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Letter

Can a late-time cosmological model based on baby universe absorption explain the z-variation of w?

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ABSTRACT

We point out that a simple late-time cosmological model where our Universe can absorb "baby universes" explains the exponential expansion of our universe without the need of a cosmological constant and leads to a z-dependence of the parameter w(z) in the equation of state. In this model w(z) is less than -1 for z sufficiently large.

1. Introduction

If one assumes that the late time cosmology is well described by the FLRW metric

$$ds^2 = -N^2(t)dt^2 + a^2(t)d\Omega_d, \qquad d\Omega_d = \sum_{i=1}^{d} dx_i^2, \tag{1}$$

the time evolution can be derived from the minisuperspace action

$$S = \int dt \Big(-\frac{3}{8}\frac{\dot{v}^2}{Nv} - Nv(\lambda + \kappa\rho(v)) \Big). \tag{2}$$

Units used are $c = \hbar = 1$, and we use

$$v(t) = \frac{1}{\kappa}a^3(t), \quad \kappa = 8\pi G, \quad G = \text{the gravitational constant}, \tag{3}$$

as variable rather than the scale factor $a(t)$. $v(t)$ is proportional to the d-dimensional spatial volume at time t and below we will just call it the spatial volume. $\rho(v)$ denotes the matter density.

$$\kappa\rho(v) = \kappa\rho_{\rm m}(v) + \kappa\rho_{\rm r}(v) = 3H_0^2\Big(\Omega_{\rm m}\frac{v_0}{v} + \Omega_{\rm r}\Big(\frac{v_0}{v}\Big)^{4/3}\Big), \tag{4}$$

where v_0 denotes the spatial volume at the present time t_0 and H_0 the Hubble constant, i.e. the Hubble parameter $H(t) = \dot{a}(t)/a(t) = \dot{v}(t)/3v(t)$ at t_0. Finally λ denotes the cosmological constant.

If p denotes the momentum conjugate to v, the Hamiltonian related to the action S given by Eq. (2) will be

$$\mathcal{H} = Nv\Big(-\frac{3}{4}p^2 + \lambda + \kappa\rho(v) \Big). \tag{5}$$

The Friedmann equation associated with (5) is

$$\frac{3}{4}p^2 = \lambda + \kappa\rho(v), \tag{6}$$

and when $t \le t_{\rm LS}$, the time of last scattering, we can ignore λ. Let $T(t)$ denote the temperature of the CMB at times $t \ge t_{\rm LS}$. Then $a(t_{\rm LS})/a(t_0) = T(t_0)/T(t_{\rm LS})$ and since we know $T(t_{\rm LS})$ from atomic physics and can measure $T(t_0)$ and H_0, we can write

$$\frac{3}{4}p^2(t_{\rm LS}) = \Big(\frac{\rho_{\rm m}(v_0)}{(1+z(t_{\rm LS}))^3} + \frac{\rho_{\rm r}(v_0)}{(1+z(t_{\rm LS}))^4}\Big), \tag{7}$$

where $z(t) = a(t_0)/a(t) - 1$ is the redshift at time t. Thus, $v(t_{\rm LS})/v_0$ is known and $p(t_{\rm LS})$ is known if $\rho_{\rm m}(v_0)$ and $\rho_{\rm r}(v_0)$ are known, and finally $t_{\rm LS}$ can be found by integrating (6) starting from $t = 0$, $z(0) = \infty$, to $z(t_{\rm LS})$. We just mention these textbook facts since we will use them in a model with a modified Friedmann equation.

In [1,2] we proposed modifications of the standard minispace Hamiltonian (5) that allow our universe to absorb other universes during its time evolution[1] In the modified models one could freely choose the distribution of spatial volumes of the absorbed universes, and we considered two limits: one where the spatial volume of an absorbed universe was infinitesimal, and one where there is a certain self-consistency between our universe and the universes absorbed. In this article we will

[1] To describe the emission instead of absorption of baby universes, one simply changes the sign of g to be negative. The mathematics of the two cases are almost identical and in this paper we focus on the case $g > 0$ and refer to baby universe absorption.

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concentrate on the latter model and below we will discuss what "self-consistency" means.

Since [1] was published the new DESI results seem to favor an equation [3] of state

$$P(z) = w(z)\,\rho(z), \tag{8}$$

where $\rho(z)$ and $P(z)$ are the energy density and the pressure of dark energy, and where w is z-dependent and $w < -1$ for z sufficiently large. Already in [1] we noted that such a behavior is a natural consequence for the kind of model we consider. We here take the opportunity to check to what extent our model can accommodate the latest DESI results.

2. The late-time cosmological model

In our model the standard minisuperspace Hamiltonian (5) is replaced by

$$\mathcal{H} = Nv\Big(-\frac{3}{4}\big(p^2 - 2g\tilde{F}(p)\big) + \lambda + \kappa\rho(v)\Big) \tag{9}$$

$$= Nv\Big(-f(p) + \kappa\rho(v)\Big), \quad f(p) = -\frac{3}{4}(p+\alpha)\sqrt{(p-\alpha)^2 + \frac{2g}{\alpha}} \tag{10}$$

where α satisfies

$$\alpha^3 - \frac{4}{3}\lambda\alpha - g = 0. \tag{11}$$

A new coupling constant g is introduced and it is assigned to the process where two universes merge to one universe (see [1] and references therein for a detailed discussion). Eq. (11) is a consistency condition that ensures that in the case where $\rho(v) = 0$, for given coupling constants g, λ, the Hartle-Hawking wave functions of the universes involved in the mergers are identical. In [1] we denoted the model with this $f(p)$ the gcdt-model, since the Eqs. (10) and (11) was first derived in a 2d quantum gravity model called generalized causal dynamical triangulations [4]. The function $\tilde{F}(p)$ has an expansion in inverse power of p and in the early universe where $v(t)$ is small and thus $\rho(v)$ and $|p|$ are large, $\tilde{F}(p)$ (and λ) will not be important.

The equations of motion for the Hamiltonian (10) are

$$\frac{\dot{v}}{v} = -f'(p), \quad \dot{p} = f(p) + \frac{1}{3}\rho_{\rm r}(v), \quad f(p) = \kappa\rho_{\rm m}(v) + \kappa\rho_{\rm r}(v), \tag{12}$$

where the last equation is the "Friedmann" equation that follows from $\partial\mathcal{H}/\partial N = 0$. Assuming that the universe expands at large t it follows that $p < -\alpha$, and from the Friedmann equation it is seen that $p(t) \to -\alpha$ for large t. Thus that the expansion is exponential for large t:

$$v(t) \propto \mathrm{e}^{-f'(-\alpha)t}, \quad f'(-\alpha) = -\frac{3}{4}\sqrt{4\alpha^2 + \frac{2g}{\alpha}} = -\sqrt{3\lambda + \frac{27\,g}{8\alpha}}. \tag{13}$$

For $g = 0$ we have the usual exponential expansion of the de Sitter universe, and increasing g will only increase this expansion in accordance with the intuition that our Universe in the model absorbs the spatial volume of other universes. In fact one can even take $\lambda = 0$ and still have an exponential expanding universe at late time, the expansion only dictated by the absorption of other universes and the rate being given by

$$v(t) \propto \mathrm{e}^{\sqrt{\frac{27}{8}}\,g^{1/3}t}. \tag{14}$$

In the following we will assume $\lambda = 0$ in our generalized minisuperspace model, such that we have exchanged the coupling constant λ with a new coupling constant g that controls the strength of the merging of universes.

For the standard ΛCDM model the value of λ is such that it plays (almost) no role for $t \le t_{\rm LS}$, the time of last scattering, as already mentioned. The same will be the case for g in our generalized model. The new coupling constant g will be so small that we can ignore it for $t \le t_{\rm LS}$, and thus we will have the same time evolution of $v(t)$ as for the ΛCDM model for $t \le t_{\rm LS}$. Given $\kappa\rho_{\rm m}(v_0)$ and $\kappa\rho_{\rm r}(v_0)$ we can solve (12) for $v(t)/v_0$ and $p(t)$, starting out at $t_{\rm LS}$ where we know the values, and integrating the equations to $v(t)/v_0 = 1$ that will then determine our present time t_0.

We thus have two models, the ΛCDM model and our multiverse model, that start out at $t_{\rm LS}$ with the same values of $v(t_{\rm LS})/v_0$ and $p(t_{\rm LS})$, and thus the same value $H(t_{\rm LS})$ of the Hubble parameter, but when integrated to the region $t \gg t_{\rm LS}$, where λ and g, respectively, are important, will behave differently. In particular the time t_0 will be different in the two models and the Hubble parameter $H(t)$ will be different, as will the Hubble constant $H_0 = H(t_0)$.

In [1,2] we made a simple analysis of this difference, by just taking the value of $\rho(v)$ that was available from the best fits to the ΛCDM model. In principle $\rho(v)$ is determined by physics at times $t < t_{\rm LS}$ and is then the same for the ΛCDM model and our model[2] By insisting that the locally measured $H_0 \approx 72.6$ km/s/Mpc (see [5]) is the correct value of H_0, the value g was fixed. We noticed then that the $H(z)$ calculated for this value of g agreed well with the low z measurements of $H(z)$. In addition, for the same value of g, we could calculate the parameter S_8, measuring the large-scale structure clustering of matter, to be $S_8 \approx 0.76$, also in agreement with local observations.

The best fit to all data, *excluding the local measurement of* H_0, using the ΛCDM model, leads to a value of λ such that $H_0 \approx 67$ km/s/Mpc and $S_8 \approx 0.83$, resulting in the so-called H_0 and S_8 tensions with local measurements. In [1,2] we formulated it as follows: assuming the local measurements of H_0 are correct our model seems to describe late time cosmology better than the ΛCDM model. In [6] a more careful analysis of the our model was performed, using the same fitting as for the ΛCDM model. The result was that the ΛCDM model fitted the data (again excluding local measurements of H_0) somewhat better[3]. However, even if we have not performed the analysis, it is relatively clear from the results [6] that *if* one was fitting with the additional constraint that $H_0 \approx 72.6$ km/s/Mpc (i.e. assuming that the local measurements give the correct value of H_0), our model would be doing as well as the ΛCDM model. With this situation in mind, we now turn to the equation of state parameter $w(z)$.

3. A z-dependent w

Using (12) we can write the generalized Friedmann equation $f(p) = \kappa\rho(v)$ as follows

$$\Big(\frac{\dot{a}(t)}{a(t)}\Big)^2 = \frac{\kappa\rho(v)}{3} + \frac{\kappa\rho_{\rm f}(v)}{3}, \quad \kappa\rho_{\rm f}(p) = \frac{1}{3}\big(f'(p)\big)^2 - f(p), \tag{15}$$

where we can view $\rho_{\rm f}(p)$ as the energy density associated to the merging of other universes with our Universe. If we define the corresponding pressure $P_{\rm f}$ by the energy conservation equation

$$\frac{d}{dt}(v\rho_{\rm f}) + P_{\rm f}\frac{d}{dt}v = 0, \tag{16}$$

we obtain the *formal equation of state parameter* $w_{\rm f}$:

$$w_{\rm f} = \frac{P_{\rm f}}{\rho_{\rm f}} = \frac{f(p)\Big(\frac{2}{3}f''(p) - 1\Big)}{\frac{1}{3}\big(f'(p)\big)^2 - f(p)} - 1. \tag{17}$$

When $t \gg t_{\rm LS}$ we can ignore $\rho_{\rm r}(v(t))$ and the only contribution to w comes from $\rho_{\rm f}$. As a function of t or $z(t)$, $w_{\rm f}(z)$ interpolates between $w_{\rm f} = -1.5$ at $z \to \infty$, i.e. $t \to 0$, and $w_{\rm f} = -1$ for $z \to -1$, i.e. $t \to \infty$ (where the universe expands exponentially). We now want to compare this behavior with the $w(z)$ extracted from the DESI data [3,7].

[2] One should note that since $\rho(z_{\rm LS})$ is determined by physics at $t \le t_{\rm LS}$, also $\rho_{\rm m}(z=0) = \rho_{\rm m}(z_{\rm LS})/(1+z_{\rm LS})^3$ is determined by physics at $t \le t_{\rm LS}$ and the same is then true for $H_0^2\Omega_{\rm m} = \kappa\rho_{\rm m}(z=0)$. Thus the analysis of the CMB should lead to a value of $H_0^2\Omega_{\rm m}$ (approximately) independent of late time cosmological models. We thus assume our value of $H_0^2\Omega_{\rm m}$ is fixed as the Planck collaboration value. If our late time cosmological model leads to a different value of H_0, this is then reflected in a different value of $\Omega_{\rm m}$.

[3] In [6] one found $\chi^2 = 4780.4$ for the ΛCDM model and $\chi^2 = 4783.5$ for the multiverse model, i.e. a $\Delta\chi^2 = 3.1$. This implies (according to standard AIC) that the probability that the multiverse model should be preferred rather than the ΛCDM model, based on the present data, is $\mathrm{e}^{-\Delta\chi^2/2} = 0.21$.

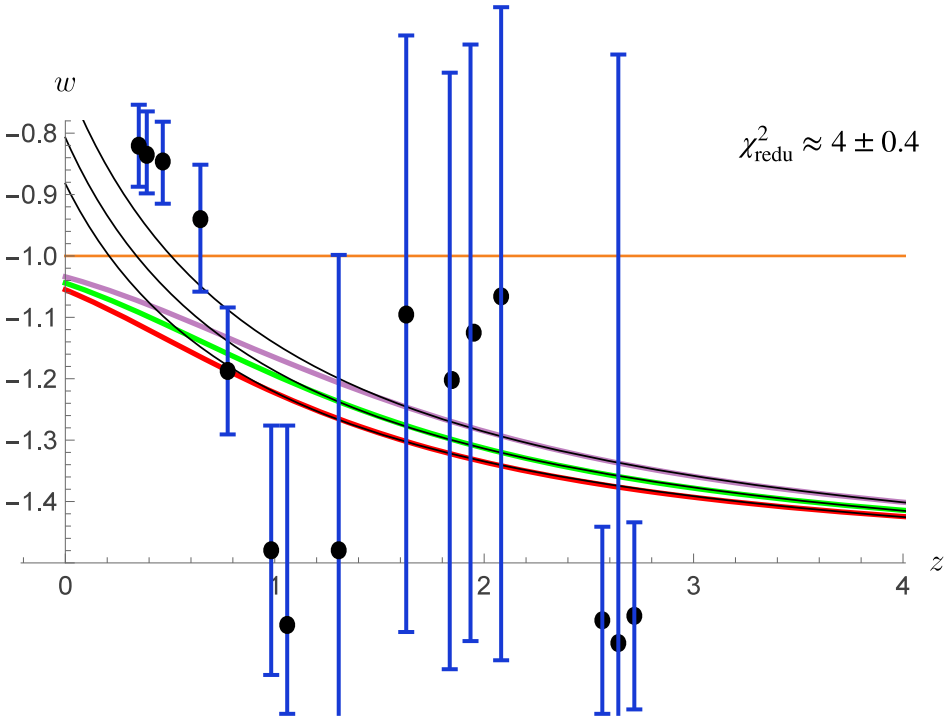


Fig. 1. The data points and error bars are imported from Lodha et al. [3], Fig. 7. The red curve is $w_{\text{f}}(z)$ for $H_0 = 72.6$ km/s/Mpc. The reduced χ^2 value comparing the curve with data results in a $\chi^2_{\text{redu}} \approx 4 \pm 0.4$. A corresponding $w(z)$ (black curve) given by (18) with $(w_0, w_a) = (-0.866, -0.695)$ is a best fit to $w_{\text{f}}(z)$ for $z > 0.8$. The green curve is $w_{\text{f}}(z)$ with the first order correction in G_{s} included in f (Eq. (A.2)), $G_{\text{s}} = -4$, and the corresponding $w(z)$ (black curve) is given by (18) with $(w_0, w_a) = (-0.789, -0.778)$. The purple curve is $w_{\text{f}}(z)$ for $G_{\text{s}} = -8$, calculated using a Padé[2/1] approximant as described in the Appendix A. The corresponding $w(z)$ (black curve) has $(w_0, w_a) = (-0.690, -0.884)$.

The DESI DR2 data and analysis [3] indicate that a z-dependent $w(z)$ in the equation of state (8) leads to a better fit to the late time data than the $w = -1$ predicted by the ΛCDM model. In addition a number of the parameterizations of $w(z)$ suggest that $w(z) < -1$ for $z > 0.5$. Values of $w(z) < -1$ have been difficult to explain in physical motivated extensions of the ΛCDM model (see [8,9] for recent discussions), and most of the parameterizations of $w(z)$ used in fits are chosen for their simplicity and are not related to any physics. Here we will just concentrate on the commonly used w_0, w_a parameterization

$$w(z) = w_0 + \frac{z}{z+1} w_a. \tag{18}$$

Fig. 1 shows $w_{\text{f}}(z)$ (the red curve), calculated using the value $H_0 = 72.6$ km/s/Mpc. In addition a best fit of the form (18) to this $w_{\text{f}}(z)$ is shown. The fit is performed for $z \in [0.8, 4]$. It is clear that the functional form (18) cannot fit $w_{\text{f}}(z)$ both for small values of $z \in [0, 0.8]$ and for $z \in [0.8, 4]$. The fit corresponds to the values $(w_0, w_a) = (-0.87, -0.70)$.

The best values of w_0, w_a extracted from the data, i.e. DESI data and other data set depend on the combination of data set. As an example of best values, the combination DESI DR1, CMB and pantheon+ data leads to Adame and et al. [7]

$$w_0 = -0.83 \pm 0.06, \qquad w_a = -0.75 \pm 0.26, \tag{19}$$

compatible with the values used in Fig. 1. However, clearly the data used in Fig. 1, which uses the Union3 data rather than pantheon+ data would lead to somewhat different (w_0, w_a) values.

From Fig. 1 we have calculated the reduced χ^2 values of our function $w_{\text{f}}(z)$ compared to the data points and error-bars and we obtain that a rough estimate leads to

$$\chi^2_{\text{redu}} \approx 4 \pm 0.4. \tag{20}$$

Thus we are far away from a perfect fit. It is clear that it is the small z data points that do not agree well with $w_{\text{f}}(z)$.

One may wonder if the $w(z)$ calculated for $H_0 = 72.6$ km/s/Mpc (the locally observed value) will change significantly if we use instead the value $H_0 = 68.72$ km/s/Mpc (the latest value published in [10] based on the ΛCDM model). However that is not the case. This is clear already from Fig. 4 in [1]. That figure shows $w(z)$ calculated for $H_0 = 73.0$ km/s/Mpc and $H_0 = 67.4$ km/s/Mpc for the gcdt-model (the model used in this article). The two curves are almost identical. For the convenience of the reader we have included a new figure (Fig. 2) that shows $w(z)$ calculated for $H_0 = 68.72$ km/s/Mpc, Again, also for the two curves where higher loop corrections are included, the differences to Fig. 1 are small, in fact so small that we preferred to use a separete figure rather than presenting the new curves on an extended Fig. 1.

One should also note that since we by assumption have $H_0^2 \Omega_{\text{m}}$ fixed (see the discussion in footnote 2), changing H_0 between $H_0 \approx 68$ km/s/Mpc and $H_0 \approx 73$ km/s/Mpc is equivalent to changing Ω_{m} approximately between 0.31 and 0.27, and all such curves of $w(z)$ lie in between the two set of curves shown in Figs. 1 and 2.

4. Inclusion of topology changes

Can one improve the agreement between $w_{\text{f}}(z)$ and the data for small z? As already remarked, our model is not unique. One can change the distribution of the spatial volume of the absorbed universes by hand. It is indeed possible to change the function $\tilde{F}(p)$ in such a way that $w_{\text{f}}(z) > -1$ for small z while still being less than -1 for larger z. In Fig. 3 we have shown the result of using

$$f(p) = \frac{3}{4}(p^2 - 2g\tilde{F}(p)) \tag{21}$$

with the following choices of $\tilde{F}(p)$

$$\tilde{F}_1(p) = \frac{1}{\sqrt{p^2 + a_1\, g^{2/3}}}, \quad \tilde{F}_2(p) = \frac{-p}{p^2 + a_2\, g^{2/3}}, \quad \tilde{F}_3(p) = \frac{-p^3}{(p^2 + a_3\, g^{2/3})^2}. \tag{22}$$

The fits have an additional parameter a_i. We have not tried to adjust the parameter to a best fit to data, but it is clear from Fig. 3 that both a_2 in $\tilde{F}_2(p)$ and a_3 in $\tilde{F}_3(p)$ can be adjust to provide a good fit. For the

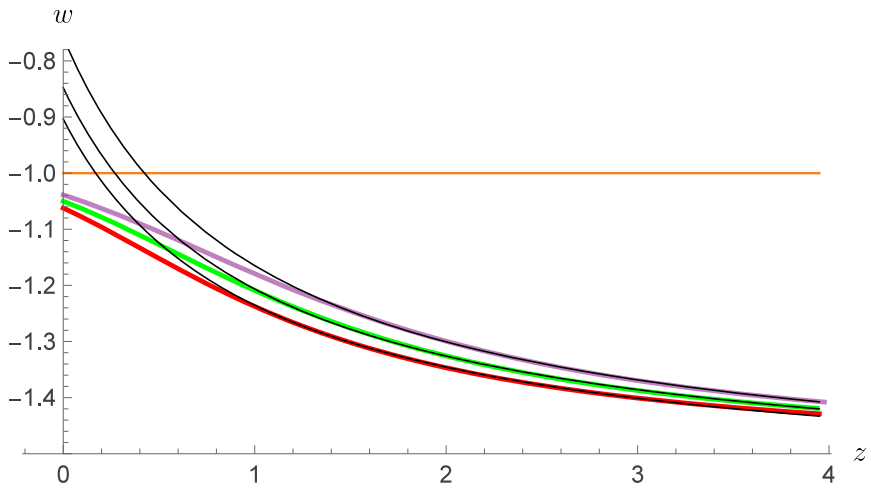


Fig. 2. The same notation as in Fig. 1. Only difference is that $H_0 = 68.72$ km/s/Mpc. The curves are almost identical to the curves in Fig. 1.

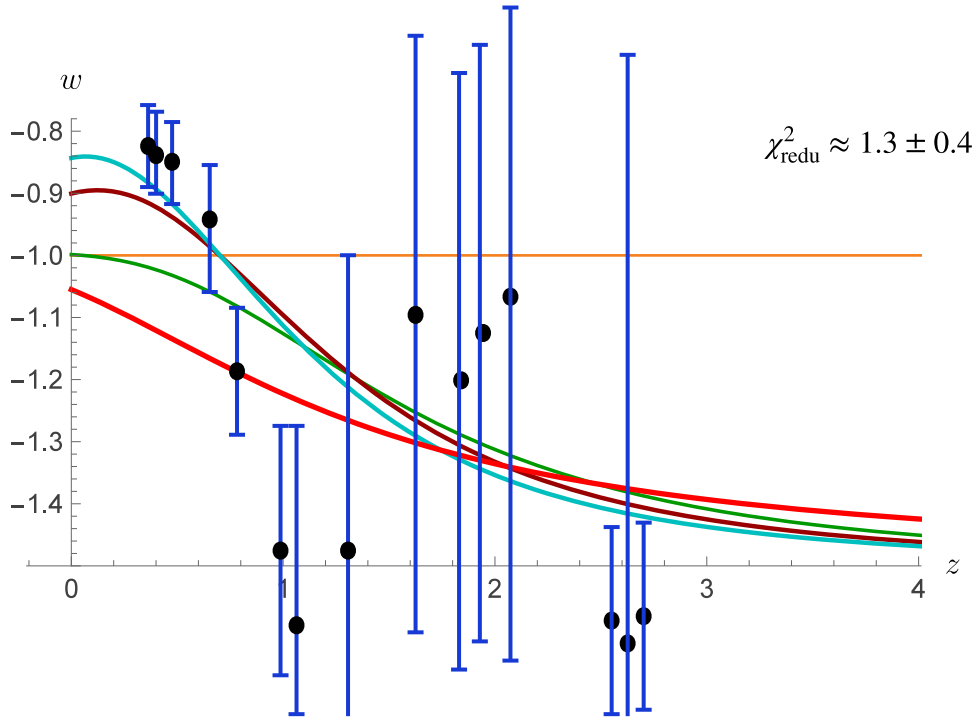


Fig. 3. As in Fig. 1 the data points are imported from Fig. 7 in [3]. The curve in red is the $w_f(z)$ curve for $f(p)$ given by (10), green curve is $\tilde{F}_1$ from Eq. (22) with $a_1 = 3$, the dark red curve $\tilde{F}_2$ with $a_2 = 1$ and the cyan curve $\tilde{F}_3$ with $a_3 = 0.4$. The χ^2_{redu} refers to the $\tilde{F}_3$ curve.

$\tilde{F}_3(p)$ used in Fig. 3 we have $\chi^2_{\text{redu}} \approx 1.3 \pm 0.4$. However, with this choice of parameters (a_2 and a_3) the Hubble parameters $H_{f_2}(z)$ and $H_{f_3}(z)$ calculated using functions f_2 and f_3 from (21) and (22) corresponding to $\tilde{F}_2$ and $\tilde{F}_3$ do not fit very well with the measured $H(z)$, as shown in Fig. 5. It might be that one can find a function $\tilde{F}$ leading to acceptable fits to both $H(z)$ and $w(z)$ data, but we have not tried to do that, the reason being that one would like some physical motivation for a given choice of $\tilde{F}$. The choice (10) was motivated by the idea that all universes, including our "own" Universe, have the same Hartle-Hawking wave function if $\rho(v) = 0$. The choice that the absorbed universes have a matter content and that the Hartle-Hawking wave functions of such universes should be identical to the Hartle-Hawking wave function of our Universe, including matter, is maybe even more "natural". Unfortunately, we do not know how to calculate the Hartle-Hawking wave function when matter is included, even in a minisuperspace approximation.

In Fig. 4 we have shown $w(z)$ calculated for the three models in (21) and (22), but using $H_0 = 68.72$ km/s/Mpc rather than the value $H_0 = 72.6$ km/s/Mpc used in Fig. 3. As in Figs. 1 and 2, this change in H_0 results in only minor changes in the $w(z)$-curves.

However, it seems natural to try to include more complicated changes of spacetime topologies in the model. Until now we have considered only the process that two universes can merge to one universe. One could include processes where the individual universes are allowed to change spacetime topologies, and one can calculate the simplest topology change corrections to the function $f(p)$ given by (10) (see [11,12]).

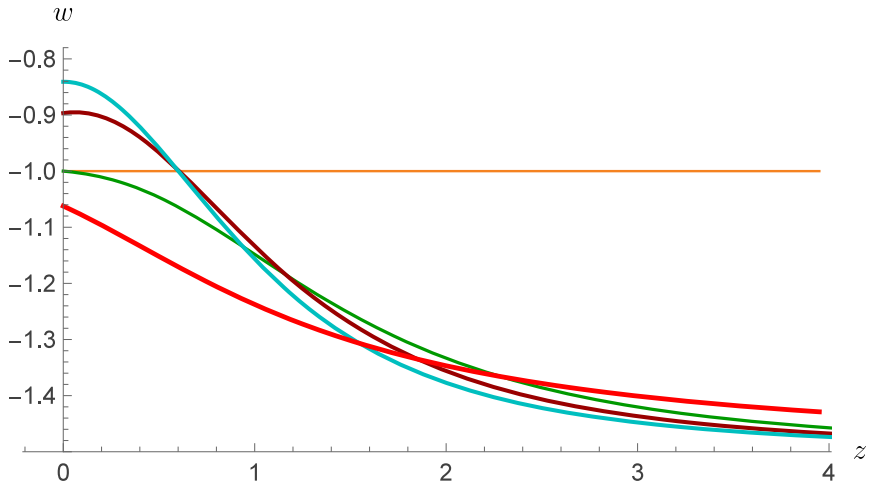


Fig. 4. The same notation as in Fig. 3. Only difference is that $H_0 = 68.72$ km/s/Mpc. The curves are almost identical to the curves in Fig. 1.

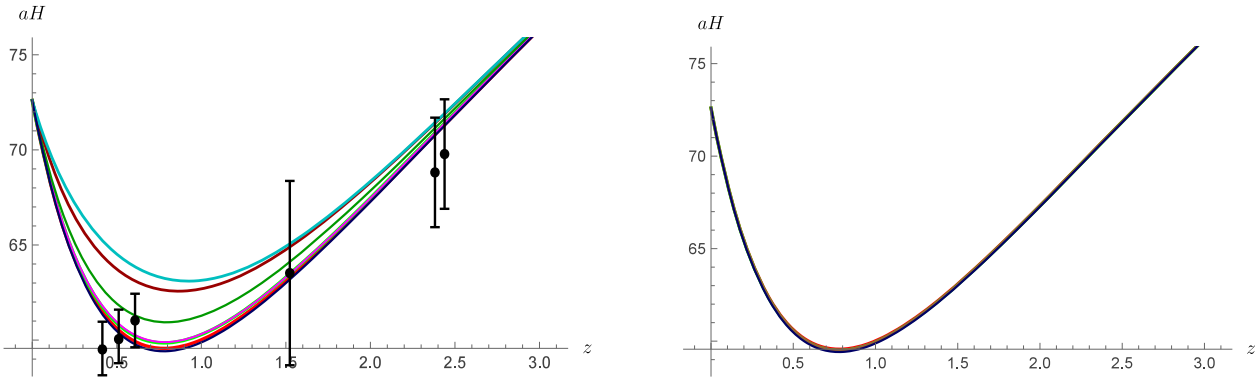


Fig. 5. The left panel shows $H(z)/(1+z)$ calculated for $f(p)$ given by (21) and (22) (the three upper curves (dark green, dark red and cyan), using $\tilde{F}_1$, $\tilde{F}_2$ and $\tilde{F}_3$ with $a_1 = 3$, $a_2 = 1$ and $a_3 = 0.4$), as well as $H(z)/(1+z)$ calculated for $f(p)$ given by (23)–(24) with $G_s = -4$ and to 0th, 1st and 2nd orders in G_s, (red, green and magenta) Also, in blue, is shown $H(z)/(1+z)$ calculated to all orders in G_s for $G_s = 1$. The $H(z)$ data the same as used in [1]. On the right panel we show $H(z)/(1+z)$ again calculated for $f(p)$ given by (23)–(24), but with $G_s = 1$. Same color labelling as for the lower 4 graphs on the left panel and in addition in brown is the calculation to 3rd order in G_s.

This introduces a new coupling constant G_s (a "string" coupling constant)

$$f(p) = -\frac{3\,s^2}{4}\Big((\hat{p}+1)\sqrt{(\hat{p}-1)^2+2} + 2s\,\tilde{F}(\hat{p}, G_s)\Big), \tag{23}$$

$$s\tilde{F}(\hat{p}, G_s) = \sum_{h=1}^{\infty} G_s^h\, s\tilde{F}^{(h)}(\hat{p}), \quad s = g^{1/3}, \quad \hat{p} = p/s. \tag{24}$$

Here h denotes the "genus" of the spacetime[4], and G_s the coupling constant associated with the change of spacetime topology from h to $h+1$. The functions $\tilde{F}^{(h)}(\hat{p})$, $h \geq 1$, fall off like $1/\hat{p}^2$ for large $|\hat{p}|$ and they are then not important for large values of z, but they could potentially change the behavior for small values of z and this is what we want. We have calculated $\tilde{F}^{(1)}$, $\tilde{F}^{(2)}$ and $\tilde{F}^{(3)}$ (see Appendix A). The expansion (24) is only an asymptotic expansion, and since we are interested in the series in the region where $|\hat{p}|$ is not much larger than 1 (small z), only the first two orders can be trusted unless we take G_s small. In Fig. 5 we show the results of calculating $H(z)$ to various orders of G_s for $G_s = -4$ and $G_s = 1$. Rather surprisingly, even for G_s as large as -4, the first and second order corrections to $H(z)$ are quite small, but one sees a clear change when one includes the third order corrections. Thus, for this value of G_s, we can only trust the expansion (24) to second order. However, one can include the third order correction by using instead the Padé[2/1] approximant. That seems to give good results even for larger values of $|G_s|$ (see the Appendix A). For $G_s = 1$ one can actually calculate $\tilde{F}(\hat{p}, G_s)$ to all orders in G_s [13] (it is the Laplace transform of the Airy function). One can then check that the Padé[2/1] approximant , using up to 3rd order in G_s works very well for this value of G_s and one can observe that for this value of G_s the function $H(z)$ is not changed much compared to $H(z)$ calculated for $G_s = 0$ (that gives a good fit to the $H(z)$ data [1,2]).

While $H(z)$ only involves the first derivative of $\tilde{F}(\hat{p}, G_s)$ wrt $\hat{p}$, $w_f(z)$ involves the second derivative, and the change as a function of G_s could be larger. However, it is not the case. The result is shown in Fig. 1 for $G_s = -4$ using the 1st order correction and for $G_s = -8$ using the Padé[2/1] approximant. As shown in Fig. A.1 in the Appendix A one can obtain a positive $w_f(z)$ for small z by taking G_s very large and negative, but such a large value is not "natural". In addition, while such values of G_s might give a better fit to the $w(z)$ data, the corresponding $H(z)$ will not give a good fit to the $H(z)$ data, i.e. we have the same situation as with the functions $\tilde{F}_2$ and $\tilde{F}_3$ above.

5. Extending the baby universe model

Until now we have considered a model where the ΛCDM model's cosmological constant λ is replaced by the coupling constant g

[4] The notion of "genus" relates to the original calculation in two-dimensional spacetime. In that case it was literally an expansion in the number of handles, h, of the two-dimensional manifold.

describing the merging of spatial universes. However, it is of course possible to enlarge the model. The simplest extension of the model is to leave both λ and g in the expression (10) for $f(p)$. This has been analyzed in [6], studying various combinations of λ and g greater than or smaller than zero. In this case it is indeed possible to find values of λ and g, such that the χ^2 value from comparing the model with data is less than the corresponding χ^2 for the best fit of the ΛCDM model. This is not surprising since already the "pure" model with $\lambda = 0$ was doing quite well (see footnote 3). However, these extended models do not really do better than the $\lambda = 0$ model when comparing the calculated $w_{\rm f}(z)$ and the $w(z)$ determined from data, and they come at the price of having two coupling constants instead of one.

Another extension of the model is to combine it with quintessence and consider adding a scalar field ϕ coupled minimally to gravity (see [14] for a review). These models in general lead to a $w(z)$ that varies with z, but it is difficult to obtain a $w(z) < -1$. Thus a combination of our model and quintessence might result in a $w(z) > -1$ for small z and $w(z) < -1$ for larger z. It is possible that one can find such an extended model that can fit all data, but the drawback is again that it involves a number of new parameters.

6. Discussion

Our late-time cosmological model has replaced the cosmological constant λ in the ΛCDM model with a new coupling constant g, and the value of g is then determined by insisting that the calculated value of the Hubble parameter $H(t)$ at the present time t_0 agrees with the locally measured value $H_0 = 72.6$ km/s/Mpc. This allows us to *calculate* $w_{\rm f}(z)$ and it has some qualitative similarity with the $w(z)$ one has extracted from the data: a $w(z)$ decreasing with increasing z and being less than -1 for sufficient large z. For $z < 0.5$ the agreement with data is not good and a simple χ^2-fit to the data leads to $\chi_{\rm red} \approx 4 \pm 0.4$. The agreement for small z could be improved a little by including spacetime topology changing corrections, but comes at the expense of the introduction of a new coupling constant. Good agreement for small z could be obtained by changing the function $\tilde{F}$ as in (21) and (22). Unfortunately we lack presently physical motivation for making such a change.

However, let us stress simplicity of the original model. It has just one coupling constant g related to late time cosmology, and it is fixed just fitting to local model-independent measurements of H_0 and T_0 (the present CMB temperature). For this value of g there is good agreement with most low z measurements[5] and it leads to an equation of state parameter $w_{\rm f}(z)$ with a z dependence sharing qualitative features with the $w(z)$ extracted from data. It explains the late time exponential expansion of the universe without the need a cosmological constant, and recent work suggests that it might be possible to explain the smallness of the coupling constant g from an underlying microscoping theory [15]. It thus seems worth to try to develop this theory further, in particular to try to understand the merging of universes in the context of ordinary GR.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

No data was used for the research described in the article.

Appendix A.

The functions $\tilde{F}^{(h)}(\hat{p})$ can be found by iteratively solving the Swinger-Dyson equations defining GCDT [11], but a better way is to use the results of topological recursion as described in [16]. At zeroth order we have ($\hat{p} \le -1$):

$$s\tilde{F}^{(0)}(\hat{p}) := \frac{1}{2}(\hat{p}+1)\sqrt{\hat{p}^2-2\hat{p}+3} + \frac{1}{2}\hat{p}^2 = -\frac{1}{\hat{p}} - \frac{3}{4}\frac{1}{\hat{p}^2} + \cdots, \tag{A.1}$$

the "genus 0" GCDT amplitude we have already used. The function $s\tilde{F}^{(1)}(\hat{p})$ can also be found in [11]:

$$s\tilde{F}^{(1)}(\hat{p}) = \frac{(-\hat{p}+3)(\hat{p}^2-2\hat{p}+9)}{72(\hat{p}^2-2\hat{p}+3)^{5/2}}, \tag{A.2}$$

when adjusted for sign changes and normalization, compared to [11]

The iteration of the Swinger-Dyson equations to next order leads to

$$s\tilde{F}^{(2)}(\hat{p}) = \frac{\left(\begin{array}{c}-17\hat{p}^9+187\hat{p}^8-1122\hat{p}^7+4522\hat{p}^6-13319\hat{p}^5+\\ 29073\hat{p}^4-45252\hat{p}^3+41040\hat{p}^2+2430\hat{p}-23814\end{array}\right)}{15552(\hat{p}^2-2\hat{p}+3)^{11/2}}. \tag{A.3}$$

The expression for $\tilde{F}^{(3)}(\hat{p})$ is quite long and is as follows

$$s\tilde{F}^{(3)}(\hat{p}) = \frac{\left(\begin{array}{c}1684\hat{p}^{15}-28628\hat{p}^{14}+257652\hat{p}^{13}-1577908\hat{p}^{12}+7282601\hat{p}^{11}\\ -26698041\hat{p}^{10}+80254609\hat{p}^9-201831221\hat{p}^8+430169949\hat{p}^7\\ -782096733\hat{p}^6+1210818861\hat{p}^5-1568258253\hat{p}^4\\ +1612434762\hat{p}^3-1150859178\hat{p}^2+424098666\hat{p}-31847094\end{array}\right)}{1679616(\hat{p}^2-2\hat{p}+3)^{17/2}}. \tag{A.4}$$

The expansion $s\tilde{F}(\hat{p}, G_{\rm s})$ in powers of $\hat{p}^{-n}$ and $G_{\rm s}^h$ is

$$\begin{aligned} s\tilde{F}(\hat{p}, G_{\rm s}) &= \sum_{h=0}^{\infty} G_{\rm s}^h s\tilde{F}^{(h)}(\hat{p}) \\ &= s\tilde{F}^{(0)}(\hat{p}) + G_{\rm s} s\tilde{F}^{(1)}(\hat{p}) + G_{\rm s}^2 s\tilde{F}^{(2)}(\hat{p}) + G_{\rm s}^3 s\tilde{F}^{(3)}(\hat{p}) + \dots \\ &= -\frac{1}{\hat{p}} - \frac{3}{4\hat{p}^2} + \frac{1}{\hat{p}^4} + \frac{3}{2\hat{p}^5} + \frac{9}{16\hat{p}^6} - \frac{2}{\hat{p}^7} + \mathcal{O}\Big(\frac{1}{\hat{p}^8}\Big) \\ &\quad - G_{\rm s}\Big(-\frac{1}{72\hat{p}^2} + \frac{1}{36\hat{p}^5} + \frac{25}{48\hat{p}^6} + \frac{2}{\hat{p}^7} + \mathcal{O}\Big(\frac{1}{\hat{p}^8}\Big)\Big) \\ &\quad - G_{\rm s}^2\Big(-\frac{17}{15552\hat{p}^2} + \frac{17}{7776\hat{p}^5} + \frac{53}{3456\hat{p}^6} + \mathcal{O}\Big(\frac{1}{\hat{p}^8}\Big)\Big) \\ &\quad - G_{\rm s}^3\Big(\frac{421}{419904\hat{p}^2} - \frac{421}{209952\hat{p}^5} - \frac{1075}{559872\hat{p}^6} + \mathcal{O}\Big(\frac{1}{\hat{p}^8}\Big)\Big) + \dots \end{aligned} \tag{A.5}$$

or, as an expansion in $\hat{p}^{-n}$:

$$\begin{aligned} s\tilde{F}(\hat{p}, G_{\rm s}) = &-\frac{1}{\hat{p}} - \Big(\frac{3}{4} - \frac{G_{\rm s}}{72} - \frac{17G_{\rm s}^2}{15552} + \frac{421G_{\rm s}^3}{419904} + \mathcal{O}(G_{\rm s}^4)\Big)\frac{1}{\hat{p}^2} + \frac{1}{\hat{p}^4} \\ &- \Big(-\frac{3}{2} + \frac{G_{\rm s}}{36} + \frac{17G_{\rm s}^2}{7776} - \frac{421G_{\rm s}^3}{209952} + \mathcal{O}(G_{\rm s}^4)\Big)\frac{1}{\hat{p}^5} \\ &+ \Big(\frac{9}{16} - \frac{25G_{\rm s}}{48} - \frac{53G_{\rm s}^2}{3456} + \frac{1075G_{\rm s}^3}{559872} + \mathcal{O}(G_{\rm s}^4)\Big)\frac{1}{\hat{p}^6} \\ &- \big(2 + 2G_{\rm s} + \mathcal{O}(G_{\rm s}^4)\big)\frac{1}{\hat{p}^7} + \mathcal{O}\Big(\frac{1}{\hat{p}^8}\Big). \end{aligned} \tag{A.6}$$

From these expansions one might think that $|G_{\rm s}| < 1$ is needed in order to trust the expansion to third order in $G_{\rm s}$. However, we find that

[5] Of course the standard ΛCDM model also has only one parameter, the cosmological constant λ, related to late time cosmology. However, fixing it simply from the local measurements of T_0 and H_0 does not lead to good agreement with other local measurements.

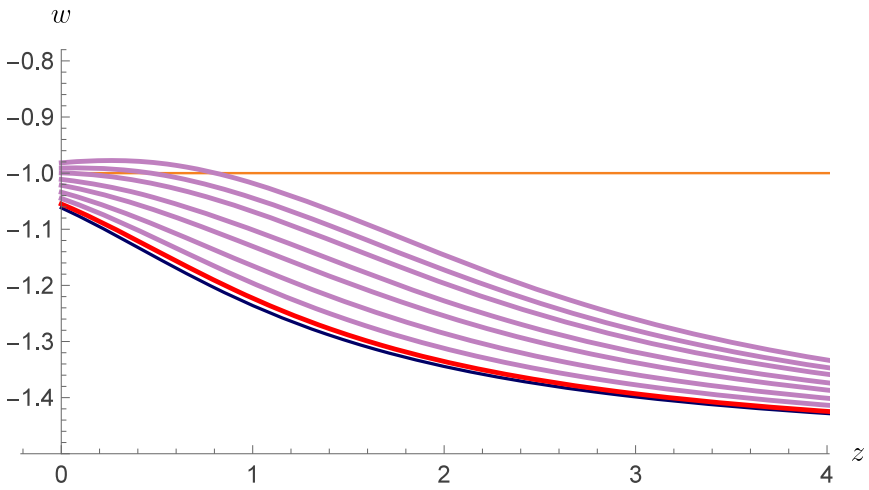


Fig. A.1. The calculated w_f for $G_s = 1, 0, -4, -8, -12, -15, -18, -20, -22$ using the Padé[2/1] approximant, the red curve being the $G_s = 0$ curve. For G_s large negative $w_f(z)$ becomes larger than -1 for small values of z.

for $|G_s| \leq 2$ there is little change in $H(z)$, $\Omega_m(z)$ and $w_f(z)$ when the higher topology corrects are included.

It is worth to note that in the case where $G_s = 1$ and one can perform the summation to all orders in G_s, one finds that there is no higher order corrections to the $\hat{p}^{-4}$ term in agreement with the expansion (A.5) to 3rd order on G_s. Also, in the exact summation when $G_s = 1$ one finds that the coefficient to the term $\hat{p}^{-7}$ is 4. This is in agreement with the expansion (A.5) and (A.6) provided all corrections of order four and higher to the $\hat{p}^{-7}$-term is zero.

For $G_s = -4$ the 3rd order correction seems unreliable, as mentioned in the main text, but using the Padé[2/1] approximant one can seemingly use much larger $|G_s|$. In Fig. A.1 we have shown calculation of $w_f(z)$ for some range of G_s using the Padé approximant.

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