Eur. Phys. J. C (2025) 85:327
https://doi.org/10.1140/epjc/s10052-025-13998-1

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Constraining string cosmology with the gravitational-wave background using the NANOGrav 15-year data set

Qin Tan[1,2,a], **You Wu**[3,b], **Lang Liu**[4,c]

[1] Department of Physics, Key Laboratory of Low Dimensional Quantum Structures and Quantum Control of Ministry of Education, Synergetic Innovation Center for Quantum Effects and Applications, Hunan Normal University, Changsha 410081, Hunan, China
[2] Institute of Interdisciplinary Studies, Hunan Normal University, Changsha, Hunan 410081, China
[3] College of Mathematics and Physics, Hunan University of Arts and Science, Changde 415000, China
[4] Faculty of Arts and Sciences, Beijing Normal University, Zhuhai 519087, China

Received: 26 December 2024 / Accepted: 21 February 2025

Abstract Multiple pulsar timing array (PTA) collaborations, including the European PTA in partnership with the Indian PTA, the North American Nanohertz Observatory for Gravitational Waves (NANOGrav), the Parkes PTA, and the Chinese PTA have recently reported strong evidence for a signal at nanohertz, potentially the first detection of the stochastic gravitational-wave background (SGWB). We investigate whether the NANOGrav signal is consistent with the SGWB predicted by string cosmology models. By performing Bayesian parameter estimation on the NANOGrav 15-year data set, we constrain the key parameters of a string cosmology model: the frequency f_s and the fractional energy density $\Omega^s_{\rm gw}$ of gravitational waves at the end of the dilaton-driven stage, and the Hubble parameter H_r at the end of the string phase. Our analysis yields constraints of $f_s = 1.2^{+0.6}_{-0.6} \times 10^{-8}$Hz and $\Omega^s_{\rm gw} = 2.9^{+5.4}_{-2.3} \times 10^{-8}$, consistent with theoretical predictions from string cosmology. However, the current NANOGrav data is not sensitive to the H_r parameter. We also compare the string cosmology model to a simple power-law model using Bayesian model selection, finding a Bayes factor of 2.2 in favor of the string cosmology model. Future pulsar timing array observations with improved sensitivity and extended frequency coverage will enable tighter constraints on string cosmology parameters.

[a] e-mail: tanqin@hunnu.edu.cn
[b] e-mail: youwuphy@gmail.com (corresponding author)
[c] e-mail: liulang@bnu.edu.cn (corresponding author)

1 Introduction

Building upon the groundbreaking detection of gravitational waves (GWs) from the coalescence of black holes and neutron stars by LIGO–Virgo–KAGRA (LVK) [1–6], the next exciting discovery may be the identification of the stochastic GW background (SGWB), which can span a wide frequency range. Pulsar timing arrays (PTAs) serve as indispensable tools for probing the nanohertz frequency band of the SGWB, offering a valuable window into the detection of GWs that originated from the early Universe.

Recently, multiple PTA collaborations, including the North American Nanohertz Observatory for GWs (NANOGrav) [7,8], the Parkes PTA (PPTA) [9,10], the European PTA (EPTA) in partnership with the Indian PTA (InPTA) [11,12], and the Chinese PTA (CPTA) [13], have independently provided strong evidence for spatial correlations that are consistent with the Hellings–Downs [14] pattern in their most recent data sets. These correlations align with the expected properties of an SGWB as predicted by the theory of general relativity. These discoveries mark a pivotal achievement in the field of GW astronomy, as they demonstrate the successful detection of GWs through the meticulous timing of exceptionally stable millisecond pulsars. For further details on PTAs, we refer readers to recent reviews [15–17].

Despite these remarkable achievements, the precise origin of the observed PTA signals remains uncertain [18,19], with hypotheses encompassing both astrophysical and cosmological sources. The diverse range of potential origins includes supermassive black hole binaries (SMBHBs) [20–24], as well as more exotic phenomena such as scalar-induced GWs [25–48], cosmic phase transitions [49–57], domain walls [58–61], cosmic strings [62–68], primordial GWs [69–73],

Published online: 21 March 2025

and modified gravities [74–81]. Another intriguing possibility is that the detected signal may be the SGWB originating from string cosmology.

The standard cosmological model [82] has achieved great success in describing the behavior of our Universe. When augmented with the inflationary epoch [83], this model offers a compelling explanation for conundrums such as the fine-tuning of initial conditions and demonstrates excellent concordance with the observed inhomogeneous structure of the Universe. However, this mechanism is not without its limitations. In most models of inflation based on a scalar field minimally coupled to gravity, the inflationary period lasts so long that the physical fluctuations corresponding to the present large-scale structure would have shrunk to scales smaller than the Planck length at the beginning of inflation. This is known as the “trans-Planckian” problem [84]. Furthermore, as we look back in time, the spacetime curvature increases, inevitably leading to an initial singularity [85,86] from the Big Bang. Additionally, our understanding of the physical nature of the inflation field remains limited due to its exotic characteristics.

Quantum effects of gravity are inescapable in the primordial Universe. Consequently, string theory may offer solutions to these challenges. The resulting string cosmology gives rise to a pre-big bang scenario [87,88] in which extra dimensions are introduced, and the small characteristic size of strings [89] circumvents the initial singularity encountered in general relativity. In this framework, the Universe can commence inflation with a large Hubble horizon, thereby resolving the trans-Planckian problem. String theory predicts the presence of a scalar dilaton field coupled to gravity, which induces an inflationary process distinct from the standard slow-roll inflation [90]. Within the framework of string cosmology, the origin of the SGWB stems from quantum fluctuations in the early Universe. Unlike standard inflationary models, string cosmology postulates a pre-big bang phase driven by the dynamics of the dilaton field and the compactification of extra dimensions. During this phase, primordial GWs are generated with distinctive spectral properties – their energy density spectrum is fundamentally altered by the accelerated contraction of the pre-big bang phase, creating a blue-tilted spectrum that contrasts with the red-tilted predictions of conventional inflation.

The gravitational sector of string cosmology contains the usual two tensor polarization modes of general relativity (plus and cross). However, the presence of extra dimensions and the dilaton field introduces additional degrees of freedom that could manifest as scalar (breathing) modes or massive tensor modes in the four-dimensional effective theory. These additional polarization states could provide a smoking gun for string-theoretic physics through future multi-messenger observations. The predicted spectral blue tilt provides a unique observational fingerprint, distinguishable from both astrophysical foregrounds (such as SMBHBs) and inflationary relics. This spectral characteristic positions SGWB observations as a critical probe of string-theoretic extensions to the Standard Model of cosmology [91–93], while potentially offering a natural explanation for the recently observed PTA signal.

In this work, we assume that the signal detected by PTAs has its origin in string cosmology and employ the PTA observations to constrain the string cosmology model. Our primary objectives are to investigate whether the PTA signal is consistent with the string cosmology model and to place constraints on the parameter space of the string cosmology model. The remainder of the paper is structured as follows. Section 2 provides an overview of the SGWB in the context of the string cosmology scenario. In Sect. 3, we outline the methodology for data analysis and present the results obtained from the NANOGrav 15-year data set. Finally, we draw conclusions in Sect. 4.

2 SGWB from string cosmology

In this section, we provide a brief overview of the SGWB in the context of the string cosmology scenario. In a typical string cosmology model, the Universe undergoes two early inflationary phases: the “dilaton-driven” stage and the “string” stage [92]. Each stage generates a SGWB, which can be characterized using the spectral function $\Omega_{\mathrm{GW}}(f)$, defined as

$$\Omega_{\mathrm{GW}}(f) = \frac{1}{\rho_{\mathrm{c}}} \frac{d\rho_{\mathrm{GW}}}{d \ln f}, \tag{1}$$

where ρ_{GW} denotes the energy density of the SGWB between frequencies f and $f+df$, and ρ_{c} is the critical energy density, given by

$$\rho_{\mathrm{c}} = \frac{3c^2 H_0^2}{8\pi G}. \tag{2}$$

According to [94,95], the spectrum of the SGWB in the string cosmology scenario can be approximated as

$$\Omega_{\mathrm{GW}}(f) = \begin{cases} \Omega_{\mathrm{gw}}^{s}(f/f_s)^3, & f < f_s, \\ \Omega_{\mathrm{gw}}^{s}(f/f_s)^{\beta}, & f_s < f < f_1, \\ 0, & f_1 < f, \end{cases} \tag{3}$$

where

$$\beta = \frac{\log\left(\Omega_{\mathrm{gw}}^{\max}/\Omega_{\mathrm{gw}}^{s}\right)}{\log\left(f_1/f_s\right)} \tag{4}$$

is the logarithmic slope of the spectrum of the SGWB produced during the string epoch. The spectrum depends on four parameters: the frequency f_s, the fractional energy density Ω_{gw}^{s} generated at the end of the dilaton-drive stage, the maximum frequency f_1 (above which no gravitational radiation is generated), and the maximum fractional energy density $\Omega_{\rm gw}^{\rm max}$ occurring at the maximum frequency f_1. Assuming no late entropy generation and a reasonable choice about the number of effective degrees of freedom, we can express f_s and $\Omega_{\rm gw}^{\rm max}$ in terms of the Hubble parameter H_r at the end of the string phase as [96]

$$f_1 = 1.3 \times 10^{10} \left(\frac{H_r}{5 \times 10^{17}\,{\rm GeV}} \right)^{1/2} \,{\rm Hz}, \tag{5}$$

and

$$\Omega_{\rm gw}^{\rm max} = 1 \times 10^{-7} h_{100}^{-2} \left(\frac{H_r}{5 \times 10^{17}\,{\rm GeV}} \right)^2, \tag{6}$$

where $h_{100} = 0.674$ [97] is the reduced Hubble constant. The spectrum is now determined by only three parameters, f_s, $\Omega_{\rm gw}^s$, and H_r, which are related to the fundamental parameters of the string cosmology model. Although some studies [98,99] consider more complex models and their resulting spectra, the results are similar to the spectrum discussed here and equally dependent on the same parameters. Therefore, the model we use captures the main features of string cosmology. In the next section, we will use data from the NANOGrav to estimate the parameters of this string cosmology model. By employing Bayesian parameter estimation techniques, we aim to provide insights into the search for observational signatures of string cosmology.

3 Data analysis and results

In this work, we utilize the NANOGrav 15-year data set [8] to estimate the parameters of the string cosmology model. The NANOGrav 15-year data set includes observations of 67 millisecond pulsars with a timing baseline $\geq$ 3 years, spanning a total time span of approximately 16.03 years [7, 8]. This extensive data set provides a unique opportunity to search for the SGWB signal and constrain cosmological models, such as string cosmology.

We specifically employ the free spectrum amplitudes obtained by the NANOGrav 15-year data set when considering spatial correlations of the Hellings–Downs pattern [14]. The Hellings–Downs correlation pattern is a distinctive signature of the SGWB, arising from the quadrupolar nature of GWs. By incorporating this pattern into our analysis, we can effectively distinguish the SGWB signal from other noise sources, such as intrinsic pulsar noise or Earth-term errors [100].

The sensitivity of a PTA's observations to the SGWB begins at a frequency of $1/T_{\rm obs}$, where $T_{\rm obs} = 16.03\,{\rm yr}$ is the observational time span. NANOGrav employs 14 frequency bins [7] in their search for the SGWB signal, covering a frequency range from 2.0×10^{-9} Hz to 2.8×10^{-8} Hz. This frequency range is particularly relevant for detecting the SGWB signal predicted by string cosmology models, as the signal is expected to have a significant contribution at frequencies around 10^{-8} Hz. In Fig. 2, we illustrate the data employed in our analyses and depict the energy density.

In our analysis, we start by utilizing the posterior data of the time delay $d(f)$ obtained from the NANOGrav 15-year data set. The time delay is related to the power spectrum $S(f)$ through the following relation:

$$S(f) = d(f)^2 T_{\rm obs}, \tag{7}$$

where $T_{\rm obs}$ represents the total observational time span. This relation allows us to calculate the power spectrum from the time delay data, which is a crucial step in determining the energy density of the SGWB.

Using the power spectrum, we can then compute the energy density of the free spectrum, $\hat{\Omega}_{\rm GW}(f)$, as

$$\hat{\Omega}_{\rm GW}(f) = \frac{2\pi^2}{3H_0^2} f^2 h_c^2(f) = \frac{8\pi^4}{H_0^2} T_{\rm obs} f^5 d^2(f), \tag{8}$$

where $H_0 = 67.4\,{\rm km\,s^{-1}\,Mpc^{-1}}$ is the Hubble constant determined by the Planck collaboration [97]. Here, the characteristic strain, $h_c(f)$, is defined as

$$h_c^2(f) = 12\pi^2 f^3 S(f). \tag{9}$$

By combining the power spectrum and the characteristic strain, we can fully characterize the SGWB and its energy density.

For each of the 14 observed frequencies f_i in the NANOGrav data set, we estimate the corresponding kernel density, $\mathcal{L}_i$, using the obtained posteriors of $\hat{\Omega}_{\rm GW}(f_i)$. The total log-likelihood is then calculated as the sum of the individual log-likelihoods for each frequency, as [27,31,32,101–103]

$$\ln \mathcal{L}(\Lambda) = \sum_{i=1}^{14} \ln \mathcal{L}_i(\Omega_{\rm GW}(f_i, \Lambda)), \tag{10}$$

where $\Lambda = \{f_s, \Omega_{\rm gw}^s, H_r\}$ represents the set of three model parameters that we aim to constrain using the NANOGrav data. These parameters are the frequency f_s and the fractional energy density $\Omega_{\rm gw}^s$ of GWs generated at the end of the dilaton-driven stage, and the Hubble parameter H_r at the end of the string phase.

To efficiently explore the parameter space and obtain posterior distributions for the model parameters, we employ the `dynesty` [104] sampler, which is a nested sampling algorithm implemented in the `Bilby` [105,106] package. Nested sampling is a powerful technique for Bayesian inference, as it allows for the computation of the evidence (marginal likelihood) and the posterior distributions simultaneously. The priors and results for the model parameters are summarized in Table 1, providing an overview of the constraints obtained from our analysis of the NANOGrav 15-year data set.

Table 1 Prior distributions, parameter constraints, and Bayesian evidence ($\mathcal{Z}$) comparison for string cosmology (SC) and power-law (PL) models using NANOGrav 15-year data set. All parameters are sampled from log-uniform priors over the specified ranges, except for $\gamma_{\rm PL}$ which uses a uniform prior. Posterior estimates are reported as median values with 90% equal-tail credible intervals. The Hubble parameter at the end of the string phase, H_r, remains unconstrained by the current data. The Bayes factor comparing string cosmology to power-law models is $\mathcal{B}_{\rm PL}^{\rm SC} = \exp[-27.985 - (-28.785)] \simeq 2.2$, indicating a slight preference for the string cosmology model over the power-law scenario

Parameter	Prior	String cosmology	Power-law
f_s [Hz]	LogUniform[10^{-10}, 10^{-5}]	$1.2^{+0.6}_{-0.6} \times 10^{-8}$	–
$\Omega^s_{\rm gw}$	LogUniform[10^{-8}, 10^{-7}]	$2.9^{+5.4}_{-2.3} \times 10^{-8}$	–
H_r [GeV]	LogUniform[10^{12}, 10^{19}]	Unconstrained	–
$A_{\rm PL}$	LogUniform[10^{-18}, 10^{-11}]	–	$7.1^{+4.8}_{-3.2} \times 10^{-15}$
$\gamma_{\rm PL}$	Uniform[0, 8]	–	$3.1^{+0.7}_{-0.6}$
ln $\mathcal{Z}$	–	− 27.985	− 28.785

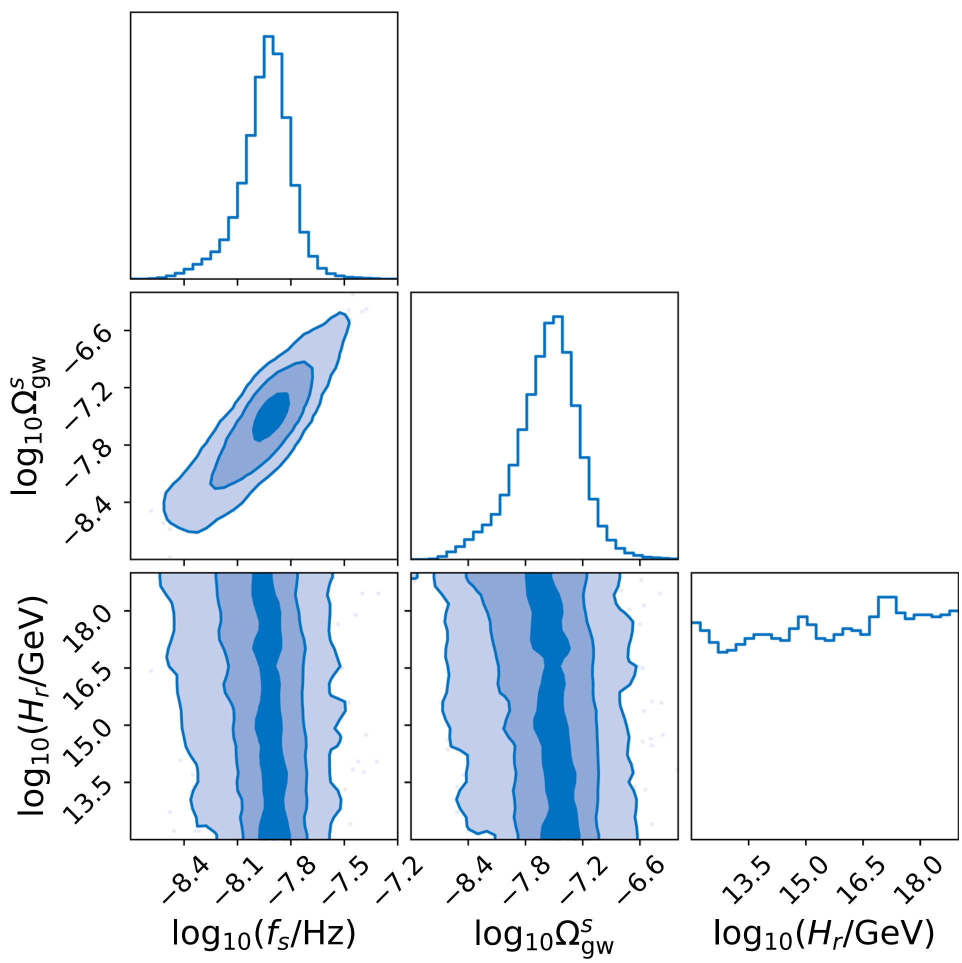

Fig. 1 Marginal posterior distributions for the string cosmology model parameters, $\Lambda = \{f_s, \Omega^s_{\rm gw}, H_r\}$, derived from the NANOGrav 15-year data set. The one-dimensional histograms show the marginalized posteriors for each parameter, while the two-dimensional plots display the joint posterior distributions, with contours delineating the 1σ, 2σ, and 3σ credible regions

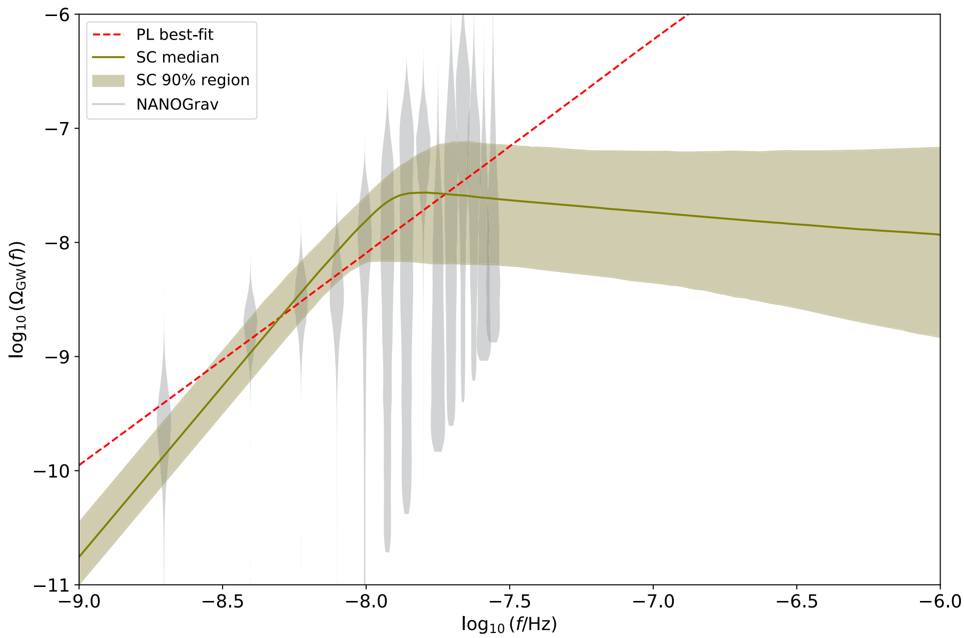

Fig. 2 PPD for the energy density spectrum of the SGWB from string cosmology (SC). The gray violins represent the free spectra obtained from the NANOGrav 15-year data set, while the olive shaded region indicates the 90% credible interval of the posterior distribution. The PPD showcases the compatibility of the string cosmology model with the observed data, highlighting the constraining power of the NANOGrav measurements on the model parameters. We also show the best-fit power-law (PL) model in red dashed line

We present the posterior distributions for the model parameters in Fig. 1. Our analysis reveals that to explain the PTA signal detected by the NANOGrav 15-year data set, the parameters should satisfy $f_s = 1.2^{+0.6}_{-0.6} \times 10^{-8}$Hz, $\Omega^s_{\rm gw} = 2.9^{+5.4}_{-2.3} \times 10^{-8}$, and H_r shows no constraints, following the prior distribution. This indicates that the current data is not sensitive to the H_r parameter. The posterior predictive distribution (PPD) of our model is shown in Fig. 2, demonstrating the agreement between our model and the observed NANOGrav data. Specifically, the PPD is consistent with the free spectrum amplitudes obtained by NANOGrav, indicating that our string cosmology model provides a good fit to the data.

To assess the relative performance of our string cosmology model compared to alternative explanations for the NANOGrav signal, we have also calculated the Bayes factor between the string cosmology model and a simple power-law model, which is commonly associated with the SGWB from SMBHBs [107,108]:

$$\Omega_{\rm GW}(f) = \frac{2\pi^2 A_{\rm PL}^2}{3H_0^2}\left(\frac{f}{f_{\rm yr}}\right)^{5-\gamma_{\rm PL}} f_{\rm yr}^2, \qquad (11)$$

where $A_{\rm PL}$ is the amplitude of the characteristic strain measured at the reference frequency $f_{\rm yr} \equiv 1\,{\rm yr}^{-1}$. The Bayes factor is defined as the ratio of the marginal likelihoods of two competing models, and it quantifies the relative support for each model given the observed data [109]. Our calculation yields a Bayes factor of 2.2 in favor of the string cosmology model over the power-law model. This suggests that the string cosmology model, which is based on string theory, provides a slightly better fit to the NANOGrav data than the SMBHB model. However, it is important to note that a Bayes factor of 2.2 is considered to be only weak evidence in favor of the string cosmology model [109], and further data from future PTA observations will be necessary to confirm or refute this preference.

4 Conclusion

String cosmology models predict a blue-tilted primordial GW background spectrum with a characteristic “knee” feature at a specific frequency f_s, corresponding to the end of the dilaton-driven inflationary stage, which distinguishes it from the red-tilted or nearly scale-invariant spectra expected from standard slow-roll inflation, and the detection of this increasing GW energy density as a function of frequency and the spectral knee feature would provide a promising smoking-gun observation favoring string cosmology over standard inflation.

While string cosmology introduces unique features in the early Universe, particularly during the inflationary epoch, it is expected to converge with standard cosmology at later times, resulting in a consistent picture of the Universe’s expansion, large-scale structure formation, and evolution, as the blue-tilted primordial GW background predicted by string cosmology does not significantly influence these processes, and the existence of black holes, formed through astrophysical processes on smaller scales, is not precluded by string cosmology and has been independently confirmed by GW observations.

In this work, we have utilized the NANOGrav 15-year data set to constrain the parameters of a string cosmology model for the SGWB. Our analysis focused on three key parameters: the frequency f_s and the fractional energy density $\Omega^s_{\rm gw}$ of

GWs generated at the end of the dilaton-driven stage, and the Hubble parameter H_r at the end of the string phase.

By employing Bayesian parameter estimation techniques, we have obtained constraints on f_s and $\Omega^s_{\rm gw}$ that are consistent with theoretical predictions from string cosmology. Specifically, we find $f_s = 1.2^{+0.6}_{-0.6} \times 10^{-8}$Hz and $\Omega^s_{\rm gw} = 2.9^{+5.4}_{-2.3} \times 10^{-8}$. These results demonstrate the ability of PTAs to probe the early Universe and constrain cosmological models. To assess the relative performance of our string cosmology model compared to alternative explanations for the NANOGrav signal, we have calculated the Bayes factor between the string cosmology model and a simple power-law model, which is commonly associated with the SGWB from SMBHBs. The Bayes factor of 2.2 in favor of the string cosmology model suggests that string theory-based models may provide a better explanation for the NANOGrav signal than SMBHBs. However, this evidence is considered weak, and further data from future PTA observations will be necessary to confirm or refute this preference conclusively.

Looking ahead, the increasing precision of PTA measurements will allow for more stringent tests of string cosmology scenarios. The constraints obtained on the string cosmology parameters f_s and $\Omega^s_{\rm gw}$ represent a significant step forward in the search for observational signatures of string cosmology. As PTAs continue to improve their sensitivities and gather more data, we can expect even more stringent constraints on these parameters and a deeper understanding of the nature of the SGWB and the early Universe.

Acknowledgements QT is supported by the National Natural Science Foundation of China (Grants no. 12405055 and no. 12347111), the China Postdoctoral Science Foundation (Grant no. 2023M741148), the Postdoctoral Fellowship Program of CPSF (Grant no. GZC20240458), and the innovative research group of Hunan Province under Grant no. 2024JJ1006. YW is supported by the National Natural Science Foundation of China under Grant no. 12405057. LL is supported by the National Natural Science Foundation of China Grant under Grant no. 12433001.

Data Availability Statement Data will be made available on reasonable request. [Author's comment: The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request].

Code Availability Statement Code/software will be made available on reasonable request. [Author's comment: The code/software generated during and/or analysed during the current study is available from the corresponding author on reasonable request].

References

1. B.P. Abbott et al. (LIGO Scientific, Virgo), Observation of gravitational waves from a binary black hole merger. Phys. Rev. Lett. **116**, 061102 (2016). https://doi.org/10.1103/PhysRevLett.116.061102. arXiv:1602.03837 [gr-qc]
2. B.P. Abbott et al. (LIGO Scientific, Virgo), GW170817: observation of gravitational waves from a binary neutron star Inspiral. Phys. Rev. Lett. **119**, 161101 (2017). https://doi.org/10.1103/PhysRevLett.119.161101. arXiv:1710.05832 [gr-qc]
3. B.P. Abbott et al. (LIGO Scientific, Virgo), GWTC-1: a gravitational-wave transient catalog of compact binary mergers observed by LIGO and Virgo during the first and second observing runs. Phys. Rev. X **9**, 031040 (2019). https://doi.org/10.1103/PhysRevX.9.031040. arXiv:1811.12907 [astro-ph.HE]
4. R. Abbott et al. (LIGO Scientific, Virgo), GWTC-2: compact binary coalescences observed by ligo and virgo during the first half of the third observing run. Phys. Rev. X **11**, 021053 (2021). https://doi.org/10.1103/PhysRevX.11.021053. arXiv:2010.14527 [gr-qc]
5. R. Abbott et al. (LIGO Scientific, VIRGO), GWTC-2.1: deep extended catalog of compact binary coalescences observed by LIGO and Virgo during the first half of the third observing run. Phys. Rev. D **109**, 022001 (2024), https://doi.org/10.1103/PhysRevD.109.022001arXiv:2108.01045 [gr-qc]
6. R. Abbott et al. (KAGRA, VIRGO, LIGO Scientific), GWTC-3: compact binary coalescences observed by LIGO and Virgo during the second part of the third observing run. Phys. Rev. X **13**, 041039 (2023). https://doi.org/10.1103/PhysRevX.13.041039. arXiv:2111.03606 [gr-qc]
7. G. Agazie et al. (NANOGrav), xThe NANOGrav 15 yr data set: evidence for a gravitational-wave background. Astrophys. J. Lett. **951**, L8 (2023). https://doi.org/10.3847/2041-8213/acdac6 arXiv:2306.16213 [astro-ph.HE] https://doi.org/10.1103/PhysRevX.13.041039
8. G. Agazie et al. (NANOGrav), The NANOGrav 15 yr data set: observations and timing of 68 millisecond pulsars. Astrophys. J. Lett. **951**, L9 (2023). https://doi.org/10.3847/2041-8213/acda9aarXiv:2306.16217 [astro-ph.HE]
9. A. Zic et al., The parkes pulsar timing array third data release. Publ. Astron. Soc. Austral. **40**, e049 (2023). https://doi.org/10.1017/pasa.2023.36. arXiv:2306.16230 [astro-ph.HE]
10. D.J. Reardon et al., Search for an isotropic gravitational-wave background with the parkes pulsar timing array. Astrophys. J. Lett. **951**, L6 (2023). https://doi.org/10.3847/2041-8213/acdd02. arXiv:2306.16215 [astro-ph.HE]
11. J. Antoniadis et al. (EPTA), The second data release from the European pulsar timing array-I. The dataset and timing analysis. Astron. Astrophys. **678**, A48 (2023). https://doi.org/10.1051/0004-6361/202346841 arXiv:2306.16224 [astro-ph.HE]
12. J. Antoniadis et al. (EPTA, InPTA:), The second data release from the European pulsar timing array—III. Search for gravitational wave signals. Astron. Astrophys. **678**, A50 (2023). https://doi.org/10.1051/0004-6361/202346844. arXiv:2306.16214 [astro-ph.HE]
13. X. Heng et al., Searching for the nano-hertz stochastic gravitational wave background with the Chinese pulsar timing array data release I. Res. Astron. Astrophys. **23**, 075024 (2023). https://

doi.org/10.1088/1674-4527/acdfa5. arXiv:2306.16216 [astro-ph.HE]
14. R.W. Hellings, G.S. Downs, Upper limits on the isotropic gravitational radiation background from pulsar timing analysis. Astrophys. J. Lett. **265**, L39–L42 (1983). https://doi.org/10.1086/183954
15. V. Domcke, Discovery opportunities with gravitational waves—TASI 2024 lecture notes. (2024). arXiv:2409.08956 [astro-ph.CO]
16. R.C. Bernardo, K.-W. Ng, Charting the nanohertz gravitational wave sky with pulsar timing arrays. (2024). arXiv:2409.07955 [astro-ph.CO]
17. J.P.W. Verbiest, S.J. Vigeland, N.K. Porayko, S. Chen, D.J. Reardon, Status report on global pulsar-timing-array efforts to detect gravitational waves. Results Phys. **61**, 107719 (2024). https://doi.org/10.1016/j.rinp.2024.107719. arXiv:2404.19529 [astro-ph.HE]
18. A. Afzal et al. (NANOGrav), The NANOGrav 15 yr data set: search for signals from new physics. Astrophys. J. Lett. **951**, L11 (2023) [note Erratum: Astrophys. J. Lett. 971, L27 (2024), Erratum: Astrophys.J. 971, L27 (2024)]. https://doi.org/10.3847/2041-8213/acdc91. arXiv:2306.16219 [astro-ph.HE]
19. J. Antoniadis et al. (EPTA, InPTA), The second data release from the European pulsar timing array—IV. Implications for massive black holes, dark matter, and the early Universe. Astron. Astrophys. **685**, A94 (2024). https://doi.org/10.1051/0004-6361/202347433. arXiv:2306.16227 [astro-ph.CO]
20. G. Agazie et al. (NANOGrav), The NANOGrav 15 yr data set: constraints on supermassive black hole binaries from the gravitational-wave background. Astrophys. J. Lett. **952**, L37 (2023). https://doi.org/10.3847/2041-8213/ace18b. arXiv:2306.16220 [astro-ph.HE]
21. J. Ellis, M. Fairbairn, G. Hütsi, J. Raidal, J. Urrutia, V. Vaskonen, H. Veermäe, Gravitational waves from supermassive black hole binaries in light of the NANOGrav 15-year data. Phys. Rev. D **109**, L021302 (2024). https://doi.org/10.1103/PhysRevD.109.L021302. arXiv:2306.17021 [astro-ph.CO]
22. Z.-Q. Shen, G.-W. Yuan, Y.-Y. Wang, Y.-Z. Wang, Dark matter spike surrounding supermassive black holes binary and the nanohertz stochastic gravitational wave background. (2023). arXiv:2306.17143 [astro-ph.HE]
23. Y.-C. Bi, W. Yu-Mei, Z.-C. Chen, Q.-G. Huang, Implications for the supermassive black hole binaries from the NANOGrav 15-year data set. Sci. China Phys. Mech. Astron. **66**, 120402 (2023). https://doi.org/10.1007/s11433-023-2252-4. arXiv:2307.00722 [astro-ph.CO]
24. E. Barausse, K. Dey, M. Crisostomi, A. Panayada, S. Marsat, S. Basak, Implications of the pulsar timing array detections for massive black hole mergers in the LISA band. Phys. Rev. D **108**, 103034 (2023). https://doi.org/10.1103/PhysRevD.108.103034. arXiv:2307.12245 [astro-ph.GA]
25. K. Inomata, K. Kohri, T. Terada, Detected stochastic gravitational waves and subsolar-mass primordial black holes. Phys. Rev. D **109**, 063506 (2024). https://doi.org/10.1103/PhysRevD.109.063506. arXiv:2306.17834 [astro-ph.CO]
26. Z.-C. Chen, C. Yuan, Q.-G. Huang, Pulsar timing array constraints on primordial black holes with NANOGrav 11-year dataset. Phys. Rev. Lett. **124**, 251101 (2020). https://doi.org/10.1103/PhysRevLett.124.251101. arXiv:1910.12239 [astro-ph.CO]
27. L. Liu, Z.-C. Chen, Q.-G. Huang, Implications for the non-Gaussianity of curvature perturbation from pulsar timing arrays. Phys. Rev. D **109**, L061301 (2024). https://doi.org/10.1103/PhysRevD.109.L061301. arXiv:2307.01102 [astro-ph.CO]
28. G. Franciolini, A. Iovino Jr., V. Vaskonen, H. Veermae, Recent gravitational wave observation by pulsar timing arrays and primordial black holes: the importance of non-Gaussianities. Phys. Rev. Lett. **131**, 201401 (2023). https://doi.org/10.1103/PhysRevLett.131.201401. arXiv:2306.17149 [astro-ph.CO]
29. S.A.H. Mansoori, F. Felegray, A. Talebian, M. Sami, PBHs and GWs from $\mathbb{T}^2$-inflation and NANOGrav 15-year data. JCAP **08**, 067 (2023). https://doi.org/10.1088/1475-7516/2023/08/067. arXiv:2307.06757 [astro-ph.CO]
30. S. Wang, Z.-C. Zhao, J.-P. Li, Q.-H. Zhu, Implications of pulsar timing array data for scalar-induced gravitational waves and primordial black holes: primordial non-Gaussianity fNL considered. Phys. Rev. Res. **6**, L012060 (2024). https://doi.org/10.1103/PhysRevResearch.6.L012060. arXiv:2307.00572 [astro-ph.CO]
31. J.-H. Jin, Z.-C. Chen, Z. Yi, Z.-Q. You, L. Liu, W. You, Confronting sound speed resonance with pulsar timing arrays. JCAP **09**, 016 (2023). https://doi.org/10.1088/1475-7516/2023/09/016. arXiv:2307.08687 [astro-ph.CO]
32. L. Liu, Z.-C. Chen, Q.-G. Huang, Probing the equation of state of the early Universe with pulsar timing arrays. JCAP **11**, 071 (2023). https://doi.org/10.1088/1475-7516/2023/11/071. arXiv:2307.14911 [astro-ph.CO]
33. Q.-H. Zhu, Z.-C. Zhao, S. Wang, X. Zhang, Unraveling the early universe's equation of state and primordial black hole production with PTA, BBN, and CMB observations. (2023). https://doi.org/10.1088/1674-1137/ad79d5. arXiv:2307.13574 [astro-ph.CO]
34. Z. Yi, Z.-Q. You, W. You, Z.-C. Chen, L. Liu, Exploring the NANOGrav signal and planet-mass primordial black holes through Higgs inflation. JCAP **06**, 043 (2024). https://doi.org/10.1088/1475-7516/2024/06/043. arXiv:2308.14688 [astro-ph.CO]
35. K. Harigaya, K. Inomata, T. Terada, Induced gravitational waves with kination era for recent pulsar timing array signals. Phys. Rev. D **108**, 123538 (2023). https://doi.org/10.1103/PhysRevD.108.123538. arXiv:2309.00228 [astro-ph.CO]
36. S. Balaji, G. Domènech, G. Franciolini, Scalar-induced gravitational wave interpretation of PTA data: the role of scalar fluctuation propagation speed. JCAP **10**, 041 (2023). https://doi.org/10.1088/1475-7516/2023/10/041. arXiv:2307.08552 [gr-qc]
37. Z. Yi, Z.-Q. You, W. You, Model-independent reconstruction of the primordial curvature power spectrum from PTA data. JCAP **01**, 066 (2024). https://doi.org/10.1088/1475-7516/2024/01/066. arXiv:2308.05632 [astro-ph.CO]
38. Z.-Q. You, Z. Yi, W. You, Constraints on primordial curvature power spectrum with pulsar timing arrays. JCAP **11**, 065 (2023). https://doi.org/10.1088/1475-7516/2023/11/065. arXiv:2307.04419 [gr-qc]
39. L. Liu, W. You, Z.-C. Chen, Simultaneously probing the sound speed and equation of state of the early Universe with pulsar timing arrays. JCAP **04**, 011 (2024). https://doi.org/10.1088/1475-7516/2024/04/011. arXiv:2310.16500 [astro-ph.CO]
40. S. Choudhury, K. Dey, A. Karde, S. Panda, M. Sami, Primordial non-Gaussianity as a saviour for PBH overproduction in SIGWs generated by pulsar timing arrays for Galileon inflation. Phys. Lett. B **856**, 138925 (2024). https://doi.org/10.1016/j.physletb.2024.138925. arXiv:2310.11034 [astro-ph.CO]
41. S. Choudhury, K. Dey, A. Karde, Untangling PBH overproduction in w-SIGWs generated by pulsar timing arrays for MST-EFT of single field inflation. (2023). arXiv:2311.15065 [astro-ph.CO]
42. G. Domènech, S. Pi, A. Wang, J. Wang, Induced gravitational wave interpretation of PTA data: a complete study for general equation of state. JCAP **08**, 054 (2024). https://doi.org/10.1088/1475-7516/2024/08/054. arXiv:2402.18965 [astro-ph.CO]
43. Z.-C. Chen, L. Liu, Can we distinguish the adiabatic fluctuations and isocurvature fluctuations with pulsar timing arrays? (2024). arXiv:2402.16781 [astro-ph.CO]
44. S. Choudhury, S. Ganguly, S. Panda, S. SenGupta, P. Tiwari, Obviating PBH overproduction for SIGWs generated by pulsar timing arrays in loop corrected EFT of bounce. JCAP

09, 013 (2024). https://doi.org/10.1088/1475-7516/2024/09/013. arXiv:2407.18976 [astro-ph.CO]
45. S. Choudhury, M. Sami, Large fluctuations and primordial black holes. (2024). arXiv:2407.17006 [gr-qc]
46. Z.-C. Chen, J. Li, L. Liu, Z. Yi, Probing the speed of scalar-induced gravitational waves with pulsar timing arrays. Phys. Rev. D **109**, L101302 (2024). https://doi.org/10.1103/PhysRevD.109.L101302. arXiv:2401.09818 [gr-qc]
47. S. Choudhury, K. Dey, S. Ganguly, A. Karde, S.K. Singh, P. Tiwari, Negative non-Gaussianity as a salvager for PBHs with PTAs in bounce. (2024). arXiv:2409.18983 [astro-ph.CO]
48. C. Han et al., Constraining inflation with nonminimal derivative coupling with the Parkes Pulsar Timing Array third data release. (2024). arXiv:2412.09755 [gr-qc]
49. A. Addazi, Y.-F. Cai, A. Marciano, L. Visinelli, Have pulsar timing array methods detected a cosmological phase transition? Phys. Rev. D **109**, 015028 (2024). https://doi.org/10.1103/PhysRevD.109.015028. arXiv:2306.17205 [astro-ph.CO]
50. P. Athron, A. Fowlie, L. Chih-Ting, L. Morris, W. Lei, W. Yongcheng, X. Zhongxiu, Can supercooled phase transitions explain the gravitational wave background observed by pulsar timing arrays? Phys. Rev. Lett. **132**, 221001 (2024). https://doi.org/10.1103/PhysRevLett.132.221001. arXiv:2306.17239 [hep-ph]
51. Z. Lei, C. Zhang, Y.-Y. Li, G. Yuchao, Y.-L.S. Tsai, Y.-Z. Fan, Mirror QCD phase transition as the origin of the nanohertz stochastic gravitational-wave background. Sci. Bull. **69**, 741–746 (2024). https://doi.org/10.1016/j.scib.2024.01.037. arXiv:2306.16769 [astro-ph.HE]
52. S. Jiang, A. Yang, J. Ma, F.P. Huang, Implication of nano-Hertz stochastic gravitational wave on dynamical dark matter through a dark first-order phase transition. Class. Quantum Gravity **41**, 065009 (2024). https://doi.org/10.1088/1361-6382/ad24c6. arXiv:2306.17827 [hep-ph]
53. Y. Xiao, J.M. Yang, Y. Zhang, Implications of nano-Hertz gravitational waves on electroweak phase transition in the singlet dark matter model. Sci. Bull. **68**, 3158–3164 (2023). https://doi.org/10.1016/j.scib.2023.11.025. arXiv:2307.01072 [hep-ph]
54. K.T. Abe, Y. Tada, Translating nano-Hertz gravitational wave background into primordial perturbations taking account of the cosmological QCD phase transition. Phys. Rev. D **108**, L101304 (2023). https://doi.org/10.1103/PhysRevD.108.L101304. arXiv:2307.01653 [astro-ph.CO]
55. Y. Gouttenoire, First-order phase transition interpretation of pulsar timing array signal is consistent with solar-mass black holes. Phys. Rev. Lett. **131**, 171404 (2023). https://doi.org/10.1103/PhysRevLett.131.171404. arXiv:2307.04239 [hep-ph]
56. H. An, S. Boye, H. Tai, L.-T. Wang, C. Yang, Phase transition during inflation and the gravitational wave signal at pulsar timing arrays. Phys. Rev. D **109**, L121304 (2024). https://doi.org/10.1103/PhysRevD.109.L121304. arXiv:2308.00070 [astro-ph.CO]
57. Z.-C. Chen, S.-L. Li, W. Puxun, Yu. Hongwei, NANOGrav hints for first-order confinement-deconfinement phase transition in different QCD-matter scenarios. Phys. Rev. D **109**, 043022 (2024). https://doi.org/10.1103/PhysRevD.109.043022. arXiv:2312.01824 [astro-ph.CO]
58. N. Kitajima, J. Lee, K. Murai, F. Takahashi, W. Yin, Gravitational waves from domain wall collapse, and application to nanohertz signals with QCD-coupled axions. Phys. Lett. B **851**, 138586 (2024). https://doi.org/10.1016/j.physletb.2024.138586. arXiv:2306.17146 [hep-ph]
59. S. Blasi, A. Mariotti, A. Rase, A. Sevrin, Axionic domain walls at pulsar timing arrays: QCD bias and particle friction. JHEP **11**, 169 (2023). https://doi.org/10.1007/JHEP11(2023)169. arXiv:2306.17830 [hep-ph]
60. E. Babichev, D. Gorbunov, S. Ramazanov, R. Samanta, A. Vikman, NANOGrav spectral index γ=3 from melting domain walls. Phys. Rev. D **108**, 123529 (2023). https://doi.org/10.1103/PhysRevD.108.123529. arXiv:2307.04582 [hep-ph]
61. S.-Y. Guo, M. Khlopov, X. Liu, W. Lei, W. Yongcheng, B. Zhu, Footprints of axion-like particle in pulsar timing array data and James Webb Space Telescope observations. Sci. China Phys. Mech. Astron. **67**, 111011 (2024). https://doi.org/10.1007/s11433-024-2445-1. arXiv:2306.17022 [hep-ph]
62. Z.-C. Chen, W. Yu-Mei, Q.-G. Huang, Search for the gravitational-wave background from cosmic strings with the parkes pulsar timing array second data release. Astrophys. J. **936**, 20 (2022). https://doi.org/10.3847/1538-4357/ac86cb. arXiv:2205.07194 [astro-ph.CO]
63. N. Kitajima, K. Nakayama, Nanohertz gravitational waves from cosmic strings and dark photon dark matter. Phys. Lett. B **846**, 138213 (2023). https://doi.org/10.1016/j.physletb.2023.138213. arXiv:2306.17390 [hep-ph]
64. J. Ellis, M. Lewicki, C. Lin, V. Vaskonen, Cosmic superstrings revisited in light of NANOGrav 15-year data. Phys. Rev. D **108**, 103511 (2023). https://doi.org/10.1103/PhysRevD.108.103511. arXiv:2306.17147 [astro-ph.CO]
65. Z. Wang, L. Lei, H. Jiao, L. Feng, Y.-Z. Fan, The nanohertz stochastic gravitational wave background from cosmic string loops and the abundant high redshift massive galaxies. Sci. China Phys. Mech. Astron. **66**, 120403 (2023). https://doi.org/10.1007/s11433-023-2262-0. arXiv:2306.17150 [astro-ph.HE]
66. S. Basilakos, D.V. Nanopoulos, T. Papanikolaou, E.N. Saridakis, C. Tzerefos, Induced gravitational waves from flipped SU(5) superstring theory at nHz. Phys. Lett. B **849**, 138446 (2024). https://doi.org/10.1016/j.physletb.2024.138446. arXiv:2309.15820 [astro-ph.CO]
67. W. Ahmed, T.A. Chowdhury, S. Nasri, S. Saad, Gravitational waves from metastable cosmic strings in the Pati–Salam model in light of new pulsar timing array data. Phys. Rev. D **109**, 015008 (2024). https://doi.org/10.1103/PhysRevD.109.015008. arXiv:2308.13248 [hep-ph]
68. S. Antusch, K. Hinze, S. Saad, J. Steiner, Singling out SO(10) GUT models using recent PTA results. Phys. Rev. D **108**, 095053 (2023). https://doi.org/10.1103/PhysRevD.108.095053. arXiv:2307.04595 [hep-ph]
69. S. Vagnozzi, Implications of the NANOGrav results for inflation. Mon. Not. R. Astron. Soc. **502**, L11–L15 (2021). https://doi.org/10.1093/mnrasl/slaa203. arXiv:2009.13432 [astro-ph.CO]
70. M. Benetti, L.L. Graef, S. Vagnozzi, Primordial gravitational waves from NANOGrav: a broken power-law approach. Phys. Rev. D **105**, 043520 (2022). https://doi.org/10.1103/PhysRevD.105.043520. arXiv:2111.04758 [astro-ph.CO]
71. S. Vagnozzi, Inflationary interpretation of the stochastic gravitational wave background signal detected by pulsar timing array experiments. JHEAp **39**, 81–98 (2023). https://doi.org/10.1016/j.jheap.2023.07.001. arXiv:2306.16912 [astro-ph.CO]
72. J.-Q. Jiang, Y. Cai, G. Ye, Y.-S. Piao, Broken blue-tilted inflationary gravitational waves: a joint analysis of NANOGrav 15-year and BICEP/Keck 2018 data. JCAP **05**, 004 (2024). https://doi.org/10.1088/1475-7516/2024/05/004. arXiv:2307.15547 [astro-ph.CO]
73. G. Ye, M. Zhu, Y. Cai, Null energy condition violation during inflation and pulsar timing array observations. JHEP **02**, 008 (2024). https://doi.org/10.1007/JHEP02(2024)008. arXiv:2312.10685 [gr-qc]
74. Z.-C. Chen, C. Yuan, Q.-G. Huang, Non-tensorial gravitational wave background in NANOGrav 12.5-year data set. Sci. China Phys. Mech. Astron. **64**, 120412 (2021). https://doi.org/10.1007/s11433-021-1797-y. arXiv:2101.06869 [astro-ph.CO]
75. W. Yu-Mei, Z.-C. Chen, Q.-G. Huang, Constraining the polarization of gravitational waves with the parkes pulsar timing array

second data release. Astrophys. J. **925**, 37 (2022). https://doi.org/10.3847/1538-4357/ac35cc. arXiv:2108.10518 [astro-ph.CO]
76. Z.-C. Chen, W. Yu-Mei, Q.-G. Huang, Searching for isotropic stochastic gravitational-wave background in the international pulsar timing array second data release. Commun. Theor. Phys. **74**, 105402 (2022). https://doi.org/10.1088/1572-9494/ac7cdf. arXiv:2109.00296 [astro-ph.CO]
77. W. Yu-Mei, Z.-C. Chen, Q.-G. Huang, Search for stochastic gravitational-wave background from massive gravity in the NANOGrav 12.5-year dataset. Phys. Rev. D **107**, 042003 (2023). https://doi.org/10.1103/PhysRevD.107.042003. arXiv:2302.00229 [gr-qc]
78. Z.-C. Chen, W. Yu-Mei, Y.-C. Bi, Q.-G. Huang, Search for non-tensorial gravitational-wave backgrounds in the NANOGrav 15-year dataset. Phys. Rev. D **109**, 084045 (2024). https://doi.org/10.1103/PhysRevD.109.084045. arXiv:2310.11238 [astro-ph.CO]
79. Y.-C. Bi, W. Yu-Mei, Z.-C. Chen, Q.-G. Huang, Constraints on the velocity of gravitational waves from the NANOGrav 15-year data set. Phys. Rev. D **109**, L061101 (2024). https://doi.org/10.1103/PhysRevD.109.L061101. arXiv:2310.08366 [astro-ph.CO]
80. W. Yu-Mei, Z.-C. Chen, Y.-C. Bi, Q.-G. Huang, Constraining the graviton mass with the NANOGrav 15 year data set. Class. Quantum Gravity **41**, 075002 (2024). https://doi.org/10.1088/1361-6382/ad2a9b. arXiv:2310.07469 [astro-ph.CO]
81. M. Calzà, F. Gianesello, M. Rinaldi, S. Vagnozzi, Merging cosmologically coupled black holes: stochastic gravitational wave background and implications for pulsar timing arrays. (2024). arXiv:2409.01801 [gr-qc]
82. A.A. Coley, G.F.R. Ellis, Theoretical cosmology. Class. Quantum Gravity **37**, 013001 (2020). https://doi.org/10.1088/1361-6382/ab49b6. arXiv:1909.05346 [gr-qc]
83. A.H. Guth, The inflationary universe: a possible solution to the horizon and flatness problems. Phys. Rev. D **23**, 347–356 (1981). https://doi.org/10.1103/PhysRevD.23.347
84. J. Martin, R.H. Brandenberger, The TransPlanckian problem of inflationary cosmology. Phys. Rev. D **63**, 123501 (2001). https://doi.org/10.1103/PhysRevD.63.123501. arXiv:hep-th/0005209
85. A. Borde, A. Vilenkin, Eternal inflation and the initial singularity. Phys. Rev. Lett. **72**, 3305–3309 (1994). https://doi.org/10.1103/PhysRevLett.72.3305. arXiv:gr-qc/9312022
86. A. Borde, A.H. Guth, A. Vilenkin, Inflationary spacetimes are incompletein past directions. Phys. Rev. Lett. **90**, 151301 (2003). https://doi.org/10.1103/PhysRevLett.90.151301. arXiv:gr-qc/0110012
87. M. Gasperini, G. Veneziano, Pre—big bang in string cosmology. Astropart. Phys. **1**, 317–339 (1993). https://doi.org/10.1016/0927-6505(93)90017-8. arXiv:hep-th/9211021
88. M. Gasperini, G. Veneziano, String theory and pre-big bang cosmology. Nuovo Cim. C **38**, 160 (2016). https://doi.org/10.1393/ncc/i2015-15160-8. arXiv:hep-th/0703055
89. V.S. Kaplunovsky, Mass scales of the string unification. Phys. Rev. Lett. **55**, 1036 (1985). https://doi.org/10.1103/PhysRevLett.55.1036
90. D.A. Linde, Inflation and quantum cosmology. Phys. Scr. T **36**, 30–54 (1991). https://doi.org/10.1088/0031-8949/1991/T36/004
91. L.P. Grishchuk, M. Solokhin, Spectra of relic gravitons and the early history of the Hubble parameter. Phys. Rev. D **43**, 2566–2571 (1991). https://doi.org/10.1103/PhysRevD.43.2566
92. R. Brustein, M. Gasperini, M. Giovannini, G. Veneziano, Relic gravitational waves from string cosmology. Phys. Lett. B **361**, 45–51 (1995). https://doi.org/10.1016/0370-2693(95)01128-D. arXiv:hep-th/9507017
93. Y. Jiang, X.-L. Fan, Q.-G. Huang, Search for stochastic gravitational-wave background from string cosmology with advanced LIGO and Virgo's O1∼O3 data. JCAP **04**, 024 (2023). https://doi.org/10.1088/1475-7516/2023/04/024. arXiv:2302.03846 [gr-qc]
94. R. Brustein, Spectrum of cosmic gravitational wave background, in *International Conference on Gravitational Waves: Sources and Detectors* (1996). arXiv:hep-th/9604159
95. B. Allen, R. Brustein, Detecting relic gravitational radiation from string cosmology with LIGO. Phys. Rev. D **55**, 3260–3264 (1997). https://doi.org/10.1103/PhysRevD.55.3260. arXiv:gr-qc/9609013
96. R. Brustein, M. Gasperini, G. Veneziano, Peak and endpoint of the relic graviton background in string cosmology. Phys. Rev. D **55**, 3882–3885 (1997). https://doi.org/10.1103/PhysRevD.55.3882. arXiv:hep-th/9604084
97. N. Aghanim et al. (Planck), Planck 2018 results. VI. Cosmological parameters. Astron. Astrophys. **641**, A6 (2020) [note Erratum: Astron. Astrophys. 652, C4 (2021)]. https://doi.org/10.1051/0004-6361/201833910. arXiv:1807.06209 [astro-ph.CO]
98. A. Buonanno, M. Maggiore, C. Ungarelli, Spectrum of relic gravitational waves in string cosmology. Phys. Rev. D **55**, 3330–3336 (1997). https://doi.org/10.1103/PhysRevD.55.3330. arXiv:gr-qc/9605072
99. M. Galluccio, M. Litterio, F. Occhionero, Graviton spectra in string cosmology. Phys. Rev. Lett. **79**, 970–973 (1997). https://doi.org/10.1103/PhysRevLett.79.970. arXiv:gr-qc/9608007
100. S.R. Taylor, The nanohertz gravitational wave astronomer. (2021). arXiv:2105.13270 [astro-ph.HE]
101. C.J. Moore, A. Vecchio, Ultra-low-frequency gravitational waves from cosmological and astrophysical processes. Nat. Astron. **5**, 1268–1274 (2021). https://doi.org/10.1038/s41550-021-01489-8. arXiv:2104.15130 [astro-ph.CO]
102. W.G. Lamb, S.R. Taylor, R. van Haasteren, Rapid refitting techniques for Bayesian spectral characterization of the gravitational wave background using pulsar timing arrays. Phys. Rev. D **108**, 103019 (2023). https://doi.org/10.1103/PhysRevD.108.103019. arXiv:2303.15442 [astro-ph.HE]
103. W. Yu-Mei, Z.-C. Chen, Q.-G. Huang, Cosmological interpretation for the stochastic signal in pulsar timing arrays. Sci. China Phys. Mech. Astron. **67**, 240412 (2024). https://doi.org/10.1007/s11433-023-2298-7. arXiv:2307.03141 [astro-ph.CO]
104. J.S. Speagle, dynesty: a dynamic nested sampling package for estimating Bayesian posteriors and evidences. Mon. Not. R. Astron. Soc. **493**, 3132–3158 (2020). https://doi.org/10.1093/mnras/staa278. arXiv:1904.02180 [astro-ph.IM]
105. G. Ashton et al., BILBY: a user-friendly Bayesian inference library for gravitational-wave astronomy. Astrophys. J. Suppl. **241**, 27 (2019). https://doi.org/10.3847/1538-4365/ab06fc. arXiv:1811.02042 [astro-ph.IM]
106. I.M. Romero-Shaw et al., Bayesian inference for compact binary coalescences with bilby: validation and application to the first LIGO–Virgo gravitational-wave transient catalogue. Mon. Not. R. Astron. Soc. **499**, 3295–3319 (2020). https://doi.org/10.1093/mnras/staa2850. arXiv:2006.00714 [astro-ph.IM]
107. A. Sesana, A. Vecchio, C.N. Colacino, The stochastic gravitational-wave background from massive black hole binary systems: implications for observations with pulsar timing arrays. Mon. Not. R. Astron. Soc. **390**, 192 (2008). https://doi.org/10.1111/j.1365-2966.2008.13682.x. arXiv:0804.4476 [astro-ph]
108. Z.-C. Chen, F. Huang, Q.-G. Huang, Stochastic gravitational-wave background from binary black holes and binary neutron stars and implications for LISA. Astrophys. J. **871**, 97 (2019). https://doi.org/10.3847/1538-4357/aaf581. arXiv:1809.10360 [gr-qc]
109. R.E. Kass, A.E. Raftery, Bayes factors. J. Am. Stat. Assoc. **90**, 773–795 (1995). https://doi.org/10.1080/01621459.1995.10476572