\text{rep}}(q) = (n-1) \delta_{\mu\nu} [\beta - 2\bar{g}^2 (T_{\hat{\chi}} - T_{\hat{\chi}+\zeta})] + (q^2 \delta_{\mu\nu} - q_\mu q_\nu) \pi(q^2), \quad (\text{B.5})$$

with the finite scalar

$$\begin{aligned} \pi(q^2) &= (n-1) \bar{g}^2 [F_{\hat{\chi}+\zeta}(q^2) - F_{\hat{\chi}}(q^2)] \\ &= -\frac{(n-1) \bar{g}^2}{48\pi^2} \left[\ln \frac{\hat{\chi} + \zeta}{\hat{\chi}} + \frac{8\zeta}{q^2} + F\left(\frac{4\hat{\chi}}{q^2}\right) - F\left(\frac{4(\hat{\chi} + \zeta)}{q^2}\right) \right] \\ F(x) &= 2 \left(1 + \frac{1}{x} \right)^{3/2} \ln(\sqrt{x} + \sqrt{1+x}). \end{aligned} \quad (\text{B.6})$$

The $1/q^2$ terms cancel between the brackets, so (B.6) is regular at $q^2 \rightarrow 0$. At $q = 0$, tree + tadpole cancel in the symmetric phase, leaving the screening mass $m_g^2 = \beta_R$ (from the singled-out replica).

For the full transverse inverse two-point function

$$\Gamma_T(p^2) = D_T^{-1}(p^2) = m_g^2 + Z^{-1} p^2 + \Gamma_T^{\text{rep}}(p^2) + \Gamma_T^{\text{RGZ}}(p^2) + \dots,$$

expanding (B.6) at small p^2 and using the ST gap gives

$$\Gamma_T^{\text{rep}}(p^2) = \alpha_1 p^2 + \mathcal{O}(p^4), \quad \alpha_1 = \frac{d}{dp^2} [(q^2 \delta_{\mu\nu} - q_\mu q_\nu) \pi(q^2)] \Big|_{q^2=0}. \quad (\text{B.7})$$

Thus the replica sector only corrects the wave-function renormalization at $p^2 = 0$. The IR matching conditions used in Section 5 read

$$\Gamma_T(0) = \beta_R, \quad \Gamma_T'(0) = Z^{-1} + \alpha_1 + \frac{d\Gamma_T^{\text{RGZ}}}{dp^2} \Big|_0, \quad (\text{B.8})$$

which fix β_R (from ST) and relate (m^2, M^2, λ^4) to (β_R, Z) once the RGZ gaps are imposed.

Appendix C. Practical rules: tree-level Feynman rules, Nielsen identity, and DR conventions

- Gluon (RGZ tree):

$$D_{\mu\nu}^{ab}(p) = \delta^{ab} P_{\mu\nu}(p) \frac{p^2 + M^2}{(p^2 + M^2)(p^2 + m^2) + \lambda^4},$$

$$P_{\mu\nu}(p) = \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}.$$

- Ghost: $G^{ab}(p) = -\delta^{ab} \frac{1}{p^2}$.
- Zwanziger fields $(\varphi, \bar{\varphi}, \omega, \bar{\omega})$ propagate with masses set by M^2 (refined sector).
- Replica superfields do not propagate as ordinary local fields; their one-loop net effect is encoded by (B.1).

Standard YM three-/four-gluon vertices; ghost-gluon vertex (Taylor kinematics non-renormalization); RGZ coupling $g f^{abc} A^h(\varphi + \bar{\varphi})$. We impose MOM conditions at $p^2 = \mu^2$:

$$D_{AA}(p^2) \Big|_{p^2=\mu^2} = D_{AA}^{\text{tree}}(\mu^2), \quad G_{c\bar{c}}(p^2) \Big|_{p^2=\mu^2} = G_{c\bar{c}}^{\text{tree}}(\mu^2),$$

fixing Z_A, Z_c and imposing $Z_g Z_c Z_A^{1/2} = 1$ in Landau gauge.

In linear covariant gauges introduce a BRST doublet (α, χ) with $s\alpha = \chi, s\chi = 0$ and couple χ to the BRST variation of a gauge-fixing insertion. Differentiating the ST identity with respect to χ and to the fields yields

$$\frac{\partial}{\partial \alpha} \Gamma_T(p^2) = \mathcal{N}(p^2; \alpha) \Gamma_T(p^2),$$

with $\mathcal{N}$ regular near the pole. Hence the pole position $p^2 = m_{\text{pole}}^2$ is α -independent to all orders; in Landau gauge ($\alpha = 0$) this reduces to the statement used in the main text.

We use $d = 4 - 2\epsilon$ and the $\overline{\text{MS}}$ scale $\bar{\mu}^2 = 4\pi e^{-\gamma_E} \mu^2$. The basic integrals used in Eqs. (B.3)-(B.4) are

$$\mu^{2\epsilon} \int \frac{d^d p}{(2\pi)^d} \frac{1}{p^2 + m^2} = -\frac{m^2}{16\pi^2} \left(\frac{1}{\epsilon} + 1 + \ln \frac{\bar{\mu}^2}{m^2} \right) + \mathcal{O}(\epsilon), \quad (\text{C.1})$$

$$\begin{aligned} \mu^{2\epsilon} \int \frac{d^d p}{(2\pi)^d} \frac{1}{(p^2 + m^2)((p-q)^2 + m^2)} &= \frac{1}{16\pi^2} \left(\frac{1}{\epsilon} + \ln \frac{\bar{\mu}^2}{m^2} + 2 \right. \\ &\left. - \sqrt{1 + \frac{4m^2}{q^2}} \ln \frac{\sqrt{1 + \frac{4m^2}{q^2}} + 1}{\sqrt{1 + \frac{4m^2}{q^2}} - 1} \right) + \mathcal{O}(\epsilon). \end{aligned} \quad (\text{C.2})$$

from which (B.3)-(B.4) follow after standard tensor reduction.

Appendix D. Unified superspace for ST and RGZ

In this appendix, we show with more details compared to Section 6 the BRST (4|1) superspace for the RGZ sector and a simple “hybrid” superspace that hosts both the ST replica block and the RGZ block side by side. Throughout, the two blocks talk to each other only through the gauge field A_μ (or its BRST-invariant composite A_μ^h). The Zwanziger localizing fields come in BRST doublets

$$s \varphi_\mu^{ab} = \omega_\mu^{ab}, \quad s \omega_\mu^{ab} = 0, \quad s \bar{\omega}_\mu^{ab} = \bar{\varphi}_\mu^{ab}, \quad s \bar{\varphi}_\mu^{ab} = 0, \quad (\text{D.1})$$

with $s^2 = 0$. This already looks like a (nilpotent) supersymmetry: each fermionic field is the BRST variation of a bosonic partner. We use a (4|1) superspace with Grassmann coordinate θ and define the BRST differential as a translation:

$$s \equiv \frac{\partial}{\partial \theta}, \quad \theta^2 = 0, \quad s^2 = 0. \quad (\text{D.2})$$

In the RGZ setup, there is no independent anti-BRST, so one Grassmann direction is enough. Package the doublets into BRST superfields

$$\Phi_\mu^{ab}(x, \theta) := \varphi_\mu^{ab}(x) + \theta \omega_\mu^{ab}(x), \quad \bar{\Phi}_\mu^{ab}(x, \theta) := \bar{\omega}_\mu^{ab}(x) + \theta \bar{\varphi}_\mu^{ab}(x), \quad (\text{D.3})$$

so that $s\Phi = \partial_\theta \Phi$ and $s\bar{\Phi} = \partial_\theta \bar{\Phi}$. The engineering dimensions and ghost numbers follow from the components:

$$[\Phi] = [\bar{\Phi}] = 1, \quad \text{gh}(\Phi) = \text{gh}(\bar{\Phi}) = 0,$$

with $\text{gh}(\omega) = +1$ and $\text{gh}(\bar{\omega}) = -1$. We work with the BRST-invariant composite field

$$A_\mu^h = h^\dagger A_\mu h + \frac{i}{g} h^\dagger \partial_\mu h, \quad \partial_\mu A_\mu^h = 0, \quad (\text{D.4})$$

where $h = e^{ig\xi}$. The transversality is enforced by a Lagrange multiplier τ and an auxiliary BRST doublet $(\eta, \bar{\eta})$, while $sA_\mu^h = 0$. The localized horizon action is written as a (4|1) superspace integral

$$\begin{aligned} S_{\text{hor}} &= \int d^4 x d\theta \left[\bar{\Phi}_\mu^{ac}(x, \theta) \mathcal{M}^{ab}(A^h) \Phi_\mu^{bc}(x, \theta) \right] \\ &\quad + \gamma^2 g f^{abc} \int d^4 x d\theta A_\mu^{h,a}(x) (\Phi_\mu^{bc} + \bar{\Phi}_\mu^{bc}), \end{aligned} \quad (\text{D.5})$$

with $\mathcal{M}^{ab}(A^h) = -\partial_\mu D_\mu^{ab}(A^h)$. Using $\int d\theta (X_0 + \theta X_1) = X_1$, one recovers the standard component form

$$\begin{aligned} &\int d^4 x d\theta \bar{\Phi} \mathcal{M} \Phi = \int d^4 x (\bar{\varphi} \mathcal{M} \varphi - \bar{\omega} \mathcal{M} \omega) \\ &\int d^4 x d\theta A^h \cdot (\Phi + \bar{\Phi}) = \int d^4 x A^h \cdot (\varphi + \bar{\varphi}). \end{aligned} \quad (\text{D.6})$$

The RGZ dimension-two pieces are BRST-invariant:

$$S_{M^2} = \frac{M^2}{2} \int d^4 x d\theta \bar{\Phi}_\mu^{ab} \Phi_\mu^{ab} = \frac{M^2}{2} \int d^4 x (\bar{\varphi}_\mu^{ab} \varphi_\mu^{ab} - \bar{\omega}_\mu^{ab} \omega_\mu^{ab}), \quad (\text{D.7})$$

and

$$S_{m^2} = \frac{m^2}{2} \int d^4 x (A_\mu^{h,a})^2, \quad (\text{D.8})$$

which needs no superspace lifting. Collecting terms, the BRST-superspace action for RGZ is

$$\begin{aligned} S_{\text{RGZ}}^{(4|1)} = & \int d^4x \left(S_{\text{FP}}[A, c, \bar{c}, b] + S_{\text{transv}}[A^h; \tau, \eta, \bar{\eta}] \right) \\ & + \int d^4x d\theta \bar{\Phi}_\mu^{ac} \mathcal{M}^{ab}(A^h) \Phi_\mu^{bc} \\ & + \gamma^2 g f^{abc} \int d^4x d\theta A_\mu^{h,a} (\Phi_\mu^{bc} + \bar{\Phi}_\mu^{bc}) \\ & + \frac{M^2}{2} \int d^4x d\theta \bar{\Phi}_\mu^{ab} \Phi_\mu^{ab} + \frac{m^2}{2} \int d^4x (A_\mu^{h,a})^2. \end{aligned} \quad (\text{D.9})$$

This is BRST-invariant by construction (translation along θ). Coupling sources to BRST variations one builds the Slavnov functional; the linearized operator includes the θ -translation and is nilpotent. The cohomology at ghost number zero is generated by BRST-invariant operators made of $F_{\mu\nu}$, A^h , and $\bar{\Phi}\Phi$. Hence the algebraic renormalizability of the RGZ block follows: no counterterm beyond those already present is allowed by the identities. It is consistent to take $(\bar{\Phi})^\dagger = \bar{\Phi}$, $(\Phi)^\dagger = \Phi$, and $\int d\theta \theta = 1$, $\int d\theta 1 = 0$. Since A^h is real and BRST-invariant, the mass term (D.8) is positive in Euclidean signature.

Now we will place the ST replica sector and the RGZ sector in a common superspace. The ST block lives along a topological pair $(\theta_T, \bar{\theta}_T)$; the RGZ block lives along a BRST coordinate θ_B . They couple only through A_μ (or A_μ^h). Use a (4|3) superspace with Grassmann coordinates

$$(\theta_B; \theta_T, \bar{\theta}_T)^1,$$

and define three nilpotent differentials

$$s_B \equiv \partial_{\theta_B}, \quad Q \equiv \partial_{\theta_T}, \quad \bar{Q} \equiv \partial_{\bar{\theta}_T},$$

with $s_B^2 = Q^2 = \bar{Q}^2 = 0$ and $[s_B, Q] = [s_B, \bar{Q}] = 0$. In the ST block, $\{Q, \bar{Q}\}$ is softly broken by ζ (topological limit at $\zeta = 0$). We keep the BRST-invariant composite

$$A_\mu^h = h^\dagger A_\mu h + \frac{i}{g} h^\dagger \partial_\mu h, \quad \partial_\mu A_\mu^h = 0,$$

imposed by $S_{\text{transv}}[A^h; \tau, \eta, \bar{\eta}]$, and $s_B A_\mu^h = 0$. Next, we introduce $n-1$ group-valued superfields

$$\mathcal{V}_k(x, \theta_T, \bar{\theta}_T) \in SU(N), \quad \mathcal{V}_k^\dagger \mathcal{V}_k = 1, \quad k = 2, \dots, n,$$

where $D_\mu \mathcal{V}_k = \partial_\mu \mathcal{V}_k + ig \mathcal{V}_k A_\mu$. Their gauge-fixing action is

$$\begin{aligned} S_{\text{ST}}[A; \mathcal{V}_k] = & \frac{1}{g^2} \sum_{k=2}^n \int d^4x d\theta_T d\bar{\theta}_T \text{tr} \left[(D_\mu \mathcal{V}_k)^\dagger (D_\mu \mathcal{V}_k) \right. \\ & \left. + 2\zeta \bar{\theta}_T \theta_T \partial_{\bar{\theta}_T} \mathcal{V}_k^\dagger \partial_{\theta_T} \mathcal{V}_k \right]. \end{aligned}$$

The usual FP part $S_{\text{FP}}[A, c, \bar{c}, b]$ plays the role of the “singled” replica. BRST covariance is $s_B \mathcal{V}_k = ig \mathcal{V}_k c$ with the standard s_B on $(A, c, \bar{c}, b)$. As in the above mentioned RGZ case, we gather Zwanziger doublets into (4|1) superfields along θ_B :

$$\Phi_\mu^{ab}(x, \theta_B) = \varphi_\mu^{ab}(x) + \theta_B \omega_\mu^{ab}(x), \quad \bar{\Phi}_\mu^{ab}(x, \theta_B) = \bar{\omega}_\mu^{ab}(x) + \theta_B \bar{\varphi}_\mu^{ab}(x),$$

with $s_B \Phi = \partial_{\theta_B} \Phi$, $s_B \bar{\Phi} = \partial_{\theta_B} \bar{\Phi}$, and $s_B A_\mu^h = 0$. The horizon + refinement sector is

$$\begin{aligned} S_{\text{RGZ}}^{(4|1)}[A^h; \Phi, \bar{\Phi}] = & \int d^4x d\theta_B \bar{\Phi}_\mu^{ac} \mathcal{M}^{ab}(A^h) \Phi_\mu^{bc} \\ & + \gamma^2 g f^{abc} \int d^4x d\theta_B A_\mu^{h,a} (\Phi_\mu^{bc} + \bar{\Phi}_\mu^{bc}) \\ & + \frac{M^2}{2} \int d^4x d\theta_B \bar{\Phi}_\mu^{ab} \Phi_\mu^{ab} + \frac{m^2}{2} \int d^4x (A_\mu^{h,a})^2, \end{aligned} \quad (\text{D.10})$$

¹ This (4|3) notation does not refer to an $N = 3$ supersymmetric space; it simply denotes the product of a BRST superspace (4|1) with coordinate θ_B and the Parisi–Sourlas topological superspace (0|2) with coordinates $(\theta_T, \bar{\theta}_T)$.

in which $\mathcal{M}^{ab}(A^h) = -\partial_\mu D_\mu^{ab}(A^h)$. Putting all pieces together (and imposing $\partial_\mu A_\mu^h = 0$):

$$\begin{aligned} S_{\text{hyb}} = & S_{\text{YM}}[A] + S_{\text{FP}}[A, c, \bar{c}, b] + S_{\text{transv}}[A^h; \tau, \eta, \bar{\eta}] + S_{\text{ST}}[A; \{\mathcal{V}_k\}; \beta, \zeta] \\ & + S_{\text{RGZ}}^{(4|1)}[A^h; \Phi, \bar{\Phi}]. \end{aligned} \quad (\text{D.11})$$

The only link between the ST and RGZ parts is A_μ (and A_μ^h). The symmetry algebra is generated by $(s_B; Q, \bar{Q})$ with

$$s_B^2 = Q^2 = \bar{Q}^2 = 0, \quad [s_B, Q] = [s_B, \bar{Q}] = 0, \quad \{Q, \bar{Q}\} \text{ soft } \propto \zeta.$$

Limits: (i) $\zeta = 0$ kills replica loops (topological limit); (ii) $\gamma = 0$, $m = M = 0$ gives back the massive FP/ST limit; (iii) $\beta \rightarrow \infty$, $\zeta \rightarrow 0$ yields pure RGZ.

Then, we define the Slavnov functional for s_B with sources (K_μ, L) coupled to BRST variations, and the topological Ward identities for $(Q, \bar{Q})$ in the replica block. The linearized operator $\mathcal{B}_\Sigma$ factorizes into two commuting pieces. At ghost number zero, the cohomology is generated by BRST-invariant polynomials in $F_{\mu\nu}$ and A^h , plus the dimension-two operators $(A^h)^2$ and $(\bar{\Phi}\Phi)$. The replica block is cohomologically trivial for local counterterms under the S_{n-1} permutation symmetry. Therefore all enabled counterterms are absorbed by multiplicative renormalizations

$$Z_A, Z_c, Z_g, Z_\beta, Z_\zeta, Z_{\gamma^2}, Z_{m^2}, Z_{M^2},$$

where the Landau (Taylor) relations $Z_g Z_c Z_A^{1/2} = 1$, $Z_\beta = Z_A^{-1}$, $Z_\zeta = Z_c^{-1}$. If one prefers a single group-valued object, define

$$\mathcal{N}(x, \theta_B, \theta_T, \bar{\theta}_T) = \exp \left\{ ig [\bar{\theta}_T c_T + \bar{c}_T \theta_T + \bar{\theta}_T \theta_T \bar{h}_T + \theta_B c] \right\} U(x),$$

with $U \in SU(N)$, $s_B \mathcal{N} = ig \mathcal{N} c$, and $(Q, \bar{Q})$ acting on the “T” components only. Then the ST functional becomes

$$S_{\text{ST}} = \frac{1}{g^2} \int d^4x d\theta_T d\bar{\theta}_T \text{tr} \left[(D_\mu \mathcal{N})^\dagger (D_\mu \mathcal{N}) + 2\zeta \bar{\theta}_T \theta_T \partial_{\bar{\theta}_T} \mathcal{N}^\dagger \partial_{\theta_T} \mathcal{N} \right],$$

where unitarity $\mathcal{N}^\dagger \mathcal{N} = 1$ kept manifest. The Zwanziger fields remain as adjoint superfields $(\Phi, \bar{\Phi})$ along θ_B .

Appendix E. Algebraic unification in pure SU(N) Yang–Mills: explicit BRST for ST – RGZ

The basic fields for this work are A_μ^a , the Faddeev–Popov triplet $c^a, \bar{c}^a, b^a$, a Stückelberg element $h(x) = e^{ig \xi^a T^a}$ used to define the gauge-invariant projection A_μ^h , the Zwanziger pairs $\varphi_\mu^{ab}, \bar{\varphi}_\mu^{ab}$ and $\omega_\mu^{ab}, \bar{\omega}_\mu^{ab}$ for the RGZ block, and n replica group elements $U_i(x) \in SU(N)$ for the ST copy averaging. To keep locality and BRST exactness in the latter weights, we add two BRST doublet pairs for each replica: (χ_i^a, ρ_i^a) , $(\bar{\chi}_i^a, \bar{\rho}_i^a)$ and (ϕ_i^a, η_i^a) , $(\bar{\phi}_i^a, \bar{\eta}_i^a)$, with $\chi, \bar{\chi}, \phi, \bar{\phi}$ bosonic and $\rho, \bar{\rho}, \eta, \bar{\eta}$ fermionic.

The unified BRST operator s is nilpotent, $s^2 = 0$, and acts as

$$\begin{aligned} s A_\mu^a = & -(D_\mu c)^a, & s c^a = & -\frac{g}{2} f^{abc} c^b c^c, \\ s \bar{c}^a = & b^a, & s b^a = & 0, \\ s h = & -ig c h, & s A_\mu^h = & 0, \\ s \varphi_\mu^{ab} = & \omega_\mu^{ab}, & s \omega_\mu^{ab} = & 0, & s \bar{\omega}_\mu^{ab} = & \bar{\varphi}_\mu^{ab}, \\ s \bar{\varphi}_\mu^{ab} = & 0, & s U_i = & -ig c U_i, & s(A_i^{U_i}) = & -D_\mu(A_i^{U_i}) c, \\ s \chi_i^a = & \rho_i^a, & s \rho_i^a = & 0, & s \bar{\rho}_i^a = & \bar{\chi}_i^a, & s \bar{\chi}_i^a = & 0, \\ s \phi_i^a = & \eta_i^a, & s \eta_i^a = & 0, & s \bar{\eta}_i^a = & \bar{\phi}_i^a, & s \bar{\phi}_i^a = & 0. \end{aligned} \quad (\text{E.1})$$

The ST target weight in the static (copy-averaged) sector is

$$\begin{aligned} \mathcal{W}_{\text{ST}}[A; U_i] \propto & \frac{\det(\mathcal{F}[A^{U_i}] + \zeta \mathbb{1})}{|\det \mathcal{F}[A^{U_i}]|} \exp \left[-\beta f(A^{U_i}) \right], \\ f(A) = & \frac{1}{2} \int d^4x A_\mu^a A_\mu^a, \end{aligned} \quad (\text{E.2})$$

with the Landau FP operator $\mathcal{F}^{ab}[A] = -\partial_\mu D_\mu^{ab}(A)$ and the usual discrete sum over copies keeping $s(i) = \text{sign det } \mathcal{F}[A^{U_i}]$.

The algebraic unification is implemented by a single gauge-fixing fermion Ψ_{uni} whose BRST variation generates both the FP Landau fixing and the ST weights. We choose

$$\begin{aligned} \Psi_{\text{uni}} = & \int d^d x \left[\bar{c}^a \partial_\mu A_\mu^a + \frac{\alpha}{2} \bar{c}^a b^a \right] \\ & + \beta \sum_{i=1}^n \mathcal{V}_i [A^{U_i}; \chi_i, \bar{\chi}_i, \rho_i, \bar{\rho}_i] \\ & + \zeta \sum_{i=1}^n D_i [A^{U_i}; \phi_i, \bar{\phi}_i, \eta_i, \bar{\eta}_i], \end{aligned} \quad (\text{E.3})$$

$$S_{\text{gf}} \equiv s \Psi_{\text{uni}}. \quad (\text{E.4})$$

For the ST Landau-functional piece we take a minimal local choice

$$\mathcal{V}_i = \int d^d x \left[\bar{\rho}_i^a \partial_\mu (A^{U_i})_\mu^a + \frac{\kappa}{2} \bar{\chi}_i^a \chi_i^a \right], \quad (\text{E.5})$$

$$\begin{aligned} s \mathcal{V}_i = & \int d^d x \left[\bar{\chi}_i^a \partial_\mu (A^{U_i})_\mu^a - \bar{\rho}_i^a \partial_\mu (D_\mu (A^{U_i}) c)^a \right] \\ & + \kappa \bar{\chi}_i^a \rho_i^a. \end{aligned} \quad (\text{E.6})$$

In the Landau limit $\alpha \rightarrow 0$, Gaussian completion over $(\bar{\chi}_i, \rho_i)$ and then $\bar{\rho}_i$ yields effectively

$$\exp \left[-\beta \frac{1}{2} \int d^d x (A^{U_i})_\mu^a (A^{U_i})_\mu^a \right], \quad (\text{E.7})$$

up to BRST-exact contact terms and constants. The parameter $\kappa > 0$ is auxiliary and cancels in BRST-invariant observables.

For the FP-determinant deformation we use a Zwanziger-like first-order representation,

$$D_i = \int d^d x \left[\bar{\eta}_i^a (F[A^{U_i}] + \zeta \mathbb{1})^{ab} \phi_i^b \right], \quad (\text{E.8})$$

$$\begin{aligned} s D_i = & \int d^d x \left[\bar{\phi}_i^a (F[A^{U_i}] + \zeta)^{ab} \phi_i^b - \bar{\eta}_i^a (F[A^{U_i}] + \zeta)^{ab} \eta_i^b \right] \\ & + \bar{\eta}_i^a (s F[A^{U_i}])^{ab} \phi_i^b. \end{aligned} \quad (\text{E.9})$$

Path integration over $(\phi, \bar{\phi})$ and $(\eta, \bar{\eta})$ gives the local, BRST-exact realization of $\det(F[A^{U_i}] + \zeta)$. If one prefers to realize the ratio $\det(F + \zeta)/\det(F)$ directly, an unshifted companion $(\bar{\phi}, \bar{\eta})$ can be added,

$$\tilde{D}_i = \int d^d x \bar{\eta}_i^a F^{ab} [A^{U_i}] \bar{\phi}_i^b, \quad (\text{E.10})$$

$$\begin{aligned} s \tilde{D}_i = & \int d^d x \left[\bar{\phi}_i^a F^{ab} [A^{U_i}] \bar{\phi}_i^b - \bar{\eta}_i^a F^{ab} [A^{U_i}] \bar{\eta}_i^b \right] \\ & + \bar{\eta}_i^a (s F[A^{U_i}])^{ab} \bar{\phi}_i^b. \end{aligned} \quad (\text{E.11})$$

though in the standard ST prescription the denominator modulus is accounted for in the discrete copy sum by $s(i)$, so this extra pair may be omitted.

The RGZ sector is written entirely in terms of the BRST-invariant field A_μ^h , hence it is strictly invariant under s :

$$\begin{aligned} S_{\text{RGZ}}[A^h, \varphi, \omega; \gamma] = & \int d^d x \left[\bar{\varphi}_\mu^{ac} \mathcal{M}^{ab}(A^h) \varphi_\mu^{bc} \right] \\ & - \int d^d x \left[\bar{\omega}_\mu^{ac} \mathcal{M}^{ab}(A^h) \omega_\mu^{bc} \right] \\ & + g \gamma^2 f^{abc} (A^h)_\mu^a (\varphi + \bar{\varphi})_\mu^{bc} \\ & + d(N^2 - 1) \gamma^4, \end{aligned} \quad (\text{E.12})$$

$$s A^h = 0, \quad (\text{E.13})$$

$$s S_{\text{RGZ}} = 0. \quad (\text{E.14})$$

The full unified action in pure Yang–Mills is

$$\begin{aligned} S_{\text{uni}} = & S_{\text{YM}}[A] + s \Psi_{\text{uni}}[A, c, \bar{c}, b; U_i, \text{aux}] \\ & + S_{\text{RGZ}}[A^h, \varphi, \omega; \gamma]. \end{aligned} \quad (\text{E.15})$$

With antifields K_μ^a and L^a coupled to $s A_\mu^a$ and $s c^a$, and optional sources for the doublets, the Slavnov functional $S(\Sigma) = 0$ takes the standard form and controls the cohomology relevant for renormalization.

The interpolation limits are smooth and remain within pure Yang–Mills. When $\gamma \rightarrow 0$ one recovers the pure ST side,

$$\gamma \rightarrow 0 \Rightarrow S_{\text{uni}} \rightarrow S_{\text{YM}} + s \Psi_{\text{uni}}, \quad (\text{E.16})$$

while for $\beta, \zeta \rightarrow 0$ one keeps the standard FP Landau gauge together with the RGZ block,

$$\begin{aligned} \beta, \zeta \rightarrow 0 \Rightarrow S_{\text{uni}} \rightarrow S_{\text{YM}} + s \int d^d x \bar{c} \partial \cdot \\ + S_{\text{RGZ}}(A^h), \end{aligned} \quad (\text{E.17})$$

and the strict Landau condition is obtained by

$$\alpha \rightarrow 0 \Rightarrow \partial_\mu A_\mu = 0. \quad (\text{E.18})$$

In this way, the ST average and the RGZ restriction are glued by the same nilpotent BRST operator acting on a common field space, which is exactly what is meant by an algebraic (rather than additive) unification.

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